Module - 3

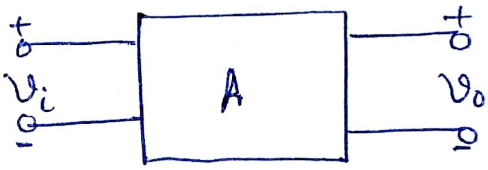
Sinusoidal Oscillators

Concept of feedback

Feedback is the process of taking a part of output signal and feeding it back to the input circuit.

Open-loop amplifier

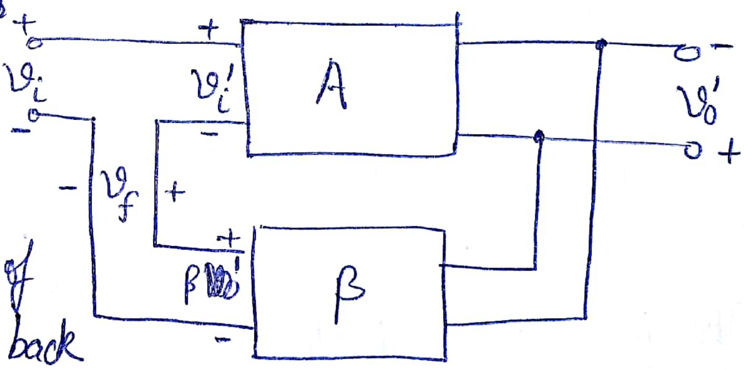
In this amplifier, the input does not know what is happening at the output. If due to some reason, the output changes, the net input remains unaffected. Such a system is called the open-loop or non-feedback system.



$$A = \frac{v_o}{v_i}$$

Closed loop amplifier

The output of the amplifier is fedback to the input through a network. This network is called a β network. A fraction $\beta v_o'$ of the output voltage is going back to the input. This changes the net input voltage to the amplifier. Thus the input is modified by the output. The input knows at every instant what the output is. Such a system is called a closed loop or feedback system.



Positive and negative feedback

Depending on the relative polarity of the signal being fedback into a circuit, one may have positive or negative feedback. Negative feedback results in decreased voltage gain, and positive feedback drives a circuit into oscillation.

If the feedback voltage v_f is in phase opposition to the input voltage v_i, then

$$v_i' = v_i - v_f$$

Since the net input decreases, output decreases from v_o to v_o', thus the gain also reduces. This type of feedback is called negative, inverse, or degenerative feedback.

If the feedback voltage is in the same phase as the external input voltage, the effective input of the amplifier is increased, therefore gain increases. This is known as positive, direct or regenerative feedback.

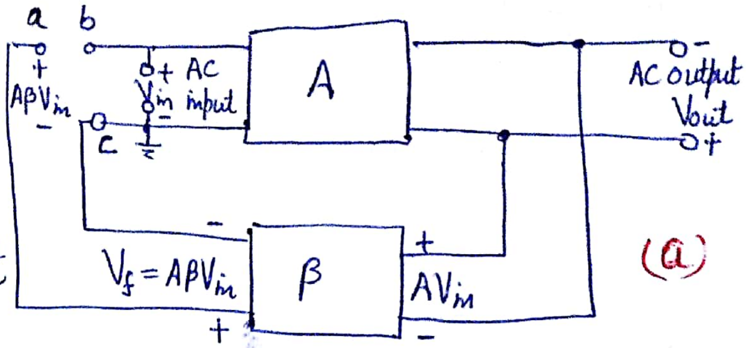
Oscillator

Any circuit that generates an alternating voltage is called an oscillator. An oscillator is the basic element of all ac signal sources and generates sinusoidal signal of known frequency and amplitude. It is one of the basic and useful instrument used in electrical and electronic measurements. For example, an oscillator finds wide applications in electronic communication equipment. Also used as clocks in digital systems such as microcomputers, in the sweep circuits found in TV sets and oscilloscopes. It covers the frequency range from a few Hz to many GHz.

Operation of Oscillator

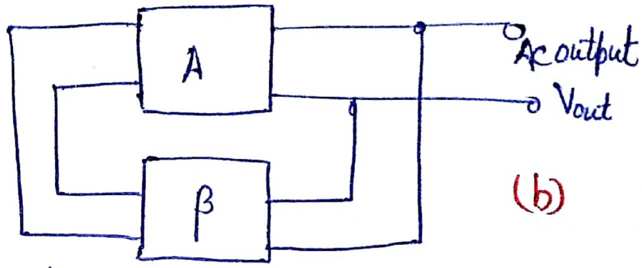
The use of positive feedback that results in a feedback amplifier having closed-loop gain A_f greater than unity and satisfies the phase condition, results in operation as an oscillator circuit. An oscillator circuit then provides a constantly varying output signal. If the output signal varies sinusoidally, the circuit is referred to as a sinusoidal oscillator and if the output voltage rises quickly to one voltage level and later drops quickly to another voltage level, the circuit is usually referred to as a pulse or square-wave generator.

In the feedback circuit (a), V_{in} is the voltage of ac input driving the input terminals bc of the amplifier having voltage gain A. The amplified output voltage is



$$V_{out} = AV_{in}$$

The feedback voltage returning to point a is given by



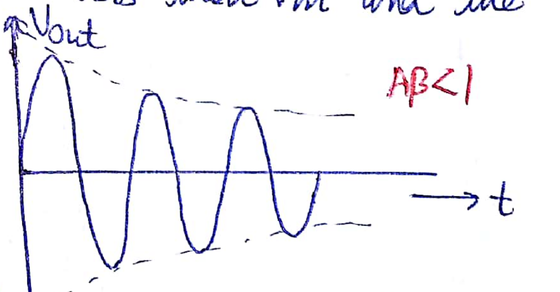
$$V_f = A\beta V_{in}$$

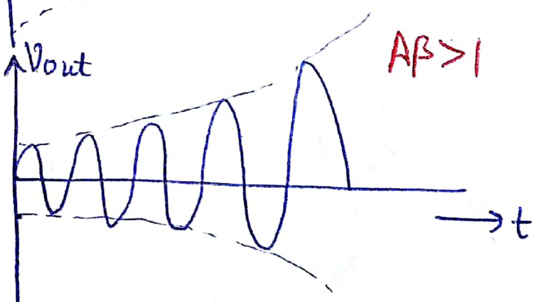
Where β is the gain of feedback network.

If the phase shift through the amplifier and feedback circuit is zero, then $A\beta V_{in}$ is in phase with the input signal V_{in} that drives the input terminals of the amplifier.

Now we connect point 'a' to point 'b' and simultaneously remove voltage source V_{in}, then feedback voltage $A\beta V_{in}$ drives the input terminals bc of the amplifier as shown in fig. (b).

