PH 113
Physics

Nuclear Shell Model

By

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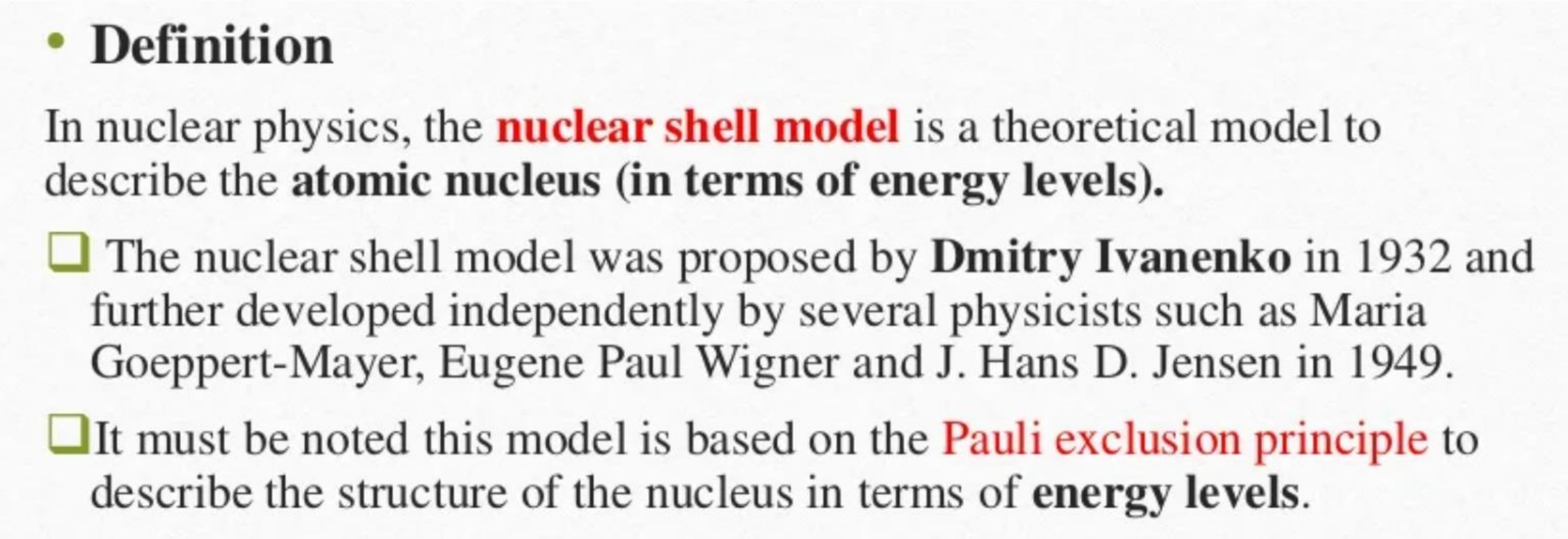
Nuclear Shell Model



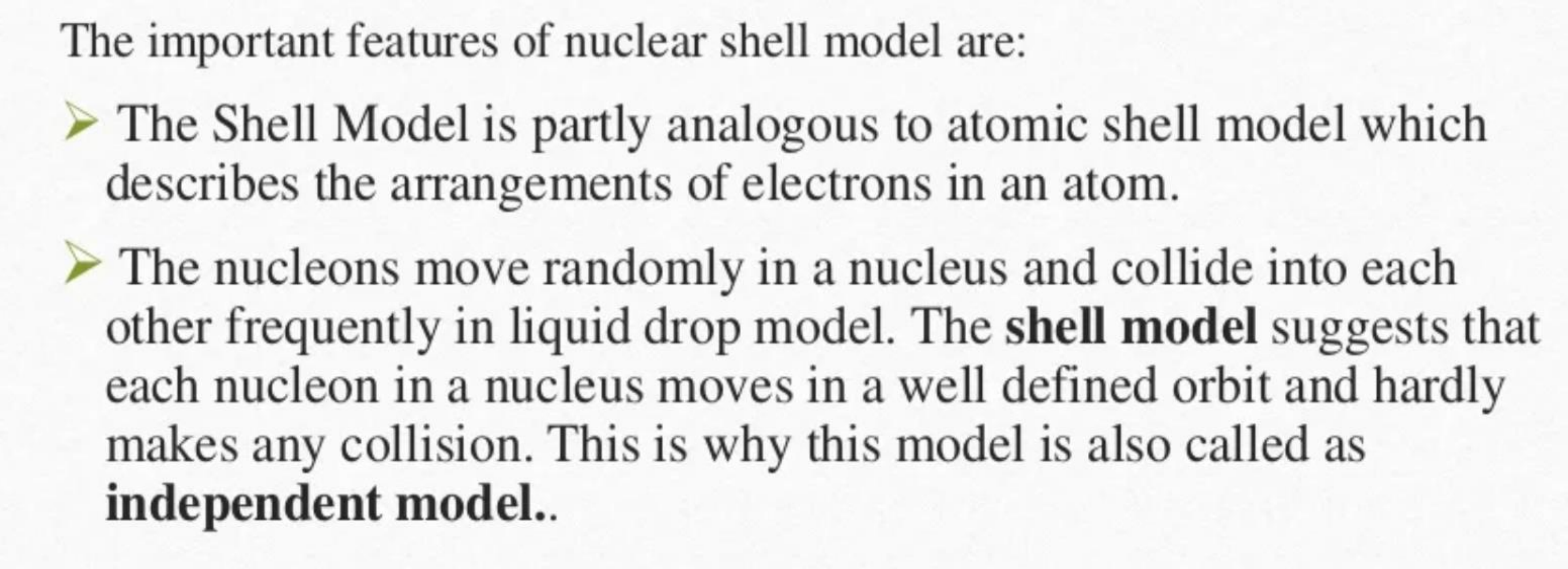
- **Definition**

In nuclear physics, the **nuclear shell model** is a theoretical model to describe the **atomic nucleus (in terms of energy levels).**

❑ The nuclear shell model was proposed by **Dmitry Ivanenko** in 1932 and further developed independently by several physicists such as Maria Goeppert-Mayer, Eugene Paul Wigner and J. Hans D. Jensen in 1949.

❑ It must be noted this model is based on the Pauli exclusion principle to describe the structure of the nucleus in terms of **energy levels**.

