4 Magnetic Circuits

4.1 INTRODUCTION

Two circuits are said to be coupled circuits when energy transfer takes place from one circuit to the other without having any electrical connection between them. Such coupled circuits are frequently used in network analysis and synthesis. Common examples of coupled circuits are transformer, gyrator, etc. In this chapter, we will discuss self and mutual inductance, magnetically coupled circuits, dot conventions and tuned circuits.

4.2 SELF-INDUCTANCE

Consider a coil of N turns carrying a current i as shown in Fig. 4.1.

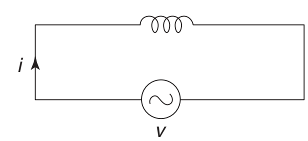


Fig. 4.1 *Coil carrying current*

When current flows through the coil, a flux ϕ is produced in the coil. The flux produced by the coil links with the coil itself. If the current flowing through the coil changes, the flux linking the coil also changes. Hence, an emf is induced in the coil. This is known as self-induced emf. The direction of this emf is given by *Lenz's law*.

We know that

$$\phi \propto i$$

$$\frac{\phi}{i} = k, \text{ a constant}$$

$$\phi = k\, i$$

Hence, rate of change of flux = $k \times$ rate of change of current

$$\frac{d\phi}{dt} = k\frac{di}{dt}$$

According to Faraday's laws of electromagnetic induction, a self-induced emf can be expressed as

$$v = -N\frac{d\phi}{dt} = -Nk\frac{di}{dt} = -N\frac{\phi}{i}\frac{di}{dt} = -L\frac{di}{dt}$$

where $L = \dfrac{N\phi}{i}$ and is called coefficient of self-inductance.

The property of a coil that opposes any change in the current flowing through it is called self-inductance or inductance of the coil. If the current in the coil is increasing, the self-induced emf is set up in such a direction so as to oppose the rise in current, i.e., the direction of self-induced emf is opposite to that of the applied voltage. Similarly, if the current in the coil is decreasing, the self-induced emf will be in the same direction as the applied voltage. Self-inductance does not prevent the current from changing, it serves only to delay the change.

4.3 MUTUAL INDUCTANCE

If the flux produced by one coil links with the other coil, placed closed to the first coil, an emf is induced in the second coil due to change in the flux produced by the first coil. This is known as mutually induced emf.

Consider two coils 1 and 2 placed adjacent to each other as shown in Fig. 4.2. Let Coil 1 has N_1 turns while Coil 2 has N_2 turns.

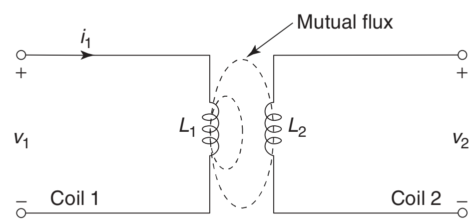


Fig. 4.2 *Two adjacent coils*

If a current i_1 flows in Coil 1, flux is produced and a part of this flux links Coil 2. The emf induced in Coil 2 is called mutually induced emf.

We know that

$$\phi_2 \propto i_1$$

$$\frac{\phi_2}{i_1} = k, \text{ a constant}$$

$$\phi_2 = k\, i_1$$

Hence, rate of change of flux = $k \times$ rate of change of current i_1

$$\frac{d\phi_2}{dt} = k\frac{di_1}{dt}$$

According to Faraday's law of electromagnetic induction, the induced emf is expressed as

$$v_2 = -N_2\frac{d\phi_2}{dt} = -N_2\, k\frac{di_1}{dt} = -N_2\frac{\phi_2}{i_1}\frac{di_1}{dt} = -M\frac{di_1}{dt}$$

where $M = \dfrac{N_2\,\phi_2}{i_1}$ and is called *coefficient of mutual inductance*.

4.4 COEFFICIENT OF COUPLING (k)

The coefficient of coupling (k) between coils is defined as fraction of magnetic flux produced by the current in one coil that links the other.

