PH113: Physics (B.Tech. I)

Instructors: Dr. S K Mukherjee and Dr. Anupam Roy

Dept of Physics, BIT Mesra.

Drop an e-mail: royanupam [AT] bitmesra [DOT] ac [DOT] in.

Section L, O & P

PH113: Physics (B.Tech. I)

Syllabus

Module-1: Physical Optics

Polarization, Malus' Law, Brewster's Law, Double Refraction, Interference in thin films (Parallel films), Interference in wedge-shaped layers, Newton's rings, Fraunhofer diffraction by single slit, Double slit. Elementary ideas of fibre optics and application of fibre optic cables. [8]

Module-2: Electromagnetic Theory

Gradient, Divergence and Curl, Statement of Gauss theorem & Stokes theorem, Gauss's law, Applications, Concept of electric potential, Relationship between E and V, Polarization of dielectrics, dielectric constant, Boundary conditions for E & D, Gauss's law in magnetostatics, Ampere's circuital law, Boundary conditions for B & H, Equation of continuity, Displacement current, Maxwell's equations. [8]

Module-3: Special Theory of Relativity

Introduction, Inertial frame of reference, Galilean transformations, Postulates, Lorentz transformations and its conclusions, Length contraction, time dilation, velocity addition, Mass change, Einstein's mass energy relation. [6]

Module-4: Quantum Mechanics

Planck's theory of black-body radiation, Compton effect, Wave particle duality, De Broglie waves, Davisson and Germer's experiment, Uncertainty principle, Brief idea of Wave Packet, Wave Function and its physical interpretation, Schrodinger equation in one-dimension, free particle, particle in an infinite square well. [9]

Module-5: Modern Physics

Laser – Spontaneous and stimulated emission, Einstein's A and B coefficients, Population inversion, Light amplification, Basic laser action, Ruby and He-Ne lasers, Properties and applications of laser radiation, Nuclear Physics- Binding Energy Curve, Nuclear Force, Liquid drop model, Introduction to Shell model, Applications of Nuclear Physics, Concept of Plasma Physics, and its applications. [9]

Text books:

1: A. Ghatak, Optics, 4th Edition, Tata Mcgraw Hill, 2009
2: Mathew N.O. Sadiku, Elements of Electromagnetics, Oxford University Press (2001)
3: Arthur Beiser, Concept of Modern Physics, 6th edition 2009, Tata McGraw- Hill
4. F. F. Chen, Introduction to Plasma Physics and controlled Fusion, Springer, Edition 2016.

Reference books: 1: Fundamentals of Physics, Halliday, Walker and Resnick

PH113: Physics (B.Tech. I)

Module 1

Module – 1 : Physical Optics

Polarization, Malus' Law, Brewster's Law, Double Refraction, Interference in thin films (Parallel films), Interference in wedge-shaped layers, Newton's rings, Fraunhofer diffraction by single slit, Double slit. Elementary ideas of fibre optics and application of fibre optic cables. [8]

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References: (1) Optics – E Hecht **(2)** Fundamentals of Optics – Jenkins & White

Class structure: 4 Lectures including 1 Tutorial per week. (8 hours ~ 2 weeks for this module!)

PH113: Physics (B.Tech. I)

Module 1

Date: 05.09.2023

Lecture: 1

Module 1 (Physical Optics)

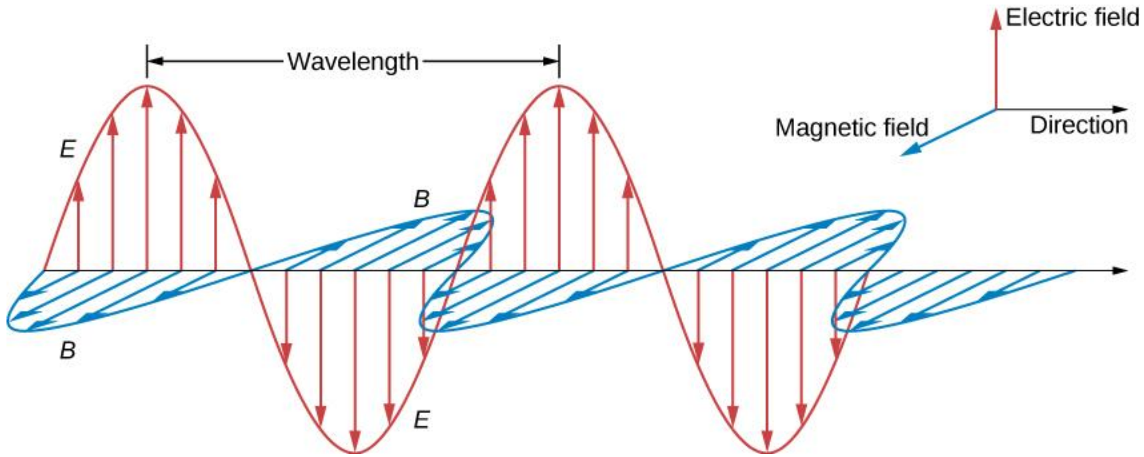
Optics

- Geometrical Optics and Physical Optics
- Geometrical Optics is useful when the objects are very *large* compared to wavelength of light, *e.g.*, lenses and mirrors.
- Physical Optics deals with situations where the objects have dimensions comparable to the wavelength.
- Several new aspects of light-matter interaction – interference, diffraction, polarization (can only be explained by Physical Optics).

Module 1 (Physical Optics)

Optics

- Two types of waves: Longitudinal and Transverse
- **Longitudinal wave**: particles of the medium moves in the direction of propagation of the wave. Example: Sound wave, expansion & compression of spring.
- **Transverse wave**: particles movement is orthogonal/perpendicular to the direction of propagation. Example: EM wave (light wave).



- Light is an electromagnetic (EM) wave – consists of electric and magnetic field vectors in perpendicular directions, and propagates in a direction perpendicular to both the fields.

Module 1 (Physical Optics)

Polarization

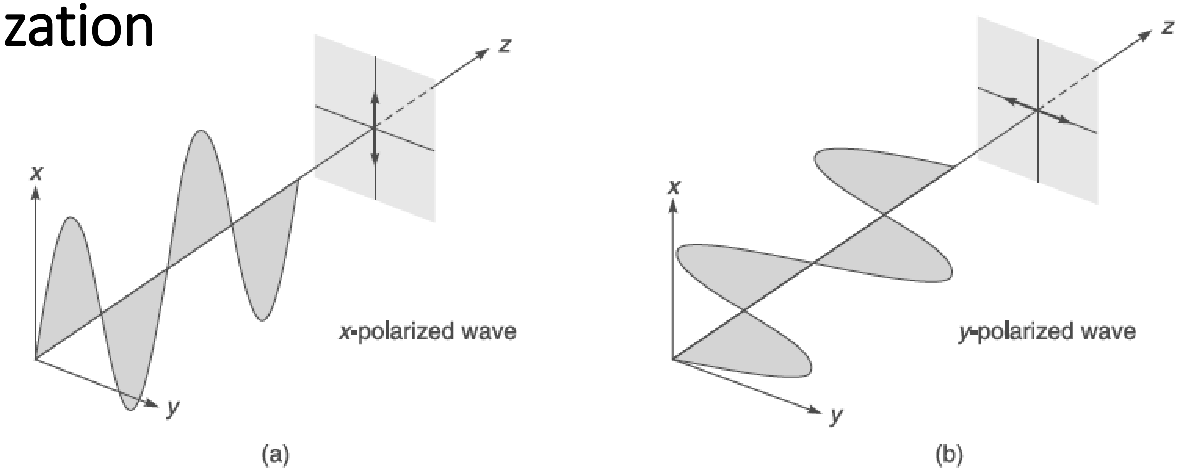


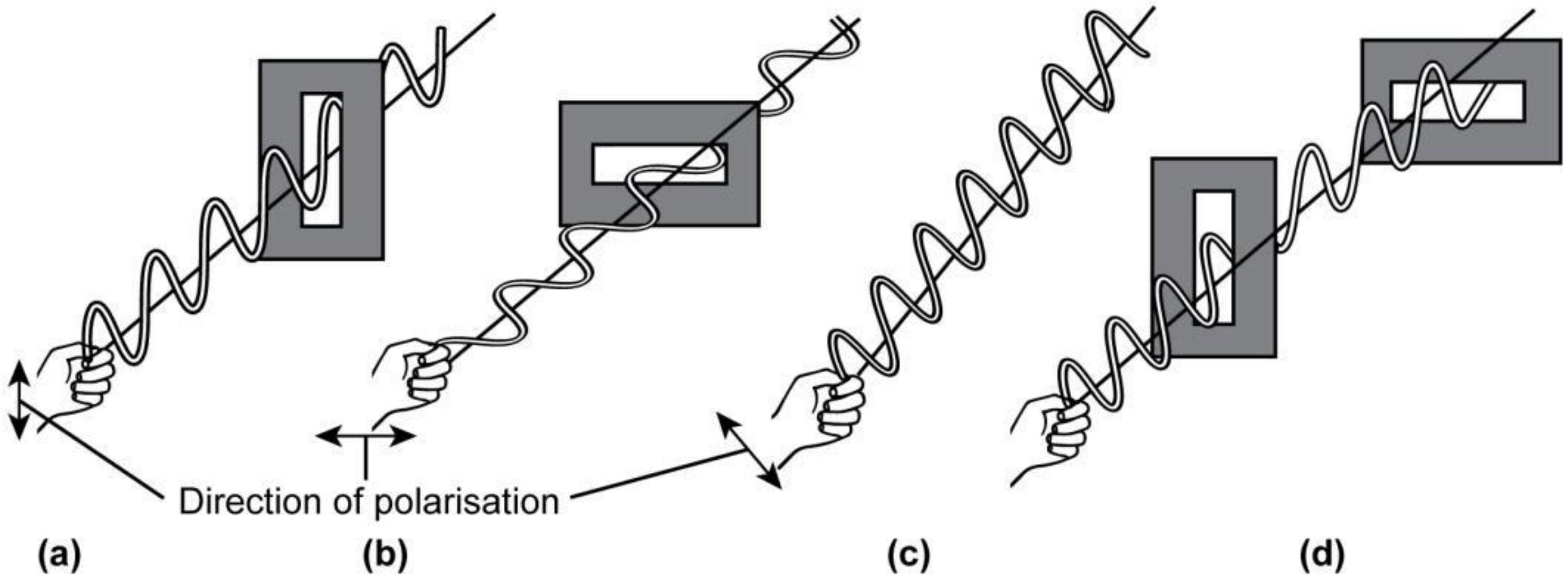
Fig. 22.1 (a) A linearly polarized wave on a string with the displacement confined to the x-z plane; (b) A linearly polarized wave on a string with the displacement confined to the y-z plane.

Direction of polarization is determined only by the oscillating direction/plane of the electric field, $\boldsymbol{E}$ (not by the magnetic field, $\boldsymbol{B}$) $(|\boldsymbol{E}| = c|\boldsymbol{B}| \gg |\boldsymbol{B}|)$

- Oscillate one end of a string up and down – a **transverse wave** is generated.
- Each point of the string executes a sinusoidal oscillation in a straight line (along the x-axis) and the wave is, therefore, known as a **linearly polarized** wave. It is also known as a **plane polarized** wave because the string is always confined to the x-z plane.
- Polarization can be defined only in case of transverse waves and not in the longitudinal wave.

Polarization

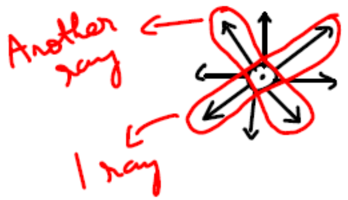
- **Polarization**: When the electric (or magnetic) field variations in a light wave are in a fixed plane, we say that the light is polarized.



Module 1 (Physical Optics)

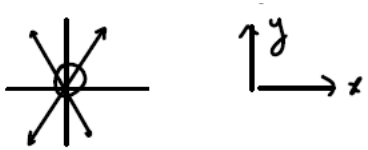
Polarization

- Light emitted from a common source like Sun or Electric bulb, the planes of vibration are random (each ray in the beam can vibrate in an arbitrary direction) – **Unpolarized** Light.

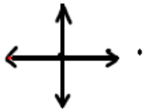


$\Rightarrow$ Unpolarized light. ($\ast$)

(Each arrow represents a plane of vibration of the electric field).



- Alternative way to represent unpolarized light: Each electric field vectors can be decomposed into x- and y-components.



$$\left.\begin{aligned} E_x &= E_x(t) \\ E_y &= E_y(t) \end{aligned}\right\} \text{Both are sinusoidal}$$

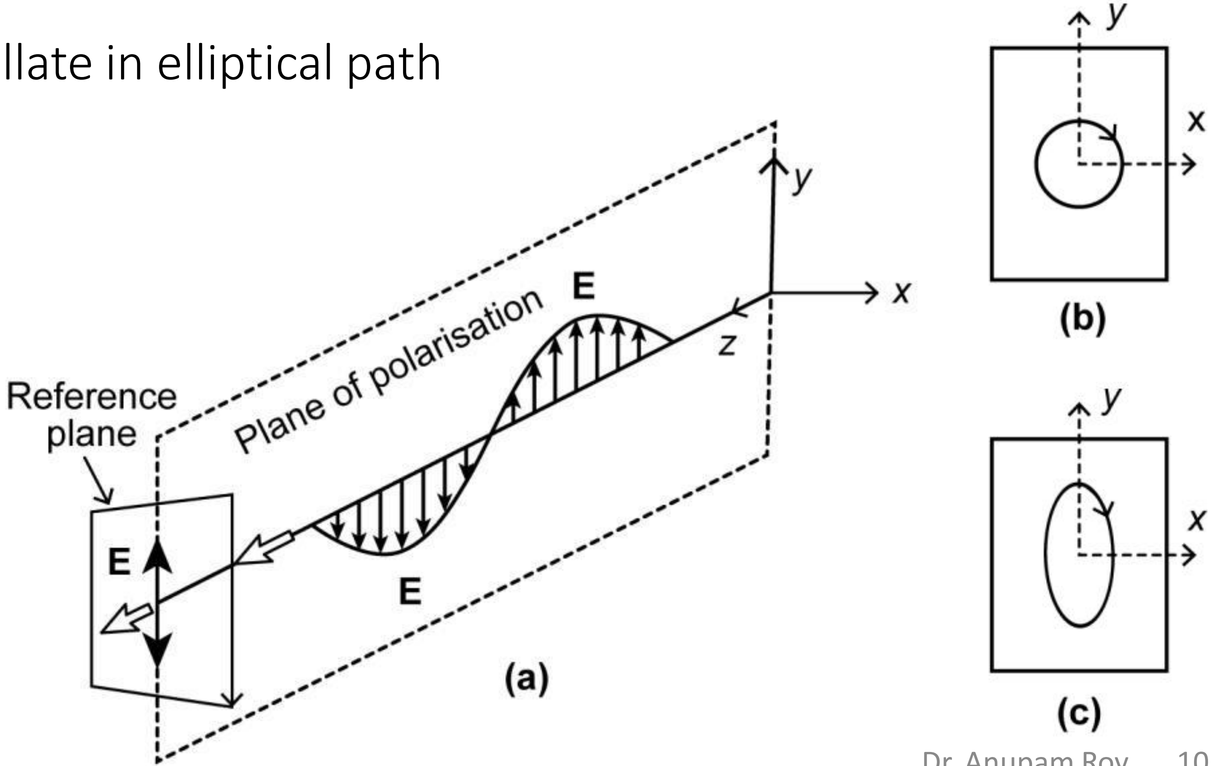
- Amplitudes of vibrations along x-direction is E_x and along y-direction is E_y.

Only $\bar{E}$-field is considered

- **Polarizer** is a device that make a unpolarized/randomly polarized light into a polarized light.

Polarization

- Types of polarization:
  - Plane or linear polarization: The electric field vector oscillate in one fixed plane
  - Circular polarization: The electric field vector oscillate in a circle. Left & Right CP
  - Elliptical polarization: The electric field vector oscillate in elliptical path



(a) Linear polarization (b) circular polarization, and (c) elliptical polarization

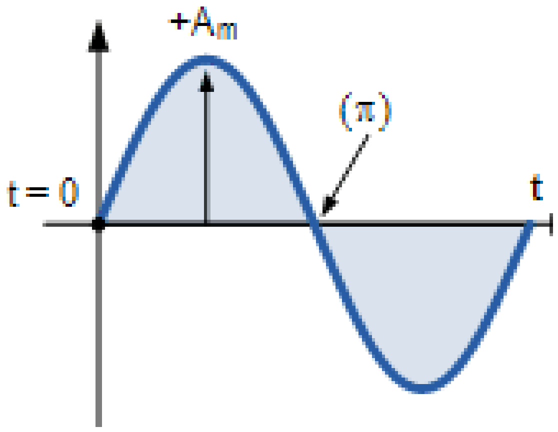
Module 1 (Physical Optics)

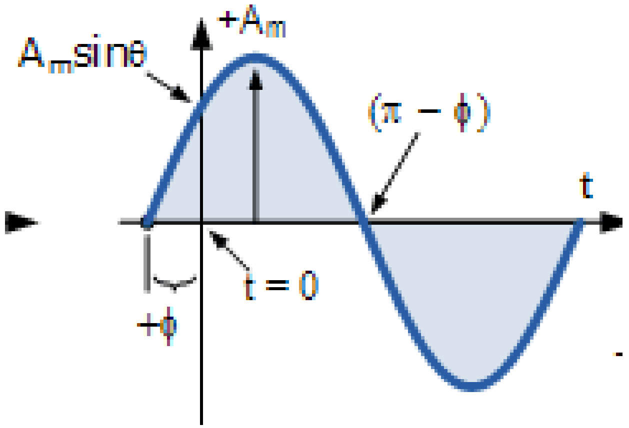
Polarization

Mathematically, the plane/linearly polarized light along x-axis is given by $\overrightarrow{E_1}(z,t) = \hat{x}\, E_0\, cos(kz - \omega t)$ (1)

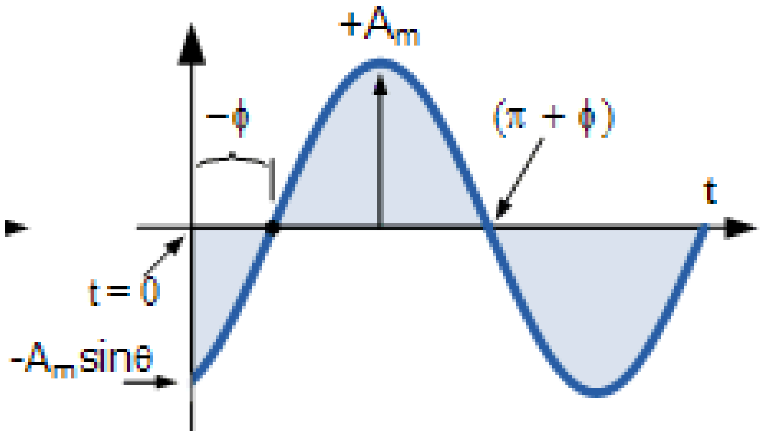
Plane/linearly polarized light along y-axis is given by $\overrightarrow{E_2}(z,t) = \hat{y}\, E_0\, cos(kz - \omega t + \varphi)$ (2)

φ is the phase difference compared to wave in (1)





Positive phase shift



Negative phase shift

Module 1 (Physical Optics)

Polarization

Superposition of two linearly polarized waves with $\boldsymbol{\varphi} = 0$

$$\vec{E}(z,t) = \overrightarrow{E_1}(z,t) + \overrightarrow{E_2}(z,t)$$

$$= \hat{x}\, E_{01}\, cos(kz-\omega t) + \hat{y}\, E_{02}\, cos(kz-\omega t) = (\hat{x}\, E_{01} + \hat{y}\, E_{02})\, cos(kz-\omega t)$$

Amplitude of the resultant wave $\sqrt{E_{01}^2 + E_{02}^2} = \sqrt{2}E_0$ (when $E_{01} = E_{02} = E_0$)

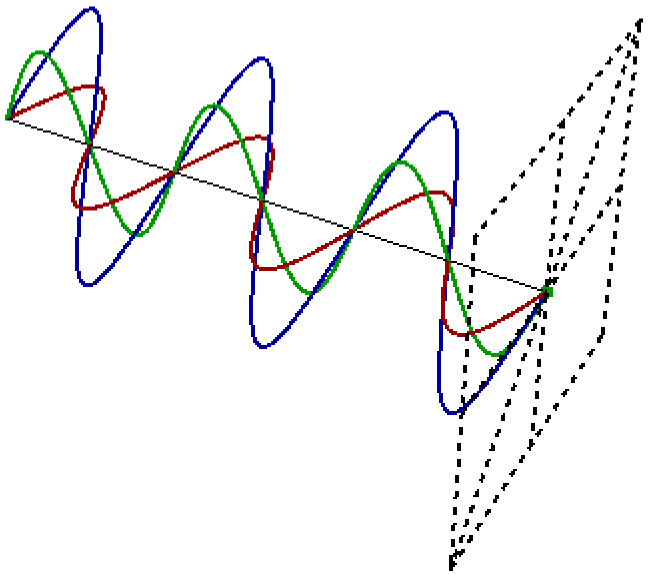
Polarization (direction of field) of the resultant wave $\theta = tan^{-1}\left(\frac{E_{02}}{E_{01}}\right) = tan^{-1}\left(\frac{E_0}{E_0}\right) = 45^o$

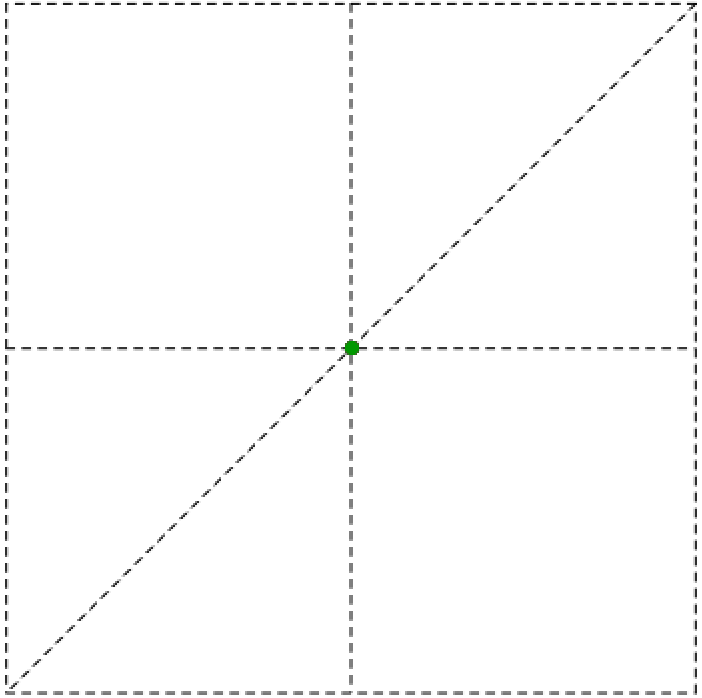
- ❑ When two in-phase linearly polarized light waves are superposed, resultant wave has fixed orientation as well as amplitude.
- ❑ In other words, any linearly polarized light can be resolved into two components: along x-axis and along y-axis.

Module 1 (Physical Optics)

Polarization

Consider uniform plane waves of same amplitude propagating along $\hat{z}$ but polarized along $\hat{x}$ and $\hat{y}$. Their superposition: $\vec{E}(z,t) \; = (\hat{x}\, E_{01} + \hat{y}\, E_{02})\, cos(kz - \omega t)$





Note the relative phase!

45° linearly polarized light !

Polarization

- What is the amplitude and direction of resultant field when two linearly polarized light with $\varphi = \pi$ superpose with each other?

- Solve

Module 1 (Physical Optics)

Polarization

Superposition of two linearly polarized waves with $E_{01} = E_{02} = E_0$ and $\varphi = \left(\pm\frac{\pi}{2}\right) = 90^o$

$$\overrightarrow{E_1}(z,t) = \hat{x}\, E_0\, cos(kz - \omega t)$$

$$\overrightarrow{E_2}(z,t) = \hat{y}\, E_0\, sin(kz - \omega t)$$

Superposed/resultant field: $\vec{E}(z,t) = \overrightarrow{E_1}(z,t) + \overrightarrow{E_2}(z,t) = E_0[\hat{x}\, cos(kz - \omega t) + \hat{y}\, sin(kz - \omega t)]$

Amplitude of the resultant field

$$|E| = \left(\vec{E}_1 \cdot \vec{E}_2\right)^{1/2}$$

$$= ([\hat{x}E_0\, cos(kz - \omega t) + \hat{y}E_0\, sin(kz - \omega t)] \cdot [\hat{x}E_0\, cos(kz - \omega t) + \hat{y}E_0\, sin(kz - \omega t)])^{1/2} = E_0$$

Resultant amplitude is constant and equal to the amplitude of the initial wave

Module 1 (Physical Optics)

Polarization

- Direction or polarization of the field

From scalar part of two initial waves

$\overrightarrow{E_1}(z,t) = \hat{x}\, E_0\, cos(kz - \omega t)$

$\overrightarrow{E_2}(z,t) = \hat{y}\, E_0\, sin(kz - \omega t)$

$$\left(\frac{E_1}{E_0}\right)^2 + \left(\frac{E_2}{E_0}\right)^2 = 1 \qquad \text{(Equation of circle)}$$

The resultant field equation rotate in a circle. The polarization is called **Circular Polarization**

Hence, superposition of **two plane/linearly polarized** wave with **equal amplitude** and **phase difference =** $\pm\frac{\boldsymbol{\pi}}{\mathbf{2}}$ leads to **circular polarization.**

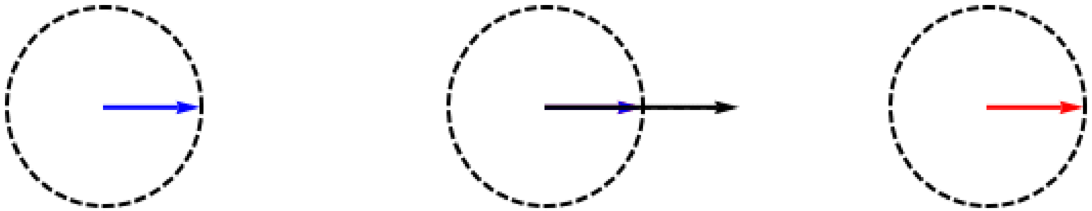
Problem: What is the difference between polarization direction when phase difference is $\pi/2$ and $-\pi/2$?
'Or' Define left handed polarized (LHP) and Right handed polarized (RHP) light.

Polarization

- Direction or polarization of the field

$$\overrightarrow{E_1}(z,t) = \hat{x}\ E_0\ cos(kz-\omega t)$$
$$\overrightarrow{E_2}(z,t) = \hat{y}\ E_0\ sin(kz-\omega t)$$

$$\overrightarrow{E_1}(z,t) = \hat{x}\ E_0\ cos(kz-\omega t)$$
$$\overrightarrow{E_2}(z,t) = -\hat{y}\ E_0\ sin(kz-\omega t)$$



Left Circular   Linear   Right Circular

Module 1 (Physical Optics)

Polarization

- Use of Polarization
  - Astronomy, Communication and RADAR applications (all radio transmitting and receiving antennas are intrinsically polarized, special use in radar), 3-D movies and polarized 3-D glasses
  - Polarization is used in infrared spectroscopy, use in ophthalmic instruments
  - In sunglasses, polarization is used to reduce glare.

Module 1 (Physical Optics)

Polarization

- If an **unpolarized** wave falls on a slit S_1 then the displacement associated with the transmitted wave is along the slit length.
- A rotation of the slit will not affect the amplitude of the transmitted wave although the plane of the polarization of transmitted wave depend on the orientation of the slit. Thus, the transmitted wave will be **linearly polarized** and slit S_1 is acting as a polarizer.
- If this polarized beam falls on another slit S_2, then by rotating S_2 we can vary the amplitude of transmitted wave. This second slit S_2 is acting as an analyzer. This slit can be used to detect the direction of polarization of the linearly polarized wave.

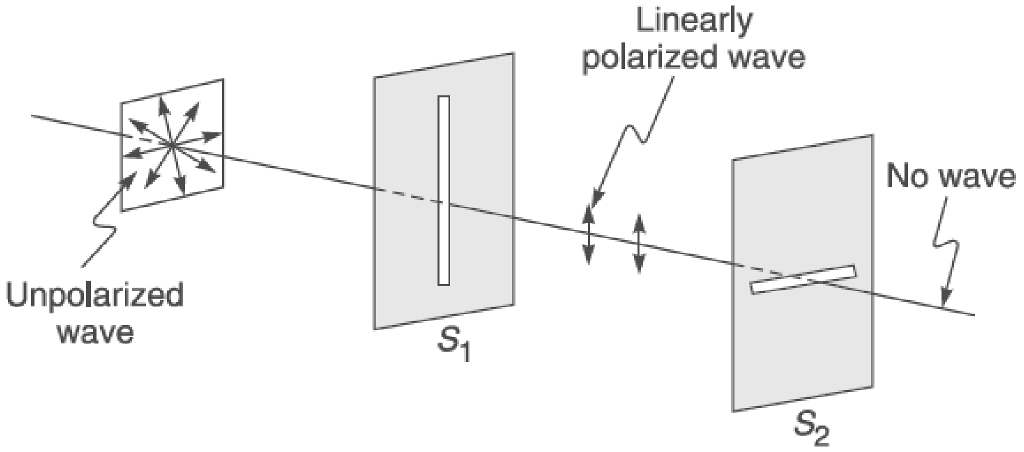


Fig. 22.4 If an unpolarized wave propagating on a string is incident on a long narrow slit S_1, then the transmitted beam will be linearly polarized and its amplitude will not depend on the orientation of S_1. If this polarized wave is allowed to pass through another slit S_2, then the intensity of the emerging wave will depend on the relative orientation of S_2 with respect to S_1.