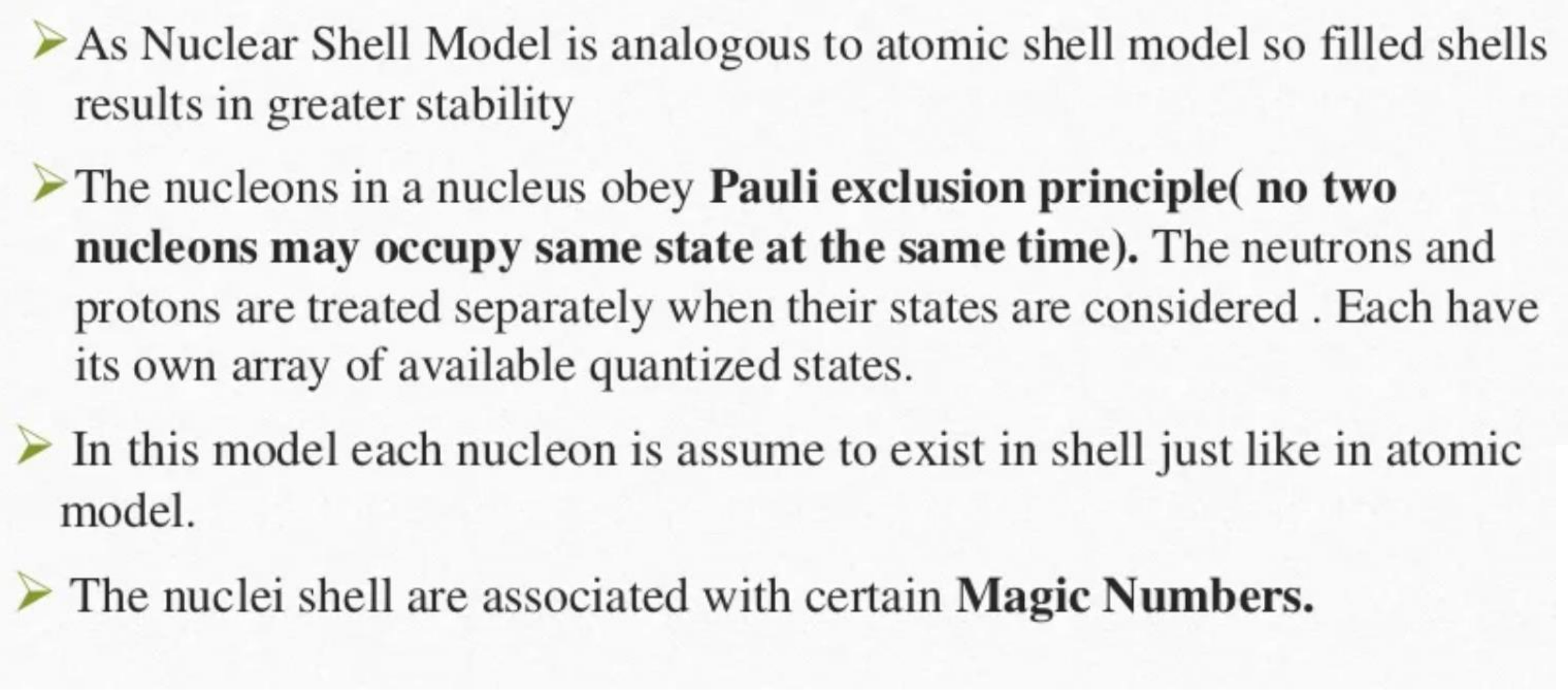
Features of Shell Model



The important features of nuclear shell model are:

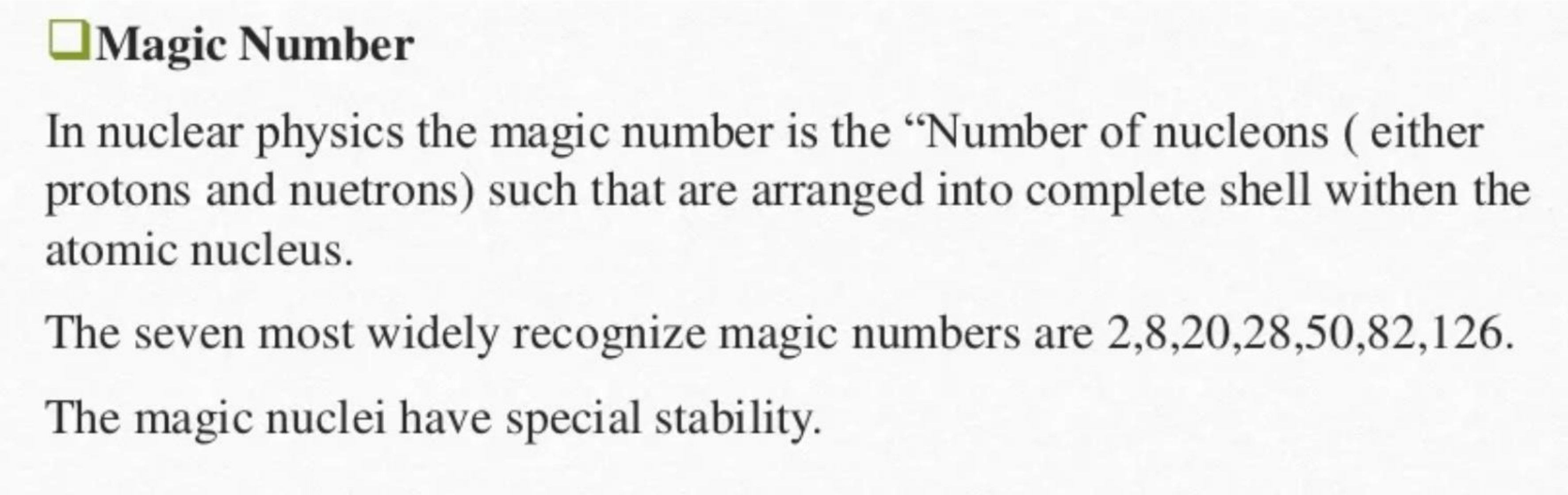
- The Shell Model is partly analogous to atomic shell model which describes the arrangements of electrons in an atom.
- The nucleons move randomly in a nucleus and collide into each other frequently in liquid drop model. The **shell model** suggests that each nucleon in a nucleus moves in a well defined orbit and hardly makes any collision. This is why this model is also called as **independent model.**.

Features of Shell Model



- As Nuclear Shell Model is analogous to atomic shell model so filled shells results in greater stability
- The nucleons in a nucleus obey **Pauli exclusion principle(no two nucleons may occupy same state at the same time).** The neutrons and protons are treated separately when their states are considered . Each have its own array of available quantized states.
- In this model each nucleon is assume to exist in shell just like in atomic model.
- The nuclei shell are associated with certain **Magic Numbers.**

