Special theory of Relativity :-

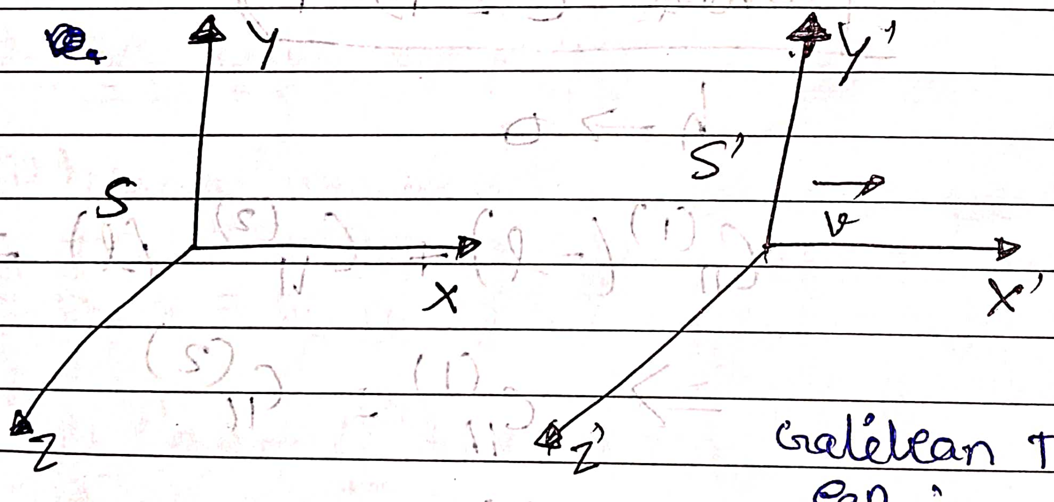
Inertial Frame of reference :-

- Newton's 1st law of motion is satisfied
- Any frame of reference that moves at a constant velocity relative to an inertial frame is itself an inertial frame.

Postulates of special Theory of relativity :-

1. The laws of physics are the same in all inertial frames of reference.
2. The speed of light in free space has the same value in all inertial frames of reference.

Galileon Transformation



Galilean Transformation eqn :-

$x' = x - vt$ — (1)

$y' = y$ — (2)

$z' = z$ — (3)

$t' = t$ — (4)

Taking derivatives of Eqn ①, ②, ③

$$V_x' = V_x - v \qquad V_x' = \frac{dx'}{dt'} \qquad V_x = \frac{dy}{dt}$$

$$V_y' = V_y \qquad V_y' = \frac{dy'}{dt'} \qquad V_y = \frac{dy}{dt}$$

$$V_z' = V_z \qquad V_z' = \frac{dz'}{dt'} \qquad V_z = \frac{dz}{dt}$$

Galilean transformation equations violate both the postulates of the special theory of relativity (STR)

- 1^{st} Postulate implies the same eqn of physics in all the inertial frames (say S & S') but the eqn of electricity & magnetism become very different when the Galilean transformation is used to convert quantities from one inertial frame to the other.

- The 2^{nd} Postulate is also violated. The speed of light in free space in the S' frame (C') & speed of light in the S frame (C) are related as:

$$C' = C - V,$$

Indicating that it is different inertial frame

Lorentz Transformation :-

$$x' = k(x - vt) \quad \text{——— (1)}$$

$y' = y$ where k does not depend on x & t but can be a fn of v.

$z' = z$

Eq (1) is written considering the following.

1. It should be linear in x & x' so that a single event in frame S corresponds to a single event S'.

2. It is simple as a simple sol^n to the problem should be explored first

As the eqn of physics must be the same in both S & S' frames to write the corresponding eqn for x, we only need to change v to $-v$ (to account for the change in the direction of the relative motion) so we can write:

$$x = k(x' + vt') \quad \text{——— (2)}$$

$$x = k[k(x - vt) + vt']$$

$$= k^2(x - vt) + kvt'.$$

$$x = k^2 x - k^2 vt + kvt'$$

$$(1-k^2)x + k^2 vt = kvt'$$

$$\boxed{t' = kt + \frac{(1-k^2)x}{kv}} \quad \text{——— (3)}$$

Initial condⁿ

$t = t' = 0$

$x = ct$ (in S frame) — (4)

$x' = ct'$ (in S' frame) — (5)