Consider two coils having number of turns N_1 and N_2 respectively. When a current i_1 is flowing in Coil 1 and is changing, an emf is induced in Coil 2.

$$M = \frac{N_2\,\phi_2}{i_1}$$

Let
$$k_1 = \frac{\phi_2}{\phi_1}$$
$$\phi_2 = k_1\ \phi_1$$
$$M = \frac{N_2\ k_1\ \phi_1}{i_1} \quad \ldots(4.1)$$

If the current i_2 is flowing in Coil 2 and is changing, an emf is induced in Coil 1,
$$M = \frac{N_1\ \phi_1}{i_2}$$

Let
$$k_2 = \frac{\phi_1}{\phi_2}$$
$$\phi_1 = k_2\ \phi_2$$
$$M = \frac{N_1\ k_2\ \phi_2}{i_2} \quad \ldots(4.2)$$

Multiplying Eqs (4.1) and (4.2),
$$M^2 = k_1 k_2 \times \frac{N_1\phi_1}{i_1} \times \frac{N_2\phi_2}{i_2} = k^2 L_1 L_2$$
$$M = k\sqrt{L_1 L_2}$$

where
$$k = \sqrt{k_1 k_2}$$

4.5 INDUCTANCES IN SERIES

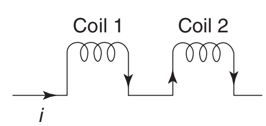


Fig. 4.3 *Cumulative coupling*

1. ***Cumulative Coupling*** Figure 4.3 shows two coils 1 and 2 connected in series, so that currents through the two coils are in the same direction in order to produce flux in the same direction. Such a connection of two coils is known as *cumulative coupling*.

Let L_1 = coefficient of self-inductance of Coil 1

L_2 = coefficient of self-inductance of Coil 2

M = coefficient of mutual inductance

If the current in the coil increases by *di* amperes in *dt* seconds then

Self-induced emf in Coil 1 $= -L_1 \dfrac{di}{dt}$

Self-induced emf in Coil 2 $= -L_2 \dfrac{di}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= -M\dfrac{di}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= -M\dfrac{di}{dt}$

Total induced emf
$$v = -(L_1 + L_2 + 2M)\frac{di}{dt} \quad \ldots(4.3)$$

If L is the equivalent inductance then total induced emf in that single coil would have been

$$v = -L\frac{di}{dt} \qquad \ldots(4.4)$$

Equating Eqs (4.3) and (4.4),

$$L = L_1 + L_2 + 2M$$

2. *Differential Coupling* Figure 4.4 shows the coils connected in series but the direction of current in Coil 2 is now opposite to that in 1. Such a connection of two coils is known as *differential coupling*.

Hence, total induced emf in coils 1 and 2.

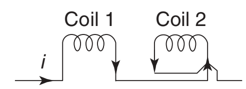


Fig. 4.4 *Differential coupling*

$$v = -L_1\frac{di}{dt} - L_2\frac{di}{dt} + 2M\frac{di}{dt} = -(L_1 + L_2 - 2M)\frac{di}{dt}$$

Coils 1 and 2 connected in series can be considered as a single coil with equivalent inductance L. The induced emf in the equivalent single coil with same rate of change of current is given by,

$$v = -L\frac{di}{dt}$$

$$-L\frac{di}{dt} = -(L_1 + L_2 - 2M)\frac{di}{dt}$$

$$L = L_1 + L_2 - 2M$$

4.6 INDUCTANCES IN PARALLEL

1. *Cumulative Coupling* Figure 4.5 shows two coils 1 and 2 connected in parallel such that fluxes produced by the coils act in the same direction. Such a connection of two coils is known as cumulative coupling.

Let L_1 = coefficient of self-inductance of Coil 1

L_2 = coefficient of self-inductance of Coil 2

M = coefficient of mutual inductance

If the current in the coils changes by di amperes in dt seconds then

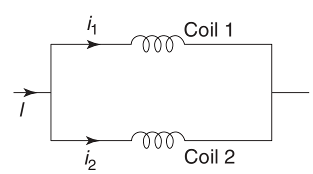


Fig. 4.5 *Cumulative coupling*

Self-induced emf in Coil 1 $= -L_1\dfrac{di_1}{dt}$

Self-induced emf in Coil 2 $= -L_2\dfrac{di_2}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= -M\dfrac{di_2}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= -M\dfrac{di_1}{dt}$

Total induced emf in Coil 1 $= -L_1\dfrac{di_1}{dt} - M\dfrac{di_2}{dt}$

Total induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt} - M\dfrac{di_1}{dt}$

As both the coils are connected in parallel, the emf induced in both the coils must be equal.