Features of Shell Model



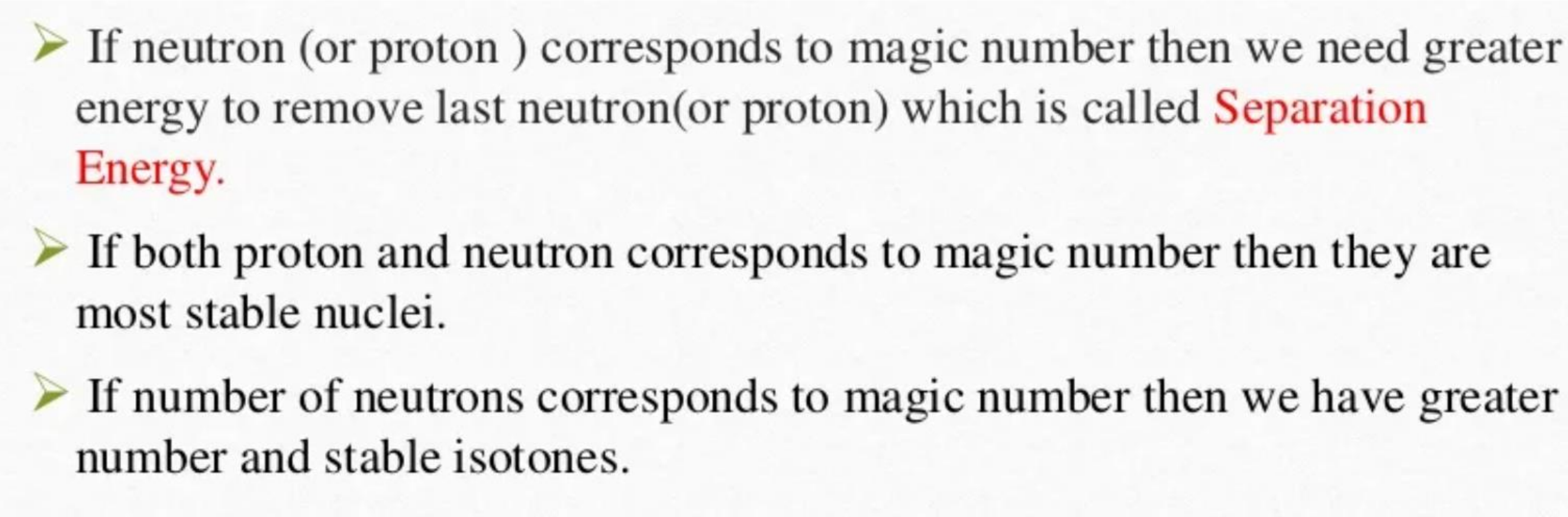
❑**Magic Number**

In nuclear physics the magic number is the “Number of nucleons (either protons and nuetrons) such that are arranged into complete shell withen the atomic nucleus.

The seven most widely recognize magic numbers are 2,8,20,28,50,82,126.

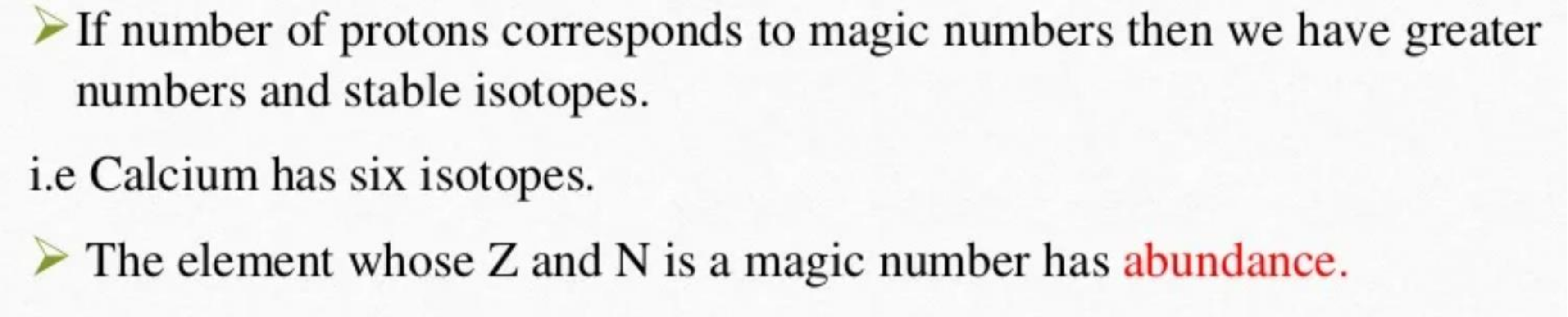
The magic nuclei have special stability.

Features of Shell Model



- If neutron (or proton) corresponds to magic number then we need greater energy to remove last neutron(or proton) which is called Separation Energy.
- If both proton and neutron corresponds to magic number then they are most stable nuclei.
- If number of neutrons corresponds to magic number then we have greater number and stable isotones.

Features of Shell Model

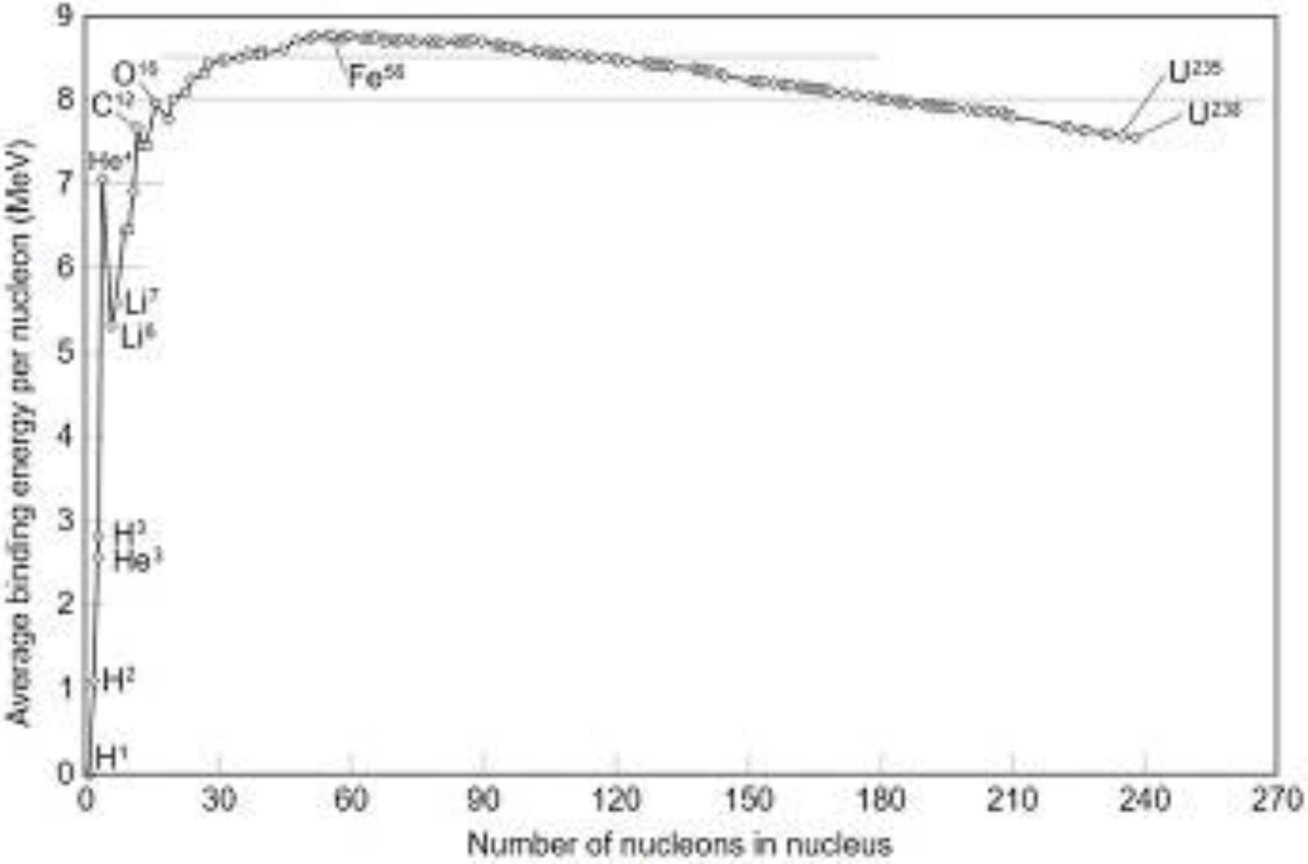


- If number of protons corresponds to magic numbers then we have greater numbers and stable isotopes.