Module 1 (Physical Optics)

Polarization

Malus' law

- This law tells us how the intensity transmitted by the analyzer varies with the angle that its plane of transmission makes with that of the polarizer (**ideal polarizer**).
- In the case of two piles of plates, the plane of transmission is the plane of incidence, and for the law of Malus to hold we must assume that the transmitted light is **completely** plane-polarized.
- The law of Malus states that the transmitted intensity varies as the square of the cosine of the angle between the two planes of transmission.

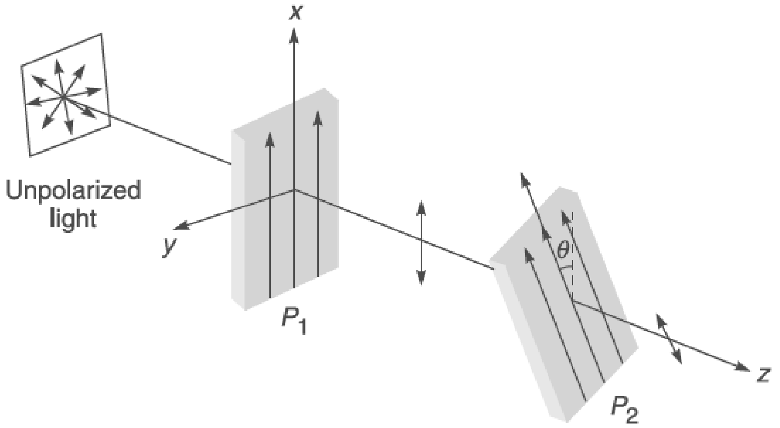
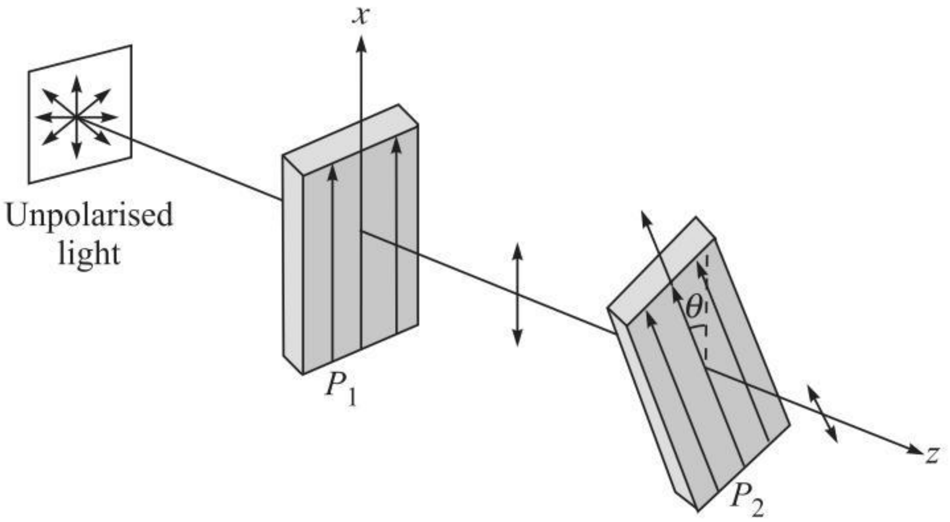


Fig. 22.6 For an unpolarized wave propagating in the +z-direction, the electric vector (which lies in the x-y plane) continues to change its direction in a random manner. If an unpolarized light beam is allowed to fall on a Polaroid, then the emerging beam will be linearly polarized; i.e., the electric vector will oscillate along a particular direction. If we place another Polaroid P_2, then the intensity of the transmitted light will depend on the relative orientation of P_2 with respect to P_1.; if the pass axis of the second polaroid P_2 makes an angle θ with the x-axis, the intensity of the emerging beam will vary as $\cos^2\theta$.

Module 1 (Physical Optics)

Polarization

Malus' law (Contd..)



An unpolarized light beam gets x-polarized after passing through the polaroid P_1, the pass axis of the second polaroid P_2 makes an angle θ with the x-axis. The intensity of the emerging beam will vary as $cos^2\theta$.

- If an x-polarized beam is passed through a polaroid P_2 whose pass axis makes an angle θ with the x-axis, then the intensity of the emerging beam will be given by,

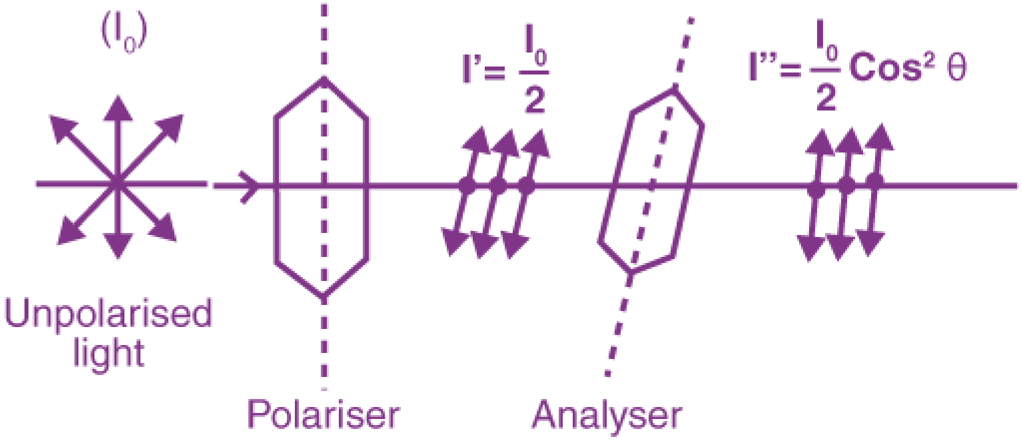
$$I = I_0 cos^2\theta$$

- where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is also along the x-axis (i.e., when $\theta = 0$). This equation ($I = I_0 cos^2\theta$) represents the Malus' law.

Module 1 (Physical Optics)

Polarization

Malus' law (Contd..)



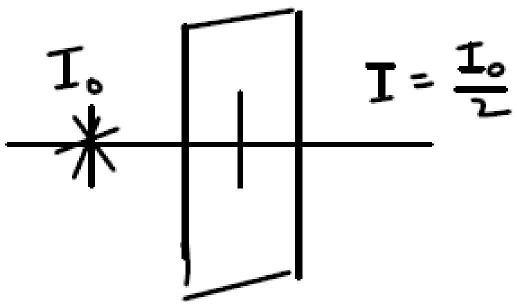
- $Intensity = (Amplitude)^2$
- If incident wave has amplitude E_0, then the outgoing wave has an amplitude of $E = E_0 cos\theta$.
- $I = E^2 = {E_0}^2 \, cos^2\theta = I_0 cos^2\theta$
- $I = I_0 cos^2\theta$

Module 1 (Physical Optics)

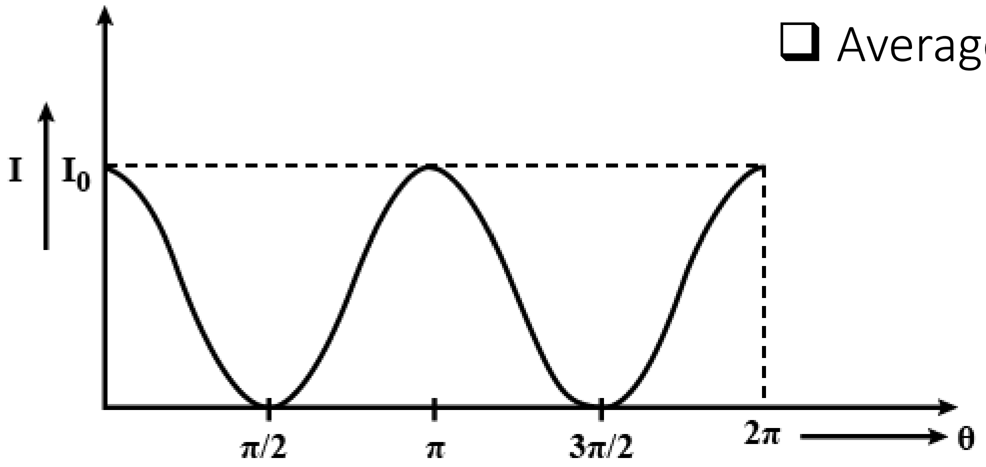
Polarization

- Production of polarized light (methods to produce linearly polarized light)

Malus' law (Contd..)



- ❑ $Intensity = (Amplitude)^2$
- ❑ For any θ, $I = I_0 cos^2\theta$
- ❑ But θ is random => $I = I_0\langle cos^2\theta \rangle = \frac{I_0}{2}$
- ❑ Average value of $cos^2\theta$ is $\frac{1}{2}$



(Using Malus law: incident light is polarized)

Module 1 (Physical Optics)

Polarization

Malus' law (Contd..)

Example:

A beam of vertically polarized light is incident on a linear polarizer. It is observed that the intensity of the light emerging out of the polarizer is 25% of the incident intensity when the angle between the transmission axis of the polarizer and the vertical is 60^o.

Solution

According to Malus law, $I = I_0 Cos^2\theta$

When the transmission axis of polarizer is 60^o, the intensity of transmitted light is,

$$I(60) = I_0 Cos^2(60) = \frac{I_0}{4} = 0.25\, I_0$$

Module 1 (Physical Optics)

Polarization

So far…

- Polarization of light is due to its transverse nature.
- Types of polarization: (1) Linear (2) circular (RHP & LHP) (3) Elliptical
- Superposition of two linearly polarized waves with phase difference 0, $\pm\pi$ leads to another linearly polarized wave.
- A linearly polarized light can be resolved into two components perpendicular to each other (along x and y).
- Superposition of two linearly polarized wave with equal amplitude and phase difference = $\pm\frac{\pi}{2}$ leads to circular polarization.
- Superposition of two linearly polarized wave with phase difference other than 0, $\pm\pi$ and $\pm\frac{\pi}{2}$ leads to formation of elliptical polarized wave.
- Malus' law provides a condition, $I = I_0 Cos^2\theta$ for an ideal polarizer.

Module 1

Question?

Module 1

Date: 08.09.2023

Lecture: 2

PH113: Physics (B.Tech. I)

Module 1

Module – 1 : Physical Optics

Polarization, Malus' Law, Brewster's Law, Double Refraction, Interference in thin films (Parallel films), Interference in wedge-shaped layers, Newton's rings, Fraunhofer diffraction by single slit, Double slit. Elementary ideas of fibre optics and application of fibre optic cables. [8]

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Module 1 (Physical Optics)

Polarization

- Production of polarized light (methods to produce linearly polarized light)

Polarization of unpolarized light:

- **Several strategies**
  - By scattering
  - By reflection
  - By double refraction
  - By absorption

Module 1 (Physical Optics)

Polarization

- Production of polarized light (methods to produce linearly polarized light)

1) Polarization by Scattering

- **If an unpolarized light is allowed to fall on a gas, then the beam scattered at 90° to the incident beam is linearly polarized.**
- It follows from the fact that the waves propagating in the y-direction are produced by the x-component of the dipole oscillations. The y-component of dipole oscillations will produce no field in the y-direction.
  - ✓ Unscattered part of the incident beam remains unpolarized.
  - ✓ The waves scattered by the gas molecules in perpendicular direction is linearly polarized.
  - ✓ Waves scattered in other directions are partially polarized.

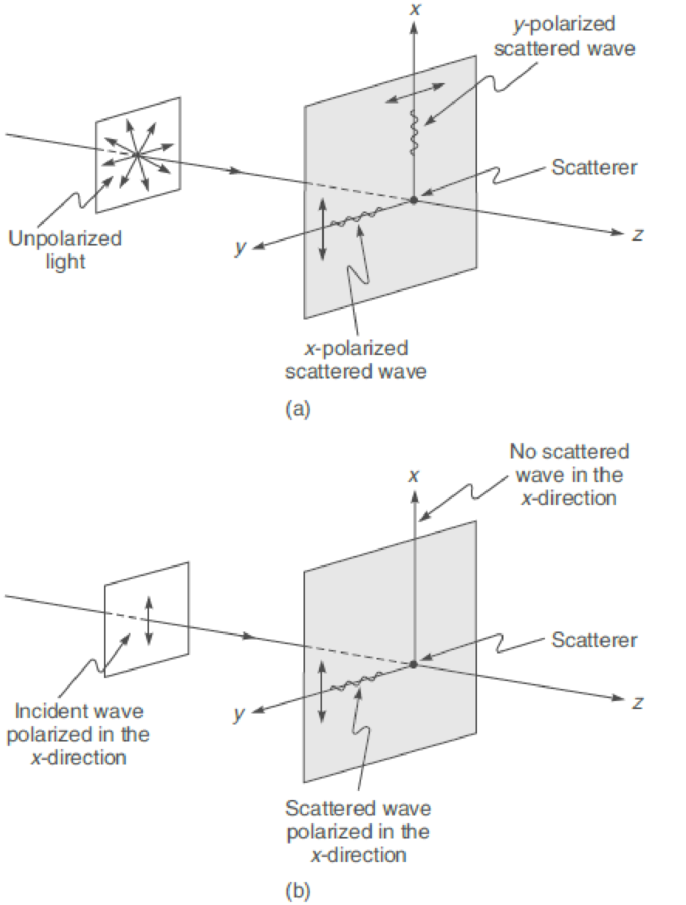


Fig. 22.16 (a) If the electromagnetic wave is propagating along the z-direction, then the scattered wave along any direction perpendicular to the z-axis will be linearly polarized. (b) If a linearly polarized wave (with its **E** oscillating along the x-direction) is incident on a dipole, then there will be no scattered wave in the x-direction.

Module 1 (Physical Optics)

Polarization

- Production of polarized light (methods to produce linearly polarized light)

2) Wire Grid Polarizer and Polaroid

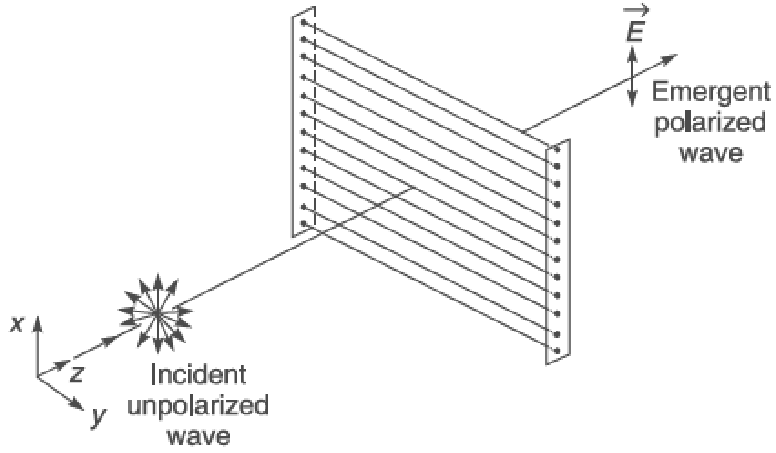


Fig. 22.8 The wire-grid polarizer.

- ❑ Consists of a regular array of parallel metallic wires (e.g., copper), placed in a plane perpendicular to the incident beam. Wires are placed closed to each other such that the spacing $\lesssim \lambda$ in order to reduce diffraction effects.
- ❑ Suppose an unpolarized EM wave incidents on the grid. The y-component of the incoming field (parallel to wires) moves the electrons of the wire and generate currents in the wire.
- ❑ This leads to Joule heating and the x-component gets converted into the heat energy. The x-component of field in not absorbed and passes through.

❑ Thus, it converts an unpolarized light into a linearly polarized with the electric vector along the x-axis.

Module 1 (Physical Optics)

Polarization

- Production of polarized light (methods to produce linearly polarized light)

2) Wire Grid Polarizer and Polaroid

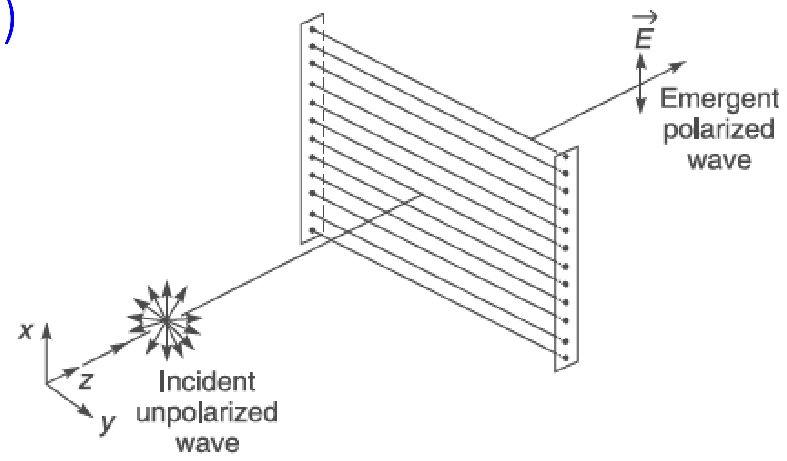


Fig. 22.8 The wire-grid polarizer.

- Note: For this system to be effective, spacing between the wires should be $\lesssim \lambda$. Remember, light waves are associated with a very small wavelength. So, fabrication of a polarizer with such a small spacings is extremely difficult.

- Instead of thin wires, one prepares sheets of long polymers chains. These atoms provide *high conductivity*.

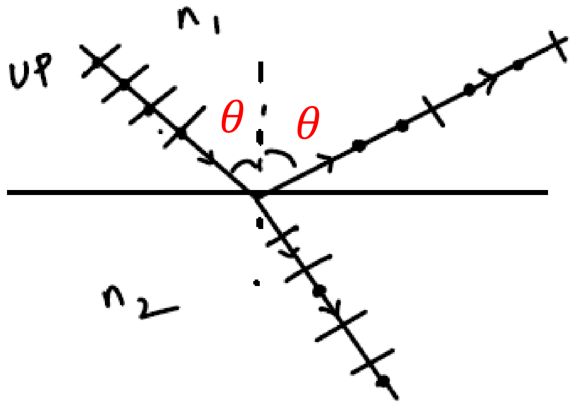
- When a light beam is incident on such a polaroid, the molecules (aligned parallel to each other) *absorb* the component of electric field (which is parallel to the alignment). It, thus, acts like the *wire grid polarizer*.

- Now the spacing between two adjacent long chain molecules is small compared to optical wavelength. So, it is very effective in producing linearly polarized light and can be used for wavelengths in visible range.

Polarization

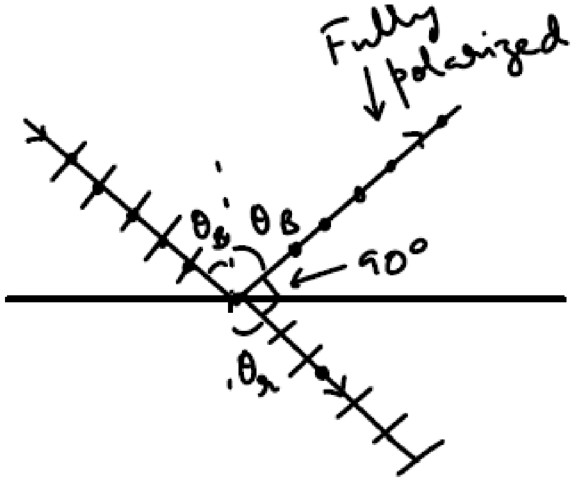
- Production of polarized light (methods to produce linearly polarized light)

3) **Polarization by reflection**: **Brewster's law**



- Consider unpolarized light to be incident at an angle θ on a dielectric like glass, as shown in Figure. There will always be a reflected ray and a refracted ray. The reflected ray is partially plane-polarized.

- Only at a certain angle, about 57° for ordinary glass (air-glass interface), it is completely **plane-polarized**. At this polarizing angle θ_B, the reflected and refracted rays are just 90° apart.



- This is **Brewster's law**, which shows that the angle of incidence for maximum polarization depends only on the refractive index. This remarkable discovery enables one to correlate polarization with the refractive index.

Module 1 (Physical Optics)

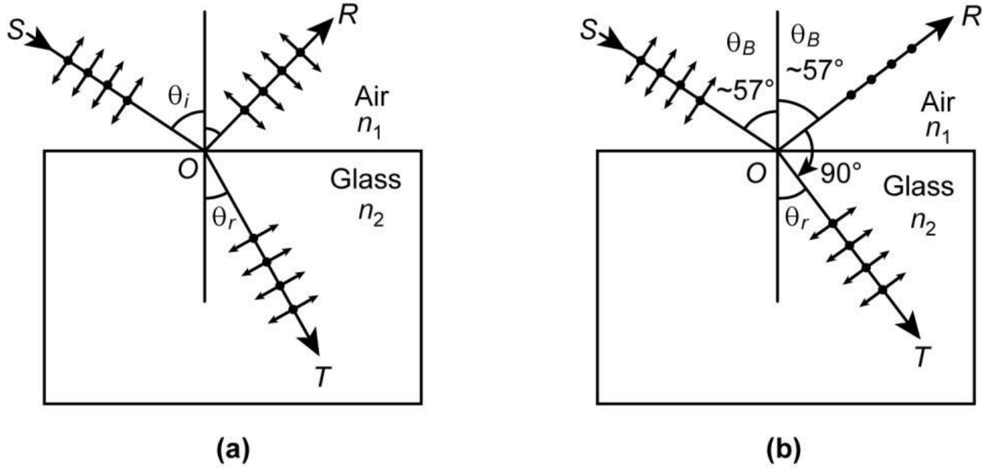
Polarization

- Production of polarized light (methods to produce linearly polarized light)

3) Polarization by reflection: Brewster's law

- When light wave travels from low density (refractive index) medium (e.g., air) to high density (refractive index) medium (e.g., water or glass), at a particular angle of incidence, the *reflected light is polarized*. The angle at which polarization occurs is known as **Brewster angle** (θ_B).

- At Brewster angle, the angle between reflected and transmitted (refracted) wave is 90°.



Unpolarized light incident (a) at angle θ_I and (b) at angle θ_B

↔ Wave polarized parallel to the plane of incidence

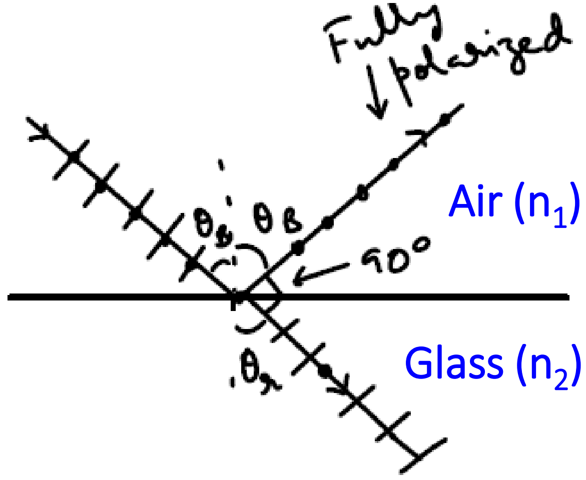
• Wave polarized perpendicular to the plane of incidence

Module 1 (Physical Optics)

Polarization

- Production of polarized light (methods to produce linearly polarized light)

3) **Polarization by reflection: Brewster's law**



- Consider unpolarized light to be incident at an angle θ on a dielectric like glass, as shown in Figure. When the reflected and refracted rays become perpendicular, the reflected ray becomes completely **plane-polarized**. The corresponding angle of incidence is called the Brewster's angle (θ_B).

- $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

- Proof: $\theta_B + \theta_r = 90°$

- **Snell's law**: $n_1 \sin\theta_B = n_2 \sin\theta_r = n_2 \sin(90° - \theta_B) = n_2 \cos\theta_B$

- $\tan\theta_B = \frac{n_2}{n_1} \quad \Rightarrow \quad \boldsymbol{\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)}$

Tangent of Brewster angle is equal to the ratio of the refractive indices of the media at whose interface incident light is reflected.

Polarization

- Production of polarized light (methods to produce linearly polarized light)

3) Polarization by reflection: Brewster's law

Example

- When the incident beam is in air ($n_1 = 1$) and the transmitting medium is glass ($n_2 = 1.5$) the Brewster angle is nearly 57°.

- For air ($n_1 = 1$) and water ($n_2 = 1.33$) interface (like the surface of a pond or a lake), θ_B is 53°. This means that when the sun is 37° above the horizontal, the light reflected by a calm pond or lake should be completely linearly polarized.

- **Problem**: A plate of flint glass (refractive index 1.67) is immersed in water (refractive index 1.33). Calculate the Brewster angle for internal as well as external reflection at an interface.

Module 1 (Physical Optics)

Polarization

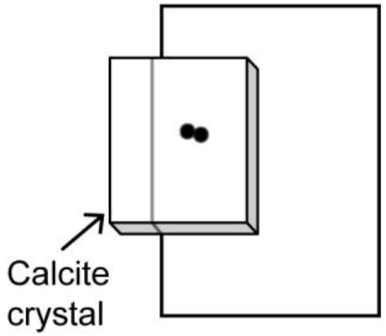
- Production of polarized light (methods to produce linearly polarized light)

4) **Polarization by double refraction (Birefringence)**

- Similar to reflection, refraction can also polarize the beam.
- Only **optically anisotropic crystals** (calcite, cellophane) polarizes the beam. (Optically anisotropic means that optical properties are not the same in all directions.)
- Light passes through optically anisotropic crystals shows "**double refraction**" or "**birefringence**".
- Materials showing double refraction are called Birefringent material.

Example:

- A dot mark on a paper appears as two dots when viewed through calcite crystal.
- Calcite crystal splits the incident beam into two due to double refraction.

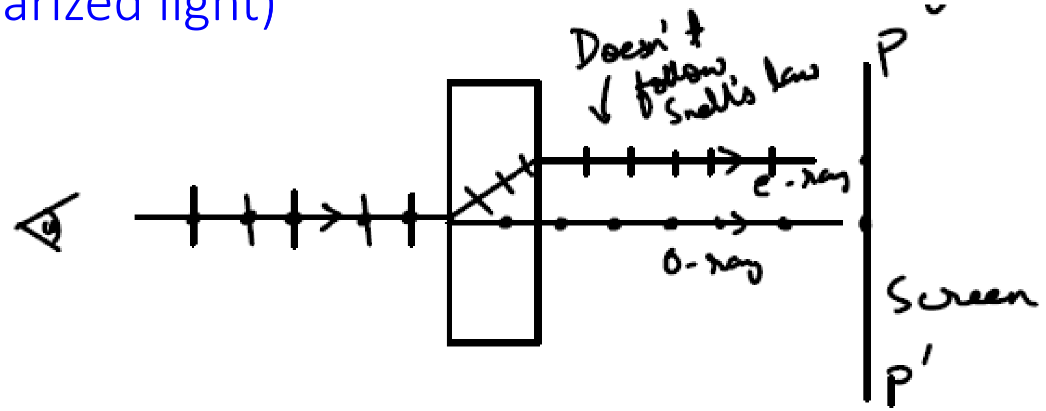


Module 1 (Physical Optics)

Polarization

- Production of polarized light (methods to produce linearly polarized light)

4) Polarization by double refraction (Birefringence)



- ❑ When an unpolarized light beam is incident on a calcite ($CaCO_3$) crystal, it splits into two beams, each of which is linearly polarized.
- ❑ The ray that follows Snell's law is called ordinary ray or o-ray.
- ❑ The ray that does not follow Snell's law is called extraordinary ray or e-ray.
- ❑ If the crystal is rotated about the incident ray as an axis, then the e-ray rotates about the o-ray.