$\downarrow$

$k(x - vt) = ckt + \frac{c(1-k^2)}{kv} cx$

solⁿ eqⁿ :- we get

$$x = \frac{(c+kv)\,kt}{k - \left(\frac{1-k^2}{kv}\right)c}$$

$$x = ct\,\frac{\left[k + \frac{v}{c}k\right]}{k - \left(\frac{1-k^2}{kv}\right)c}$$

$$x = ct\left[\frac{1+\frac{v}{c}}{\left(1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}\right)}\right] \quad — (6)$$

Factor multiplied with ct in RHS of Eqⁿ 6 must be equal to (4)

$$\frac{1+\frac{v}{c}}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}} = 1$$

$$\Rightarrow \boxed{k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Lorentz Transformation :-

$$x' = k(x - vt)$$

$$\Rightarrow x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = kt + \frac{(1-k^2)x}{kv}$$

$$\boxed{t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$v << c$ then

Galileon Transformation recovered.

for wave
nension

Inverse Lorentz Transformation.

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$\boxed{t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Module-3

Lorentz Transformation.

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Inverse Lorentz Transformation

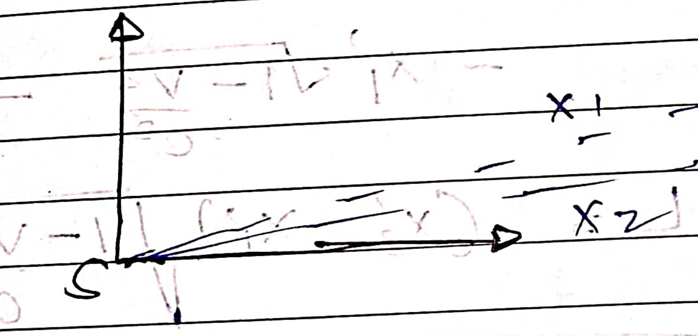
$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

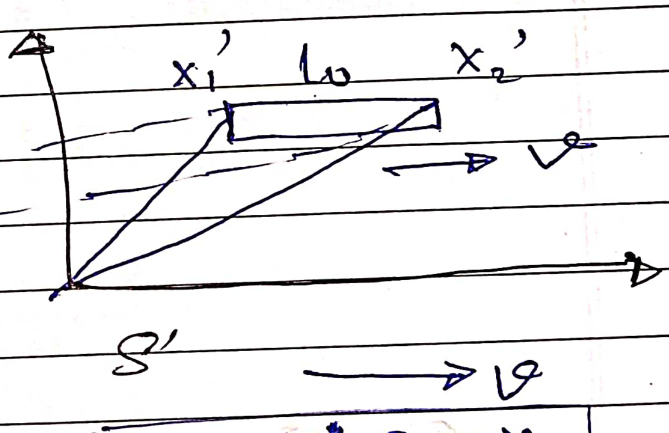
$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Length contraction :-





Proper Length

$\boxed{\text{w.r.t } S'}$ $\quad L_0 = x_2' - x_1'$

$\boxed{\text{w.r.t } S}$ $\quad L = x_2 - x_1$

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_1 = x_1'\sqrt{1-\frac{v^2}{c^2}} + vt \quad \text{— (1)}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_2 = x_2'\sqrt{1-\frac{v^2}{c^2}} + vt \quad \text{— (2)}$$

$$L = x_2 - x_1$$

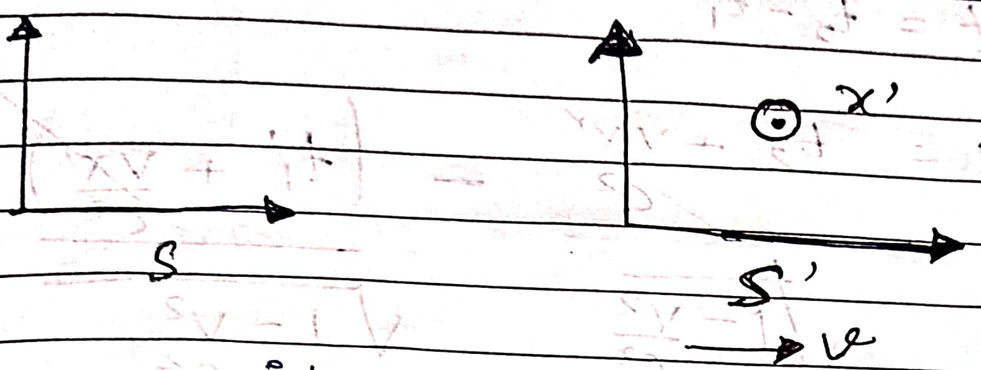
$$= x_2'\sqrt{1-\frac{v^2}{c^2}} + \cancel{vt}$$

$$- x_1'\sqrt{1-\frac{v^2}{c^2}} - \cancel{vt}$$

$$L = (x_2' - x_1')\sqrt{1-\frac{v^2}{c^2}}$$

$$\boxed{L = L_0\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} \leq 1$$



consider a clock in S' frame at position x'. when the observer in S' finds the time to be t_1', an observer in S finds it to be t_1