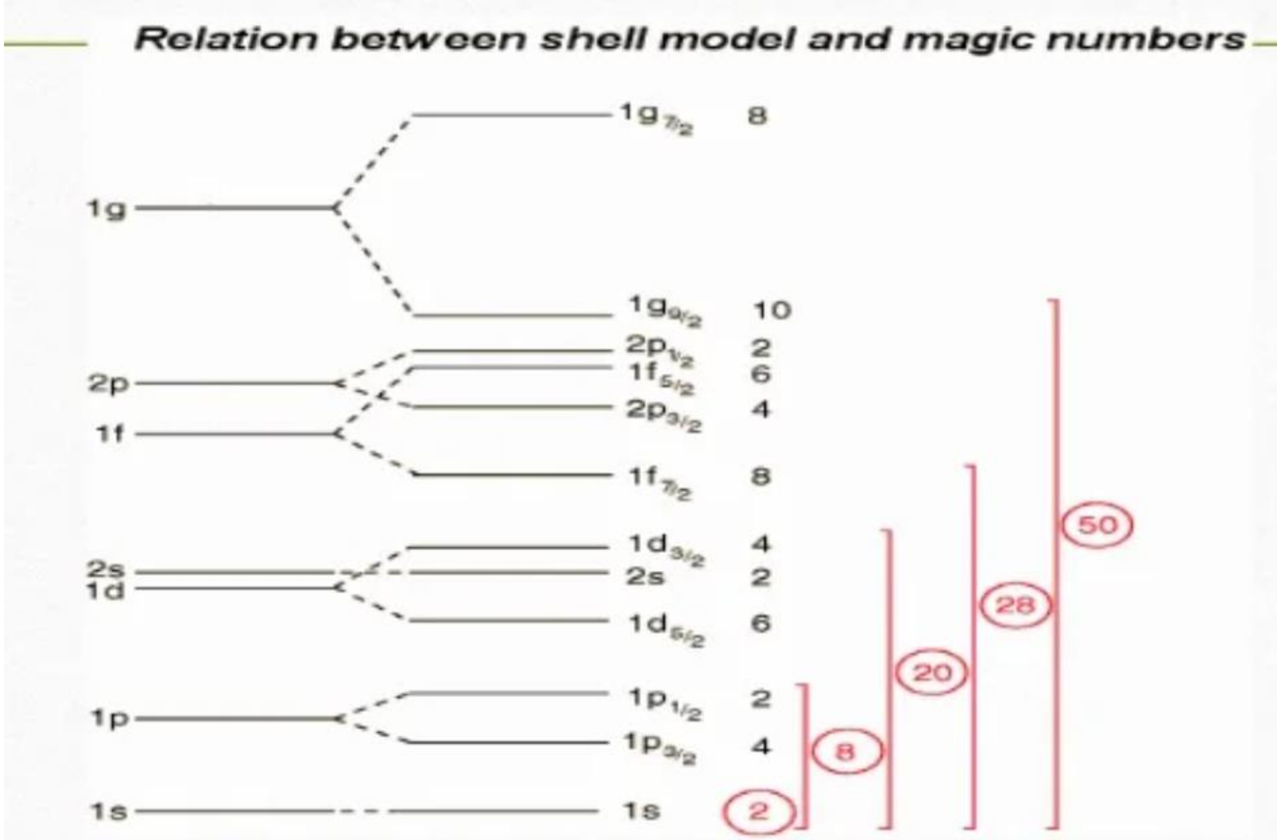
i.e Calcium has six isotopes.

- The element whose Z and N is a magic number has abundance.

B. E Curve

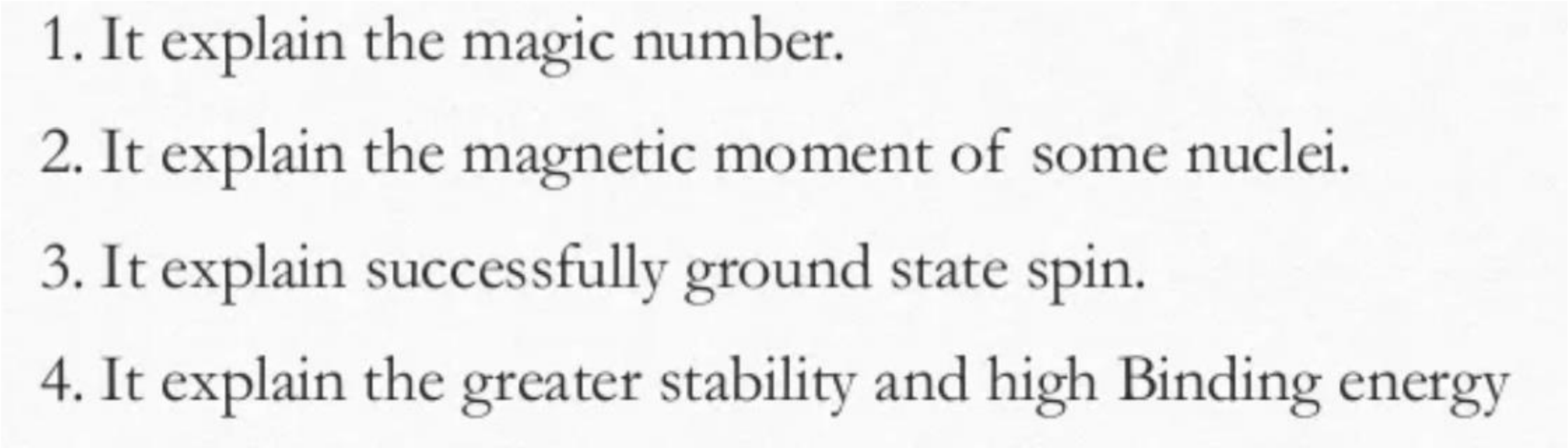

Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
0
1
2
3
4
5
6
7
8
9
0
30
60
90
120
150
180
210
240
270
H1
H2
H3
He3
He4
Li6
Li7
C12
O16
Fe56
U235
U238

Nuclear Shell Structure



Relation between shell model and magic numbers

Merits of Shell Model



1. It explain the magic number.
2. It explain the magnetic moment of some nuclei.
3. It explain successfully ground state spin.
4. It explain the greater stability and high Binding energy

Limitation of Shell Model

- The first limitation that one notices is that there is a difference between shell-model wave functions and the real state of the nucleus.
- Apart from this, the large value of the quadrupole moment seen in many nuclei is hard to explain using this method.
- The strong-spin orbit interaction cannot be applied using this method.
- The shell model has limited applications and cannot be applied to heavy nuclei.

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Nuclear Fission and Fusion

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Binding Energy

A binding energy is generally the energy required to disassemble a whole system into separate parts. It is known the sum of separate parts has typically a higher potential energy than a bound system, therefore the bound system is more stable. A creation of bound system is often accompanied by subsequent energy release.