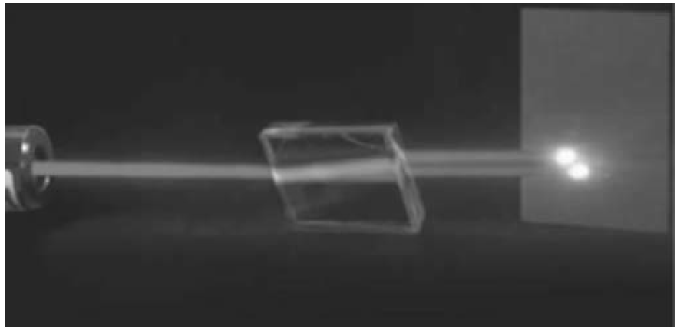
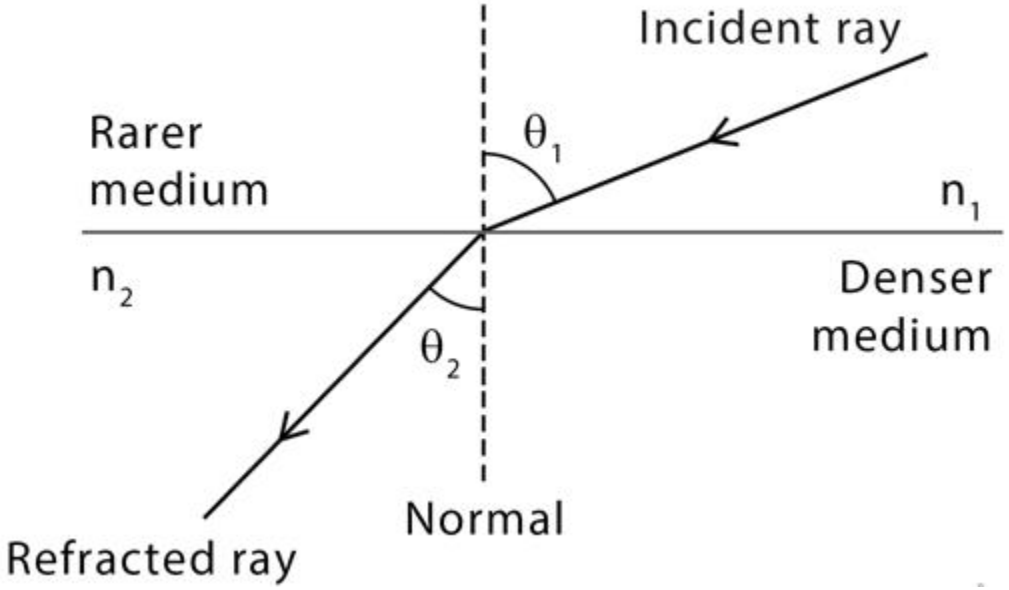


Fig. 22.13 When an unpolarized light beam is incident normally on a calcite crystal, it usually splits up into two linearly polarized beams. Photograph courtesy Professor V Lakshminarayanan and adapted from Ref. 22.16. A color photo appears as Fig. 30 in prelim pages.

Remember: Snell's law

- Snell's law gives a relationship between the angles of incidence (θ_1) and refraction (θ_2) when a ray of light travels from a rarer medium of refractive index (n_1) to a denser medium of refractive index (n_2)
- $n_1 sin\theta_1 = n_2 sin\theta_2$



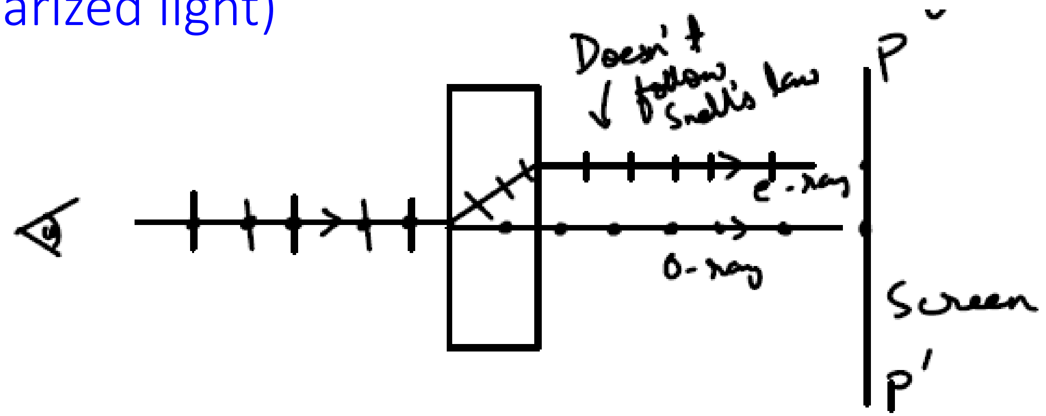
Module 1 (Physical Optics)

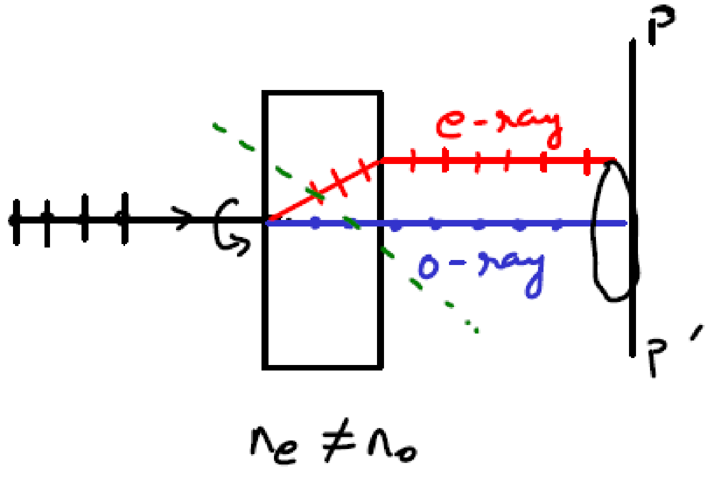
Polarization

- Production of polarized light (methods to produce linearly polarized light)

4) Polarization by double refraction (Birefringence)

- The velocity of e-ray is different from o-ray.
- If $v_o < v_e$, then the crystal is **negative** (for example, calcite).
- If $v_o > v_e$, then the crystal is **positive** (for example, quartz).
- There is a particular direction, called the **Optic Axis**, along which there is no splitting and $v_o = v_e$ (only one ray is obtained that follows the Snell's law).
- **Optic axis**: A special directions in a birefringent crystal along which two refractive indices are equal and both o- and e-rays in the direction of optic axis propagate with same velocity.

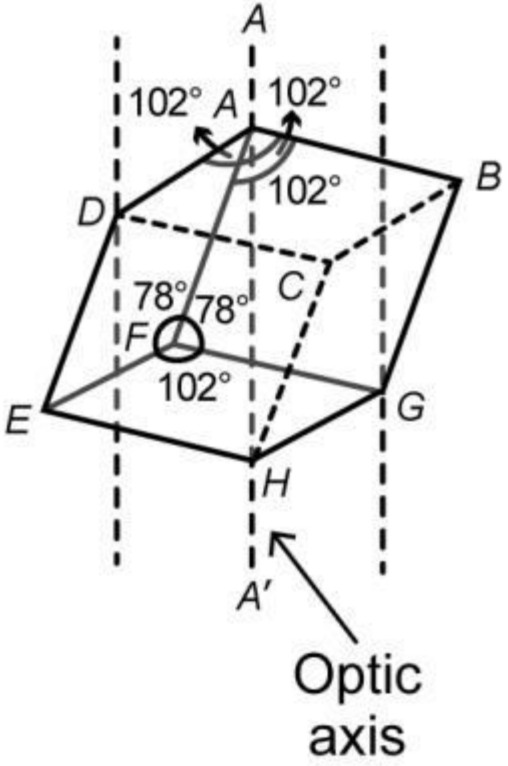




Polarization

- Production of polarized light (methods to produce linearly polarized light)

4) Polarization by double refraction (Birefringence)



- ❑ Optic axis (Example): Calcite crystal normally used in the shape of rhombohedra bounded by six parallelograms with angles 102° and 78°. At corners A and H, angle of each of the three faces is obtuse and these points are known as **blunt corners**.

- ❑ A line passing through one of the blunt corners and is equally inclined to all the three edges meeting at that point gives the direction of **optic axis**. Any line parallel to this line is also known as optic axis.

❑ Birefringent crystals with only one optic axis are called **uniaxial crystals** (e.g., Calcite, Quartz). Crystals having two optic axes are called **biaxial crystals** (e.g., Mica).

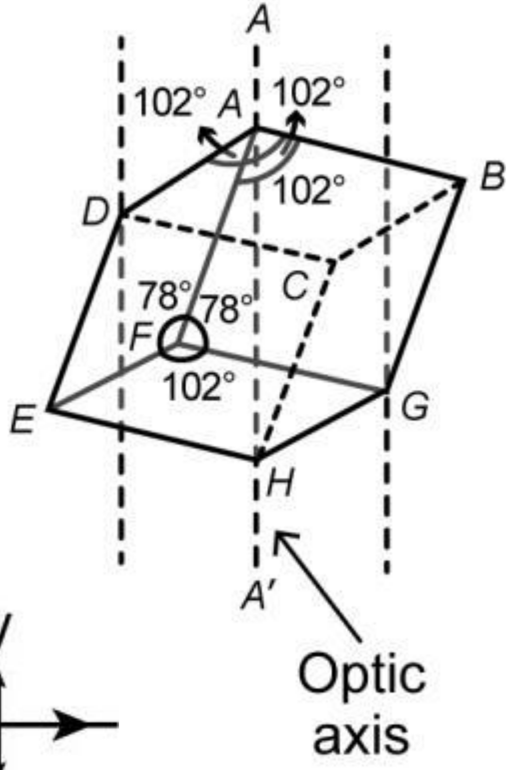
❑ When the refractive index for o-ray (n_o) is more than that for the e-ray (n_e) – negative uniaxial crystal. ($n_o > n_e$). For positive uniaxial crystal, ($n_o < n_e$).

Module 1 (Physical Optics)

Polarization

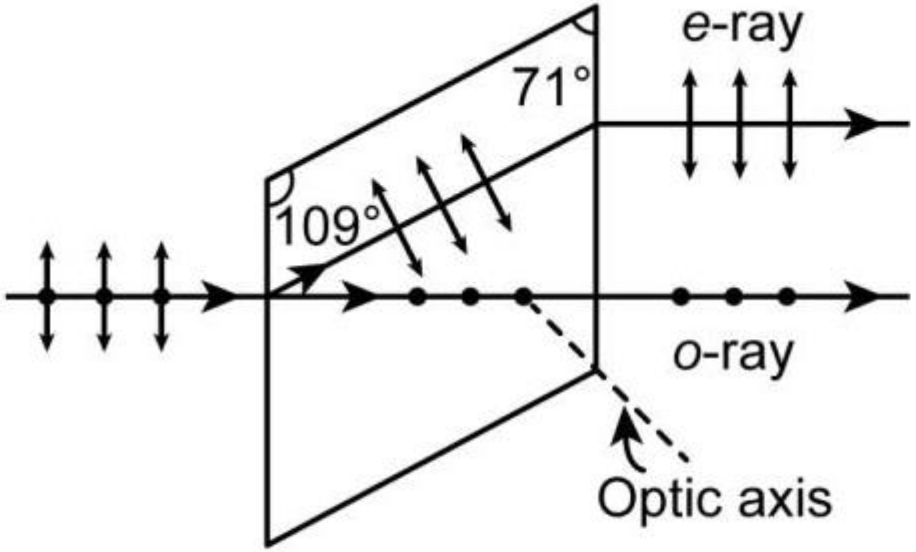
- Production of polarized light (methods to produce linearly polarized light)

4) Polarization by double refraction (Birefringence)



- The birefringent crystal splits an unpolarized light into e-ray and o-ray.
- the electric field of e-ray vibrate in the plane containing the optic axis and electric field of o-ray vibrates perpendicular to the optic axis.

- Therefore, due to double refraction, the unpolarized light beam splits into two components which are plane polarized.



Questions?

PH113: Physics (B.Tech. I)

Module 1

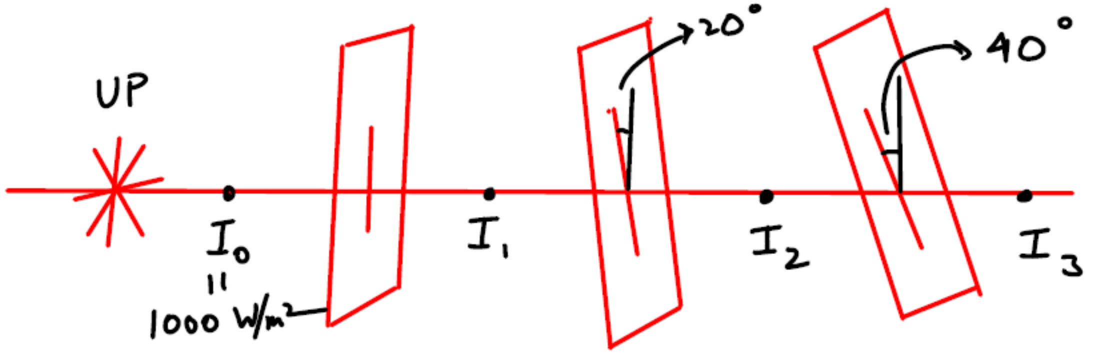
Date: 11.09.2023

Lecture: 3 (Tutorial Class)

Module 1

Malus's Law

Q.



$$I_1 = \frac{I_0}{2}$$

$$I_2 = \frac{I_0}{2}\cos^2(20°)$$

$$I_{out} = I_3 = I_2\cos^2(20°) = \frac{I_0}{2}\cos^4(20°) \simeq 390\ W/m^2$$

What happens to I_{out} if the second polarizer is removed?

$I_{out} \simeq 293\ W/m^2$.

Module 1

PH113 / Assignment - Module 1

(Dated: September 6, 2023)

1. **Malus Law.**

   (a) Consider a geometry of sequence of two polarizers rotated at a relative angle of 45° from each other. What would be resulting amplitude and intensity if the incident unpolarized (light) beam had the intensity I_0?

   (b) Similar to above, consider a sequence of three polarizers at relative angles of 20° to each other. Also, the first polarizer is aligned along the y-axis. Compute the intensities I_i $(i = 1, 2, 3)$ after each polarizer if the intensity of the y-polarized source is given to be $I_0 = 1000\,\mathrm{W/m^2}$.

   (c) What happens to the final intensity if the second polarizer is removed?

2. Brewster angle

On a calm surface of a pond or a lake (interface between air and water), for which position of the sun would the reflected light from the interface be completely linearly polarized. The refractive indexes for air and water should be taken as $n_{\text{air}} = 1$ and $n_{\text{water}} = 1.33$.

Comment: This situation is, in fact, realized at the beaches or lakes.

Module 1

3. A plate of flint glass (refractive index 1.67) is immersed in water (refractive index 1.33). Calculate the Brewster angle for internal as well as external reflection at an interface.

Module 1

Date: 12.09.2023

Lecture: 4

PH113: Physics (B.Tech. I)

Module 1

Module – 1 : Physical Optics

Polarization, Malus' Law, Brewster's Law, Double Refraction, Interference in thin films (Parallel films), Interference in wedge-shaped layers, Newton's rings, Fraunhofer diffraction by single slit, Double slit. Elementary ideas of fibre optics and application of fibre optic cables. [8]

Text book:

T1: A. Ghatak, Optics, 6th Edition, 2017, McGraw-Hill Education (India) Pvt. Ltd.

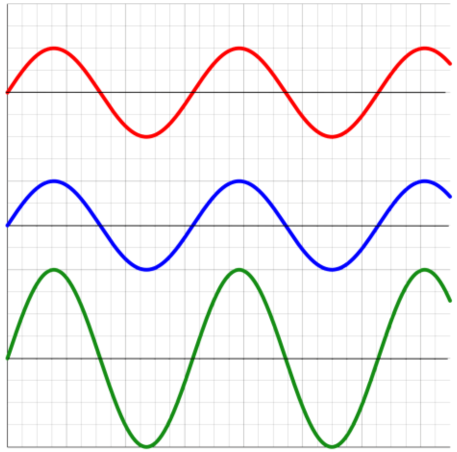
References: (1) Optics – E Hecht **(2)** Fundamentals of Optics – Jenkins & White

Class structure: 4 Lectures including 1 Tutorial per week. (8 hours ~ 2 weeks for this module!)

Interference

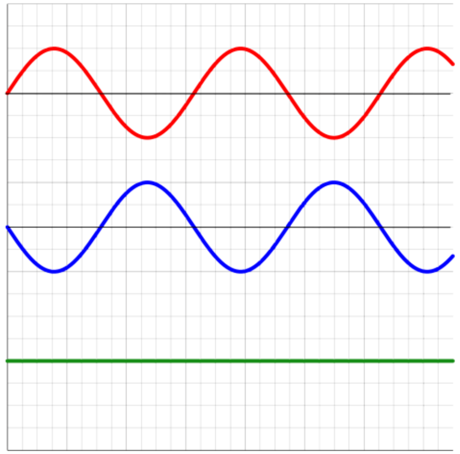
- Superposition of two coherent wave (definite phase relation) (ideally from the same source) to form periodic variation (fringes) in the intensity.
- Fringe with highest intensity is due to constructive interference.
- Fringe with lowest intensity is due to destructive interference.
- Interference can be produced by (a) division of wavefront and (b) division of amplitude.

Constructive



Wave amplitudes add in constructive interference

Destructive



Wave amplitudes cancel to give zero output due to destructive interference

- Consider two waves as
  - $y_1 = A\cos\omega t$
  - $y_2 = A\cos\ (\omega t + \varphi)$
- Principle of superposition:
  - $y = y_1 + y_2 = A\cos\omega t + A\cos\ (\omega t + \varphi)$

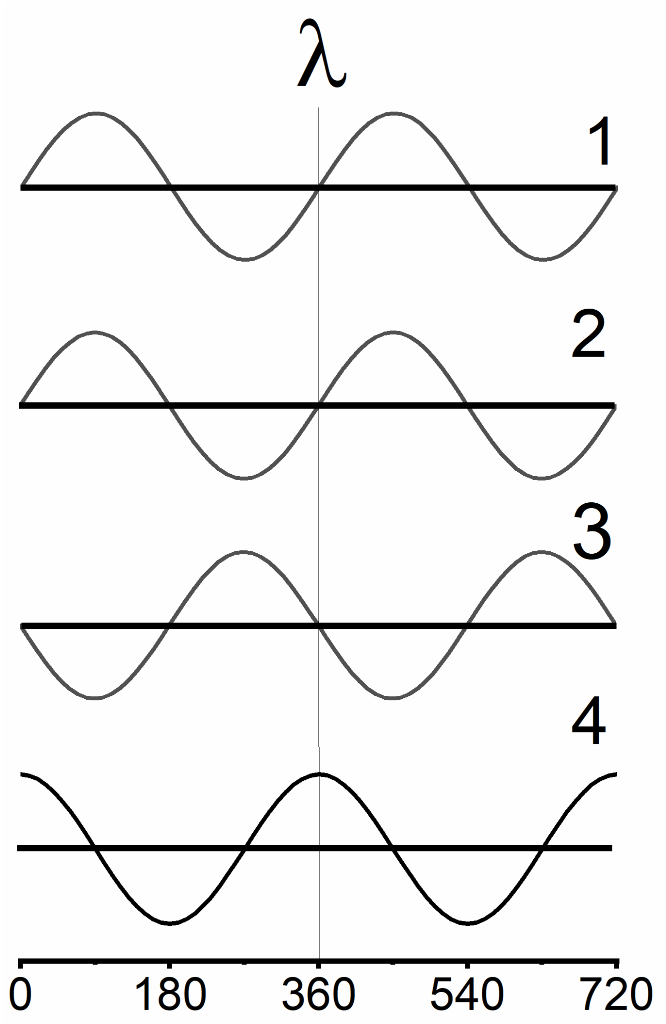
Interference by a plane parallel film

Constructive and destructive interferences

- Consider two waves as
  - $y_1 = A\cos\omega t$ and $y_2 = A\cos\ (\omega t + \varphi)$
- Principle of superposition: $y = y_1 + y_2 = A\cos\omega t + A\cos\ (\omega t + \varphi)$
- Remember: **path difference of $\boldsymbol{\lambda}$ (the wavelength) corresponds to a phase difference of $\boldsymbol{2\pi}$.**
- For constructive interference, the phase difference between y_1 and y_2 is $\varphi = 2n\pi,\ n = 0, 1, 2, \dots\dots$, *i.e.*, the optical path difference (OPD) is $l = n\lambda$
- For destructive interference, the phase difference is $\varphi = (2n+1)\pi,\ n = 0, 1, 2, \dots\dots$, *i.e.*, the optical path difference (OPD) is $l = (2n+1)\frac{\lambda}{2} = (n + \frac{1}{2})\lambda$

Module 1 (Physical Optics)

Interference by superposition of waves

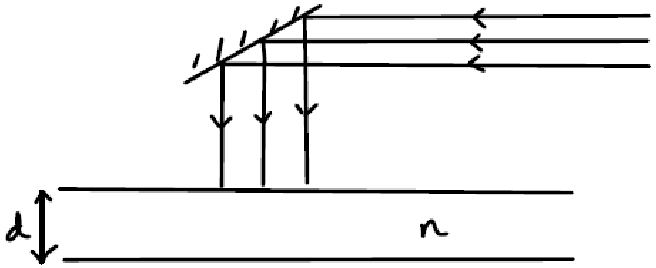


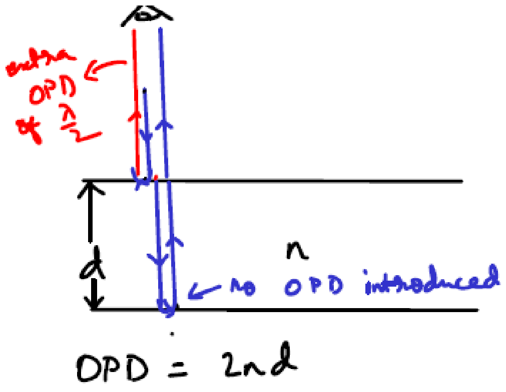
- Constructive interference: 1 + 2
  - Path diff.: 0, λ, 2λ, …., **$n\lambda$**
- Destructive interference: 1 + 3
  - Path diff.: $\lambda/2$, $3\lambda/2$, …., **$(2n+1)\lambda/2$**
- Partial constructive interference: 1 + 4
  - Path diff.: $\lambda/4$ ….
- $Phase\ difference = \frac{2\pi}{\lambda} \times Path\ Difference$

Module 1 (Physical Optics)

Interference by a plane parallel film (normal incidence)

- The film has parallel surface, and incident rays are normal to the surface





- For constructive interference, we should have:

  $2nd = m\lambda, m = 0, 1, 2, \ldots\ldots$

- For destructive interference, we should have:

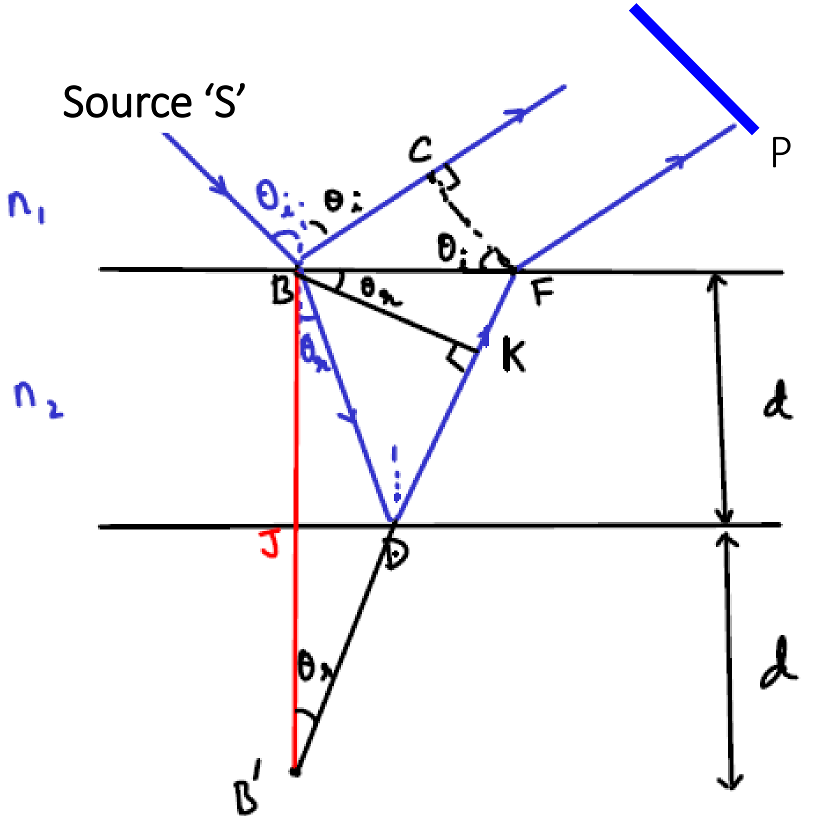
  $2nd = (m + \frac{1}{2})\lambda, m = 0, 1, 2, \ldots\ldots$

- The above relations are incorrect, because due to reflection *from a rarer medium to a denser medium*, a phase difference of π (or an optical path difference, OPD of $\frac{\lambda}{2}$) is introduced.
- Optical path difference, OPD = actual distance × refractive index of medium
- The correct relations are:
  - For constructive interference, we should have: $2nd = (m + \frac{1}{2})\lambda, m = 0, 1, 2, \ldots\ldots$
  - For destructive interference, we should have: $2nd = m\lambda, m = 0, 1, 2, \ldots\ldots$

Module 1 (Physical Optics)

Interference by a plane parallel film (oblique incidence)

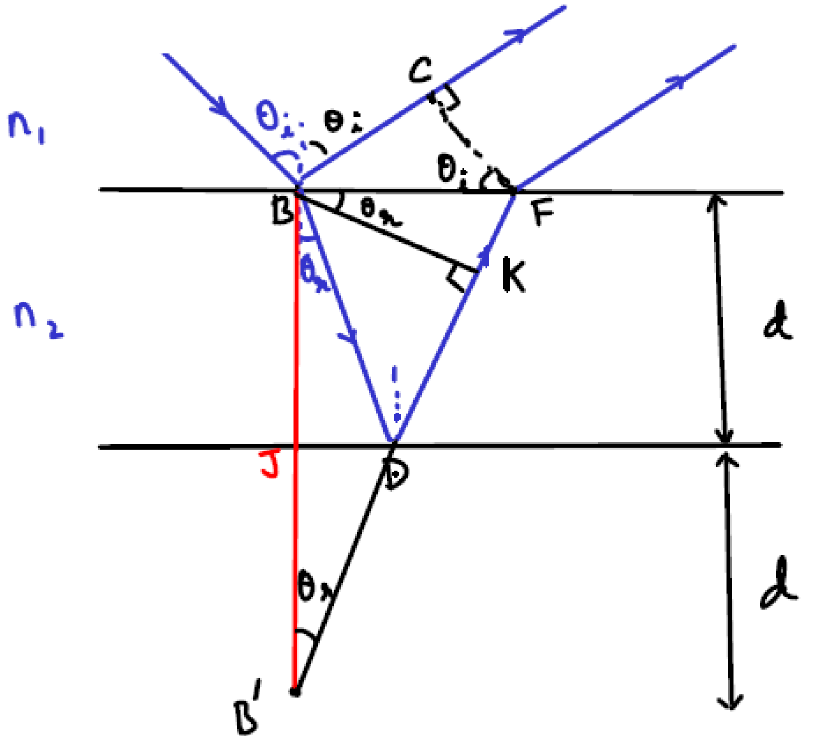
- The film has parallel surface, and incident rays are at certain angle to the surface.
- Light from a monochromatic source 'S' falling on a thin film creates two set of parallel rays.
- Wave reflected from the upper surface of the film interferes with the wave reflected from the lower surface.



- Each set, when collected through a lens can make constructive or destructive interference pattern (at P) depending upon the phase relation between them.