In case $A\beta$ is less than unity, $A\beta V_{in}$ is less than V_{in} and the output signal will die out. If $A\beta$ is greater than unity, the output signal will build up. If $A\beta$ is equal to unity, $A\beta V_{in}$ equals V_{in} and the output signal is a steady sinewave. In this case the circuit supplies its own input signal and produces a sinusoidal output.

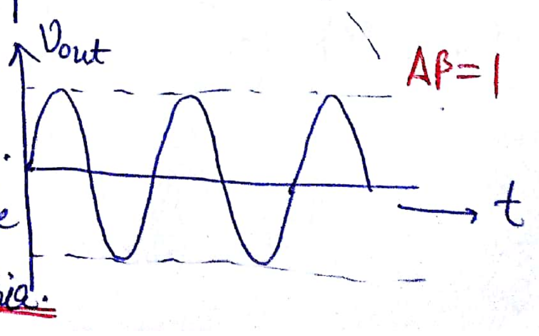




Certain conditions are required to be fulfilled for sustained oscillations and these conditions are that

(i) the loop gain of the circuit must be equal to or slightly greater than unity.

(ii) the phase shift around the circuit must be zero.

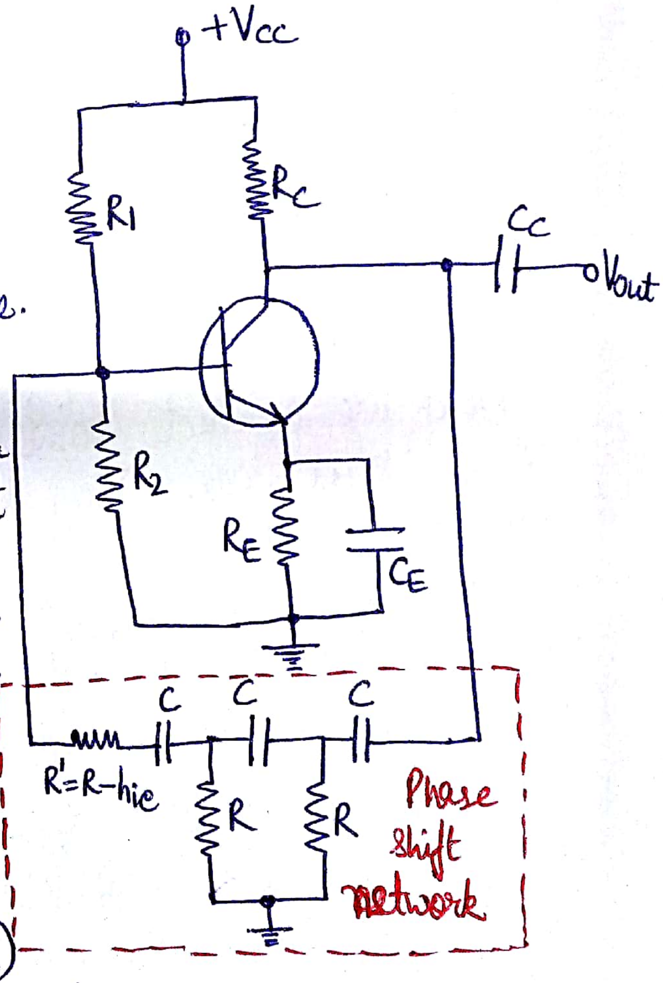


These conditions are called Barkhausen criteria.

Types of Oscillators

Type of oscillator	Frequency Ranges
Wein Bridge	1 Hz – 1 MHz
Phase Shift	1 Hz – 10 MHz
Hartley	10 KHz – 100 MHz
Colpitt's	10 KHz – 100 MHz
Crystal	Fixed Frequency

① Phase Shift Oscillator



The circuit arrangement of a phase-shift oscillator using NPN transistor in CE configuration is shown in figure. The voltage divider $R_1 - R_2$ provides dc emitter base bias, R_E and C_E combination provides temperature stability and prevent ac signal degeneration and collector resistor R_C controls the collector voltage. The oscillator output voltage is capacitively coupled to the load by C_C.

The output of the feedback network is loaded appreciably by the relatively small input resistance (h_{ie}) of the transistor, ∴ Voltage-shunt feedback is used for a transistor phase-shift oscillator. In this circuit, the feedback signal is coupled through the feedback resistor R' in series with the amplifier stage input resistance h_{ie}. The value of R' should be such that when added with amplifier stage input resistance h_{ie}, it is equal to R i.e., $R' + h_{ie} = R$.

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Operation: The circuit is set into oscillations by any random variation caused in the base current, that may be either due to noise inherent in the transistor or minor variation in voltage of dc power supply. This variation in base current is amplified in collector circuit. The output of the amplifier is supplied to an R-C feedback network. The RC network produces a phase shift of 180° between output and input voltages. Since CE amplifier produces a phase reversal of the input signal, total phase shift becomes 360° or 0° which is essential for regeneration or for sustained oscillations. The output of this network is thus in the same phase as the originally assumed input to the amplifier and is applied to the base terminal of the transistor. Thus sustained variation in collector current between saturation and cut off are obtained.

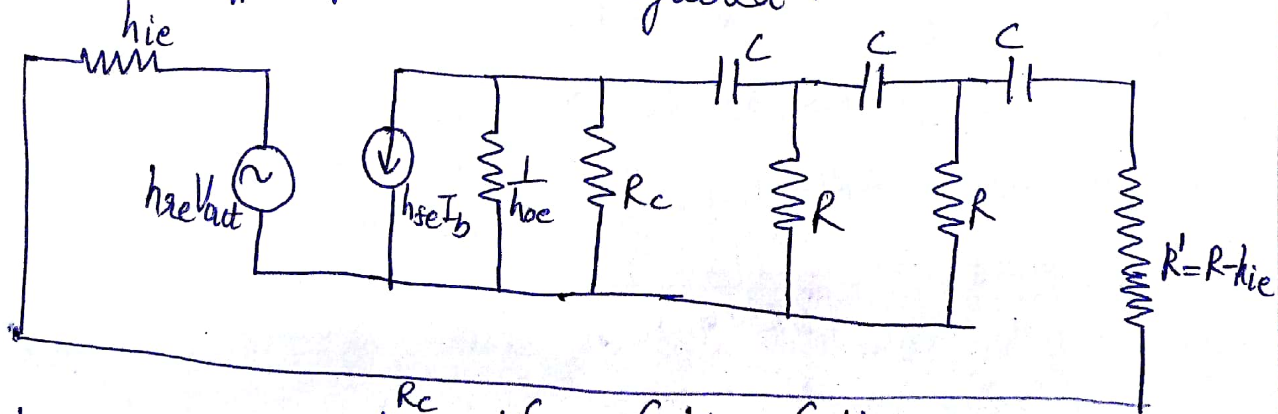
Frequency of Oscillation

The equivalent circuit for the analysis of a transistor phase-shift oscillator is shown in figure(a). The equivalent circuit is simplified if the following assumptions are made:

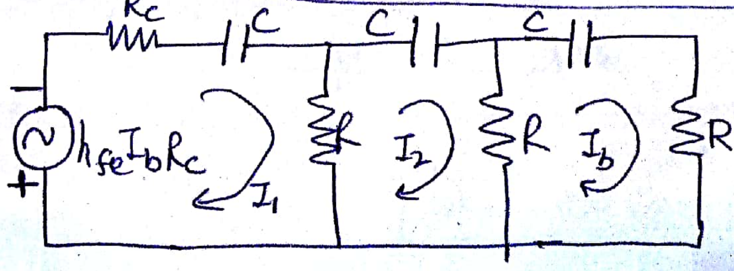
(i) h_{re} of the transistor is negligibly small and therefore $h_{re}V_{out}$ is omitted from the circuit.