Nuclear Binding Energy

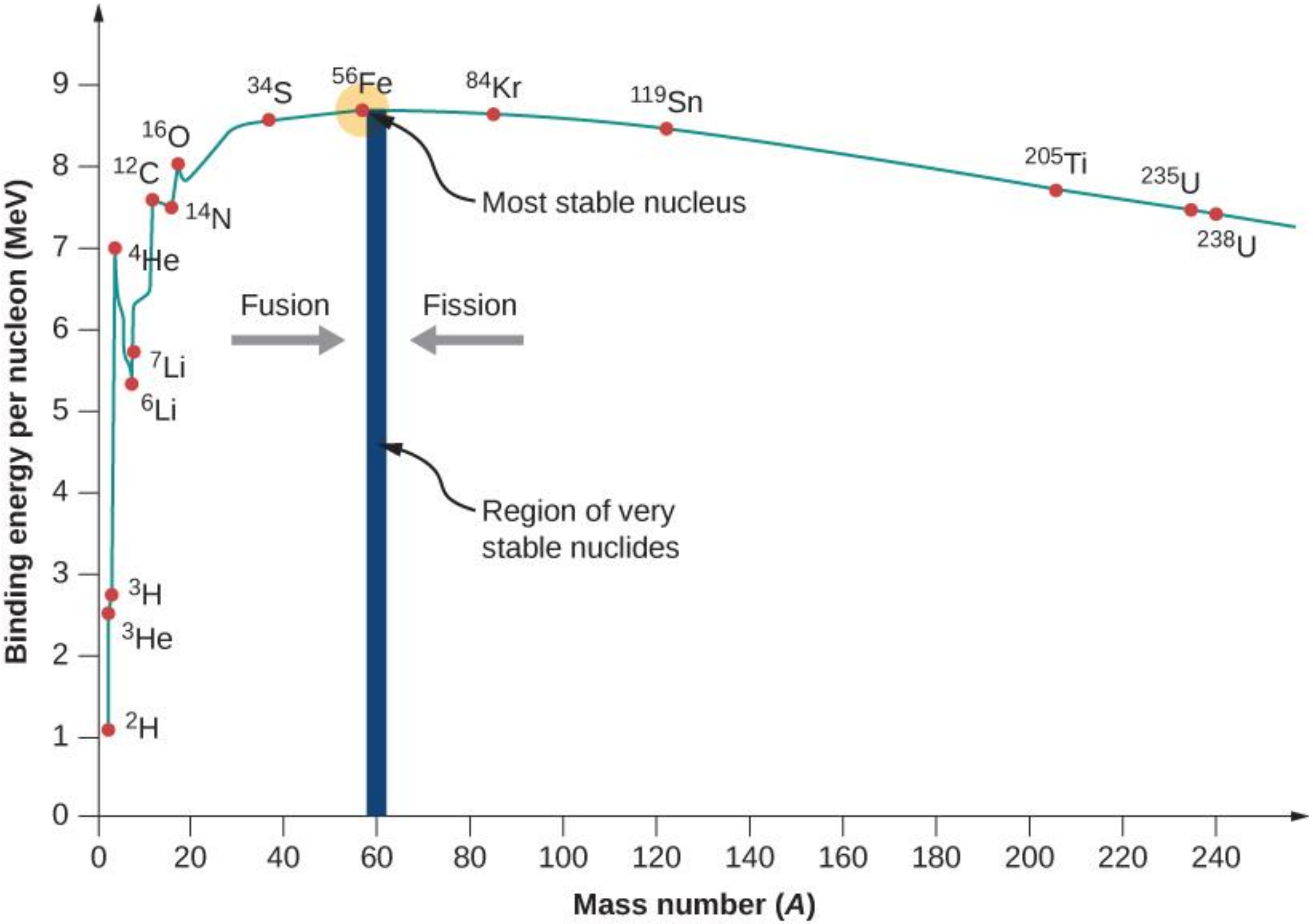
The component parts of nuclei are neutrons and protons, which are collectively called nucleons. **The mass of a nucleus is always less than the sum masses of the constituent protons and neutrons when separated**. The difference is a measure of the nuclear binding energy which holds the nucleus together. According to the Einstein relationship ($E=m.c^2$) this binding energy is proportional to this mass difference and it is known as **the mass defect**.

Nuclear Binding Curve

If the splitting releases energy and the fusion releases the energy, so where is the breaking point? For understanding this issue it is better to relate the binding energy to one nucleon, to obtain **nuclear binding curve**. The binding energy per one nucleon is not linear. There is a peak in the binding energy curve in the region of stability near **iron** and this means that either the breakup of heavier nuclei than iron or the combining of lighter nuclei than iron will yield energy.