Module 1 (Physical Optics)

Interference by a plane parallel film (oblique incidence)

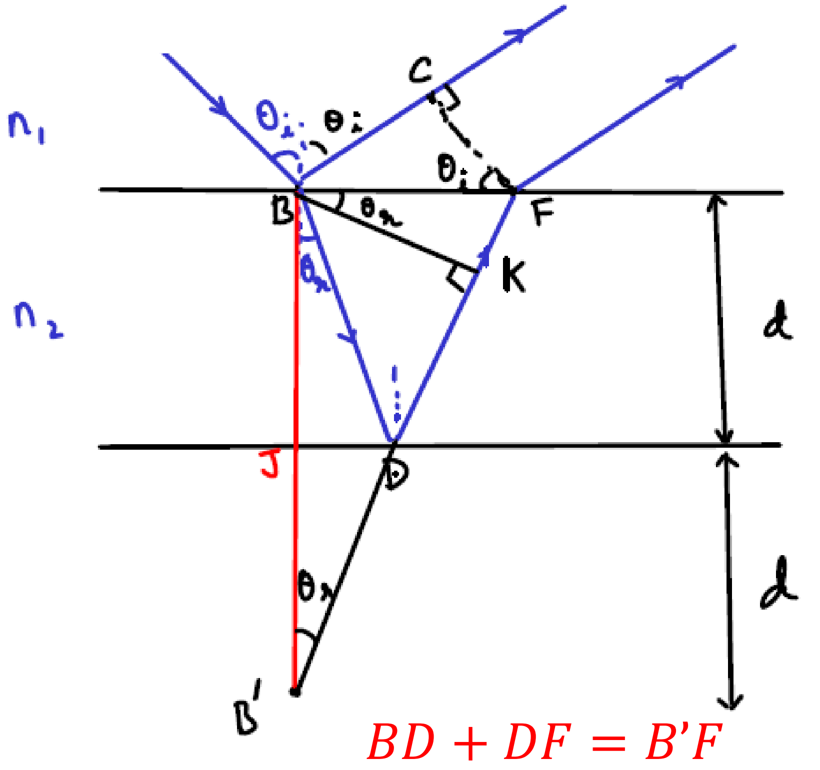


$BD + DF = B'F$

- Optical path difference (OPD) can only be introduced in the region BCFD.
- Remember: OPD = actual distance × refractive index of medium
- $OPD = n_2(BD + DF) - n_1 BC$
- $\angle BFC = \theta_i$ and $\angle FBK = \theta_r$
- From $\Delta BFC,\ BC = BF\ sin\theta_i$ ....Eqn (1)
- From $\Delta BKF,\ KF = BF\ sin\theta_r$ ....Eqn (2)
- Substitute Eqn. 2 in Eqn. 1: $BC = \frac{KF}{sin\theta_r}\ sin\theta_i = KF\ \frac{n_2}{n_1}$ (using Snell's law) ....Eqn (3)

Module 1 (Physical Optics)

Interference by a plane parallel film (oblique incidence)



- Optical path difference: $OPD\ (\Delta) = n_2(BD + DF) - n_1 BC$
- From $\Delta BFC,\ BC = BF\ sin\theta_i$ [Eqn (1)] and from ΔBKF, $KF = BF\ sin\theta_r$ [Eqn (2)]
- Substituting Eqn. 2 in Eqn. 1, we get:
- $BC = \frac{KF}{sin\theta_r}\ sin\theta_i = KF\ \frac{n_2}{n_1}$ (using Snell's law) ....[Eqn (3)]
- OPD: $\Delta = n_2(BD + DF) - n_1 BC \quad = n_2\ B'F - n_1\ BC$

$$= n_2\ B'F - n_1\left(\frac{n_2}{n_1}\ KF\right) \quad = n_2\ (B'F - KF)$$

$$= n_2\ B'K = n_2\ BB'cos\theta_r = n_2\ 2d\ cos\theta_r.$$ (cosine law)

Module 1 (Physical Optics)

Interference by a plane parallel film (oblique incidence)

- Optical path difference: $OPD = 2n_2 d\ cos\theta_r$ **(cosine law)**
- Due to reflection from a rarer medium to a denser medium, a phase difference of π (or an optical path difference of $\lambda/2$) is introduced.
- Ray reflected from the top surface of film undergoes a phase change of π (or path change $\frac{\lambda}{2}$). So, total path difference is $2n_2 d\ cos\theta_r + \frac{\lambda}{2}$
- Condition for **destructive interference** is: $2n_2 d\ cos\theta_r + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda$ for $m = 0, 1, 2, \ldots.$
- Or, $2n_2 d\ cos\theta_r = m\lambda$ (**condition for minima**)
- Condition for constructive interference is: $2n_2 d\ cos\theta_r - \frac{\lambda}{2} = m\lambda$ for $m = 0, 1, 2, \ldots.$
- Or, $2n_2 d\ cos\theta_r = (m + \frac{1}{2})\lambda$ (**condition for maxima** in interference pattern)

Module 1 (Physical Optics)

Interference by a plane parallel film (oblique incidence)

Example

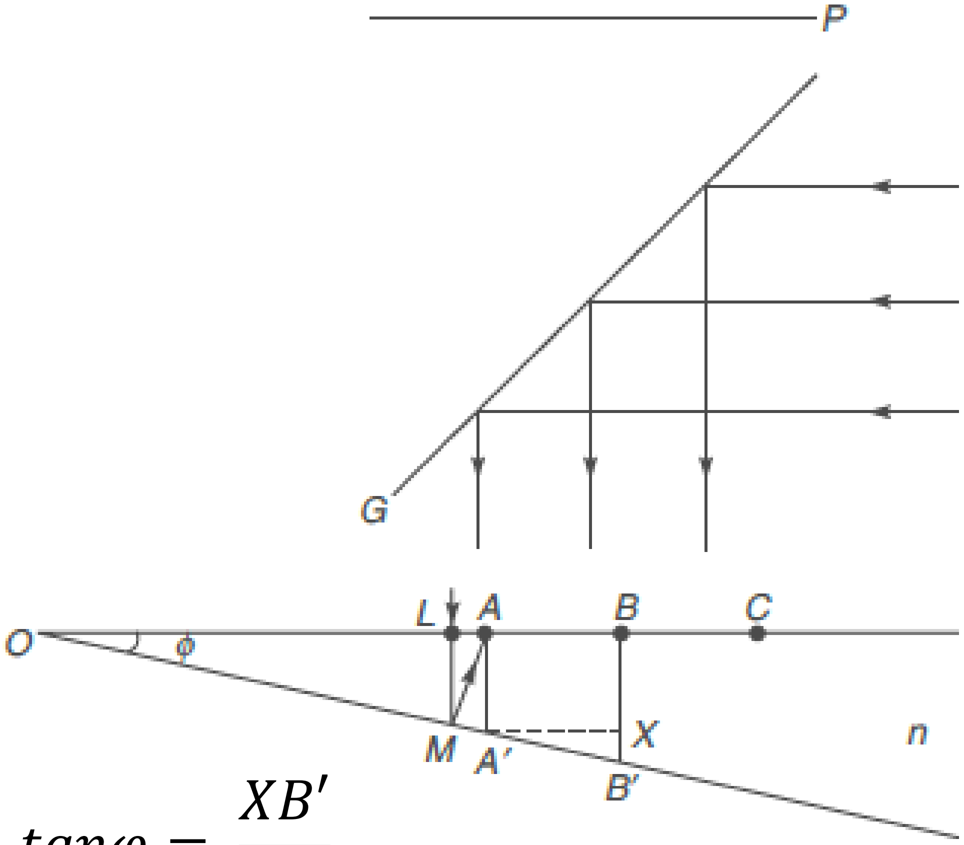
- A thin film of thickness 4 x 10^{-5} cm is illuminated by white light normal to its surface. Its refractive index is 1.5. Of what color will the thin film appear in reflected light?

Solution

- Color in reflection will appear due to constructive interference.
- Condition for constructive interference is, $2nd\ cos\theta_r = (m + \frac{1}{2})\lambda$
- By putting the values of $n\ (= 1.5)$, $\theta_r\ (= 0°)$ and d (= 4 x 10−5 cm), and solving, $\lambda = \frac{2400}{(2m+1)}\ nm$
- Taking $m = 0, 1, 2, 3$, we get $\lambda = 2400, 800, 480, 343.1\ \ nm$
- Wavelengths 2400, 800 and 343.1 nm are not in visible region. 480 nm is in blue spectral region. So, the color appear in interference is Blue.

Module 1 (Physical Optics)

Interference by a wedge-shaped film (two non-parallel reflecting planes)



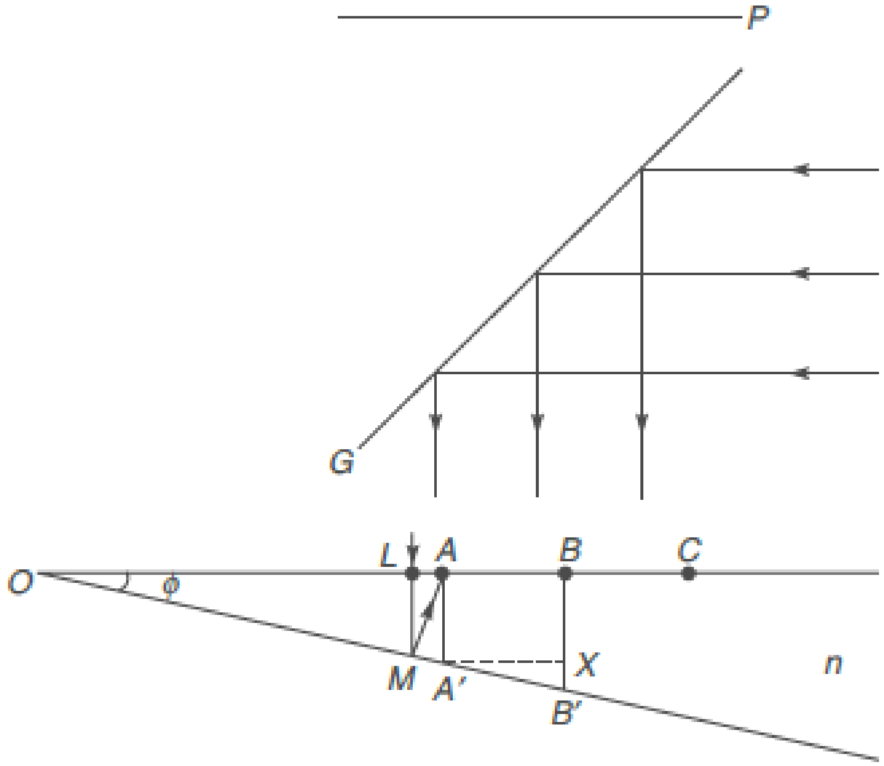
$$tan\varphi = \frac{XB'}{XA'}$$

$$XB' = BB' - AA' = XA'tan\varphi$$

- So far, we have assumed the film to be uniform thickness.
- Now, we will discuss the interference pattern produced by a film of varying thickness.
- For example, wedge shaped film, where two surfaces (top and bottom) are not parallel.
- Two rays are normally incident on the upper surface, and they get reflected from the lower surface *almost normally*.
- The path difference is still given by the **cosine law**. However, for a small angle φ and almost a normal incidence, $cos\varphi \approx 1$.

Module 1 (Physical Optics)

Interference by a wedge-shaped film (two non-parallel reflecting planes)



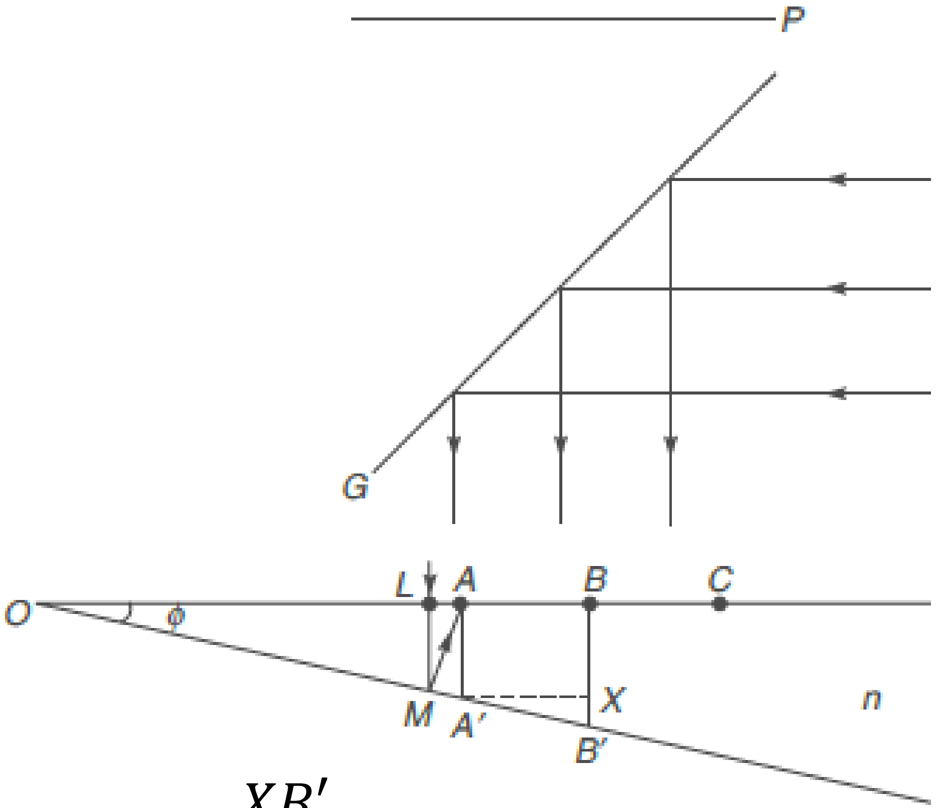
$$tan\varphi = \frac{XB'}{XA'}$$

$$XB' = BB' - AA' = XA'tan\varphi$$

- Consider a bright fringe at A. The optical path difference (OPD), $\Delta = n(LM + MA) \approx 2nAA'$ $(\because AA' \simeq LM \simeq MA)$
- For the next bright fringe (at B), OPD, $\Delta \approx 2nBB'$
- From the condition of bright fringe,
- $2nAA' = (m + \frac{1}{2})\lambda$ (set $\theta \to 0$ in $2nd\ cos\theta = (m + \frac{1}{2})\lambda$)
- So, condition for maxima is $2nd = (m + \frac{1}{2})\lambda$ (consider $AA' = d$)
- Similarly, condition for minima is $2nd = m\lambda$

Module 1 (Physical Optics)

Interference by a wedge-shaped film (two non-parallel reflecting planes)



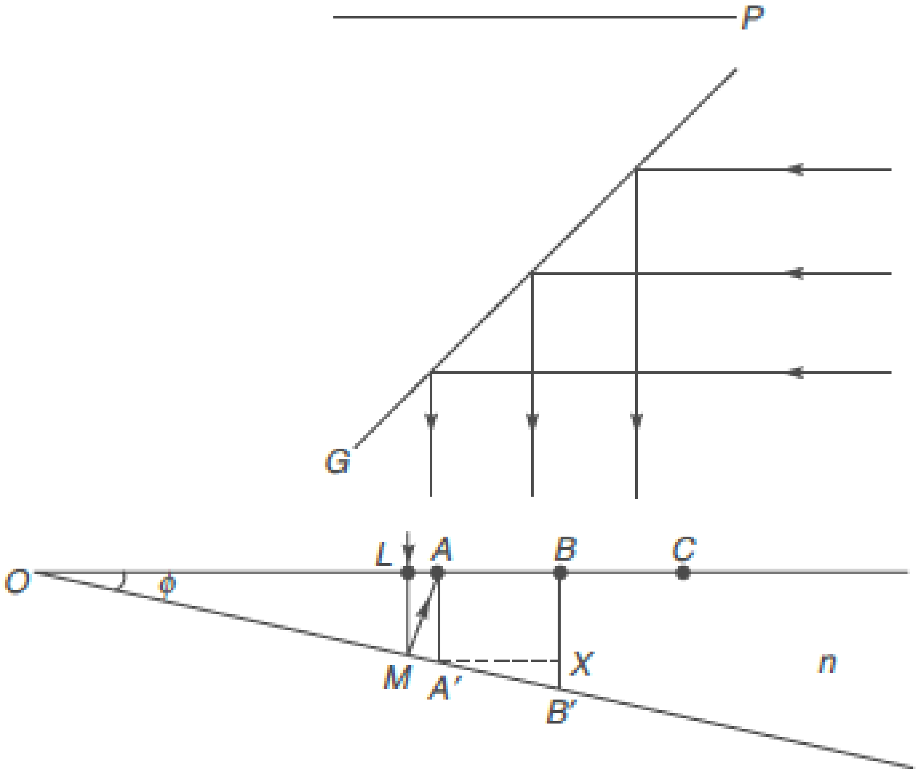
$$tan\varphi = \frac{XB'}{XA'}$$

$$XB' = BB' - AA' = XA' tan\varphi$$

- Consider a bright fringe at A. The optical path difference (OPD), $\Delta = n(LM + MA) \approx 2nAA'$ $(\because AA' \simeq LM \simeq MA)$
- For the next bright fringe (at B), OPD, $\Delta \approx 2nBB'$
- From the condition of bright fringe,
- $2nAA' = (m + \frac{1}{2})\lambda$ (set $\theta \rightarrow 0$ in $2nd\ cos\theta = (m + \frac{1}{2})\lambda$)
- $2nBB' = \left(m + 1 + \frac{1}{2}\right)\lambda = \left(m + \frac{3}{2}\right)\lambda$
- Subtracting, $2n(BB' - AA') = \lambda$
- Using the relation, $XB' = BB' - AA' = XA' tan\varphi$, we get
- $2n\,(XA' tan\varphi) = \lambda$

Module 1 (Physical Optics)

Interference by a wedge-shaped film (two non-parallel reflecting planes)



$$tan\varphi = \frac{XB'}{XA'}$$

$$XB' = BB' - AA' = XA'tan\varphi$$

- $2n\,(XA'tan\varphi) = \lambda$
- Now, $XA' = width\ of\ the\ fringe$ (distance between one bright/dark fringe to the next one)
- Let's say, $XA' \equiv \beta$
- Hence, $2n\,(XA'tan\varphi) = 2n\beta\ tan\varphi\ = \lambda$
- In the limit $\varphi \to 0$, $tan\varphi \simeq \varphi$ and we have $\boldsymbol{\beta = \frac{\lambda}{2n\varphi}}$
- This is the relation for the **fringe width**
- β represents the fringe width and we have assumed φ to be very small. Such fringes are commonly referred to as **fringes of equal thickness**.

Module 1 (Physical Optics)

Interference by a wedge-shaped film (two non-parallel reflecting planes)

- In the limit $\varphi \rightarrow 0$, $tan\varphi \simeq \varphi$ and we have $\boldsymbol{\beta = \frac{\lambda}{2n\varphi}}$
- This is the relation for the **fringe width (β)** – width of the bright fringes or that of the dark fringes.
- Fringe width depends on the wavelength (λ) of incident light, the refractive index of the film (n) and the angle of wedge (φ).
- To observe a bright or dark fringe of a particular order for a wedge-shaped film with small wedge angle (φ) and normal incidence of light, thickness (d) must be constant.
- For a wedge-shaped film, d remains constant only along lines parallel to the thin edge of the wedge. The bright and dark fringes are obtained as straight lines parallel to the thin edge of the wedge.
- At the thin edge, $d = 0$ and therefore path difference = $\frac{\lambda}{2}$, (the condition of minimum intensity) and the edge of the wedge-shaped film will be dark.

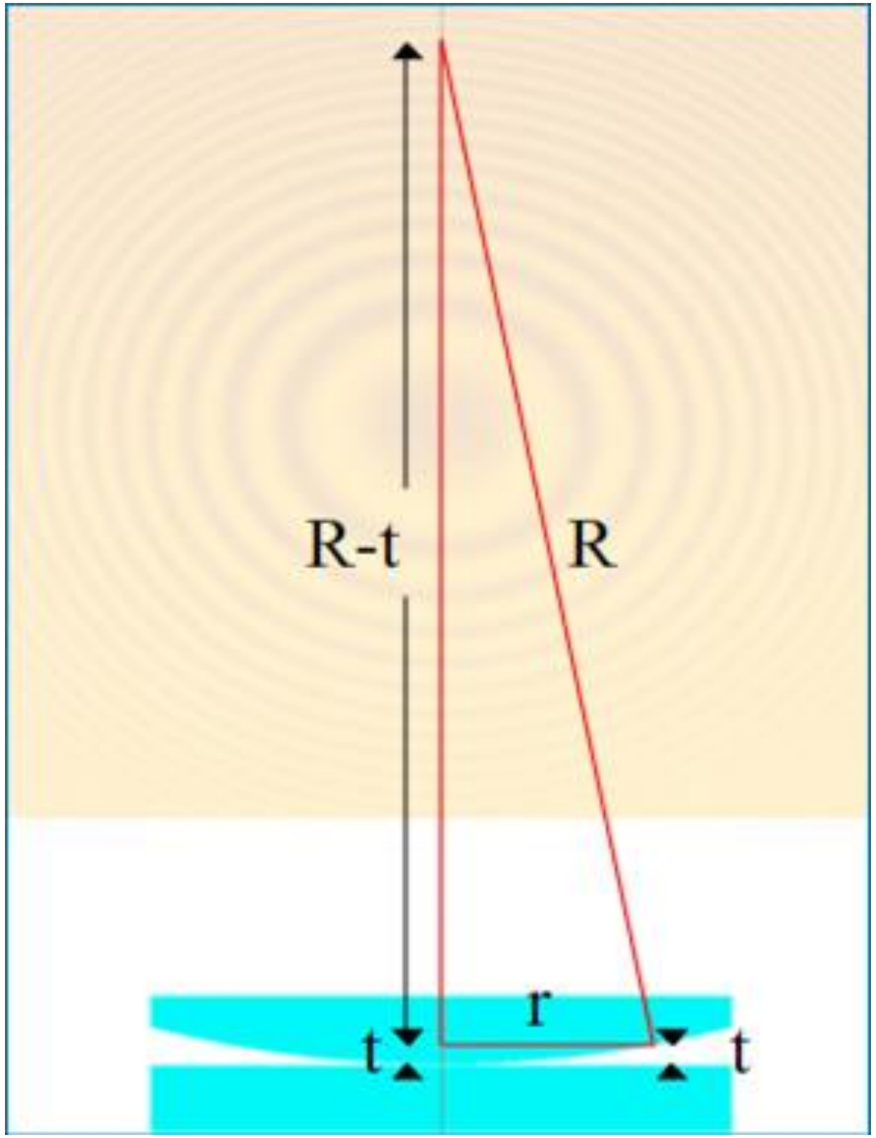
Module 1 (Physical Optics)

Thin Film Interference

- Interference from parallel reflecting plates (normal incidence)
- Interference from parallel reflecting plates (oblique incidence)
- Interference from non-parallel reflecting plates (wedge-shaped planes)

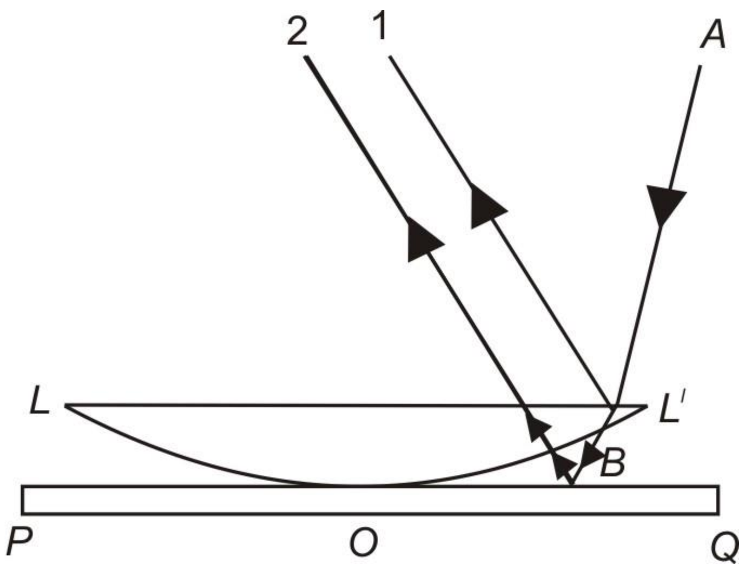
Newton's Rings

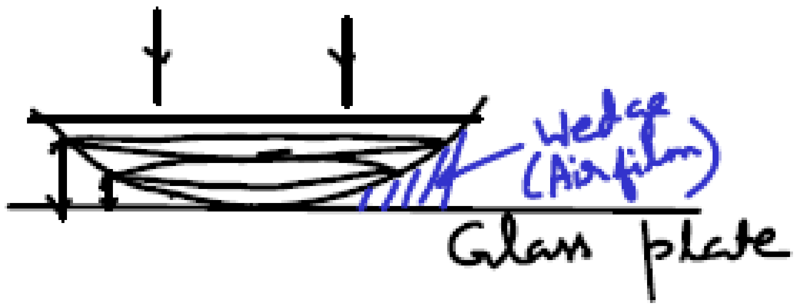
- There is a glass plate, on which a plano-convex lens is placed with the plane surface facing upward.
- Consider a monochromatic light incident on the lens.
- Then we obtain circular fringes, called "fringes of equal thickness".
- Light reflected from the curved surface of the lens and from the glass plate interfere to produce these fringes.



Module 1 (Physical Optics)

Newton’s Rings

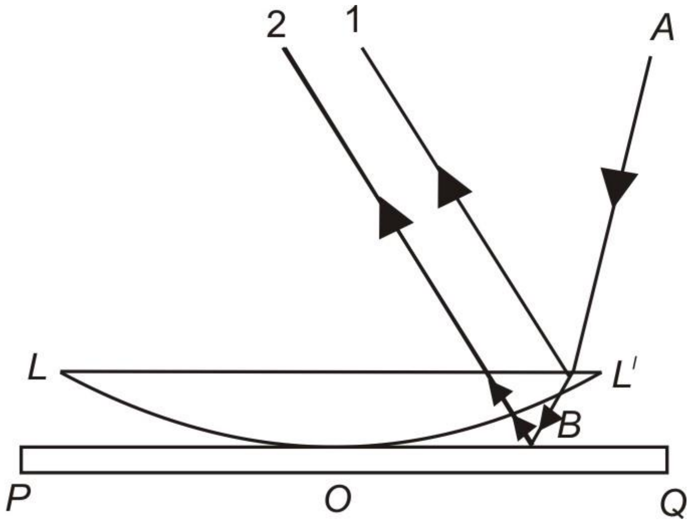


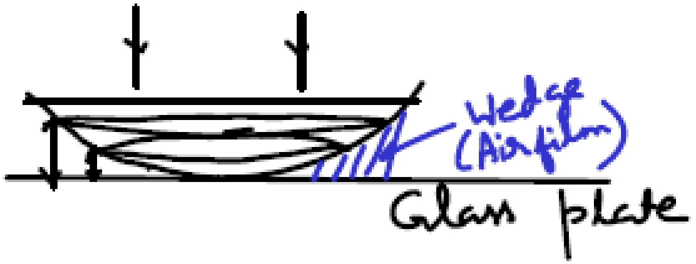


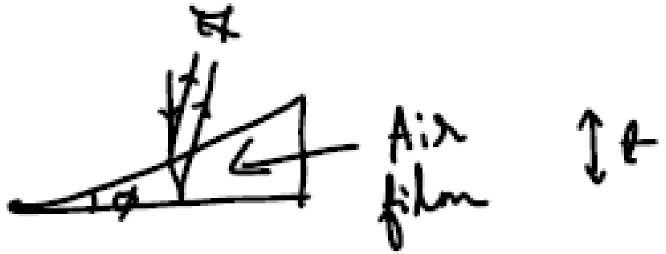
- When a curved surface (Convex surface of a plano-convex lens) LOL’ is placed on a flat surface POQ, a wedge (with upper surface curved) forms QOL’.
- At the point of contact, the thickness of air film is Zero
- Light wave from source A falling on this surface results in reflected ray 1 and refracted-reflected ray 2.
- Interference of two waves results in concentric ring of dark and bright fringes called “Newton’s Ring”.
- At the point of contact, the fringe is completely dark, as it was in case of wedge-shaped film.