(ii) h_{oe} of the transistor is very small i.e., $\frac{1}{h_{oe}}$ is much larger than R_C. Thus the effect of h_{oe} can be neglected.

(a)



Simplified equivalent circuit (b)



Making above ~~of~~ assumptions and replacing current source by equivalent voltage source, the simplified equivalent circuit is shown in figure (b)

Applying Kirchhoff's voltage law to the three loops, we have

$$\left(R+R_c+\frac{1}{j\omega C}\right)I_1 - RI_2 + h_{fe}I_b R_c = 0$$

$$-RI_1 + \left(2R+\frac{1}{j\omega C}\right)I_2 - RI_b = 0$$

$$0 - RI_2 + \left(2R+\frac{1}{j\omega C}\right)I_b = 0$$

As the currents I_1, I_2 and I_b are non-vanishing, the determinant of the coefficients of I_1, I_2 and I_b must be zero.

$$\begin{vmatrix} (R+R_c-jX_c) & (-R) & (h_{fe}R_c) \\ (-R) & (2R-jX_c) & (-R) \\ 0 & (-R) & (2R-jX_c) \end{vmatrix} = 0 \qquad \because X_c = \frac{1}{\omega C}$$

$$(R+R_c-jX_c)\left[(2R-jX_c)^2 - R^2\right] + R\left[(-R)(2R-jX_c) - h_{fe}R_c(-R)\right] = 0$$

$$(R+R_c-jX_c)(3R^2-X_c^2-j4RX_c) - R\left[2R^2-jRX_c-h_{fe}R_cR\right] = 0$$

$$R^3+R^2R_c(3+h_{fe}) - 5RX_c^2 - R_cX_c^2 - 6jR^2X_c - j4RR_cX_c + jX_c^3 = 0$$

Equating the imaginary component of the above equation to zero we have

$$6R^2X_c + 4RR_cX_c - X_c^3 = 0$$

$$X_c = \sqrt{6R^2+4RR_c}$$

$$2\pi f C = \frac{1}{\sqrt{6R^2+4RR_c}}$$

$$f = \frac{1}{2\pi RC\sqrt{6+\frac{4R_c}{R}}}$$

If $R=R_c$, then $$\boxed{f = \frac{1}{2\pi RC\sqrt{10}}}$$

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Equating the real component to zero, we have

$$R^3 + R^2 R_c(3+h_{fe}) - X_c^2(5R+R_c) = 0$$

$$R^3 + R^2 R_c(3+h_{fe}) - (6R^2 + 4RR_c)(5R+R_c) = 0$$

$$-29R^3 - 23R^2 R_c + h_{fe} R^2 R_c - 4RR_c^2 = 0$$

$$\frac{-29R}{R_c} - 23 + h_{fe} - 4\frac{R_c}{R} = 0$$

$$\boxed{h_{fe} = 23 + 29\frac{R}{R_c} + 4\frac{R_c}{R}}$$

For the loop gain to be greater than unity, the requirement of the current gain of the transistor is found to be

$$h_{fe} > 23 + 29\frac{R}{R_c} + 4\frac{R_c}{R}$$

If $R = R_c$, then $h_{fe} > (23+29+4)$ i.e. > 56

The phase shift oscillator is well suited to the range of frequencies from several hertz to several hundred kilohertz (20Hz to 200KHz), and so includes the audio frequency range (upto 20KHz).

Advantages

1. It is cheap and simple circuit as it contains resistors and capacitors (not bulky and expensive high value inductors).
2. It provides good frequency stability.
3. It is much simpler than the Wein bridge oscillator circuit because it does not need negative feedback and the stabilization arrangements.
4. The output is sinusoidal and is quite distortion free.
5. They have a wide frequency range (Hz to KHz)
6. They are particularly suitable for low frequencies, of the order of 1 KHz.

Disadvantages

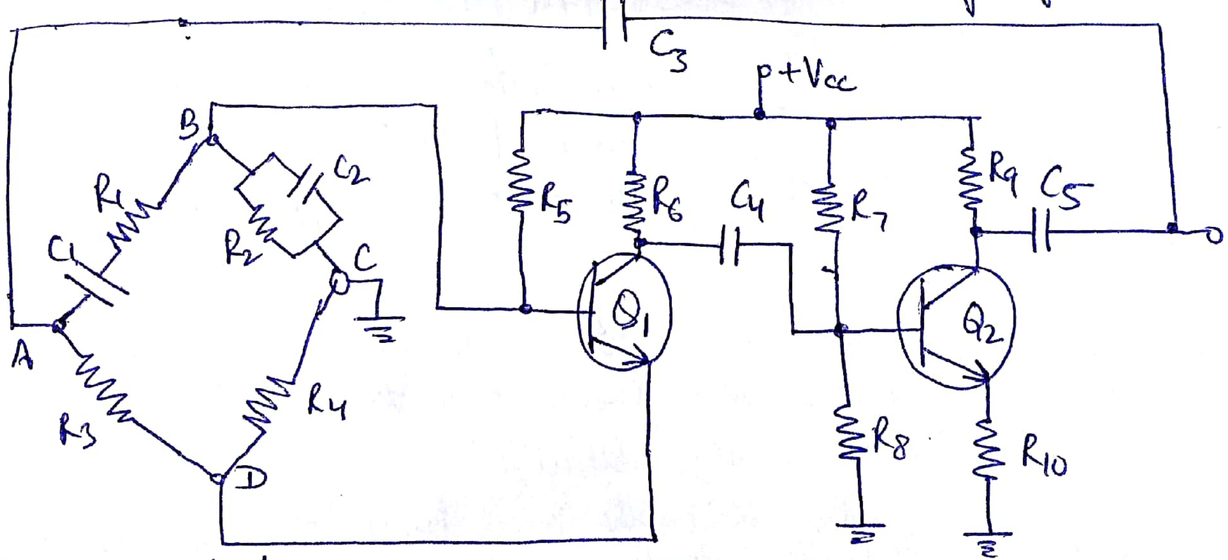
1. The output is small. It is due to smaller feedback.
2. It is difficult for the circuit to start oscillations as the feedback is usually small.