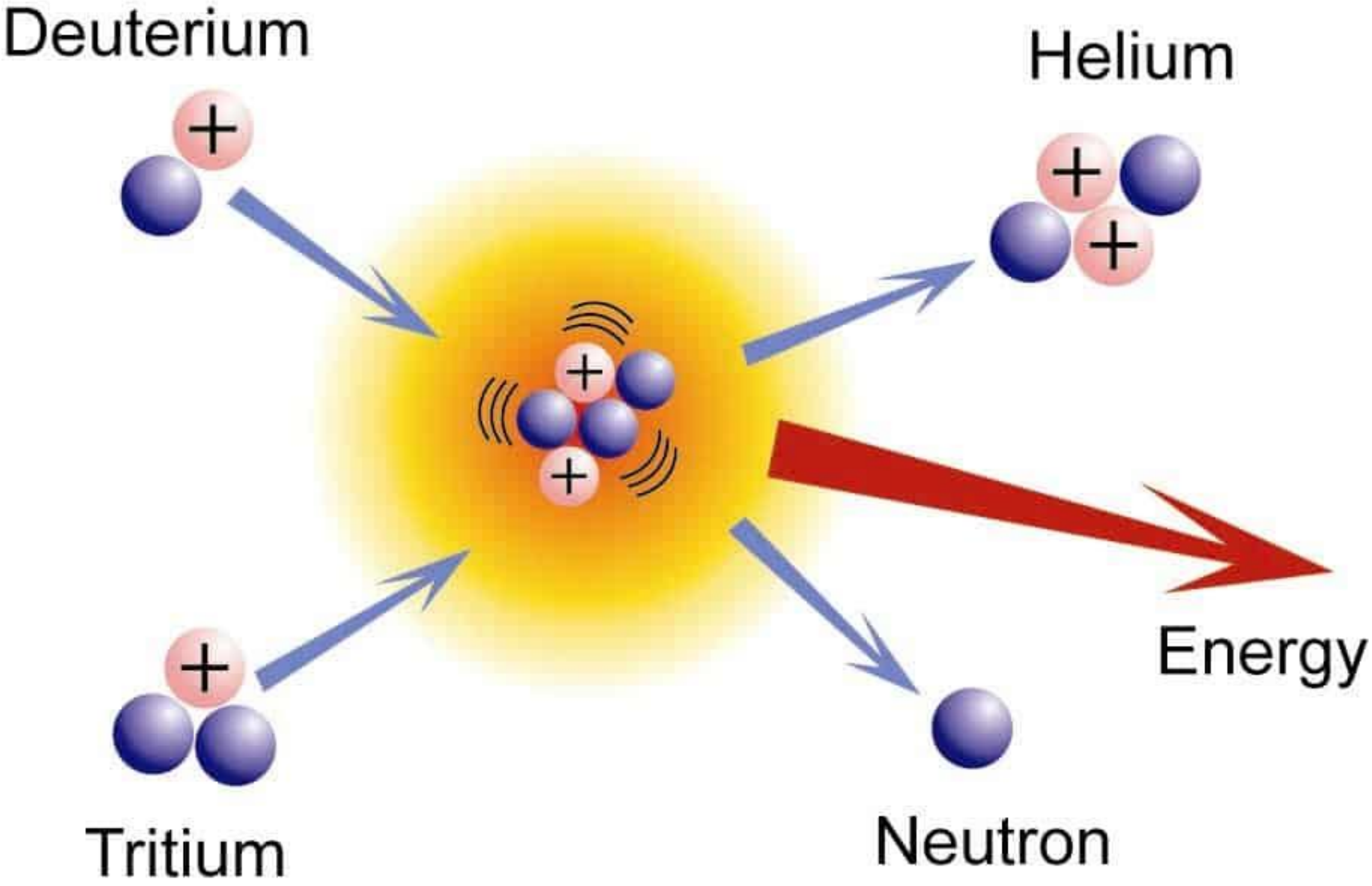
Nuclear Binding Curve

The reason the trend reverses after iron peak is the growing positive charge of the nuclei. The electric force has greater range than strong nuclear force. While the strong nuclear force binds only close neighbors the electric force of each proton repels the other protons.


Binding energy per nucleon (MeV)
0
1
2
3
4
5
6
7
8
9
Mass number (A)
0
20
40
60
80
100
120
140
160
180
200
220
240
2H
3He
3H
4He
6Li
7Li
12C
14N
16O
34S
56Fe
84Kr
119Sn
205Ti
235U
238U
Most stable nucleus
Region of very stable nuclides
Fusion
Fission

Nuclear Fusion

In nuclear physics, **nuclear fusion** is a nuclear reaction in which two or more atomic nuclei collide at a very high energy and fuse together into a new nucleus, e.g. helium. If light nuclei are forced together, they will fuse with a yield of energy because the mass of the combination will be less than the sum of the masses of the individual nuclei. If the combined nuclear mass is less than that of iron at the peak of the **binding energy curve**, then the nuclear particles will be more tightly bound than they were in the lighter nuclei, and that decrease in mass comes off in the form of energy according to the Albert Einstein relationship.


Deuterium
Helium
Energy
Tritium
Neutron

Nuclear Fusion

Fusion reactions have an energy density many times greater than nuclear fission and fusion reactions are themselves millions of times more energetic than chemical reactions.

The fusion power offers the opportunity of an almost inexhaustible source of energy for future, but it the fusion technology presents a real scientific and engineering challenges.

Deuterium-Tritium Fusion

The fusion reaction of deuterium and tritium is particularly interesting because of its potential of providing energy for the future.

3T (d, n) 4He

The reaction yields ~17 MeV of energy per reaction.

Nuclear Fission

Nuclear fission refers to the splitting of an atomic nucleus into two or more lighter nuclei. This process can occur through a nuclear reaction or through radioactive decay. Nuclear fission reactions often release a large amount of energy, which is accompanied by the emission of neutrons and gamma rays (photons holding huge amounts of energy, enough to knock electrons out of atoms).

Types of Nuclear Fission

- **Spontaneous Fission:** Nuclei consisting of protons, electrostatic force of repulsion between them exceeds the nuclear binding force hence nuclei undergoes spontaneous fission
- **Induced Fission:** When neutron is bombarded on heavy nucleus it makes nucleus unstable this unstable nucleus undergoes fission and split into two fragment. Such fission is called as induced fission.

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Liquid Drop Model

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Models of Nuclear Structure

- Why is the binding energy per nucleon almost constant?
- Why do certain nuclei emit α-and β-particles through these particles do not exist inside the nucleus?
- Why are the nuclei containing 2, 8, 20, 28, 50, 82 nucleons most stable?
- Like any other quantum mechanical system, a nucleus also exist in its excited states. The most stable state is the ground state in which the nuclei are generally found.

Models of Nuclear Structure

- The precise nature of the forces acting in the nucleus is unknown.
- Nuclear model are resorted to for investigation and theoretical prediction of its properties.

Models of Nuclear Structure

- Such models may be based on
  - the extrinsic analogy between the properties of atomic nuclei and those of a liquid drop
  - the electron shell of an atoms etc.
- The corresponding models are called the liquid drop model and shell model.

Liquid Drop Model

- The Liquid Drop Model was proposed by the Neils Bohr who observed that there are certain marked similarities between an atomic nucleus and a liquid drop.

- In the Liquid Drop Model, the forces acting in the nucleus are assumed to be analogical to the molecular forces in a droplet of some liquid.

Assumptions

- The nucleus is supposed to be spherical in shape in the stable state, just like as liquid drop is spherical due to the symmetrical surface tension forces.
- The forces of surface tension acts on the surface of the liquid drop. Similarly, there is a potential barrier at the surface of the nucleus.
- The density of a liquid drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.

Assumptions

- The intermolecular forces in a liquid are short range. Similarly the nuclear forces are short range forces.
- The molecules evaporate from liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.
- When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and nucleus breaks up into two smaller nuclei.

Semi-Empirical Formula

Using the model of the nucleus as a liquid drop, we attempt to calculate nuclear binding energies.

There are five contributions to the binding energy:

- The volume energy is proportional to the number of nucleons.
- The surface energy is the reduction in the binding energy due to nuclear surface effects.
- The Coulomb energy is the repulsion between protons.

- The asymmetry energy is a reduction in binding energy which occurs when there are more neutrons than protons, and vice versa.
- The pairing energy is somewhat of a "fudge factor" which accounts for stability of even-even and even-odd nuclei, and the reduction of stability for odd-odd nuclei.

Volume Energy

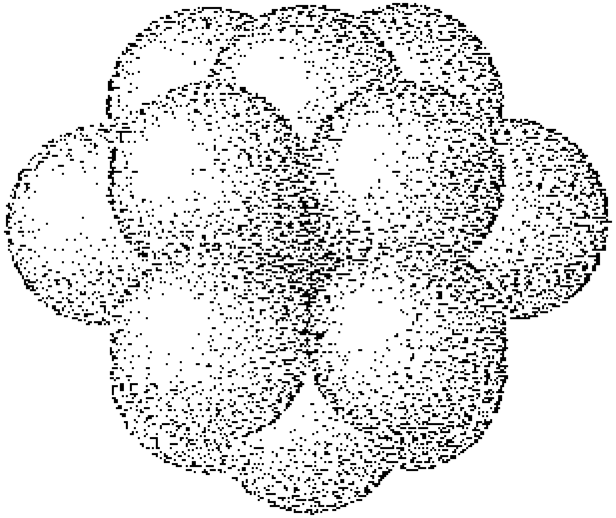
Volume Energy

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbors.

Energy associated with each nucleon-nucleon bond = U

Because each bond energy is shared by two nucleons therefore
each has a binding energy of ½ U.

When an assembly of spheres of the same size is packed together
into the smallest volume, each interior sphere has 12 other spheres
In contact with it



In a tightly packed assembly of identical spheres, each interior sphere is in contact with 12 others

Volume Energy

Hence each interior nucleon in a nucleus has a binding energy of

$$=12(1/2U) = 6U.$$

If all A nucleons in a nucleus were in the interior, the total binding Energy would be

$$E_V = 6AU$$

Or

$$\boxed{E_V = a_1 A}$$

Volume energy

Surface Energy

Actually, of course, some nucleons are on the surface of every Nucleus and therefore have fewer than 12 neighbors.