$$-L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} = -L_2 \frac{di_2}{dt} - M\frac{di_1}{dt}$$

$$L_1 \frac{di_1}{dt} - M\frac{di_1}{dt} = L_2 \frac{di_2}{dt} - M\frac{di_2}{dt}$$

$$(L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \qquad \text{...(4.5)}$$

Now,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 - M}{L_1 - M} + 1\right)\frac{di_2}{dt}$$

$$= \left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)\frac{di_2}{dt} \qquad \text{...(4.6)}$$

If L is the equivalent inductance of the parallel combination then the induced emf is given by

$$v = -L\frac{di}{dt}$$

Since induced emf in parallel combination is same as induced emf in any one coil,

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right)$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt} \qquad \text{...(4.7)}$$

Substituting Eq. (4.6) in Eq. (4.7),

$$\left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt}$$

$$L = \frac{L_1\left(\dfrac{L_2 - M}{L_1 - M}\right) + M}{\dfrac{L_1 + L_2 - 2M}{L_1 - M}}$$

$$= \frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 + L_2 - 2M}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

2. *Differential Coupling* Figure 4.6 shows two coils 1 and 2 connected in parallel such that fluxes produced by the coils act in the opposite direction. Such a connection of two coils is known as differential coupling.

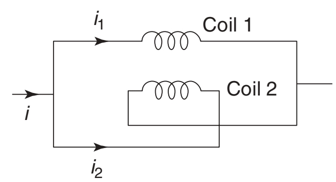


Fig. 4.6 *Differential coupling*

Self-induced emf in Coil 1 $= -L_1 \dfrac{di_1}{dt}$

Self-induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= M \dfrac{di_2}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= M \dfrac{di_1}{dt}$

Total induced emf in Coil 1 $= -L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

Total induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt}$

As both the coils are connected in parallel, the emf induced in the coils must be equal.

$$-L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$L_1 \frac{di_1}{dt} + M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_2}{dt}$$

$$(L_1 + M)\frac{di_1}{dt} = (L_2 + M)\frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 + M}{L_1 + M}\right)\frac{di_2}{dt} \qquad ...(4.8)$$

Now,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 + M}{L_1 + M}\right)\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 + M}{L_1 + M} + 1\right)\frac{di_2}{dt}$$

$$= \left(\frac{L_1 + L_2 + 2M}{L_1 + M}\right)\frac{di_2}{dt} \qquad ...(4.9)$$

If L is the equivalent inductance of the parallel combination then the induced emf is given by

$$v = -L\frac{di}{dt}$$

Since induced emf in parallel combination is same as induced emf in any one coil,

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} - M\frac{di_2}{dt}\right)$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right)\frac{di_2}{dt} - M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right) - M\right]\frac{di_2}{dt} \qquad \text{...(4.10)}$$

Substituting Eq. (4.9) in Eq. (4.10),

$$\left(\frac{L_1+L_2+2M}{L_1+M}\right)\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right) - M\right]\frac{di_2}{dt}$$

$$L = \frac{L_1\left(\frac{L_2+M}{L_1+M}\right) - M}{\frac{L_1+L_2+2M}{L_1+M}}$$

$$= \frac{L_1L_2 + L_1M - L_1M - M^2}{L_1+L_2+2M}$$

$$= \frac{L_1L_2 - M^2}{L_1+L_2+2M}$$

Example 4.1 *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on relative directions of currents in the two coils. If one of the coils has a self-inductance of 0.2 H, find (a) mutual inductance, and (b) coefficient of coupling.*

Solution

$$L_1 = 0.2\,\text{H}, \quad L_{\text{diff}} = 0.1\text{ H}, \quad L_{\text{cum}} = 0.6\text{ H}$$

(a) Mutual inductance

$$L_{\text{cum}} = L_1 + L_2 + 2M = 0.6 \qquad \text{...(i)}$$

$$L_{\text{diff}} = L_1 + L_2 - 2M = 0.1 \qquad \text{...(ii)}$$

Adding Eqs (i) and (ii),

$$2(L_1 + L_2) = 0.7$$

$$L_1 + L_2 = 0.35$$

$$L_2 = 0.35 - 0.2 = 0.15\text{ H}$$

Subtracting Eqs (ii) from Eqs (i),

$$4M = 0.5$$
$$M = 0.125 \text{ H}$$

(b) Coefficient of coupling

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.125}{\sqrt{0.2 \times 0.15}} = 0.72$$