3. The frequency stability is not as good as that of Wein-bridge oscillator.
4. It needs high voltage (12V) battery so as to develop sufficiently large feedback voltage.

② Wien Bridge Oscillator

It is one of the most popular type of oscillators used in audio and sub-audio frequency ranges (20-20KHz). This type of oscillator is simple in design, compact in size, and stable in its frequency output. Its output is relatively free from distortion and its frequency can be varied easily. The maximum frequency output of a typical Wien bridge oscillator is only about 1MHz.

It employs two transistors, each producing a phase shift of 180°; and thus producing a total phase-shift of 360° or 0°.



It is essentially a two stage amplifier with an RC bridge circuit. RC bridge circuit (Wien bridge) is a lead-lag network. The phase-shift across the network lags with increasing frequency and leads with decreasing frequency. By adding Wien bridge feedback network, the oscillator becomes sensitive to a signal of only one particular frequency. This particular frequency is that at which Wien bridge is balanced and for which the phase shift is 0°. If the Wien bridge feedback network is not employed, and the output of transistor Q_2 is feedback to transistor Q_1 for providing regeneration required for producing oscillations, the transistor Q_1 will amplify signals over a wide range of frequencies and thus direct coupling would result in poor frequency stability. Thus by employing Wien-bridge feedback network frequency stability is increased.

In the bridge circuit R_1 in series with C_1, R_3, R_4 and R_2 in parallel with C_2 form the four arms.

The bridge will be balanced only when

$$R_3\left[\frac{R_2}{1+j\omega C_2 R_2}\right] = R_4\left[R_1 - \frac{j}{\omega C_1}\right]$$

$$R_2 R_3 = R_4(1+j\omega C_2 R_2)\left(R_1 - \frac{j}{\omega C_1}\right)$$

$$R_2 R_3 - R_4 R_1 - \frac{C_2}{C_1} R_2 R_4 + \frac{j R_4}{\omega C_1} - j\omega C_2 R_2 R_1 R_4 = 0$$

Separating real and imaginary parts, we have

$$R_2 R_3 - R_4 R_1 - \frac{C_2}{C_1} R_2 R_4 = 0$$

$$\boxed{\frac{C_2}{C_1} = \frac{R_3}{R_4} - \frac{R_1}{R_2}}$$

and $\frac{R_4}{\omega C_1} - \omega C_2 R_2 R_1 R_4 = 0$

$$\omega^2 = \frac{1}{C_1 C_2 R_1 R_2}$$

$$\omega = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}}$$

If $C_1 = C_2 = C$ and $R_1 = R_2 = R$, then

$$\boxed{f = \frac{1}{2\pi RC} \quad \text{and} \quad R_3 = 2R_4}$$

In this arrangement the output of the second stage is supplied back to the feedback network and the voltage across the parallel combination C_2R_2 is fed to the input of the first stage. Transistor Q_1 serves as an oscillator and amplifier whereas the transistor Q_2 as an inverter to cause a phase shift of 180°. The circuit uses positive and negative feedbacks. The positive feedback is through R_1, C_1, R_2, C_2 to transistor Q_1 and negative feedback is through the voltage divider to the input of transistor Q_1. Resistors R_3 and R_4 are used to stabilize the amplitude of the output.

The two transistors Q_1 and Q_2 thus cause a total phase shift of 360° and ensure proper positive feedback. The negative feedback is provided in the circuit to ensure constant output over a range of frequencies.

The amplifier voltage gain, $A = \dfrac{R_3+R_4}{R_3} = \dfrac{V_o}{V_i} = \dfrac{R_3}{R_4}+1 = 3$

$\therefore \beta = \dfrac{1}{3}$ $\qquad \because R_3 = 2R_4$

Operation: The circuit is set in oscillation by any random change in base current of transistor Q_1, that may be due to noise inherent in the transistor or variation in voltage of dc supply. This variation in base current is amplified in collector circuit of transistor Q_1 but with a phase shift of 180°. The output of transistor Q_1 is fed to the base of second transistor Q_2 through capacitor C_4. Now it is still further amplified and twice phase-reversed signal appears at the collector of the transistor Q_2. Now the output signal will be in phase with the signal input to the base of transistor Q_1. A part of the output signal at transistor Q_2 is fedback to the input points of the bridge circuit (points A-C). A part of this feedback signal is applied at emitter resistor R_4 where it produces degenerative effect or negative feedback. Similarly, a part of the feedback signal is applied across the base-bias resistor R_2 where it produces regenerative effect or positive feedback. At the rated frequency, effect of regeneration is made slightly more than that of degeneration so as to obtain sustained oscillations.

Advantages :

1. It provides a stable low distortion sinusoidal output over a wide range of frequency.
2. The frequency of oscillation can be easily varied by varying capacitances C_1 and C_2 simultaneously.
3. The overall gain is high because of two transistors.

Disadvantages :

1. The circuit needs two transistors and a large number of other components.

Ex. Design the R-C elements of a Wien Bridge oscillator for operation at $f = 15\,KHz$.

Sol^n Let $R_1 = R_2 = R$ and $C_1 = C_2 = C$

Let $R = 200\,K\Omega$

$$f = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi f R} = \frac{1}{2\pi \times 15 \times 10^3 \times 200 \times 10^3} = 53\,pF$$

$R_3 = 2R_4$

Let $R_3 = 400\,K\Omega \Rightarrow R_4 = 200\,K\Omega$

Ex. Find the frequency of oscillations of a Wien bridge oscillator with $R = 20\,K\Omega$ and $C = 1{,}000\,pF$.