Surface energy $\propto$ surface area

$\propto 4\pi R^2$

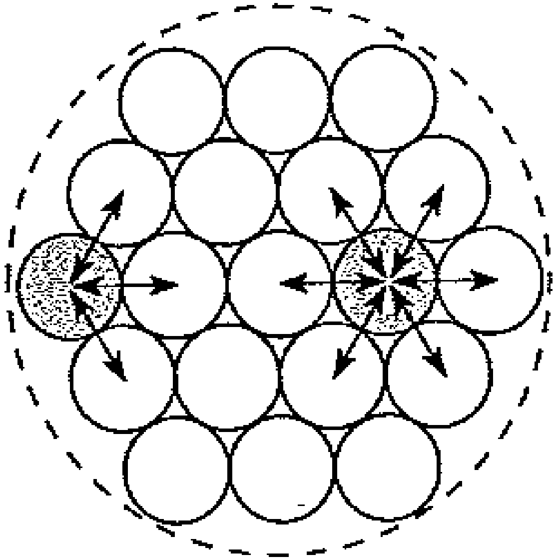
If $R = R_0 A^{1/3}$ then

Surface energy $\propto 4\pi R_0^2 A^{2/3}$

Or

Surface energy $= - a_2 A^{2/3}$

Surface energy $\propto A^{2/3}$



A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. The larger the nucleus, the smaller the proportion of nucleons at the surface

Coulomb Energy

The repulsion between each pair of protons in a nucleus also contribute toward decreasing its binding energy.

The coulomb energy E_C of a nucleus is the work that must be done To bring together Z protons from infinity into a spherical aggregate The size of the nucleus.

$$V = -\frac{e^2}{4\pi\varepsilon_0 r}$$

Since there are Z(Z-1)/2 pairs of protons,

$$E_C = \frac{Z(Z-1)}{2} V = -\frac{Z(Z-1)e^2}{8\pi\varepsilon_0}\left(\frac{1}{r}\right)_{av}$$

Coulomb Energy

Where $\left(\frac{1}{r}\right)_{av}$ is the value of 1/r averaged over all proton pairs.

If the protons are uniformly distributed throughout a nucleus of radius R, $\left(\frac{1}{r}\right)_{av}$ is proportional to 1/R and hence $1/A^{1/3}$

Therefore

$$\boxed{E_C = -a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}}}$$

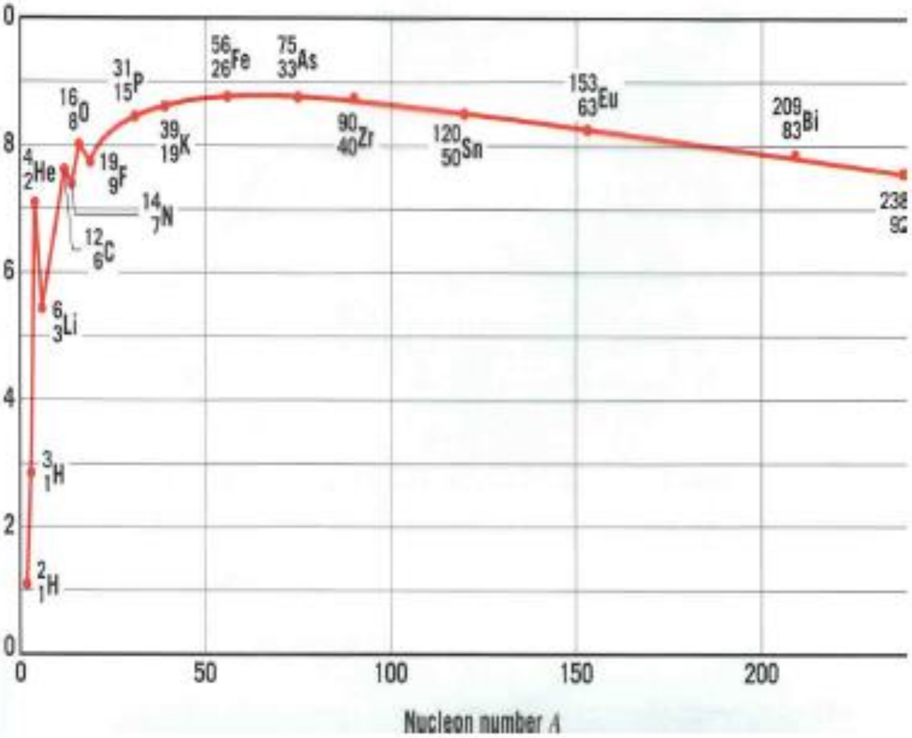
The Coulomb energy is negative because it arises from an effect that opposes nuclear stability.

The total binding energy E_b of a nucleus

$$E_b = E_v + E_S + E_C = \underbrace{a_1 A}_{\text{Volume energies}} - \overbrace{a_2 A^{\frac{2}{3}}}^{\text{Surface energies}} - \underbrace{a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}}}_{\text{Coulomb energies}}$$

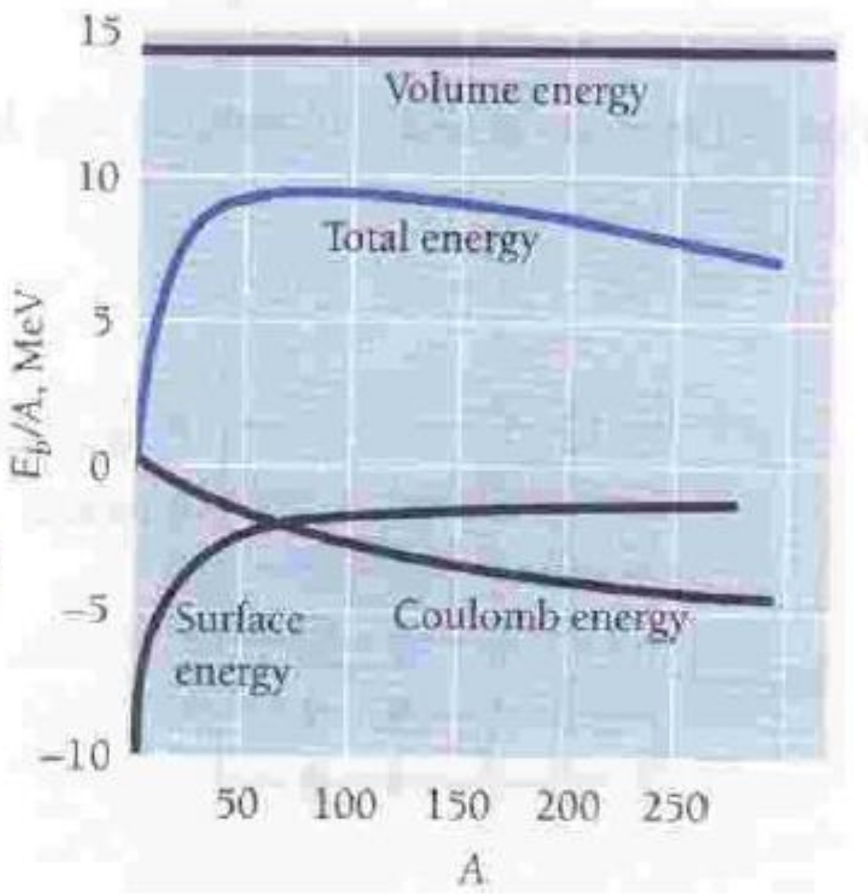
The binding energy per nucleon is

$$\frac{E_{BE}}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{\frac{4}{3}}}$$



Empirical binding energy per nucleon curve

theoretical binding energy per nucleon curve (using Liquid drop model concept)



Correction to the Formula

The binding energy formula can be improved by taking into account two effects that do no fit into the simple liquid drop model, but which make sense in terms of a model that provides for nuclear energy levels.