Inverse Lorentz Transformation

$$t_1 = \frac{t_1' + \frac{Vx'}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

There is another event which occurs at t_2' w.r.t an observer in S' such that $\boxed{t_0 = t_2' - t_1'}$

t_0 : Proper time interval.

$$t_2 = \frac{t_2' + Vx'/c^2}{\sqrt{1 - \frac{V^2}{c^2}}}$$

Time interval b/w same event w.r.t an observer in S frame

$$\boxed{t = t_2 - t_1}$$

$$t = t_2 - t_1$$

$$t = \frac{t_2' + \cancel{\frac{vx'}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\left(t_1' + \cancel{\frac{vx'}{c^2}}\right)}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{(t_2' - t_1')}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Any time interval w.r.t S frame will appear dilated.

Velocity addition

w.r.t S frame

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

w.r.t S' frame

$$v_x' = \frac{dx'}{dt'}, \quad v_y' = \frac{dy'}{dt'}, \quad v_z' = \frac{dz'}{dt'}$$

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \frac{dt' + \frac{v\,dx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$V_x = \frac{dx}{dt}$$

$$V_x = \frac{dx' + v\,dt'}{dt' + \frac{v\,dx'}{c^2}}$$

$$V_x = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

$$\boxed{V_x = \frac{V_x' + v}{1 + \frac{v}{c^2}V_x'}}$$

$$V_y = \frac{dy}{dt} = \frac{dy'}{dt} = \frac{dy'}{dt' + \frac{v\,dx'}{c^2}}\sqrt{1 - \frac{v^2}{c^2}}$$

$x' = c$

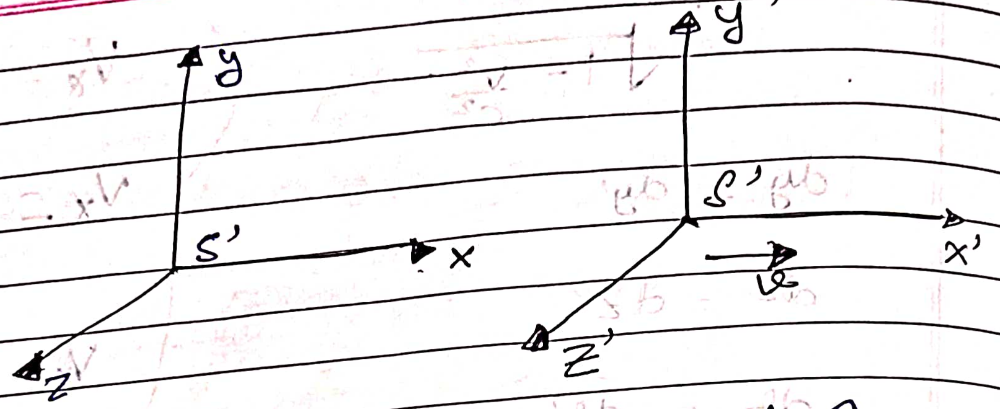
$x = c$

$$\boxed{V_y = \frac{V_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2}V_x'}}$$

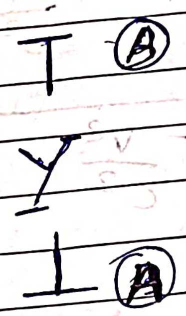
Similarly

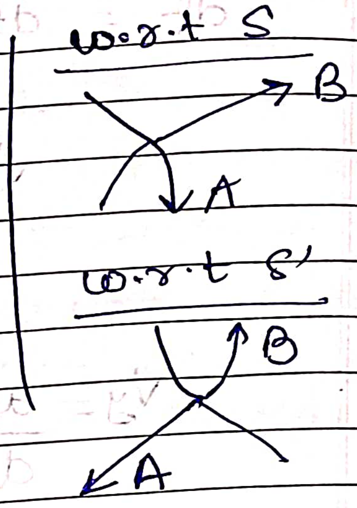
$$V_z = \frac{dz}{dt} \Rightarrow \boxed{V_z = \frac{V_z'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2}V_x'}}$$

Relativistic Momentum



(Collision is elastic)





Before collision

A is at rest w.r.t: S frame.
B is at rest w.r.t S' frame.