Sol^n

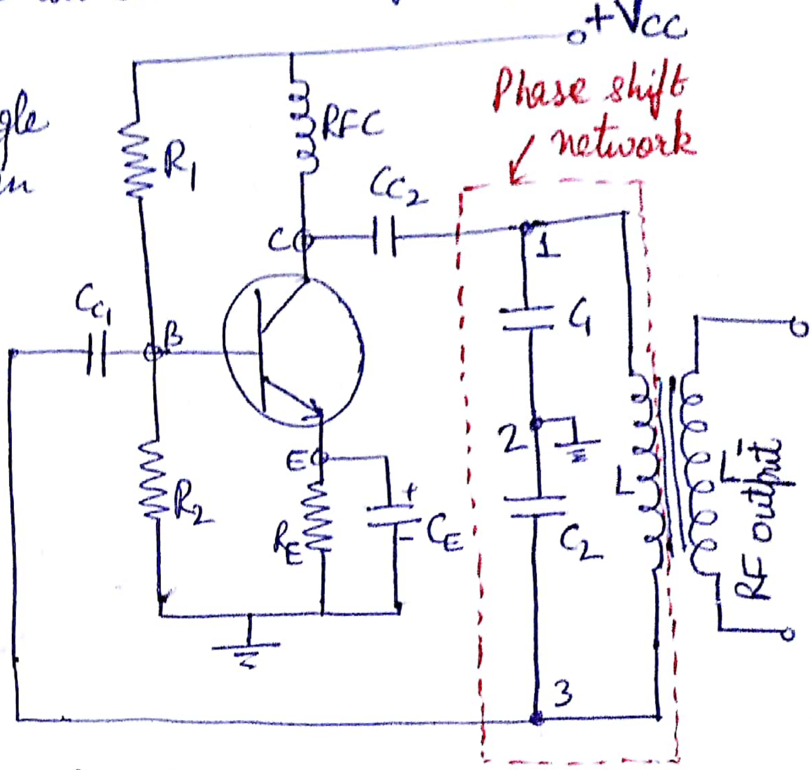
$R = R_1 = R_2 = 20\,K\Omega = 20{,}000\,\Omega$

$C = C_1 = C_2 = 1{,}000\,pF = 1 \times 10^{-9}\,F$

Frequency of oscillation, $f = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi \times 20{,}000 \times 1 \times 10^{-9}} = 7.96\,KHz$

③ Colpitt's Oscillator

This oscillator is widely used in commercial signal generators upto 100 MHz.



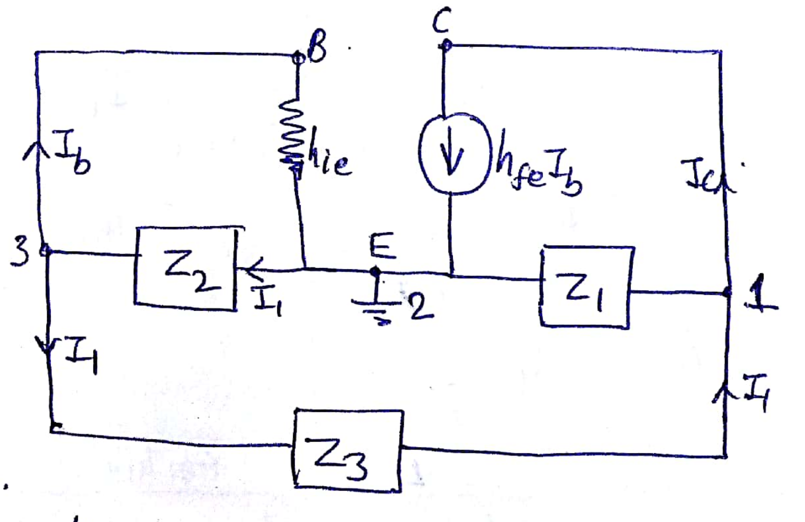
It is basically consists of a single stage inverting amplifier and an L-C phase shift network. The two series capacitors C_1 and C_2 form the potential divider used for providing the feedback voltage — the voltage developed across capacitor C_2 provides the regenerative feedback required for sustained oscillations. Parallel combination of R_E and C_E along with resistors R_1 and R_2 provides the stabilized self bias. The collector supply voltage V_{CC} is applied to the collector through a radio-frequency choke (RFC) which permits an easy flow of direct current but at the same time it offers very high impedence to the high frequency currents. The presence of coupling capacitor C_{C_2} in the output circuit does not permit the dc currents to go to the tank circuit (the flow of dc current in a tank circuit reduces its Q). The radio-frequency energy developed across RFC is capacitively coupled to the tank circuit through the capacitor C_{C_2}. The output of the phase-shift L-C network is coupled from the junction of L and C_2 to the amplifier input at base through coupling capacitor C_{C_1}, which blocks dc but provides path to ac. Transistor itself produces a phase-shift of 180° and another phase shift of 180° is provided by the capacitive feedback. Thus a total phase shift of 360° is obtained which is an essential condition for developing oscillations. The output voltage is derived from a secondary winding L' coupled to the inductance L.

Working: When the collector supply voltage V_{cc} is switched on, the capacitors C_1 and C_2 are charged. These capacitors C_1 and C_2 discharge through the coil L, setting up oscillations of frequency $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1} + \frac{1}{LC_2}}$. The oscillations across C_2 are applied to the base-emitter junction and appear in the amplified form in the collector circuit. This amplified output in the collector circuit is supplied to the tank circuit in order to meet the losses. Thus, the tank circuit is getting continuously energy from the circuit to make up for the losses occurring in it, and, therefore ensure undamped oscillations.

Frequency of Oscillation

General theory for Colpitts and Hartley oscillators:
The equivalent circuit is drawn with the following two assumptions:
(i) h_{re} of transistor is negligibly small and therefore, the feedback source $h_{re}V_{out}$ is negligible.
(ii) h_{oe} of the transistor is very small i.e. the output resistance $\frac{1}{h_{oe}}$ is very large and thus omitted from the equivalent circuit.

Let us determine the load impedence between output terminals 1 and 2. Here Z_2 and h_{ie} are in parallel and their resultant impedence is in series with impedence Z_3. The equivalent impedence is in parallel with impedence Z_1.



Thus load impedence between output terminals is given as

$$Z_L = Z_1 \,||\, [Z_3 + (Z_2 \,||\, h_{ie})]$$

$$= Z_1 \,||\, \left[Z_3 + \frac{Z_2 h_{ie}}{Z_2 + h_{ie}}\right]$$

$$= Z_1 \,||\, \left[\frac{Z_3(Z_2 + h_{ie}) + Z_2 h_{ie}}{Z_2 + h_{ie}}\right]$$

$$= Z_1 \| \left[\frac{h_{ie}(Z_2+Z_3)+Z_2Z_3}{Z_2+h_{ie}}\right]$$

or $$\frac{1}{Z_L} = \frac{1}{Z_1} + \frac{Z_2+h_{ie}}{h_{ie}(Z_2+Z_3)+Z_2Z_3}$$

$$= \frac{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_2Z_3}{Z_1[h_{ie}(Z_2+Z_3)+Z_2Z_3]}$$

or $$Z_L = \frac{Z_1[h_{ie}(Z_2+Z_3)+Z_2Z_3]}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_2Z_3}$$

The voltage gain of a CE amplifier without feedback is

$$A = \frac{-h_{fe}Z_L}{h_{ie}}$$

The output voltage between the terminals 1 and 2 is given as

$$V_{out} = \left[Z_3 + \frac{Z_2 h_{ie}}{Z_2+h_{ie}}\right]I_1 = \left[\frac{h_{ie}(Z_2+Z_3)+Z_2Z_3}{Z_2+h_{ie}}\right]I_1$$