Module 1 (Physical Optics)

Newton's Rings



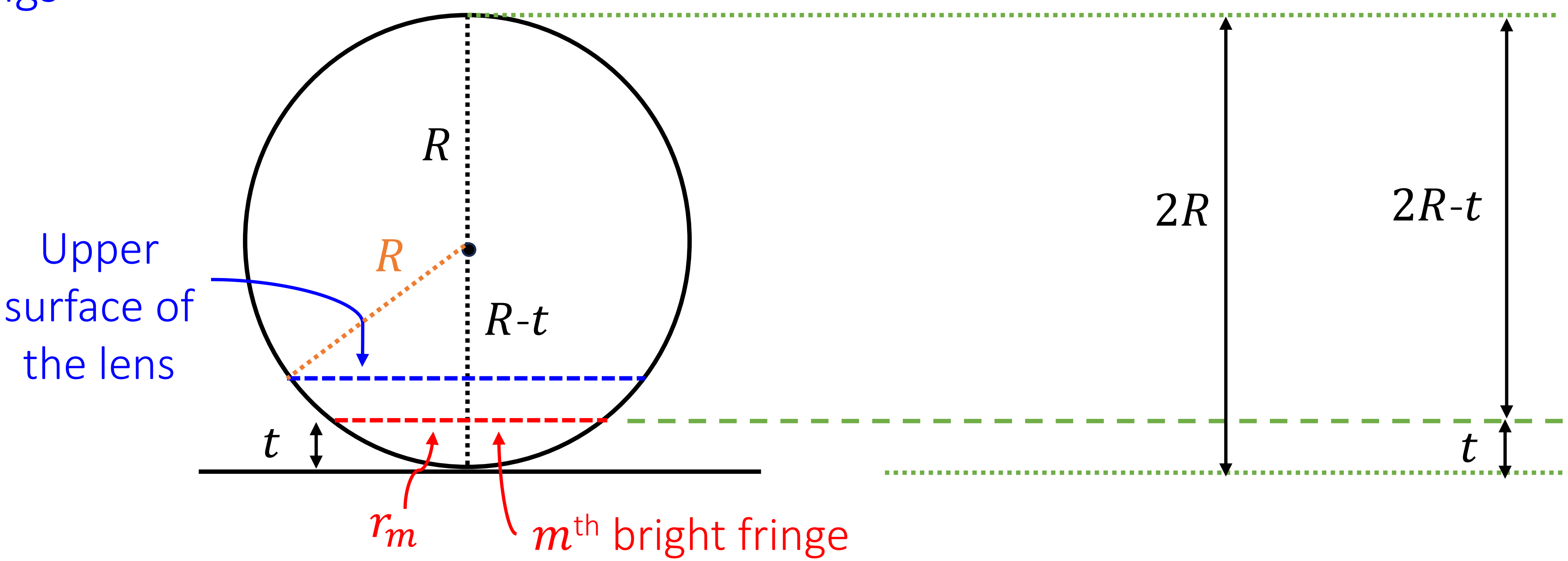




- When a curved surface (Convex surface of a plano-convex lens) LOL' is placed on a flat surface POQ, a wedge (with upper surface curved) forms QOL'.
- **Recall small (wedge) angle approximation.** At the point of contact, the fringe is completely dark, as in case of wedge-shaped film.
- For near normal incidence ($\varphi \to 0$), the optical path difference is $2nt$ ($n =$ refractive index of the lens, glass plate; see Fig).
- Condition for maxima (bright): $2nt = (m + \frac{1}{2})\lambda$
- Similarly, condition for minima (dark): $2nt = m\lambda$

Module 1 (Physical Optics)

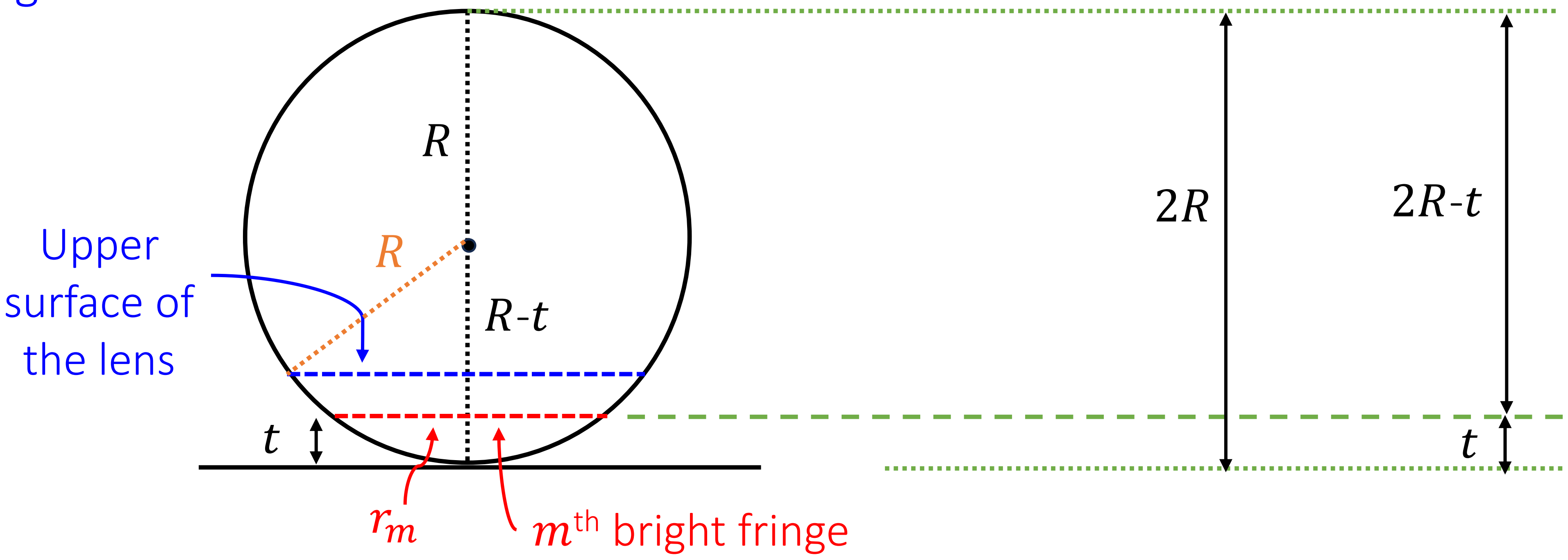
Newton's Rings



- Upper surface of the lens (labeled). Radius of curvature of the curved surface of the lens is R.
- Line in red represents m^{th} bright fringe of radius r_m and t is the thickness of air film where m^{th} film is formed.

Module 1 (Physical Optics)

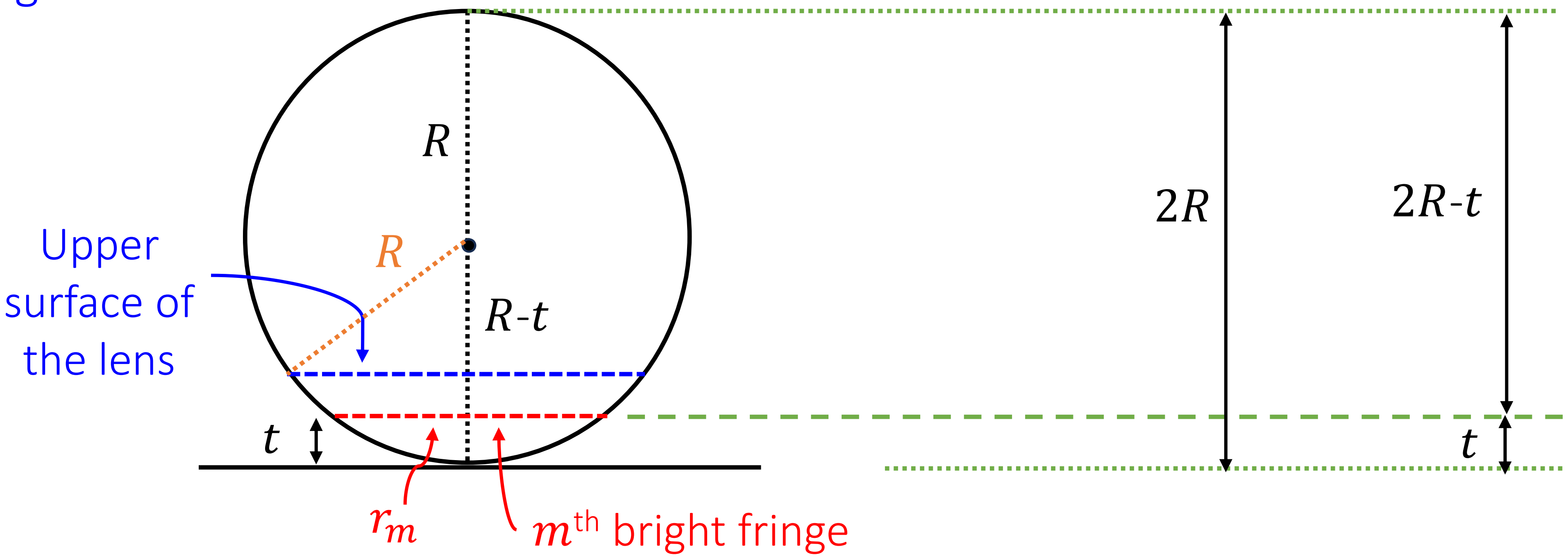
Newton's Rings



- Using the Pythagoras theorem: $R^2 = {r_m}^2 + (R-t)^2$.
- Hence, ${r_m}^2 = R^2 - (R-t)^2 = R^2 - (R^2 + t^2 - 2Rt) = 2Rt - t^2 = t(2R - t) \simeq 2Rt$. (since $2R >> t$)

Module 1 (Physical Optics)

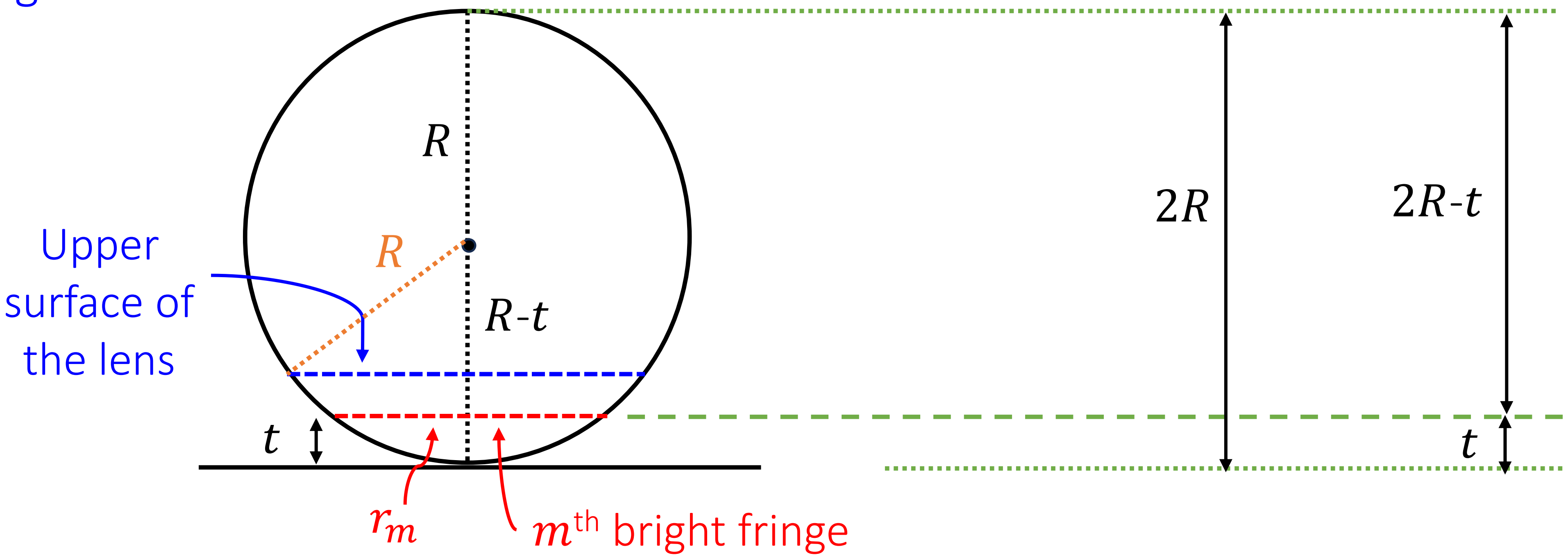
Newton's Rings



- Using the Pythagoras theorem: $R^2 = {r_m}^2 + (R-t)^2$. $\Rightarrow$ ${r_m}^2 = 2Rt$.
- For the **dark fringe:** $2nt = m\lambda$ $\Rightarrow$ $2t = \frac{m\lambda}{n}$
- Hence, ${r_m}^2 = R.2t = \frac{m\lambda R}{n}$ $\Rightarrow$ $r_m = \sqrt{\frac{m\lambda R}{n}}$

Module 1 (Physical Optics)

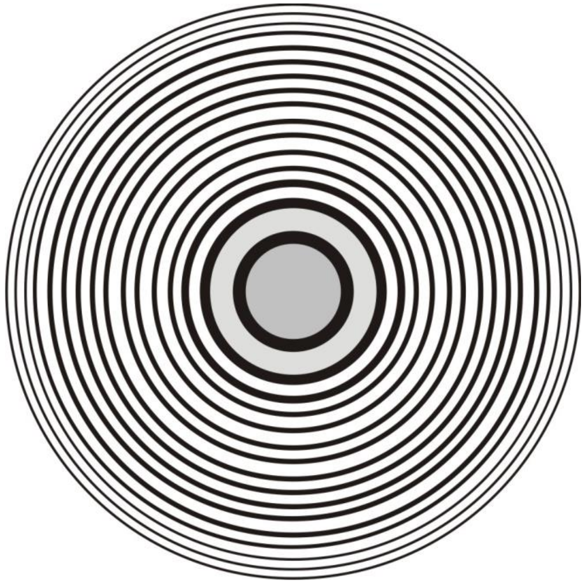
Newton's Rings



- For the **dark fringe:** $r_m = \sqrt{\frac{m\lambda R}{n}}$
- Similarly, for the bright fringe: $r_m = \sqrt{\frac{\left(m+\frac{1}{2}\right)\lambda R}{n}}$
- Note: for the air film, $n = 1$.

Module 1 (Physical Optics)

Newton's Rings



- Radii of the rings vary as the square-root of odd natural numbers ($n = 0,1,2,3 \dots..$).
- The rings will get closer and closer as their order increases.
- Radii of Newton's rings depend on the wavelength of light used.
- Wavelength of light can be determined by measuring the radius (diameter) of ring and radius of curvature (R) of lens.

Module 1 (Physical Optics)

Newton's Rings

- **Homework**
- In terms of the diameter of the film: $D_m = 2r_m$
- Calculate the diameter of the m^{th} bright ring, $D^2{}_m$
- Calculate the diameter of the $(m+p)^{\text{th}}$ bright ring, $D^2{}_{m+p}$
- Subtracting these two one can express the wavelength in terms of diameters: $\lambda = \frac{D^2{}_{m+p} - D^2{}_m}{4pR}$

Module 1

Tutorial

4. **Double refraction.**

A thin quartz crystal of thickness t has been cut so that the optical axis is $\parallel$ to the surface of the crystal. For sodium light ($\lambda = 589\,\mathrm{nm}$), the refractive index of the crystal is 1.55 for e-ray and 1.54 for o-ray. If two beams of light polarized $\parallel$ and $\perp$ O.A., respectively, start through the crystal in-phase, how thick must the crystal be if they emergy out of the crystal at and angle of $\pi/2$.

Module 1

Next Class

Questions?

PH113: Physics (B.Tech. I)

Module 1

Date: 15.09.2023

Lecture: 5

Module 1

Module – 1 : Physical Optics

Polarization, Malus' Law, Brewster's Law, Double Refraction, Interference in thin films (Parallel films), Interference in wedge-shaped layers, Newton's rings, Fraunhofer diffraction by single slit, Double slit. Elementary ideas of fibre optics and application of fibre optic cables. [8]

Text book:

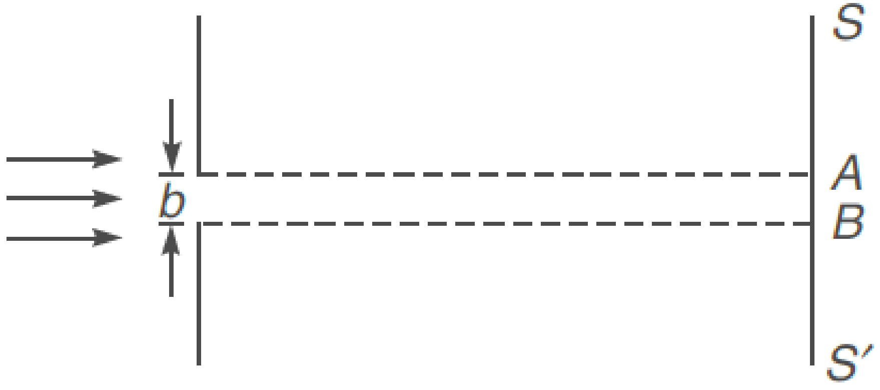
T1: A. Ghatak, Optics, 6th Edition, 2017, McGraw-Hill Education (India) Pvt. Ltd.

References: (1) Optics – E Hecht **(2)** Fundamentals of Optics – Jenkins & White

Class structure: 4 Lectures including 1 Tutorial per week. (8 hours ~ 2 weeks for this module!)

Module 1 (Physical Optics)

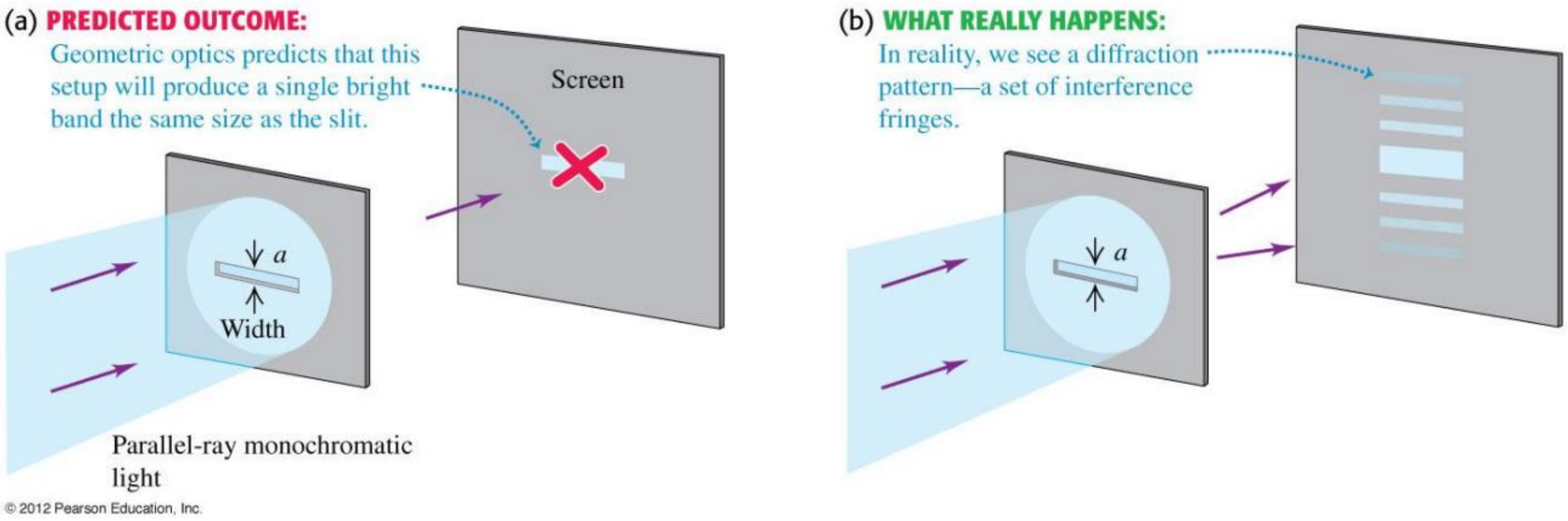
Diffraction



- Consider a plane wave incident on a narrow slit of width *b*.
- What should we expect according to the geometrical optics?
- That only the region *AB* of the screen (*SS'*) to be bright and rest of the screen will be dark.
- Now, if the slit-width is comparable to the wavelength, then the intensity in the *AB* region is not uniform, and there is also some intensity inside the geometrical shadow.
- Smaller the slit-width, larger is the amount of energy reaching the geometrical shadow.
- *This spreading out of a wave when it passes through a narrow opening is usually referred to as diffraction and the intensity distribution on the screen is known as the diffraction pattern.*
- This spreading out decreases with decrease in wavelength.

Module 1 (Physical Optics)

Diffraction



- This is the phenomenon of bending of light as it passes the edge of an object
- Most prominent when aperture $\lesssim \lambda$. Diffraction always occurs but it's effects are noticeable only when the wavelength is comparable to the size of the diffracting object.
- The phenomena is caused by the interference of a large number of waves.

Interference vs Diffraction

- Interference is due to superposition of two separate wavefronts originating from the same coherent source.
- Diffraction is due to superposition of secondary wavelets originating from a wavefront.
- The width of interference fringes may or may not be same.
- Bands observed in diffraction are never of same width.
- In interference, all bright fringes bears same intensity.
- In diffraction, intensity of bright bands usually decreases with increase in order.
- Intensity distribution of an interference fringe is uniform.
- Diffraction bands does not have uniform intensity distribution.

Diffraction

- Types of diffraction (based on the distance of source and observation screen from the diffracting aperture or obstacle):
  - Fresnel diffraction (either the source and or the screen, or both, are at finite distance from the aperture)
  - Fraunhofer diffraction (source and screen are at infinite distance from the aperture)

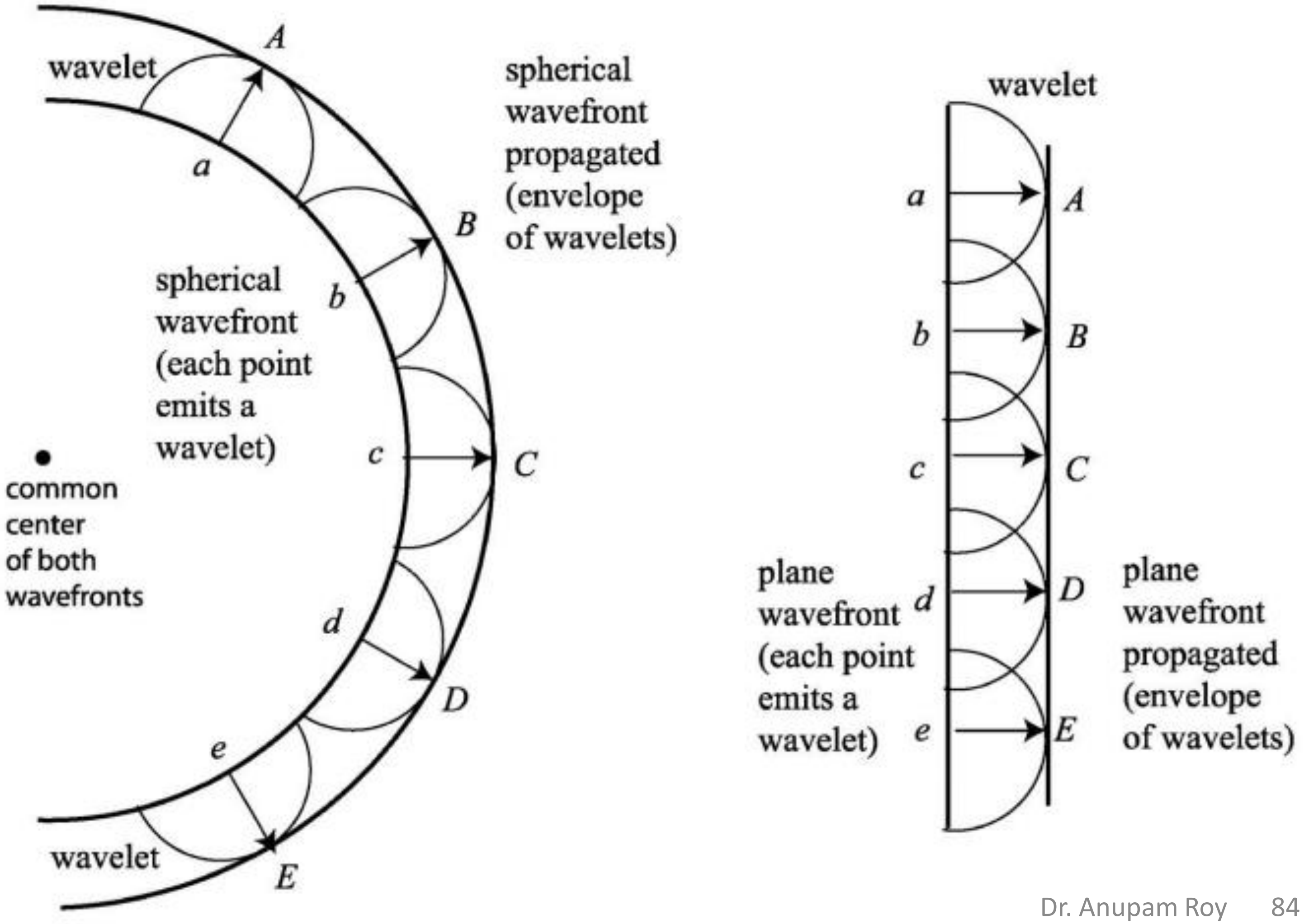
Diffraction

- In Fresnel diffraction
  - Source of light or the screen or both are, in general, at a finite distance from the diffracting obstacles.
  - Superposing wavefronts are not plane, but either cylindrical or spherical.
- In Fraunhofer diffraction
  - Source and screen are at infinite distance from the aperture. This condition is achieved by putting suitable lenses between the source, the obstacles and the Screen.
  - It is based on the concepts of secondary wavelets given by Huygens and their superposition.
  - Superposing wavefronts are plane wavefronts.

Module 1 (Physical Optics)

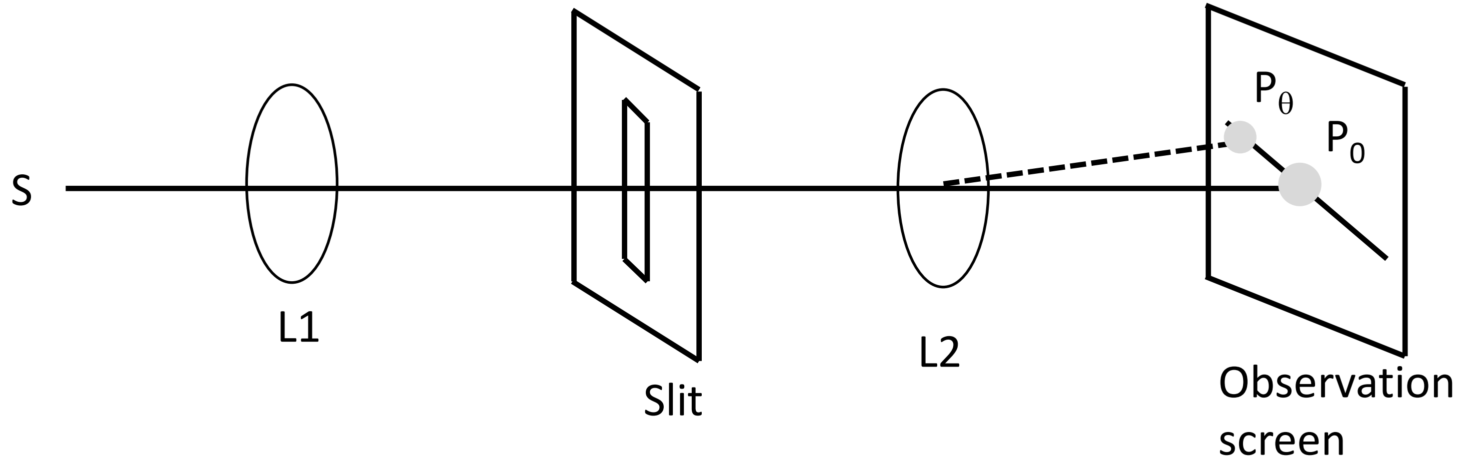
Diffraction: Diffraction from a single slit (Point source)

- Recall Huygen's Principle: Each point on a wave front can be thought of as a new source of wavelets, and the development of the new wave front at a later time is determined by the propagation of these wavelets.
- Wavefront is a surface on which all waves have the same phase.



Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

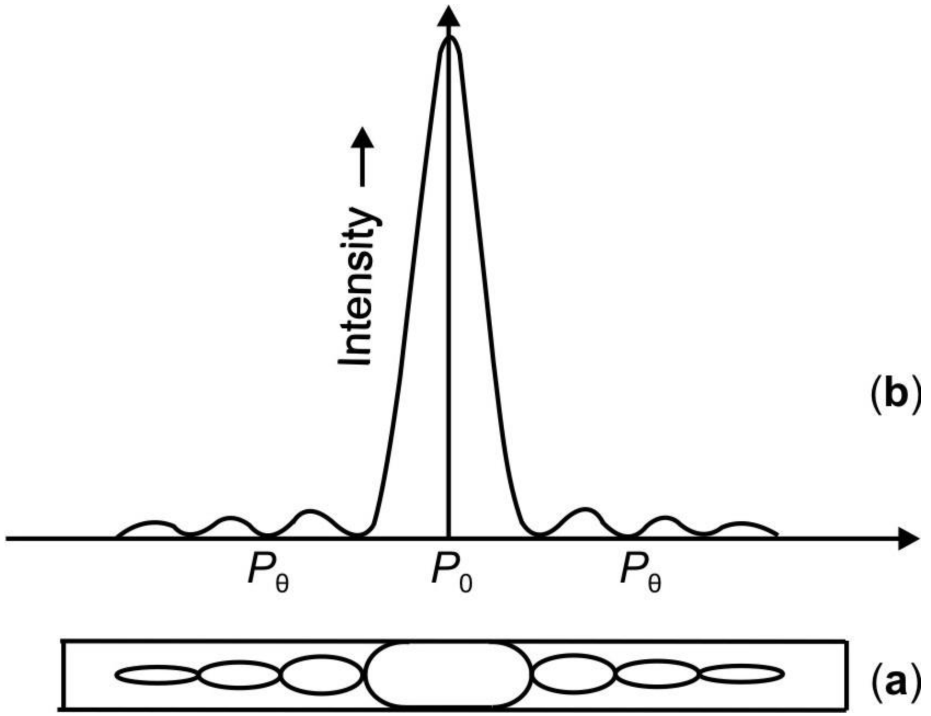


- A point source (S) of monochromatic light is placed in the focal plane of a converging lens L1, so that a plane wave is incident on a long narrow slit.
- A converging lens L2 is placed on the other side of the slit.
- The observation screen is placed at the second focal point of this lens (L2).

Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

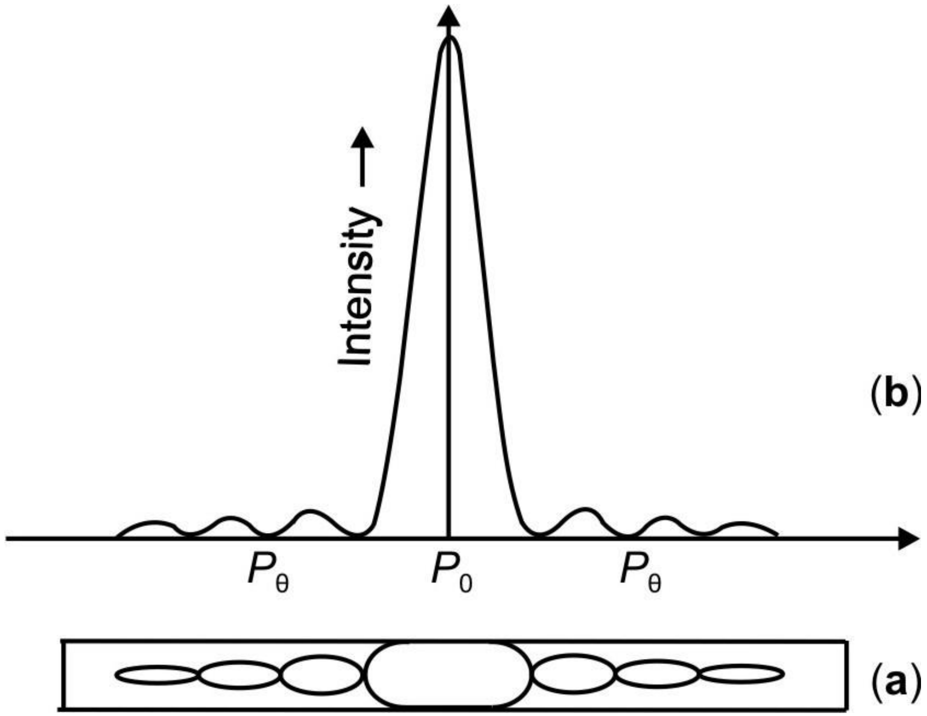
- Diffraction pattern consists of a horizontal streak of light along a line perpendicular to the length of the slit (see Figure a).
- Horizontal pattern is a series of bright spots. The intensity of the central spot is maximum and its peak is located at P_0, which lies at the intersection of the axis of L1 and L2 with the observation screen is the brightest (see Figure b).
- On the either side of this spot, few more bright spots symmetrically situated with respect to P_0 are observed.



Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

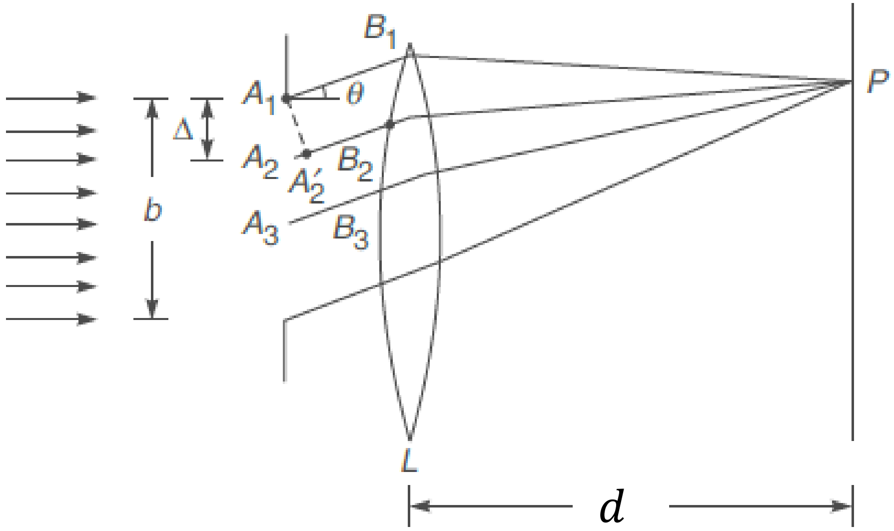
- Intensities of other spots, on either side of the central spot, decrease rapidly as we move away from P_0.
- The central maximum is called **principal maxima** and other maxima are called **secondary maxima**. The width of the central spot is twice the width of other spots.
- The central peak is symmetrical. But on either side of the central maximum, secondary maxima are asymmetrical. In fact, the positions of the secondary maxima are slightly shifted towards the observation point P_0 .

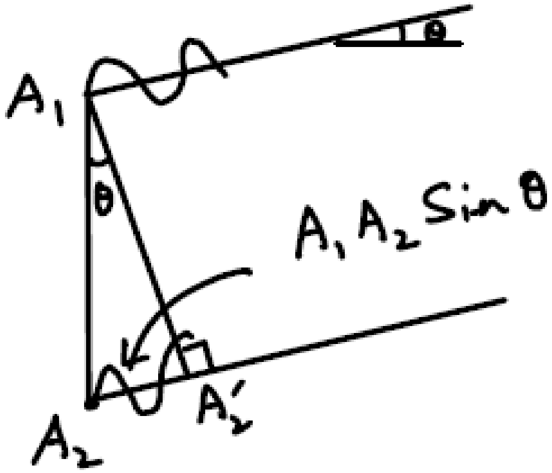


Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

- When a plane wavefront falls on the diffracting slit, each point of the aperture such as A_1, A_2, A_3,...... becomes a source of secondary wavelets, which propagate in the direction of the point P_θ.

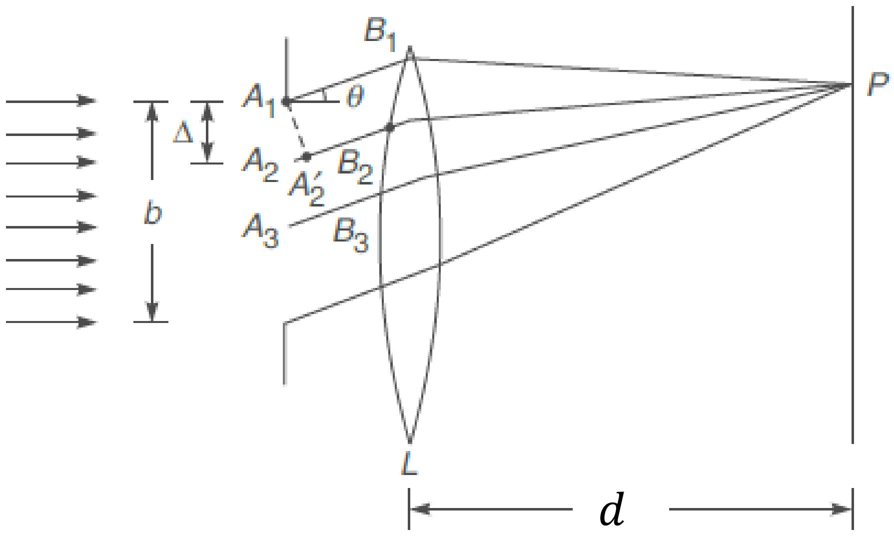




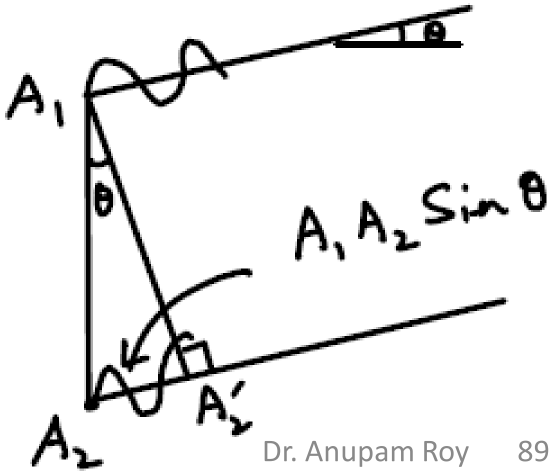
- Optical paths are same for rays passing through a lens and getting focused to a single point.
- Let $A_1A_2 = A_2A_3 = \ldots\ldots = \Delta$
- Path difference between two consecutive rays = $\Delta\, sin\theta$.
- So, the phase difference must be $\varphi = \frac{2\pi}{\lambda}\Delta\, sin\theta$.

Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)



- Suppose the 1st ray (starting at A_1) be $a\cos\omega t$.
- Then the 2nd ray (starting at A_2) is $a\cos(\omega t-\varphi)$.
- Similarly, the other rays are $a\cos(\omega t-2\varphi)$, $a\cos(\omega t-3\varphi)$, …….etc.
- All the waves superpose on a point P on the screen. So the net field at P will be:
- $E=a\,[cos\omega t+\cos(\omega t-\varphi)+\cos(\omega t-2\varphi)+\ \ldots\ldots\ldots..]$
- We are considering n sources A_1, A_2, ……, A_n, with a gap Δ of between consecutive sources. At the end, we will set $n\to\infty$ and $\Delta\to 0$ such that $n\Delta=b$ and the sources are continuous.



Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

- $E = a\,[cos\omega t + \cos(\omega t - \varphi) + \cos(\omega t - 2\varphi) + \;\ldots\ldots\ldots..]$

$$= a\, Re\,[e^{i\omega t} + e^{i(\omega t - \varphi)} + e^{i(\omega t - 2\varphi)} + \cdots]$$

- Use de Moivre's theorem: $e^{i\theta} = \cos\theta + i\sin\theta$
- Hence, $E = a\, Re\,[e^{i\omega t}\{1 + e^{-i\varphi} + e^{-2i\varphi} + \cdots\ldots + e^{-(n-1)i\varphi}\}]$

$$= a\, Re\left[e^{i\omega t}\,.\,\frac{1 - e^{-in\varphi}}{1 - e^{-i\varphi}}\right] = a\, Re\left[e^{i\omega t}\,.\,\frac{e^{-in\frac{\varphi}{2}}\left(e^{in\frac{\varphi}{2}} - e^{-in\frac{\varphi}{2}}\right)}{e^{-i\frac{\varphi}{2}}\left(e^{\frac{i\varphi}{2}} - e^{-\frac{i\varphi}{2}}\right)}\right]$$

- Use de Moivre's theorem: $e^{i\theta} - e^{-i\theta} = (\cos\theta + i\sin\theta) - (\cos\theta - i\sin\theta) = 2i\sin\theta$
- Now, look at the terms within the first bracket:

Diffraction: Diffraction from a single slit (Point source)

- Hence, $E = a\,Re\left[e^{i\omega t}\,.\,\frac{e^{-in\frac{\varphi}{2}}\left(e^{in\frac{\varphi}{2}} - e^{-in\frac{\varphi}{2}}\right)}{e^{-i\frac{\varphi}{2}}\left(e^{\frac{i\varphi}{2}} - e^{-\frac{i\varphi}{2}}\right)}\right] = a\,Re\left[e^{i\omega t}\,.\,e^{-\frac{i(n-1)\varphi}{2}}\,.\,\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}\right]$

$$= a\,Re\left[e^{i\left\{\omega t-\frac{(n-1)\varphi}{2}\right\}}\,.\,\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}\right]$$

$$\Rightarrow\quad E = a\,\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}\cos\left[\omega t - \frac{(n-1)\varphi}{2}\right]$$

$\Rightarrow\quad E = E_0\cos\left[\omega t - \frac{(n-1)\varphi}{2}\right]$, where $E_0 = a\,\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}$

- E_0 is the amplitude of the resulting field.

Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

- $E = E_0 \cos\left[\omega t - \frac{(n-1)\varphi}{2}\right]$, where $E_0 = a\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}$
- But, $\varphi = \frac{2\pi}{\lambda}\Delta\, sin\theta$
- Hence, $\frac{n\varphi}{2} = \frac{n}{2}.\frac{2\pi}{\lambda}\Delta\, sin\theta = \frac{\pi}{\lambda}\,(n\Delta)\sin\theta = \frac{\pi}{\lambda}\, b\sin\theta$ (using $n\Delta = b$ when $n \to \infty$ and $\Delta \to 0$)
- Using the above we get, where $E_0 = a\frac{\sin\left(\frac{\pi}{\lambda}b\sin\theta\right)}{\sin\left(\frac{\pi}{\lambda}\Delta\, sin\theta\right)}$
- Since, $\Delta \to 0$ we can rewrite $\sin x \approx x$ for $x \to 0$
- Hence, $E_0 = a\frac{\sin\left(\frac{\pi}{\lambda}b\sin\theta\right)}{\frac{\pi}{\lambda}\Delta\, sin\theta}$

Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

- $E = E_0 \cos\left[\omega t - \frac{(n-1)\varphi}{2}\right]$, where the amplitude of the resulting field, $E_0 = a\,\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}$.
- Using, $\frac{n\varphi}{2} = \frac{\pi}{\lambda}\, b \sin\theta$ ($n\Delta = b$ when $n \to \infty$ and $\Delta \to 0$)
- Using the above we get, where $E_0 = a\,\frac{\sin\left(\frac{\pi}{\lambda} b \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda}\Delta\, sin\theta\right)} \approx a\,\frac{\sin\left(\frac{\pi}{\lambda} b \sin\theta\right)}{\frac{\pi}{\lambda}\Delta\, sin\theta}$

  Since, $\Delta \to 0$, $\sin x \approx x$ for $x \to 0$
- Or, $E_0 = a\,\frac{\sin\left(\frac{\pi}{\lambda} b \sin\theta\right)}{\frac{\pi}{\lambda}\Delta\, sin\theta} \times \frac{n}{n} = na\,\frac{\sin\left(\frac{\pi}{\lambda} b \sin\theta\right)}{\frac{\pi}{\lambda} b\, sin\theta}$ (Why? Because $n\Delta = b$)
- This can be expressed as: $E_0 = A\,\frac{\sin\beta}{\beta}$ where $na = A$ and $\beta = \frac{\pi}{\lambda}\, b\, sin\,\theta$ (in the limit $n \to \infty$ and $a \to 0$ the product na tends to a finite limit)

Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)

- Now, $E = E_0 \cos\left[\omega t - \frac{(n-1)\varphi}{2}\right] \simeq E_0 \cos[\omega t - \frac{n\varphi}{2}]$ ($\because$ for $n \rightarrow \infty, n-1 = n$)
- Hence, $E = A \frac{\sin\beta}{\beta}\cos[\omega t - \beta]$

- **Intensity pattern on the screen**: (Intensity is given as square of amplitude)
- $I = |E_0|^2 = (A\frac{\sin\beta}{\beta})^2 = I_0 \frac{sin^2\beta}{\beta^2}$ (where $I_0 = A^2$)

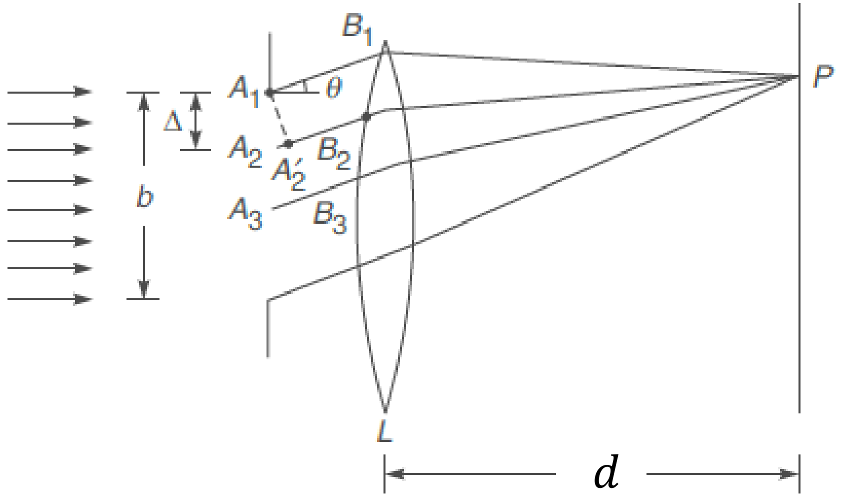
Module 1 (Physical Optics)

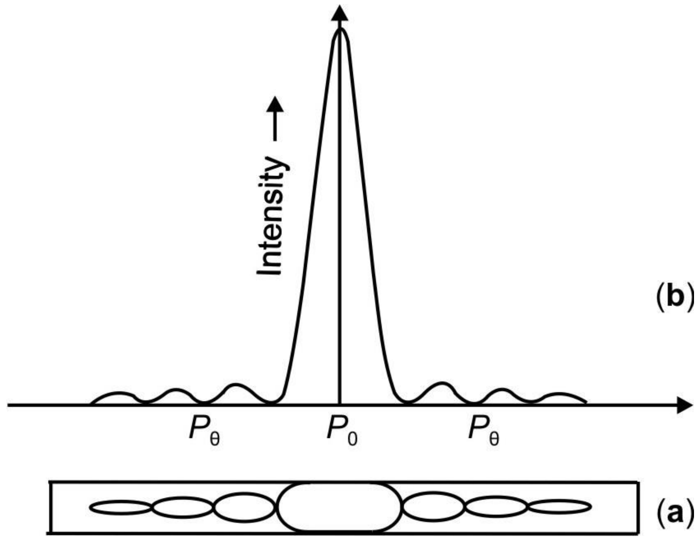
Diffraction: Diffraction from a single slit (Point source)

- Maxima and Minima of the intensity: $I = I_0 \frac{sin^2\beta}{\beta^2}$

- $\beta = 0$ will give the **central maximum** where $I = I_0 = A^2$ $\quad (\because \lim_{\beta\to 0}\left[\frac{sin\beta}{\beta}\right] = 1)$

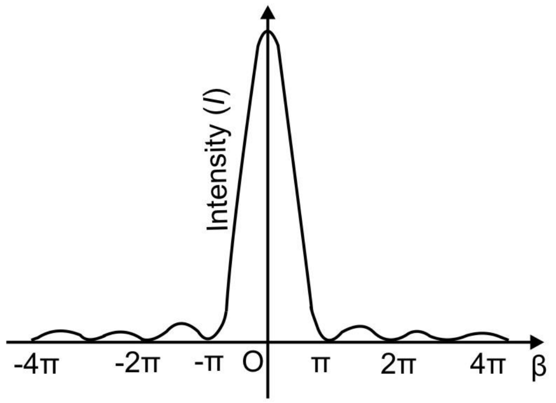
- This implies that all diffracted waves arrive in phase at the point P_0 (at $\theta = 0$) and interfere constructively. A^2 is then the value of the maximum intensity at the centre of the diffraction pattern.





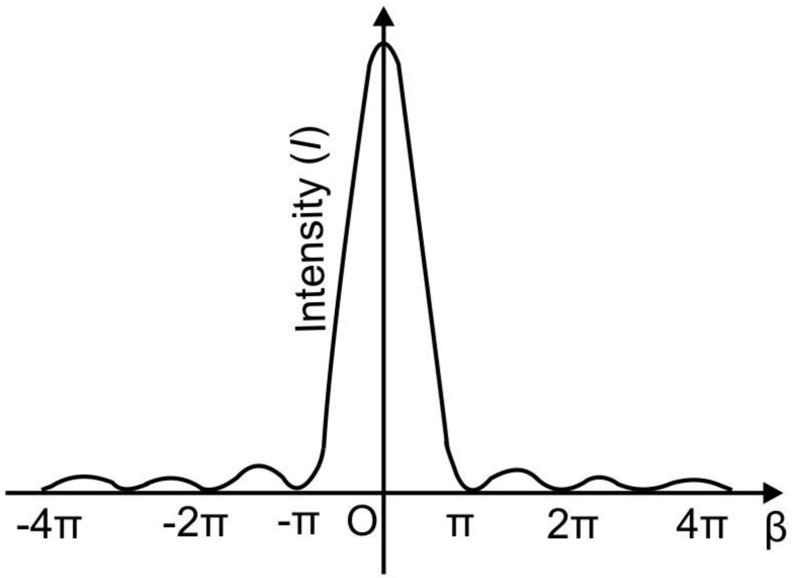
Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)



- Maxima and Minima of the intensity: $I = I_0 \dfrac{sin^2\beta}{\beta^2}$
- **Position of secondary maxima and minima**:
- $\beta = 0$ will give the **central maximum** where $I = I_0 = A^2$ $\left(\because \lim_{\beta\to 0}\left[\dfrac{sin\beta}{\beta}\right] = 1\right)$
- The intensity is maximum for $\theta = 0$ and $I_{\theta=0} = I_0 = A^2$
- The intensity gradually falls on either side of the principal maximum and becomes zero when $\beta = \pm\pi$.
- The angular half width of principal maximum is from $\beta = 0$ to $\beta = \pi$.
- The second minimum on either side occurs at $\beta = \pm 2\pi$.

Module 1 (Physical Optics)

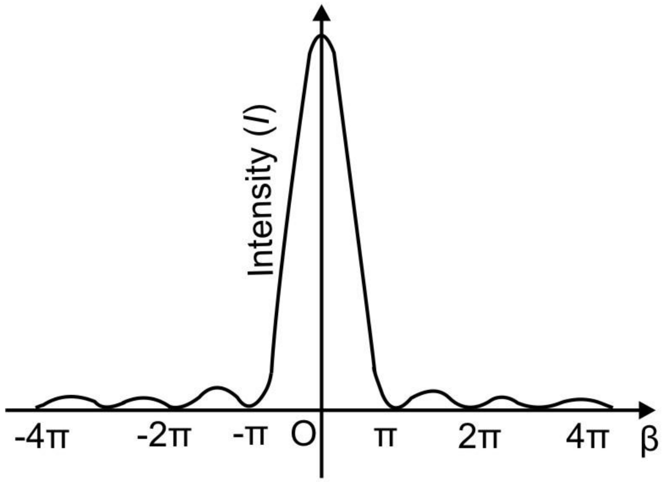


Diffraction: Diffraction from a single slit (Point source)

- Maxima and Minima of the intensity: $I = I_0 \dfrac{sin^2\beta}{\beta^2}$
- **Position of secondary maxima and minima**:
- $\beta = 0$ will give the **central maximum** where $I = I_0 = A^2$
- The intensity is maximum for $\theta = 0$ and $I_{\theta=0} = I_0 = A^2$
- General condition for **minima**: $\beta = m\pi$ where $m = \pm1, \pm2, \pm3, \ldots\ldots$

  Or, $b\, sin\, \theta = m\lambda$ where $m = \pm1, \pm2, \pm3, \ldots\ldots$ (since $\beta = \dfrac{\pi}{\lambda}\, b \sin\theta$)
- This is the condition for minima. The location of minima, determined by θ depends on the wavelength of light (λ) and the slit width (b).

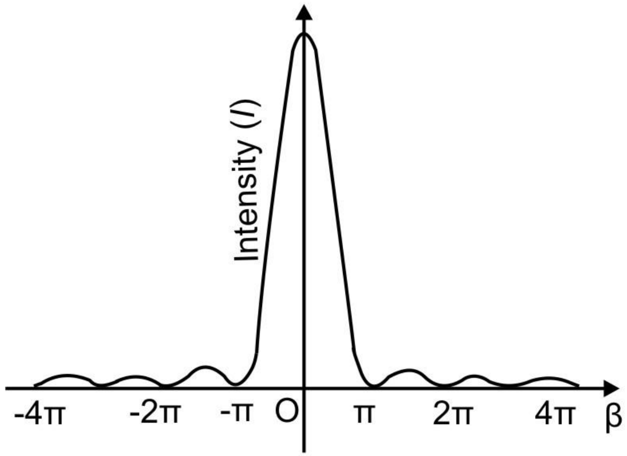
Module 1 (Physical Optics)



Diffraction: Diffraction from a single slit (Point source)

- Maxima and Minima of the intensity: $I = I_0 \frac{sin^2\beta}{\beta^2}$
- **Position of secondary maxima and minima**:
- $\beta = 0$ will give the **central maximum** (for $\theta = 0$) and $I_{\theta=0} = I_0 = A^2$
- General condition for **minima**: $b\, sin\, \theta = m\lambda$ where $m = \pm 1, \pm 2, \pm 3, \ldots..$
- Location of minima, determined by θ depends on wavelength of light (λ) and the slit width (b).
- The first minimum occurs at $\theta = \pm \sin^{-1}\left(\frac{\lambda}{b}\right)$. The second minimum occurs at $\theta = \pm \sin^{-1}\left(\frac{2\lambda}{b}\right)$ and so on ....
- Note: $\sin\theta \ngtr 1$, so the maximum value of m is the integer which is less than (and closest to) $\frac{b}{\lambda}$.

Module 1 (Physical Optics)

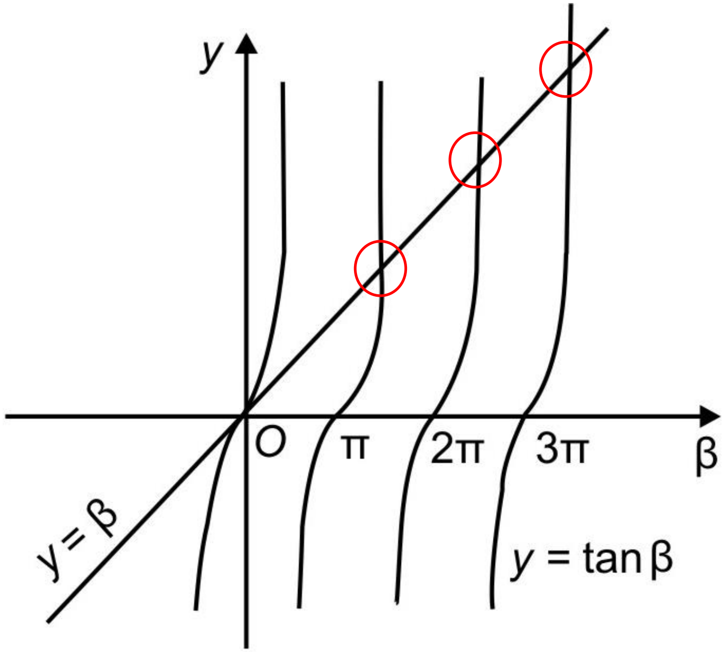


Diffraction: Diffraction from a single slit (Point source)

- Maxima and Minima of the intensity: $I = I_0 \dfrac{sin^2\beta}{\beta^2}$
- **Direction and position of secondary maxima**:
- To determine the positions of maxima, differentiate the equation $I = I_0 \dfrac{sin^2\beta}{\beta^2}$ with respect to β and set it equal to zero.
- That means: $\dfrac{dI}{d\beta} = 0$
- $\dfrac{dI}{d\beta} = \dfrac{d}{d\beta}\left(I_0 \dfrac{sin^2\beta}{\beta^2}\right) = I_0 \left[\dfrac{2\sin\beta\cos\beta}{\beta^2} - \dfrac{2sin^2\beta}{\beta^3}\right] = \sin\beta\,(\beta - \tan\beta)$
- We will set $\dfrac{dI}{d\beta} = 0$. That means, $\sin\beta\,(\beta - \tan\beta) = 0$

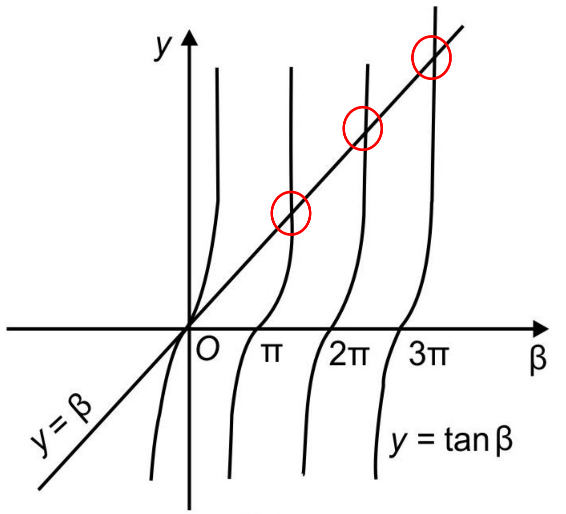
Module 1 (Physical Optics)

Diffraction: Diffraction from a single slit (Point source)



- Maxima and Minima of the intensity: $I = I_0 \frac{sin^2\beta}{\beta^2}$
- **Direction and position of secondary maxima**:
- $\frac{dI}{d\beta} = 0 \quad \Rightarrow \quad \sin\beta\,(\beta - \tan\beta) = 0$
- The condition $\sin\beta = 0$, or, $\beta = m\pi$ $(m \neq 0)$ correspond to minima.
- The transcendental equation $\tan\beta = \beta$ gives the condition for maxima.
- $\beta = 0$ will give the **central maximum**. The other roots can be found by determining the points of intersections of the curves $y = \beta$ and $y = \tan\beta$. The intersections occur at $\beta = 1.43\pi$, $\beta = 2.46\pi$, etc., and are known as the first maximum, the second maximum, etc.

Module 1 (Physical Optics)



Diffraction: Diffraction from a single slit (Point source)

- Maxima and Minima of the intensity: $I = I_0 \frac{sin^2\beta}{\beta^2}$
- **Direction and position of secondary maxima**:
- Note: The intensity maxima do not fall exactly midway between two minima. For example, the first secondary maximum occurs at $\beta = 1.43\pi$ (rather than 1.50π). Similarly, the second maximum occurs at $\beta = 2.46\pi$ (rather than 2.50π) and so on.
- This means that the intensity curves for secondary maxima are **asymmetrical** and the positions of maxima are slightly shifted towards the centre of the pattern.
- Intensity of second maxima is $I = I_0 \left[\frac{sin\beta}{\beta}\right]^2 = I_0 \left[\frac{\sin(1.43\pi)}{(1.43\pi)}\right]^2 = I_0(0.0496)$
- Intensity of the first secondary maxima (nearest to central peak) is ~ 4.96% of the central maximum.