Asymmetry Energy

This effect occurs when the neutrons in a nucleus outnumber the protons, which means that higher energy levels have to be occupied than would be case if N and Z were equal.

Asymmetry Energy

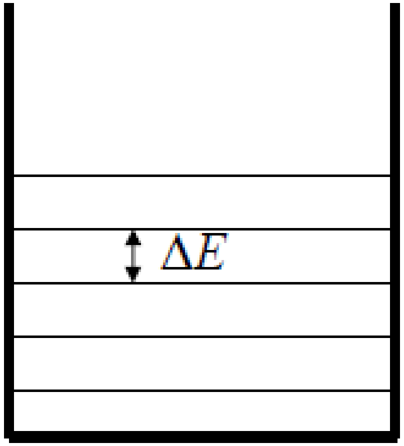
Imagine that the neutrons and protons are in a potential well (the nucleus).

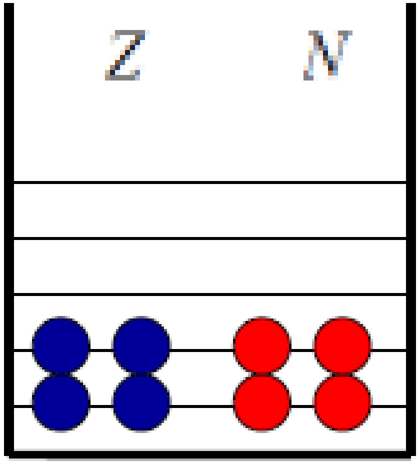
We will assume that the well has approximately equally spaced energy levels.

Let the spacing be ΔE.

Neutrons and protons are fermions (spin ½ particles).

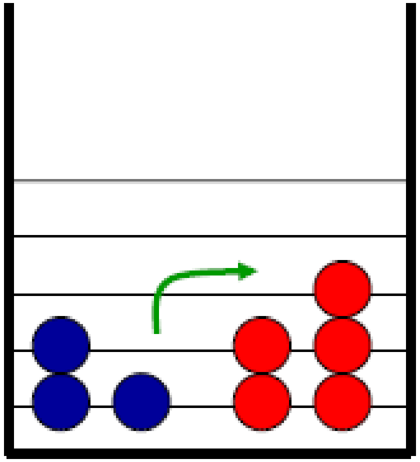
So the Pauli exclusion principle allows only 2 p and 2 n in each level.



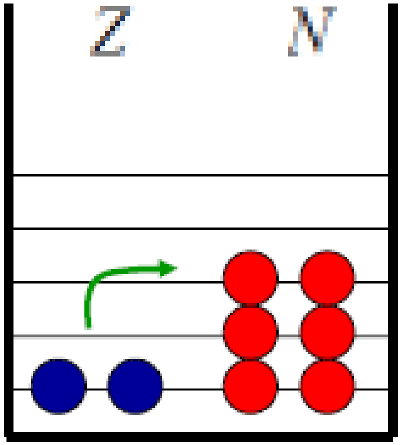


For a nucleus with a given A, *the lowest energy will be when* $N = Z$.

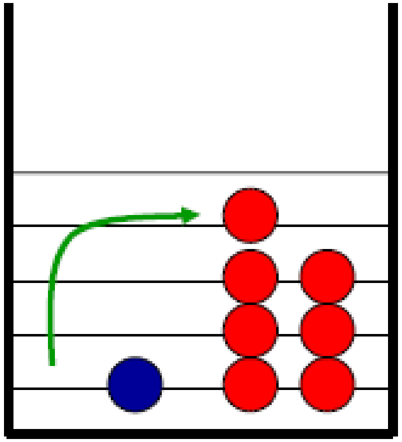
($Z - N = 0$)



If we change a proton to a neutron we increase the energy by $1\Delta E$ from Z-$N = 0$ *case.*
(Now Z-$N = 2$)



If we change another proton to a neutron … we increase the energy by ΔE *for a total change of* $2\Delta E$ *from* $Z\text{-}N = 0$ *case.*
(Now $Z - N = 4$)



Another proton to a neutron…
we increase the energy by $3\ \Delta E$
for a total change of $5\ \Delta E$
from $Z - N = 0$ *case.*
(Now $Z - N = 6$)

Continuing this, we can find the energy shift for a change in *Z - N or N - Z.*

$Z-N$ or $N-Z$	Total Energy Shift (ΔE)	$\frac{(N-Z)^2}{8}$
2	1	0.5
4	2	2
6	5	4.5
8	8	8
10	13	12.5
12	18	18
14	25	24.5
16	32	32

We can approximate this energy increase by $\frac{(N-Z)^2}{8}\Delta E$

Asymmetry Energy

It has been found that ΔE decreases with A like A^{-1}. Therefore we can write the asymmetry term as

$$E_a = -a_4 \frac{(N-Z)^2}{A} = -a_4 \frac{(A-2Z)^2}{A}$$

Pairing Energy

The last correction term arises from the tendency of proton pairs and neutron pairs to occur. Even-even nuclei are the most stable and hence higher binding energies than would otherwise expected. Thus such nuclei appears as peaks in the empirical curve of binding energy per nucleon. At the other extreme, odd- odd nuclei have both unpaired protons and neutrons and have relatively low binding energies.

The pairing energy E_p is positive for even-even nuclei, 0 for odd-even nuclei and even-odd nuclei, and negative for odd-odd nuclei. It seems to vary with A as $A^{-3/4}$. Hence

$$E_p = (\pm, 0)\, a_5/A^{3/4}$$

Semi-Empirical Binding Energy Formula

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm, 0) \frac{a_5}{A^{3/4}}$$

A set of coefficients that gives a good fit with the data is as follows:

$a_1 = 14.1$ MeV $a_2 = 13.0$ MeV $a_3 = 0.595$ MeV

$a_4 = 19.0$ MeV $a_5 = 33.5$ MeV

Merits of Liquid Drop Model

- The liquid drop model accounts for many of the salient features of nuclear matter, such as the observed binding energies of nuclei and their stability against α and β disintegration as well as nuclear fission.

- The calculation of atomic masses and binding energies can done with good accuracy with the liquid drop model.

Demerits of Liquid Drop Model

- It fails to explain the extra stability of certain nuclei, where the number of protons or neutrons in the nucleus are 2, 8, 20, 28, 50, 82, or 126.
- It fails to explain the measured magnetic moments of many nuclei
- it also fails to explain the spin of nuclei
- It could not explain the excited states in most of the nuclei.