The voltage feedback to the input terminals 2 and 3 is given as

$$V_f = \frac{Z_2 h_{ie}}{Z_2+h_{ie}} I_1$$

$$\therefore \beta = \frac{V_f}{V_{out}} = \frac{Z_2 h_{ie}}{h_{ie}(Z_2+Z_3)+Z_2Z_3}$$

Also, $A\beta = 1$

$$\frac{-h_{fe}Z_L}{h_{ie}} \cdot \frac{Z_2 h_{ie}}{h_{ie}(Z_2+Z_3)+Z_2Z_3} = 1$$

$$\frac{h_{fe}Z_1[h_{ie}(Z_2+Z_3)+Z_2Z_3]}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_2Z_3} \cdot \left[\frac{Z_2}{h_{ie}(Z_2+Z_3)+Z_2Z_3}\right] = -1$$

$$\frac{h_{fe}Z_1Z_2}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_2Z_3} = -1$$

$$h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_2Z_3=-h_{fe}Z_1Z_2$$

$$\boxed{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2(1+h_{fe})+Z_2Z_3=0}$$

This is the general equation for the oscillator.

Colpitt's oscillator

$$Z_1=\frac{1}{j\omega C_1}=\frac{-j}{\omega C_1} \quad ; \quad Z_2=\frac{1}{j\omega C_2}=\frac{-j}{\omega C_2} \quad \text{and} \quad Z_3=j\omega L$$

$$h_{ie}\left[\frac{-j}{\omega C_1}-\frac{j}{\omega C_2}+j\omega L\right]+\left[\frac{-j}{\omega C_1}\cdot\frac{-j}{\omega C_2}\right](1+h_{fe})+\left[\frac{-j}{\omega C_2}\right]j\omega L=0$$

$$-j\,h_{ie}\left[\frac{1}{\omega C_1}+\frac{1}{\omega C_2}-\omega L\right]-\frac{1+h_{fe}}{\omega^2 C_1 C_2}+\frac{L}{C_2}=0$$

Equating the imaginary component to zero,

$$h_{ie}\left[\frac{1}{\omega C_1}+\frac{1}{\omega C_2}-\omega L\right]=0$$

$$\frac{1}{\omega C_1}+\frac{1}{\omega C_2}=\omega L$$

$$\omega^2=\frac{C_1+C_2}{L C_1 C_2} \Rightarrow \omega=\sqrt{\frac{C_1+C_2}{L C_1 C_2}}=\sqrt{\frac{1}{LC_1}+\frac{1}{LC_2}}$$

or $\boxed{f=\frac{1}{2\pi}\sqrt{\frac{1}{LC_1}+\frac{1}{LC_2}}}$

Equating the real component to zero,

$$\frac{1+h_{fe}}{\omega^2 C_1 C_2}=\frac{L}{C_2}$$

$$1+h_{fe}=\omega^2 L C_1=\frac{C_1+C_2}{L C_1 C_2}\times L C_1=\frac{C_1+C_2}{C_2}=1+\frac{C_1}{C_2}$$

$$\boxed{h_{fe}=\frac{C_1}{C_2}}$$

$$A\beta\geq 1 \Rightarrow A\geq\frac{1}{\beta}\geq\frac{1}{h_{fe}} \Rightarrow \boxed{A\geq\frac{C_2}{C_1}}$$

Ex In a transistor Colpitt's oscillator, $L = 100\mu H$, $L_{RFC} = 0.6mH$, $C_1 = 0.001\mu F$, $C_2 = 0.01\mu F$ and $C_c = 10\mu F$. Determine (i) operating frequency (ii) feedback fraction (iii) minimum gain to sustain oscillations

Sol^n^

(i) $$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1} + \frac{1}{LC_2}} = \frac{1}{2\pi}\sqrt{\frac{1}{10^{-4} \times 0.001 \times 10^{-6}} + \frac{1}{10^{-4} \times 0.01 \times 10^{-6}}}$$

$$= 528 \text{ KHz}$$

(ii) $$\beta = h_{fe} = \frac{C_1}{C_2} = \frac{0.001 \times 10^{-6}}{0.01 \times 10^{-6}} = 0.1$$

(iii) $$A_{min} = \frac{1}{\beta} = \frac{1}{0.1} = 10$$

4) Hartley Oscillator

It is widely used as a local oscillator in radio receivers. Hartley oscillator circuit is similar to Colpitts oscillator circuit, except that phase-shift network consists of two inductors L_1 and L_2 and a capacitor C instead of two capacitors and one inductor. The output of the amplifier is applied across inductor L_1 and the voltage across inductor L_2 forms the feedback voltage. The coil L_1 is inductively coupled to coil L_2, the combination functions as an auto-transformer. However, because of direct connection, the junction of L_1 and L_2 cannot be directly grounded. Instead, another capacitor C_1 is used.

There exists mutual inductance between coils L_1 and L_2 because the coils are wound on the same core, their net effective inductance is given by the equation

$$L = L_1 + L_2 + 2M$$

Frequency of Oscillation:

$$h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 (1 + h_{fe}) + Z_2 Z_3 = 0$$

Here $Z_1 = j\omega L_1 + j\omega M$; $Z_2 = j\omega L_2 + j\omega M$; $Z_3 = \frac{1}{j\omega C} = -\frac{j}{\omega C}$

$$h_{ie}\left[(j\omega L_1+j\omega M)+(j\omega L_2+j\omega M)-\frac{j}{\omega C}\right]+(j\omega L_1+j\omega M)(j\omega L_2+j\omega M)(1+h_{fe})+(j\omega L_2+j\omega M)\left(\frac{-j}{\omega C}\right)=0$$

$$j\omega h_{ie}\left[L_1+L_2+2M-\frac{1}{\omega^2 C}\right]-\omega^2(L_2+M)\left[(L_1+M)(1+h_{fe})-\frac{1}{\omega^2 C}\right]=0$$