Module 1

Date: 19.09.2023

Lecture: 6

Module 1 (Physical Optics)

Diffraction: Two-Slit interference (Young's double slit experiment)

- Optical path difference between the two rays reaching at point P is $d \sin\theta$.

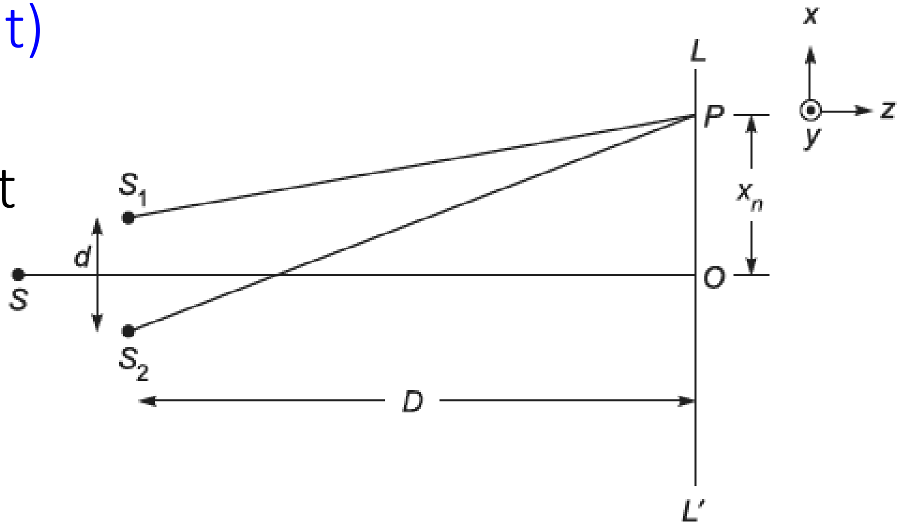
- Phase difference, $\varphi = \frac{2\pi}{\lambda}\, d\, sin\theta$.

- Total amplitude at P:

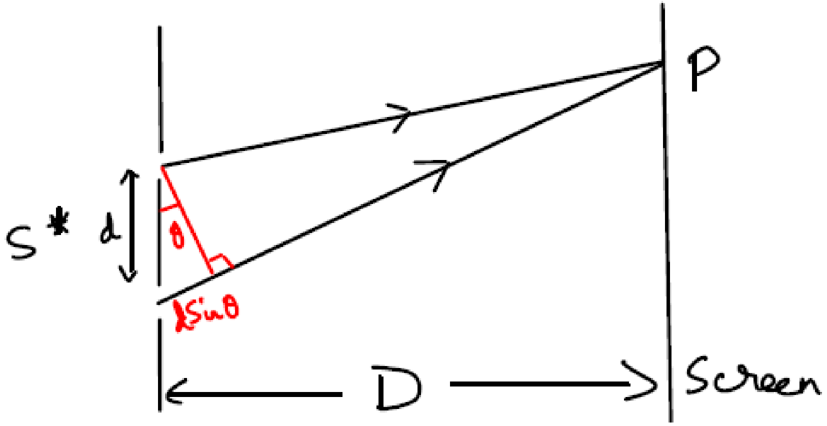
$$E_p = a\cos\omega t + a\cos(\omega t - \varphi) = 2a\cos\left(\omega t - \frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)$$

- Let, $\frac{\varphi}{2} = \frac{\pi}{\lambda}\, d\, sin\theta \equiv \gamma$

- Then, $E_p = 2a\cos(\omega t - \gamma)\cos\gamma = (2a\cos\gamma)\cos(\omega t - \gamma)$



Arrangement for producing Young's interference pattern



Module 1 (Physical Optics)

Diffraction: Two-Slit interference (Young's double slit experiment)

- Intensity distribution for two-slit interference:

$$I = |E_{p0}|^2 = (2a\cos\gamma)^2$$

$$\Rightarrow I = 4\, I_0\, \cos^2\gamma \quad (\text{where } I_0 = a^2)$$

- Recall the intensity expression for single-slit diffraction: $I = I_0 \dfrac{sin^2\beta}{\beta^2}$

- Next topic: Intensity expression for two-slit diffraction: $I = 4\, I_0 \dfrac{\sin^2\beta}{\beta^2} \cos^2\gamma$

- Note: For the two-slit diffraction, the intensity distribution is a combination of intensity patterns of single-slit diffraction and two-slit interference.

Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- ✓ Fraunhofer **diffraction** pattern produced **by a single slit** of width b and found that the intensity distribution consists of maxima and minima.

- ❑ Now, we will study the Fraunhofer **diffraction** pattern produced **by two parallel slits** (each of width b) separated by a distance d.

- ❑ In this case, the resultant intensity distribution is a product of the single-slit diffraction pattern and the interference pattern produced by two point sources separated by a distance d.

Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- Assumption: two slits are identical in all respect.
- Source slit and the lens are symmetrically placed relative to two slits.
- We assume that the slits consist of a large number of **equally spaced** point sources and that each point on the slit is a source of **Huygens' secondary wavelets**.

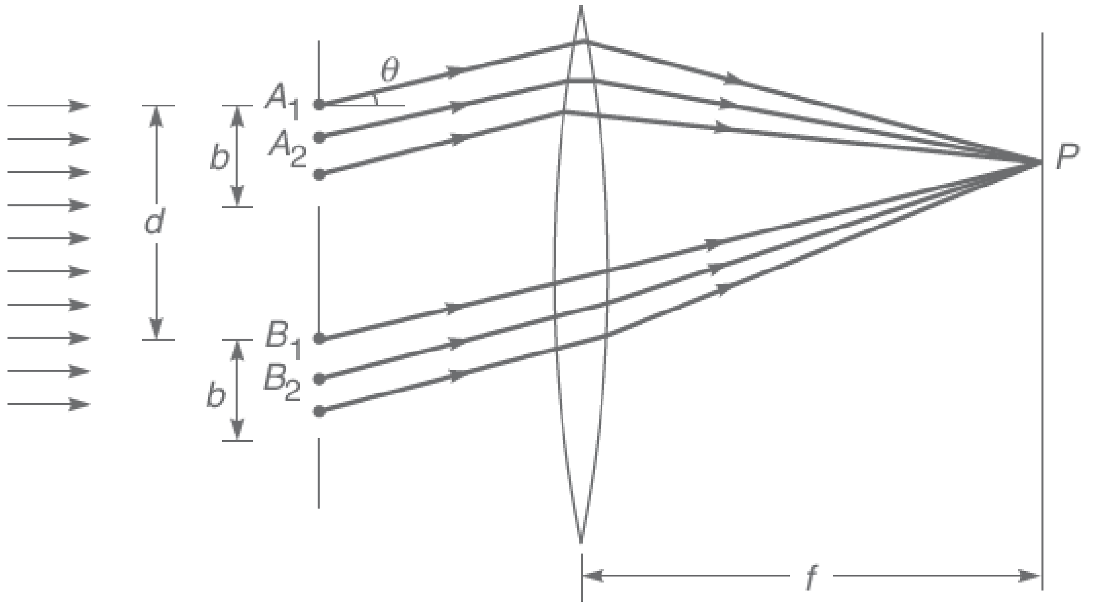


Fig. 18.29 Fraunhofer diffraction of a plane wave incident normally on a double slit.

- Let the point sources be at A_1, A_2, A_3, ……(in the 1st slit) and B_1, B_2, B_3, …… (in the 2nd slit).
- We assume that the distance between two consecutive points in either of the slits is Δ.

Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- Let's consider that the diffracted rays make an angle θ with the normal to the plane of the slits.
- The path difference between two consecutive rays reaching the point P will be $\Delta \sin\theta$.
- The field produced by the 1st slit at the point P will, therefore, be given by

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

- Similarly, the 2nd slit will produce a field at P:

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi)$$

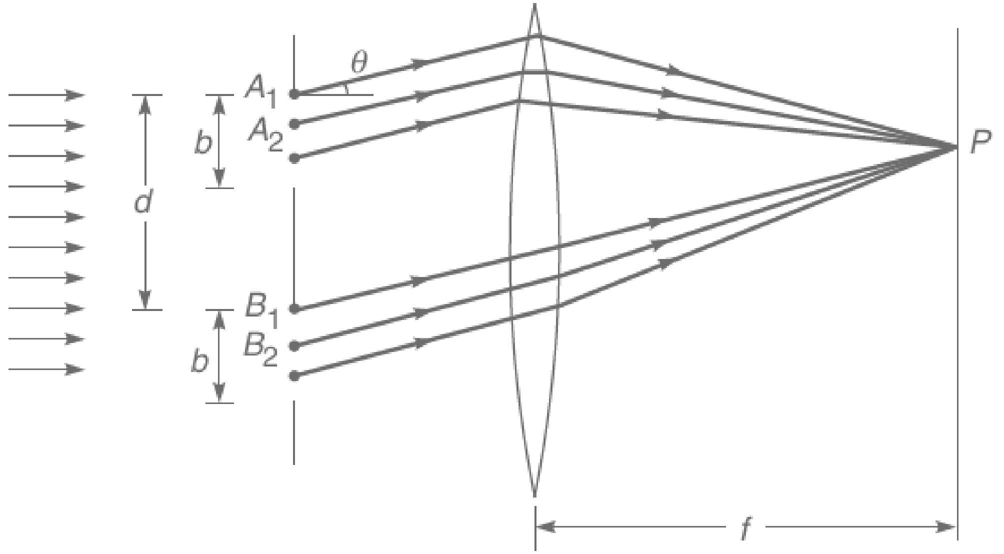


Fig. 18.29 Fraunhofer diffraction of a plane wave incident normally on a double slit.

Where, $\beta = \frac{\pi}{\lambda}\, b\, sin\theta$ (θ is the diffraction angle).

Phase difference between the rays coming from pair of corresponding points on the slits (e.g., A_1 and B_1, A_2 and B_2, etc.) is: $\Phi = \frac{2\pi}{\lambda}\, d\, sin\theta$.

Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- Hence, the resultant field

$$E = E_1 + E_2 = A\,\frac{sin\beta}{\beta}\,[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi)]$$

- This represents the interference of two waves each of amplitude $A\frac{sin\beta}{\beta}$ and differing in phase by Φ.

- The above equation can be rewritten as

$$E = 2A\,\frac{sin\beta}{\beta}\cos\left(\frac{\Phi}{2}\right)\cos\left(\omega t - \beta - \frac{\Phi}{2}\right)$$

Where $\Phi = \frac{2\pi}{\lambda}\, d\, sin\theta$

$$\Rightarrow \frac{\Phi}{2} = \frac{\pi}{\lambda}\, d\, sin\theta \quad \equiv \gamma$$

and $\beta = \frac{\pi}{\lambda}\, b\, sin\theta$.

- The intensity distribution will be of the form

$$I = |E_0|^2 = 4\, I_0\, \frac{\sin^2\beta}{\beta^2}\cos^2\gamma \qquad (\text{where } I_0 = A^2)$$

Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- Intensity distribution is of the form: $I = 4\, I_0\, \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$

- $I_0\, \frac{\sin^2 \beta}{\beta^2}$ represents the intensity distribution produced by one of the slits.

- Note: the intensity distribution is a product of two terms: the first term ($\frac{\sin^2 \beta}{\beta^2}$) represents the diffraction pattern produced by a single slit of width b and the second term ($\cos^2 \gamma$) represents the interference pattern produced by two point sources separated by a distance d.

- Note: if the slit widths are very small (so that there is almost no variation of the $\frac{\sin^2 \beta}{\beta^2}$ term with θ) then one simply obtains the Young's interference pattern.

Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- Maxima and Minima

- Intensity distribution is of the form: $I = 4\,I_0\,\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$. Intensity, $I = 0$ when
  - $\beta = \pi, 2\pi, 3\pi, \dots\dots$ $(\because \sin\beta = 0)$ This is the condition for **diffraction minima**
  - $\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots\dots$ $(\because \cos\gamma = 0)$ This is the condition for **interference minima**
  - Corresponding angles of diffraction are given by the following equations:

$$b\sin\theta = m\lambda; \qquad (m = 1, 2, 3, \dots)$$

and

$$d\sin\theta = \left(n + \frac{1}{2}\right)\lambda; \qquad (n = 1, 2, 3, \dots)$$

  - Interference maxima occurs when (approximately) $\gamma = 0, \pi, 2\pi, 3\pi, \dots\dots$ Or when $d\,sin\theta = 0, \lambda, 2\lambda, \dots\dots$

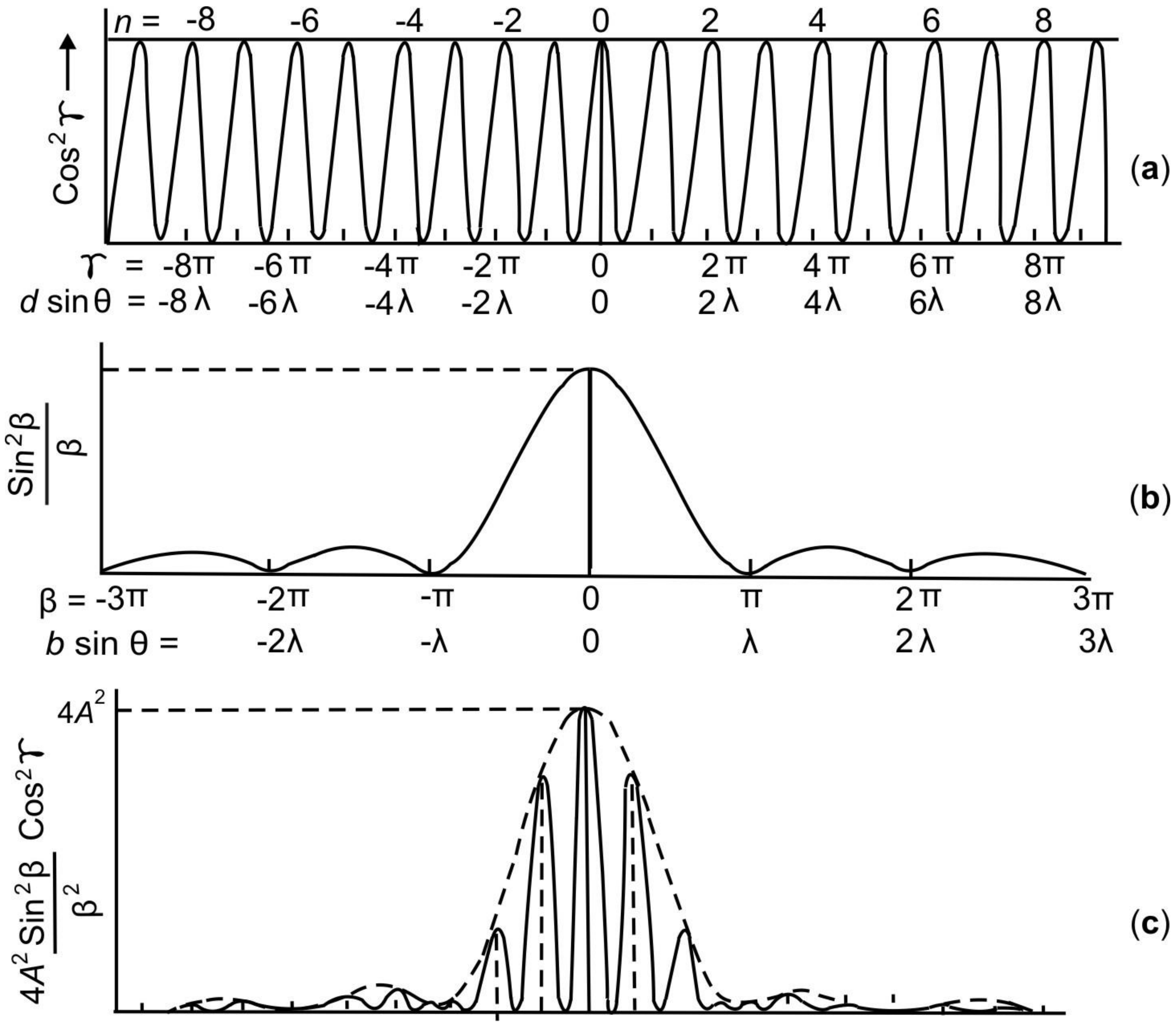
Module 1 (Physical Optics)

Diffraction: Two-Slit Fraunhofer Diffraction

- Intensity distribution is of the form:

$$I = 4\, I_0 \,\frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

- Graphical representation



Diffraction: Two-Slit Fraunhofer Diffraction

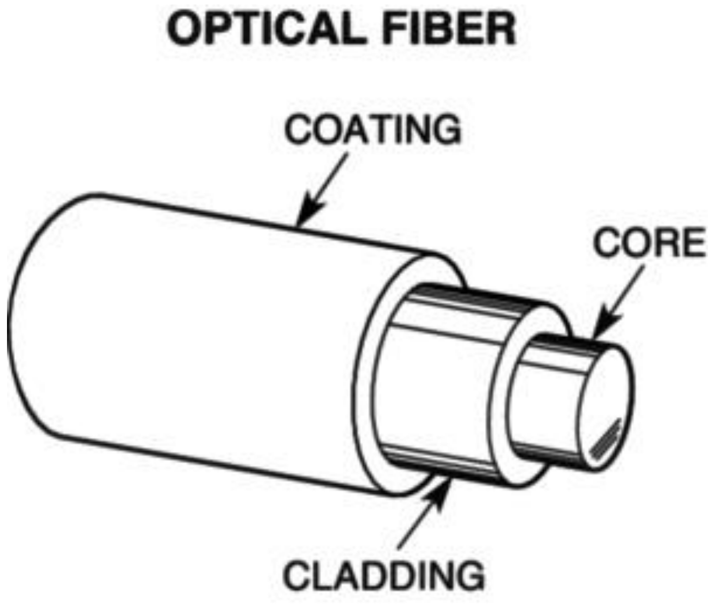
- Maxima and Minima
- Intensity distribution is of the form: $I = 4\,I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$.
- **Missing orders**
  - A maximum may not occur at all if θ corresponds to a diffraction minimum, i.e., if $b\sin\theta = \lambda, 2\lambda, 3\lambda, \dots \dots$

Module 1 (Physical Optics)

Optical Fibre

- Optical fibre are optical waveguides which allows the transmission of light wave.
    - *Idea of using light waves for communication: Graham Bell (1980) – photophone.*

- Optical fibre works on the law of **total internal reflection**.

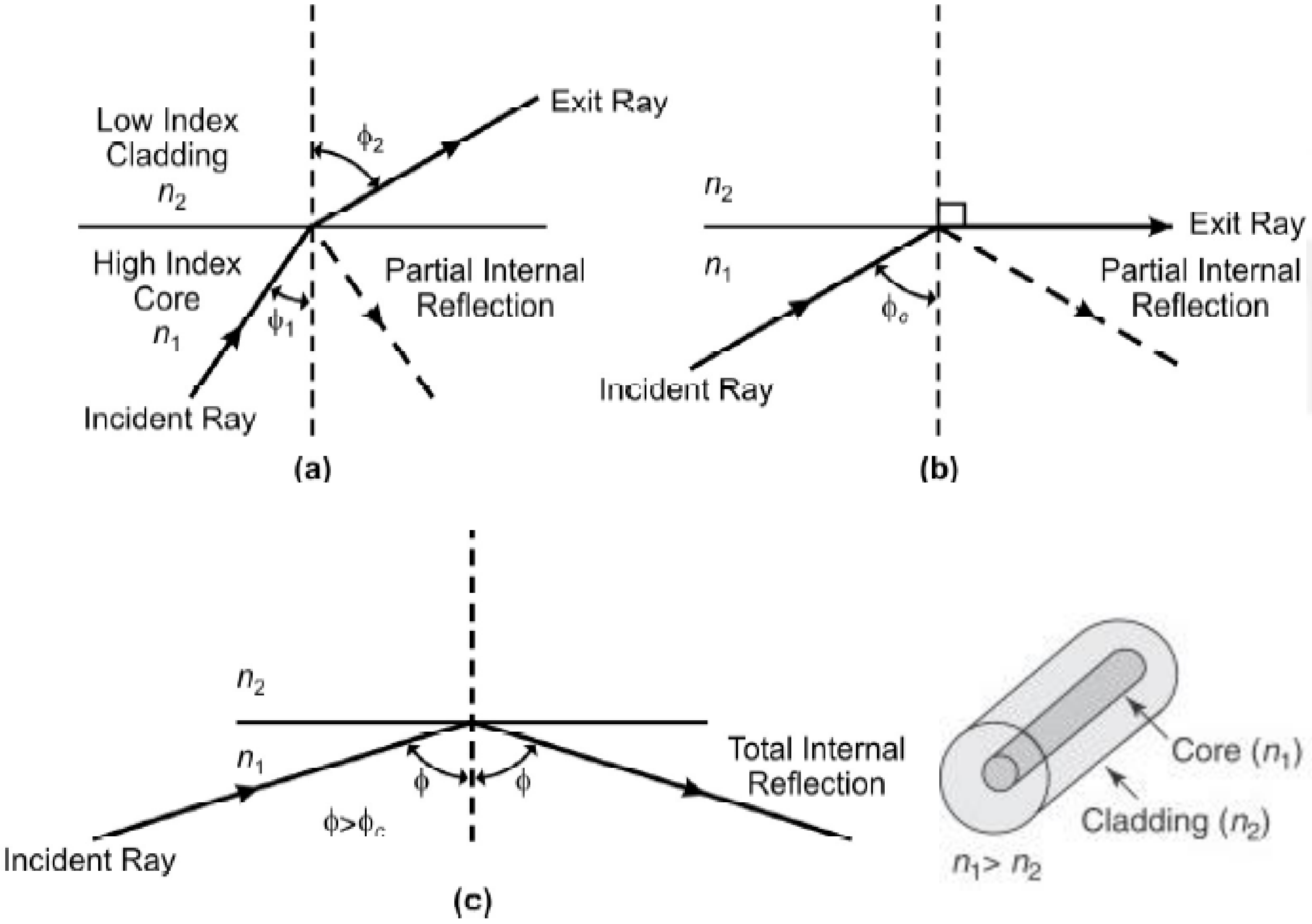


- The optical fibre consists of two concentric glass (silica) cylinders.
- The inner cylinder is called the core, while the outer is known as cladding.
- The diameter of the core can be in the range of 2-50 μm and the cladding has typical diameter of 125 μm.

Module 1 (Physical Optics)

Total Internal Reflection

- When the angle of incidence is greater than the critical angle, the light is reflected back into the originating dielectric medium with high efficiency.
- **Snell's law**:
- $n_1 \sin\phi_1 = n_2 \sin\phi_2$
- At the critical angle: $\phi_1 = \phi_c$ and $\phi_2 = \frac{\pi}{2}$
- $\sin\phi_c = {}^{n_2}/_{n_1}$



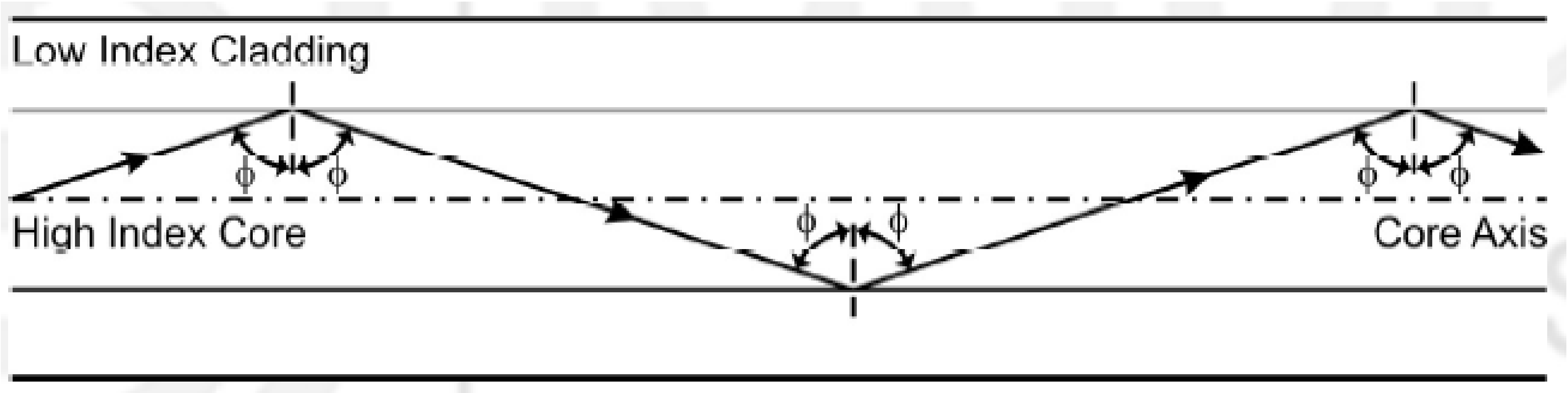
Module 1 (Physical Optics)

Optical Fibre as a Communication Medium: Application

- High information capacity of fibre
- Low attenuation and dispersion
- Space saving
- Low cross-talk
- Immunity to electromagnetic interference (EMI)
- Higher security
- Ease of expansion
- Easy maintainability

Optical Fibre

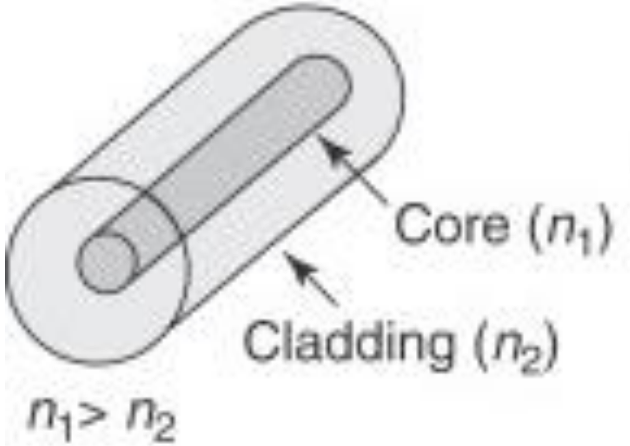
- Remember: refractive indices for the core and cladding are n_1 and n_2, respectively, with $n_1 > n_2$. This condition is necessary for the **total internal reflection**.

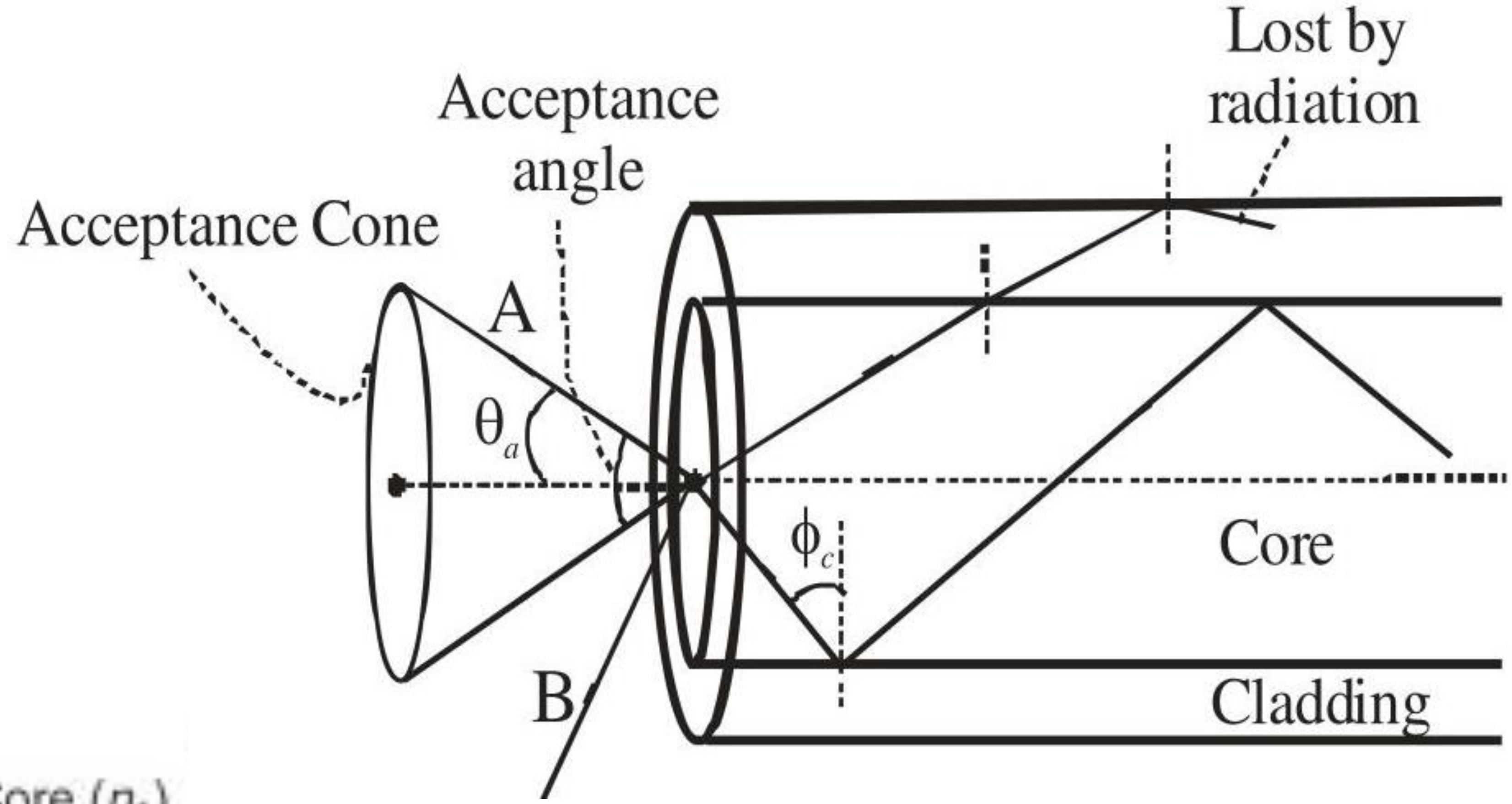


Module 1 (Physical Optics)

Optical Fibre

- ❑ **Acceptance angle**
- ❑ The maximum angle θ_a to the axis at which light may enter the fibre in order to be propagated is referred as the **acceptance angle** for the fibre.