At some instant A is thrown in +y direction with velocity V_A (w.r.t S) and B is thrown in $-y$ direction with velocity V_B' (w.r.t S')

So that $V_A = V_B'$

After collision

A rebounds in $-y$ direction with speed V_A while B rebounds in $+y'$ direction with speed V_B'.

If the particles are thrown from position 'Y' apart, an observer in S finds the

Schrodinger wave eqn

...ve motion

collision happens at $y = \frac{Y}{2}$ & an observer in S' will find the collision to happen at $y' = \frac{Y}{2}$.

The round trip time T_0 for A is measured w.r.t S

$$T_0 = \frac{Y}{V_A}$$

The round trip time T_0 for B is measured w.r.t S'

$$T_0 = \frac{Y}{V_B'}$$

Let's consider the round trip for B w.r.t S be T

$$T = \frac{Y}{V_B} \quad \text{— (1)}$$

[V_B : velocity of B w.r.t S frame]

$$\boxed{T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad \text{— (2)}$$

For momentum conservation

$$P_A = P_B$$

$$m_A V_A = m_B V_B$$

From (1) & (2)

$$V_B = \frac{Y}{T_0}$$

The momentum will be conserved only if $\boxed{m_B = \frac{m_A}{\sqrt{1 - \frac{v^2}{c^2}}}}$

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If $V_A' + V_B' << c$

In the limit $V_A' = 0$ & if m is the mass of A in S. In limit, $V_B' = 0$ & if $m(v)$ is the mass of B in S,

$$\boxed{m(v) = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

m: proper mass or rest mass

NOTE: Idea of Relativistic mass is not good as no clear definition can be given.

Relativistic momentum $\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$

conservation of momentum holds in STR. (Special Theory of Relativity)

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic second law :-

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(\gamma m\vec{v})$$

Mass & energy :

If an object is displaced by a distance s in presence of force $\vec{F}$, Kinetic energy can be defined as :

$$KE = \int_0^s \vec{F}\cdot\vec{ds} = \int_0^s F\cdot ds$$

$$= \int_0^s \frac{d}{dt}(\gamma m v)\, ds$$

$$= u\int v - \int\left(\frac{du}{dx}\int v\right)$$

$$= \int_0^v d(\gamma m v)\, v$$

$$= \int_0^v v\, d\left[\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

Integration by part

$$KE = \left[\frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}}\right]_0^v - \int_0^v \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\, dv$$

$$KE = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} + \left[mc^2\sqrt{1-\frac{v^2}{c^2}}\right]_0^u$$

$$KE = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} + mc^2\sqrt{1-\frac{v^2}{c^2}} - mc^2$$

$$KE = \frac{\cancel{mv^2} + mc^2\left[\frac{c^2-\cancel{v^2}}{c^2}\right]}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$$

$$KE = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$$

$$\boxed{KE = (\gamma - 1)mc^2}$$

γmc^2 is interpreted as the total energy E.

$$E = \gamma mc^2 = (\gamma-1)mc^2 + mc^2$$

$$E = mc^2 + KE$$

if $KE = 0$

$E = E_0 = mc^2$ = Rest-mass energy

or

Rest Energy.

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$: Total Energy

Show that

$E^2 = (mc^2)^2 + p^2c^2$

Total Energy

$$E^2 = \left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 = \frac{m^2c^4}{1-\frac{v^2}{c^2}} = \frac{m^2c^6}{c^2-v^2}$$

$$p^2c^2 = \left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 c^2 = \frac{m^2v^2c^4}{c^2-v^2}$$

$$E^2 - p^2c^2 = \frac{m^2c^4}{\cancel{c^2-v^2}}\left[\cancel{c^2-v^2}\right]$$

$$E^2 - p^2c^2 = (mc^2)^2$$

$$\boxed{E^2 = (mc^2)^2 + p^2c^2}$$ Hence Proved

For massless particles

$E = PC$

if $m=0$ & $v=c$, $E = \frac{0}{0}$, $P = \frac{0}{0}$. Thus,

E & P can have any values. The massless particles can have energy & momentum provided they travel with speed & light.