Equating the imaginary part to zero,

$$h_{ie}\left[L_1+L_2+2M-\frac{1}{\omega^2 C}\right]=0$$

$$L_1+L_2+2M-\frac{1}{\omega^2 C}=0$$

$$\omega=\frac{1}{\sqrt{C(L_1+L_2+2M)}}$$

$$\boxed{f=\frac{1}{2\pi\sqrt{C(L_1+L_2+2M)}}}$$

Equating the real component to zero,

$$(L_1+M)(1+h_{fe})-\frac{1}{\omega^2 C}=0$$

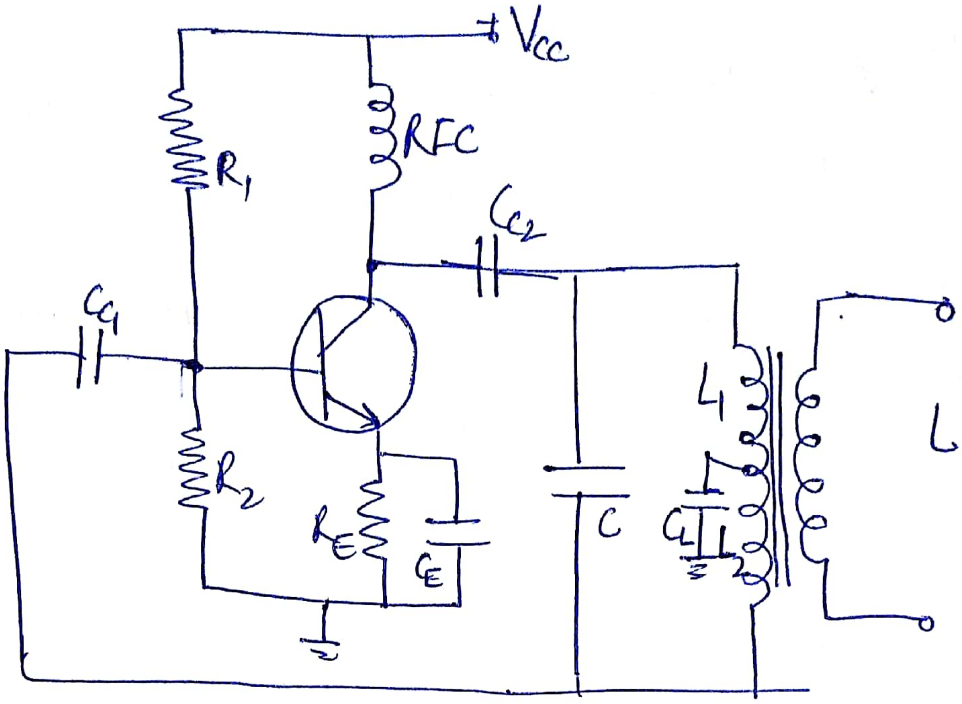
$$1+h_{fe}=\frac{1}{\omega^2 C(L_1+M)}=\frac{L_1+L_2+2M}{L_1+M}=1+\frac{L_2+M}{L_1+M}$$

$$\beta=\boxed{h_{fe}=\frac{L_2+M}{L_1+M}}$$

$$A\beta\geq 1$$

$$A\geq\frac{1}{\beta}\geq\frac{L_1+M}{L_2+M}$$

Hartley oscillator



Ex. In a Hartley oscillator, the tank circuit has the capacitance of 100 pF. The value of inductance between the collector and tapping point is 30 mH and the value of inductance between the tapping point and the transistor base is 1×10^{-8} H. Determine the frequency of oscillations. Neglect the mutual inductance.

Soln

$$L = L_1 + L_2 + 2M = 30 \times 10^{-3} + 1 \times 10^{-8} + 0 \cong 0.03\,H$$

$$C = 100\,pF = 1 \times 10^{-10}\,F$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0.03 \times 10^{-10}}} = 91.9\,kHz.$$

Ex. Determine the oscillation frequency of a transistor Hartley oscillator with circuit values $L_1 = 1\,mH$, $L_2 = 100\,\mu H$, $M = 50\,\mu H$ and $C = 100\,pF$.

Soln

$$L = L_1 + L_2 + 2M = 1 \times 10^{-3} + 100 \times 10^{-6} + 2 \times 50 \times 10^{-6} = 1200\,\mu H$$

$$C = 10^{-10}\,F$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1200 \times 10^{-6} \times 10^{-10}}} = 459\,kHz.$$

⑤ Crystal Oscillators

In crystal oscillators, the usual electrical resonant circuit is replaced by a mechanically vibrating crystal. The crystal (quartz) has a high degree of stability in holding constant at whatever frequency the crystal is originally cut to operate.

A quartz crystal exhibits a very important property known as piezo-electric effect. When a mechanical pressure is applied across the faces of the crystal, a voltage proportional to the applied mechanical pressure appears across the crystal.

Conversely, when a voltage is applied across the crystal surfaces, the crystal is distorted by an amount proportional to the applied voltage. An alternating voltage applied to a crystal causes it to vibrate at its natural frequency.

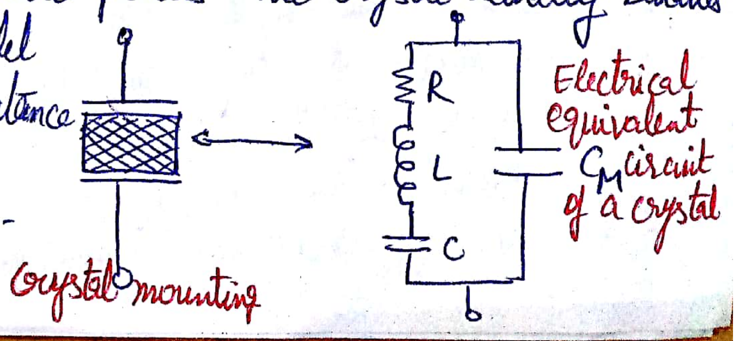
Besides quartz, the other substances that exhibit the piezo-electric effect are Rochelle salt and Tourmaline. Rochelle salt exhibits the greatest piezo-electric effect, but its applications are limited to manufacture of microphones, headsets and loudspeakers. It is because the Rochelle salt is mechanically the weakest and strongly affected by moisture and heat.

Tourmaline is most rugged but shows the least piezo-electric effect.

Quartz is inexpensive and readily available in nature. It is mainly the quartz crystal that is used in radio frequency (RF) oscillators.

For use in electronic oscillators, the crystal is suitably cut and then mounted between two metal plates. The crystal actually behaves as a series RLC circuit in parallel with C_M where C_M is the capacitance of the mounting electrodes.

The equivalent crystal Q is high - typically 20,000.

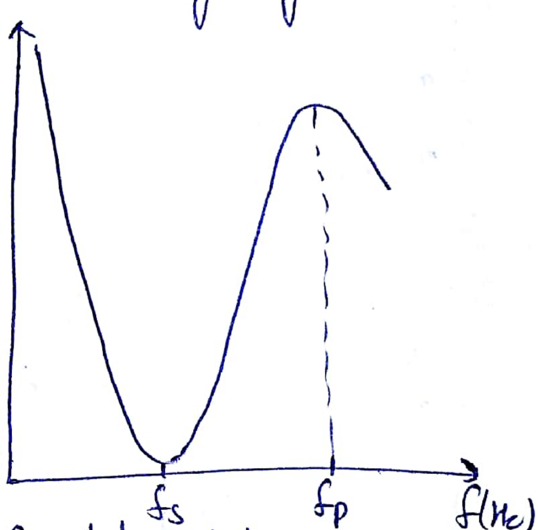


Because of presence of C_M, the crystal has two resonant frequencies. One of these is the series resonant frequency f_s at which $2\pi fL = \frac{1}{2\pi fC}$ and in this case the crystal impedence is very low. The other is parallel resonance frequency f_p which is due to parallel resonance of capacitance C_M and the reactance of the series circuit. In this case crystal impedence is very high.