Module 1 (Physical Optics)

Types of Optical Fibre

- On basis of core diameter:
  - Single-mode (SM) fibre
  - Multi-mode (MM) fibre
- On basis of refractive indices of core and clad:
  - Step index single mode (SI SM) fibre: uniform refractive indices for both core (n_1) and cladding (n_2) with $n_1 > n_2$. The core is very narrow (typically 2 to 10 μm) and supports only a single mode propagation.
  - Step index multimode (SI MM) fibre: uniform refractive indices for both core (n_1) and cladding (n_2) with $n_1 > n_2$ as in case of single mode fibre, but the core diameter is above 50 μm to support many modes simultaneously.
  - Graded index multimode (GRIN MM) fibre: core with refractive index highest at the centre and gradually decreasing till it is the same as that of the cladding at core-cladding boundary.

PH113: Physics (B.Tech. I)

Module 1

Module – 1 : Physical Optics

Polarization, Malus' Law, Brewster's Law, Double Refraction, Interference in thin films (Parallel films), Interference in wedge-shaped layers, Newton's rings, Fraunhofer diffraction by single slit, Double slit. Elementary ideas of fibre optics and application of fibre optic cables. [8]

Text book:

T1: A. Ghatak, Optics, 6th Edition, 2017, McGraw-Hill Education (India) Pvt. Ltd.

References: **(1)** Optics – E Hecht **(2)** Fundamentals of Optics – Jenkins & White

Module 1

PH113 / Assignment - Module 1

(Dated: September 6, 2023)

Module 1

PH113 / Assignment - Module 1

1. **Malus Law.**

   (a) Consider a geometry of sequence of two polarizers rotated at a relative angle of 45° from each other. What would be resulting amplitude and intensity if the incident unpolarized (light) beam had the intensity I_0?

- After the 1st polarizer: $E_1 = E_0 \cos(45°)$
- The related intensity: $I_1 = \frac{I_0}{2}$
- Remember: $E_0 = \sqrt{I_0}$
- After the 2nd polarizer: $E_1 = E_0 \cos(45°)$. The related intensity: $I_2 = \left(\frac{I_0}{2}\right)\cos^2(45°)$

Module 1

PH113 / Assignment - Module 1

1. **Malus Law.**

   (b) Similar to above, consider a sequence of three polarizers at relative angles of 20° to each other. Also, the first polarizer is aligned along the y-axis. Compute the intensities I_i $(i = 1, 2, 3)$ after each polarizer if the intensity of the y-polarized source is given to be $I_0 = 1000\,\mathrm{W/m^2}$.

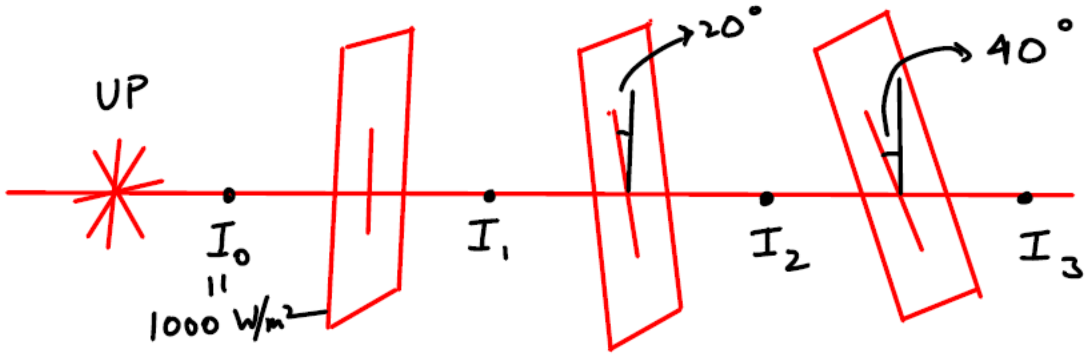
- Initial beam is polarized along the y-axis.
- So, it does not affect the intensity of a y-polarized light.
- All subsequent polarizers have the same effect: $I_i = (I_0)\cos^{2(i-1)}(20^\circ)$ where $i = 1, 2, \ldots . n$.

Module 1

PH113 / Assignment - Module 1

1. **Malus Law.**

   (c) What happens to the final intensity if the second polarizer is removed?



$I_1 = \frac{I_0}{2}$

$I_2 = \frac{I_0}{2} \cos^2(20°)$

$I_{out} = I_3 = I_2 \cos^2(20°) = \frac{I_0}{2} \cos^4(20°) \simeq 390 \text{ W/m}^2$

What happens to I_{out} if the second polarizer is removed?

$I_{out} \simeq 293 \text{ W/m}^2$.

Intensity should decrease if the second polarizer is removed since we'll now work with $\cos^2(40°)$.

Module 1

2. **Brewster angle**

On a calm surface of a pond or a lake (interface between air and water), for which position of the sun would the reflected light from the interface be completely linearly polarized. The refractive indexes for air and water should be taken as $n_{\text{air}} = 1$ and $n_{\text{water}} = 1.33$.

Comment: This situation is, in fact, realized at the beaches or lakes.

- Brewster's law: $\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$ where $n_2 > n_1$.
- Here $n_2 = n_{water}$ and $n_1 = n_{air}$
- Find the angle.
- $\theta_B = 53°$. Therefore, the sun should be $37°$ above the ground (horizontal).

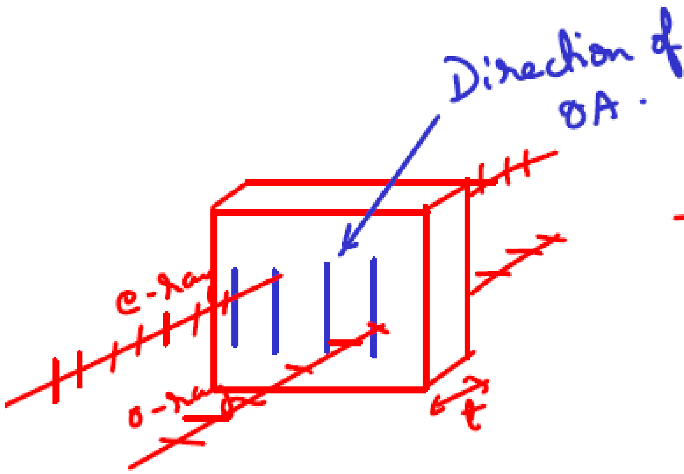
Module 1

3. A plate of flint glass (refractive index 1.67) is immersed in water (refractive index 1.33). Calculate the Brewster angle for internal as well as external reflection at an interface.

Hint: The external surface is formed between air/water, and the internal surface is formed between water/glass.

Use: $\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$ where $n_2 > n_1$

PH113 / Assignment - Module 1



4. **Double refraction.**

A thin quartz crystal of thickness t has been cut so that the optical axis is $\parallel$ to the surface of the crystal. For sodium light ($\lambda = 589\,\text{nm}$), the refractive index of the crystal is 1.55 for e-ray and 1.54 for o-ray. If two beams of light polarized $\parallel$ and $\perp$ O.A., respectively, start through the crystal in-phase, how thick must the crystal be if the emerging rays have a phase difference of $\pi/2$.

Solution. Time taken by the e-ray to cross the thickness t: $t_e = t/v_e = t/(c/n_e) = n_e t/c$ with $n_e = 1.55$.

Similarly, time taken by the o-ray: $t_o = n_o t/c$.

Time spent by the e-ray inside the crystal is longer. In the time difference $t_e - t_o$, the o-ray travels in vaccum/air.

Therefore, the path difference between the two rays when they emerge from the other side:

$$c(t_e - t_o) = c(\frac{n_e t}{c} - \frac{n_o t}{c}) = (n_e - n_o)t$$

Corresponding phase difference:

$$\phi = \frac{2\pi}{\lambda}(n_e - n_o)t\,.$$

Equating this (p.d) to $\pi/2$, we get:

$$t = \frac{\lambda}{4(n_e - n_o)} = \frac{589\,\text{nm}}{4 \times 0.01} = 14.7\,\mu\text{m}\,.$$

PH113 / Assignment - Module 1

5. Plane waves and their superposition

(a) Plot $y = y_0 \cos(\omega t - kx + \phi)$ for (i) $\omega = 1\ \mathrm{s}^{-1}$, (ii) $\omega = 0.5\ \mathrm{s}^{-1}$ and $\phi = 0, \pm\pi/4$, $\pm\pi/2$, $\pm\pi$, $\pm 2\pi$. What are the units of ϕ, ω and k?

(b) Consider superposition of two plane waves of equal amplitude and in-phase ($\phi = 0$), polarized along $\hat{x}$ and $\hat{y}$, respectively. What would be resultant? (Find the amplitude and polarization of the resultant wave).

(c) In the above, consider the phase-difference between the two waves to be fixed at $\phi = \pi$. How does this affect the resultant wave?

(d) Show that the the amplitude of the wave obtained by superposition of two waves of equal amplitude and a phase difference of $\phi = \pi/2$ is equal to the amplitude of the initial waves. What does the wavefront of the superposed wave look like?

What happens for $\phi = -\pi/2$?

PH113 / Assignment - Module 1

- Hint: Q. 5(b)

- $E_{res} = E_1 + E_2 = E_0\hat{x}\cos(\omega t - kx) + E_0\hat{y}\cos(\omega t - kx) = E_0\,(\hat{x} + \hat{y})\cos(\omega t - kx)$

- Therefore, the amplitude of the resultant wave is $\sqrt{2}E_0$, obtained using $|\boldsymbol{E_{res}}| = \sqrt{(\boldsymbol{E_{res}} \cdot \boldsymbol{E_{res}})}$ and the polarization direction is $(\hat{x} + \hat{y})$), i.e., $\pi/4$ away from the x-axis.

- Hint: Q. 5(c)

- Due to the phase π, one of the wave will pick up a $-$ sign (minus sign).

Module 1 (Physical Optics)

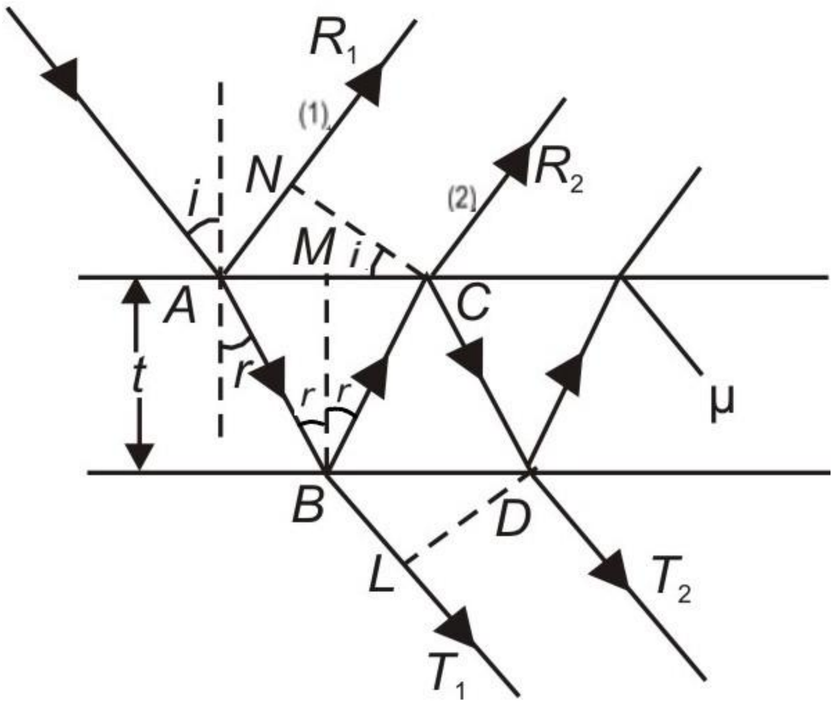
PH113 / Assignment - Module 1

6. **Interference**

Prove the cosine law: the path difference between the interfering beams in a (parallel) thin film geometry at oblique incidence is $2\mu t \cos\theta_r$, where μ is the refractive index of the denser medium, t is the thickness of the medium, and θ_r is the angle of refraction with respect to normal.

- Interference by a plane parallel film (oblique incidence): We already solved it considering a denser medium (with refractive index n_2) and a rarer medium (with refractive index n_1).
- Here, consider the rarer medium to be air, hence, $n_1 = 1$. Consider the refractive index of the denser medium to be $n_2 = \mu$.

PH113 / Assignment - Module 1



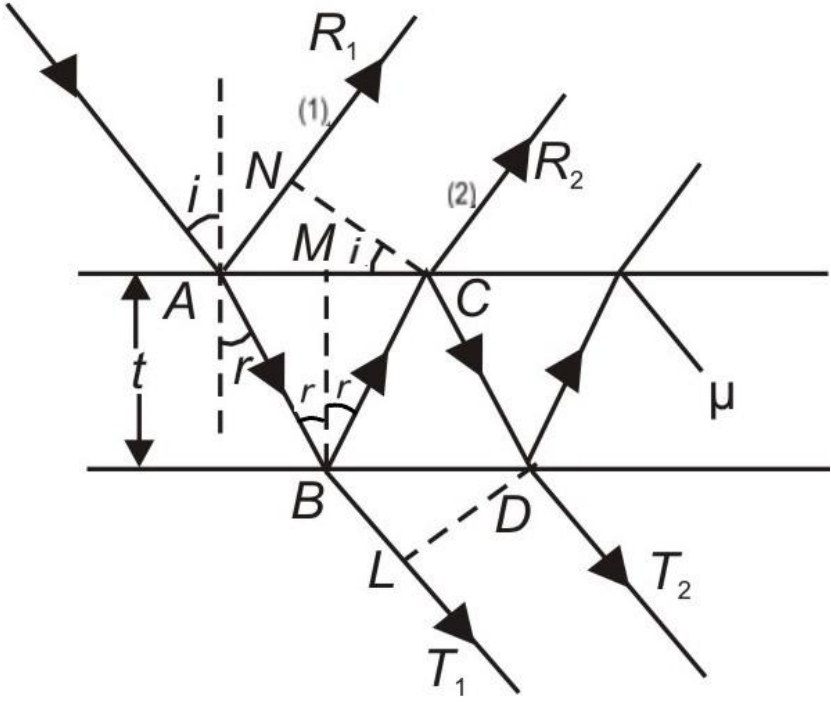
- Consider the geometry shown in Figure.
- Path difference between the reflected beams from the top surface (R_1) and the bottom surface R_2 is:
- $p.d. = \mu(AB + BC) - AN$
- $AB = BC = \frac{BM}{Cos(\theta_r)} = \frac{t}{Cos(\theta_r)}$
- $AN = AC\ Sin(\theta_i)$
- $AC = AM + MC = BM \tan(\theta_r) + BM \tan(\theta_r) = 2t \tan(\theta_r)$
- So, $AN = 2t\ \frac{Sin(\theta_r)}{Cos(\theta_r)}\ \sin(\theta_i)$

Module 1 (Physical Optics)

PH113 / Assignment - Module 1

- $AB = BC = \frac{BM}{Cos(\theta_r)} = \frac{t}{Cos(\theta_r)}$
- Using Snell's law: $AN = 2t\,\frac{Sin(\theta_r)}{Cos(\theta_r)}\,\mu\sin(\theta_r) = 2\mu t\,\frac{Sin^2(\theta_r)}{Cos(\theta_r)}$
- With AB, BC and AN, the path difference is,
- $p.d. = \mu(AB + BC) - AN$

$$= \mu\left(\frac{t}{\cos(\theta_r)} + \frac{t}{\cos(\theta_r)}\right) - 2\mu t\,\frac{Sin^2(\theta_r)}{Cos(\theta_r)} = 2\mu t\cos(\theta_r)$$



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7. A thin film of thickness 4×10^{-5} cm is illuminated by white light normal to its surface. Its refractive index is 1.5. Of what color will the thin film appear in reflected light?

- The color in reflection will appear due to constructive interference.
- The condition for constructive interference is, $2\mu t\cos(\theta) = (n + \frac{1}{2})\lambda$ where θ is the angle of incidence with respect to the normal.
  - By putting $\theta = 0$, $\mu = 1.5$ and $t = 4 \times 10^{-5}$ cm, we get
  - the values of n, θ and d, and solving, $\lambda = \frac{2400}{(2n+1)}\ nm$
  - Taking $n = 0, 1, 2, 3$, we get $\lambda = 2400, 800, 480, 343.1\ nm$, respectively.
  - Wavelength 2400, 800 and 343.1 nm are not in the visible region. Only 480 nm is in the visible spectral region (blue). So, the color appear in interference is Blue.

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8. White light is reflected normally from a uniform oil film ($n = 1.33$). An interference maximum for 6000 Å and a minimum for 4500 Å with no minimum in between, are observed. Calculate the thickness of the film.

- The condition for interference maxima in the light reflected normally from the film of thickness t is: $2\mu t = (n + \frac{1}{2})\lambda$
- For the minima: $2\mu t = n\lambda$
- Use $\mu = 1.33$ and respective wavelengths for maxima and minima.
- Solve it
- Answer: $t = 3383$ Å

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9. **Interference from wedge-shaped surfaces**

Light of wavelength $\lambda = 6300$ Å falls normally on a thin wedge-shaped film ($n = 1.5$). There are ten bright and nine dark fringes over the length of the film. By how much does the film thickness change over this length?

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- The condition for constructive interference is, $2\mu t\cos(\theta) = n\lambda$ where θ is the angle of incidence with respect to the normal.
- Suppose the film thickness changes from t to $t+\delta t$ over the length.
- Let n be the order of the dark fringe appearing at one end of the film. The order of dark fringe at the other end will be $(n+9)$.
- Therefore, we have:
  - $2\mu t\cos(\theta) = n\lambda$
  - $2\mu\,(t+\delta t)\cos(\theta) = (n+9)\lambda$

From the above two equations, we get $2\mu\delta t\cos(\theta) = 9\lambda \quad => \quad \delta t = \dfrac{9\lambda}{2\mu\cos\theta}$

- Use $\cos(\theta) = 1$ for the normal incidence and find the answer. (Answer: 1.89×10^{-4} cm)

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10. Two glass plates 12 cm long touch at one end, and are separated by a wire 0.048 mm in diameter at the other. How many bright fringes will be observed over the 12 cm distance in the light ($\lambda = 6800$ Å) reflected normally from the plates? (Take $\mu = 1$).

Let t be the thickness of the wire and l is the length of the wedge. The wedge angle, $\theta = t/l$ (in radian)

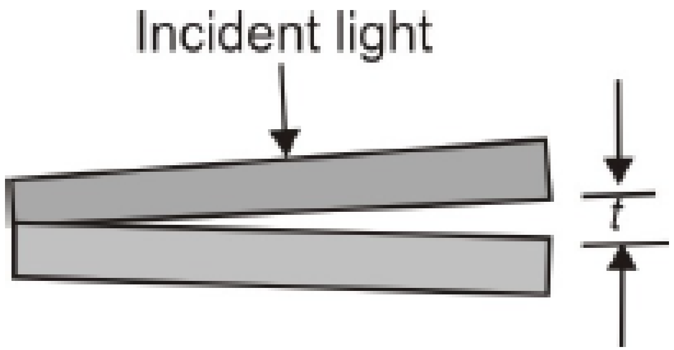
Now, the fringe width is $\beta = \lambda/2\theta$. Or, $\beta = \lambda l/2t$

Since N fringes are seen, $l = N\beta$.

Thus, $\beta = N\beta\lambda/2t$ => $N = 2t/\lambda$

Given: $\lambda = 6800$ Å, and $t = 0.048\ nm$

Answer: $N = 141$ (Be careful with the units)



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11. **Newton's rings**

(a) What is an efficient (and presumably error-free) way to obtain λ?

(b) Show that in the Newton's rings arrangement, if the plano-convex lens is slowly lifted vertically upwards, the fringes will move inwards.

(c) Newton's rings are formed in reflected light of wavelength 5895 Å with a liquid between the plane and curved surfaces. The diameter of the 5^{th} ring is 0.3 cm and the radius of curvature of the curved surface is 100 cm. Calculate the refractive index of the liquid, when the 5^{th} ring is i) bright, ii) dark.

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- For near normal incidence (and considering points very close to the point of contact) the optical path difference between the two waves is very nearly equal to $2\mu t$, where μ is the refractive index of the film and t the thickness of the film.
- Condition for maxima: $2\mu t = \left(n + \frac{1}{2}\right)\lambda; \quad n = 0,1,2, \dots \dots$
- Condition for minima: $2\mu t = n\lambda$

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- Recall: ${r_n}^2 = 2Rt$. Hence, $2t = {r_n}^2/R$
- Condition for maxima (bright fringe): $2\mu t = \left(n + \frac{1}{2}\right)\lambda$. For an air film. $\mu = 1$. Hence, $2t = \left(n + \frac{1}{2}\right)\lambda$
- So, we get: $\frac{{r_n}^2}{R} = \left(n + \frac{1}{2}\right)\lambda \Rightarrow {r_n}^2 = \left(n + \frac{1}{2}\right)R\lambda$
- In terms of the diameter of the ring, $D_n = 2r_n$
- The diameter of the n^{th} bright ring is, $D_n^2 = 2(2n + 1)R\lambda$
- If D_{n+p} is the diameter of $(n + p)$-th bright fringe, $D_{n+p}^2 = 2[2(n + p) + 1]R\lambda$
- Subtracting above two equations, $D_{n+p}^2 - D_n^2 = 4\mathrm{p}R\lambda$
- Hence, $\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$

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Q. 11(a)

The best way would be to compute the difference between $(n + p)$ and n-th ring:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$ (typically p ≈ 10)

Here, D_n is the diameter of the n-th ring.

Q. 11(b)

Example 15.2 in Ajoy Ghatak's Optics book (Sixth edition).

Q. 11(c)

i) The diameter D_n of the nth bright ring is given by

$$D_n^2 = \frac{2(2n+1)\lambda R}{\mu}$$

$$\therefore \quad \mu = \frac{2(2n+1)\lambda R}{D_n^2}$$

Here $n = 4$, $\lambda = 5895 \times 10^{-8}$ cm, $R = 100$ cm and $D_n = 0.3$ cm.

$$\therefore \quad \mu = \frac{2(8+1) \times 5895 \times 10^{-8} \times 100}{(0.3)^2} = 1.18$$

ii) The diameter of the nth dark ring is given by

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$\therefore \quad \mu = \frac{4n\lambda R}{D_n^2} = \frac{4 \times 5 \times 5895 \times 10^{-8} \times 100}{(0.3)^2} = 1.31.$$

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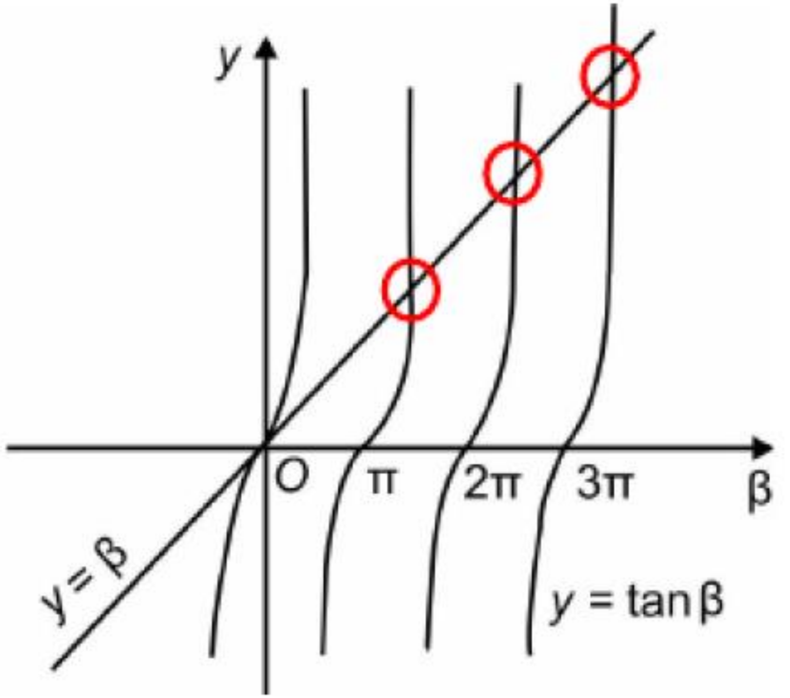
12. Solution to the transcendental equation $\tan(\beta) = \beta$



- Transcendental equation generally do not permit analytical solutions.
- One can obtain a solution graphically.
- Plot $y = \beta$ and $y = \tan(\beta)$ and find the point of intersection (marked red circle in the figure).
- $x = 0$ is a trivial solution.
- Other solution(s) are: $x = 1.43\pi, 2.46\pi, 3.47\pi, \ldots$

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13. **Diffraction from a single slit.**

Sound wave has a speed of 330 m/s, and suppose it's frequency is 100 Hz. What would be the typical aperture size to experience appreciable diffraction?

- Use: velocity = frequency X wavelength. That means, $v = f\lambda$
- So, $\lambda = v/f = 330/100\, m = 3.3\, m$.
- Note: Diffraction is observed if the aperture is at least of the size of λ.

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14. **Diffraction from 2-slit arrangement.**

    Explicitly derive the conditions for maxima/minima. Note and plot the intensity profile.

15. **Computational/Simulation exercise(s).**

    Plot the resulting intensity patterns from (i) a single slit diffraction; (ii) double slit diffraction.

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Additional Questions

A Newton's rings arrangement is used with a source emitting two wavelengths: $\lambda_1 = \mathbf{4.5 \times 10^{-5}}$ cm and $\lambda_2 = \mathbf{4.5 \times 10^{-5}}$ cm and it is found that the n-th dark ring due to λ_1 coincides with the $(n+1)$-th dark ring due to λ_2. If the radius of curvature of the curved surface is 90 cm, find the diameter of the n-th dark ring for λ_1.

- ❑ Condition for the minima (dark fringes) in a Newton's ring: $D_n^2 = 4nR\lambda$

Where D_n is the diameter of the n-th ring, R is the radius of curvature and λ is the wavelength of the light.

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Additional Questions

A Newton's rings arrangement is used with a source emitting two wavelengths: $\lambda_1 = 4.5 \times 10^{-5}$ cm and $\lambda_2 = 4.5 \times 10^{-5}$ cm and it is found that the n-th dark ring due to λ_1 coincides with the $(n+1)$-th dark ring due to λ_2. If the radius of curvature of the curved surface is 90 cm, find the diameter of the n-th dark ring for λ_1.

- In this case, $D_n = D_{n+1} \;\Rightarrow\; D_n^2 = D_{n+1}^2$

$$\Rightarrow \quad 4nR\lambda_1 = 4(n+1)R\lambda_2$$

$$\Rightarrow n = \frac{4R\lambda_2}{4R(\lambda_1 - \lambda_2)} = \frac{\lambda_2}{(\lambda_1 - \lambda_2)}$$ (Calculate n) (Answer: $n = 3$)

- Use $n = 3$ to calculate D_3 $(= 25.45 \times 10^{-2}$ cm)

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Additional Questions

In the experimental set up for obtaining Fraunhofer diffraction pattern of a vertical slit of width 0.3 mm, the focal length of lens kept between the slit and the screen is 30 cm. The slit is illuminated with yellow sodium light which is a doublet. You may take λ = 6000 Å. Calculate (a) the diffraction angles and positions of the first, second and third minima, and (b) the positions of the first, second and third secondary maxima on either side of the central spot.

Suppose, the slit widths are changed to 0.2 mm, 0.1 mm and 0.06 mm. Calculate the positions of the first and second minima.

- ❑ Solve it

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Additional Questions

In the experimental set up for single slit, consider that the slit width b has values b = 10λ, 5λ and λ. Calculate the spread of the central maximum for each value of slit width.

- ❑ Solve it

Module 1

Next Class: Module 2

Questions?