Series resonant frequency, $f_s = \frac{1}{2\pi\sqrt{LC}}$

Parallel resonant frequency, $f_p = \frac{1}{2\pi}\sqrt{\frac{1+C/C_M}{LC}}$

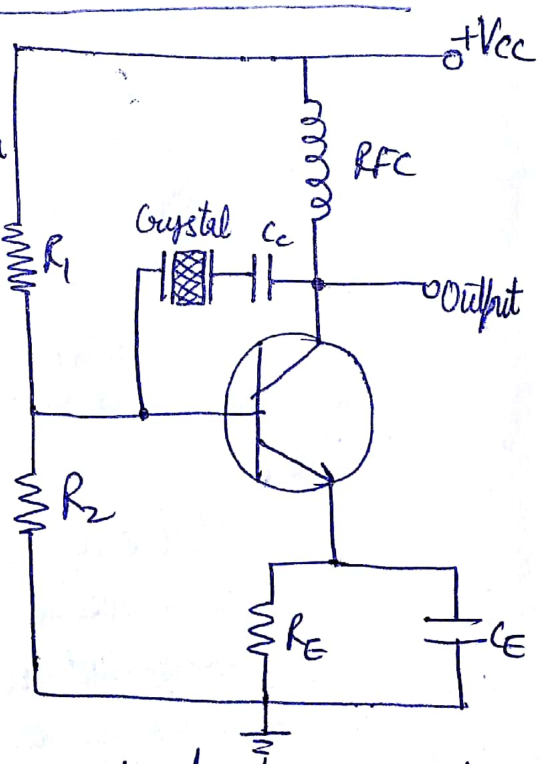
To stabilize the frequency of an oscillator, a crystal may be operated at either its series or parallel resonant frequency.



Crystal impedence vs freq curve

Oscillator with Crystal Operating in Series Resonance

Crystal can be connected as a series element in a feedback path for operation in the series resonant mode. The crystal impedence is the smallest and the amount of positive feedback is the largest.

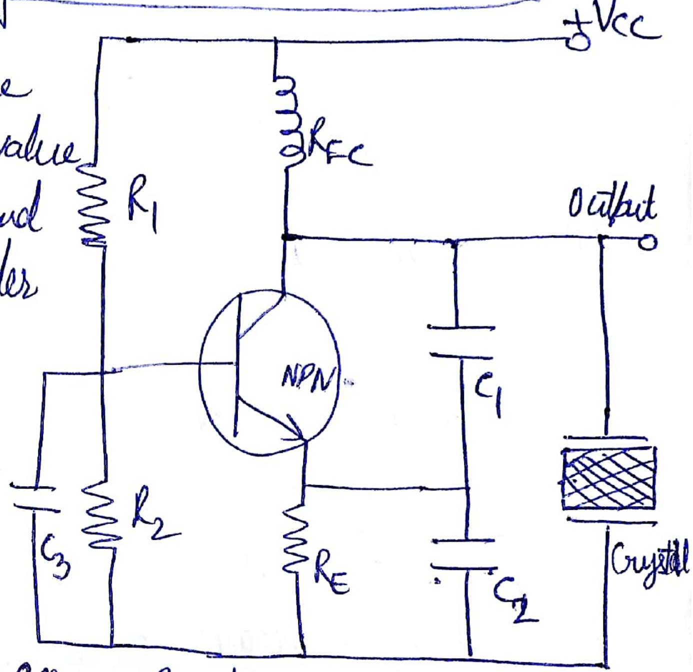


Resistors R_1, R_2 and R_E provide a voltage divider stabilized dc bias circuit, the capacitor C_E provides ac bypass of the emitter resistor R_E and the radio frequency coil (RFC) provides for dc bias and decouples any ac signal from affecting the output signal. The voltage feedback signal from the collector to the base is maximum when the crystal impedence is minimum.

The coupling capacitor C_c has negligible impedence at the circuit operating frequency but blocks any dc between collector and base. The circuit is called the Pierce Crystal. Variations in supply voltage, transistor parameters, etc. have no effect on the circuit operating frequency which is held stabilized by the crystal.

Oscillator with crystal Operating in Parallel Resonance

The parallel-resonant impedence of a crystal is of a maximum value so it is connected in parallel. C_1 and C_2 form a capacitive voltage divider which returns a portion of the output voltage to the transistor emitter. Transistor NPN combined with R_1, R_2, RFC, and R_E, constitutes a common base circuit. Capacitor C_3 provides an ac short circuit across R_2 to ensure that the transistor base remains at a fixed voltage level. As the output voltage increases positively, the emitter voltage also increases, and since the base voltage is fixed, the base emitter voltage is reduced. The reduction in V_{BE} causes collector current I_C to diminish and this in turn causes the collector voltage V_C to increase positively. Thus, the circuit is applying its own input, and a state of oscillation exists. The crystal in parallel with C_1 and C_2 permits maximum voltage feedback from the collector to emitter when its impedence is maximum. The oscillation frequency is stabilized at the parallel resonant frequency of the crystal.



Advantages

1. It is very simple circuit as it does not need any tank circuit other than crystal itself.
2. Different oscillation frequencies can be had by simply replacing one crystal with another.
3. The Q-factor, which is a measure of the quality of resonance circuit of a crystal, is very high. The Q-factor of a crystal may range from 10^4 to 10^6 whereas the LC circuit may have a Q-factor only of the order of 100.
4. Most crystals will maintain frequency drift to within a few cycles at 25°C. For greater frequency stability, the crystal is often contained in an insulated enclosure termed as crystal oven in which the temperature is thermostatically controlled.

Disadvantages

1. Have a very limited tuning range. They are used for frequencies exceeding 100KHz.
2. The crystal oscillators are fragile (easily broken) and therefore can only be used in low power circuits.

Ex The parameters of a crystal oscillator equivalent circuit are $L_s = 0.8H$; $C_s = 0.08\,pF$, $R_s = 5K\Omega$ and $C_p = 1.0\,pF$. Determine the frequencies (resonance) f_s and f_p.

Sol^n^ Series resonance frequency, $f_s = \dfrac{1}{2\pi\sqrt{L_s C_s}} = \dfrac{1}{2\pi\sqrt{0.8 \times 8 \times 10^{-14}}}$

$= 629\,KHz$

Parallel resonance frequency, $f_p = \dfrac{1}{2\pi}\sqrt{\dfrac{1 + C_s/C_p}{L_s C_s}}$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{1 + 8\times10^{-14}/1\times10^{-12}}{0.8 \times 8\times10^{-14}}} = 654\,KHz.$