Example 4.2 *Two coils with a coefficient of coupling of 0.6 between them are connected in series so as to magnetise in (a) same direction, and (b) opposite direction. The total inductance in the same direction is 1.5 H and in the opposite direction is 0.5 H. Find the self-inductance of the coils.*

Solution

$$k = 0.6, \quad L_{\text{diff}} = 0.5 \text{ H}, \quad L_{\text{cum}} = 1.5 \text{ H}$$

$$L_{\text{diff}} = L_1 + L_2 - 2M = 0.5 \qquad \text{...(i)}$$
$$L_{\text{cum}} = L_1 + L_2 + 2M = 1.5 \qquad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii),

$$4M = 1$$
$$M = 0.25 \text{ H}$$

Adding Eq. (i) and (ii),

$$2(L_1 + L_2) = 2$$
$$L_1 + L_2 = 1 \qquad \text{...(iii)}$$
$$k = \frac{M}{\sqrt{L_1 L_2}}$$
$$0.6 = \frac{0.25}{\sqrt{L_1 L_2}}$$
$$L_1 L_2 = 0.1736 \qquad \text{...(iv)}$$

Solving Eqs (iii) and (iv),

$$L_1 = 0.22 \text{ H}$$
$$L_2 = 0.78 \text{ H}$$

Example 4.3 *Two coils having self-inductances of 4 mH and 7 mH respectively are connected in parallel. If the mutual inductance between them is 5 mH, find the equivalent inductance.*

Solution

$$L_1 = 4 \text{ mH}, \quad L_2 = 7 \text{ mH}, \quad M = 5 \text{ mH}$$

For cumulative coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4 \times 7 - (5)^2}{4 + 7 - 2(5)} = 3 \text{ mH}$$

For differential coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{4 \times 7 - (5)^2}{4 + 7 + 2(5)} = 0.143 \text{ mH}$$

Example 4.4 *Two inductors are connected in parallel. Their equivalent inductance when the mutual inductance aids the self-inductance is 6 mH and it is 2 mH when the mutual inductance opposes the self-inductance. If the ratio of the self- inductances is 1:3 and the mutual inductance between the coils is 4 mH, find the self-inductances.*

Solution

$$L_{\text{cum}} = 6 \text{ mH}, \quad L_{\text{diff}} = 2 \text{ mH}, \quad \frac{L_1}{L_2} = 1.3, \quad M = 4 \text{ mH}$$

For cumulative coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$6 = \frac{L_1 L_2 - (4)^2}{L_1 + L_2 - 2(4)}$$

$$6 = \frac{L_1 L_2 - 16}{L_1 + L_2 - 8} \qquad \text{...(i)}$$

For differential coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$2 = \frac{L_1 L_2 - (4)^2}{L_1 + L_2 + 8}$$

$$2 = \frac{L_1 L_2 - 16}{L_1 + L_2 + 8} \qquad \text{...(ii)}$$

From Eqs (i) and (ii),

$$2(L_1 + L_2 + 8) = 6(L_1 + L_2 - 8)$$

$$L_1 + L_2 + 8 = 3L_1 + 3L_2 - 24$$

$$L_1 + L_2 = 16$$

But

$$\frac{L_1}{L_2} = 1.3$$

$$1.3\ L_2 + L_2 = 16$$

$$2.3\ L_2 = 16$$

$$L_2 = 6.95 \text{ mH}$$

$$L_1 = 1.3\ L_2 = 9.035 \text{ mH}$$

4.7 DOT CONVENTION

Consider two coils of inductances L_1 and L_2 respectively connected in series as shown in Fig. 4.7. Each coil will contribute the same mutual flux (since it is in a series connection, the same current flows through L_1 and L_2) and hence, same mutual inductance (M). If the mutual fluxes of the two coils aid each other as shown in Fig 4.7 (a), the inductances of each coil will be increased by M, i.e., the inductance of coils will become $(L_1 + M)$ and $(L_2 + M)$. If the mutual fluxes oppose each other as shown in Fig. 4.7 (b), inductance of the coils will become $(L_1 - M)$ and $(L_2 - M)$. Whether the two mutual fluxes aid to each other or oppose will depend upon the manner in which coils are wound. The method described above is very inconvenient because we have to include the pictures of the coils in the circuit. There is another simple method of defining the directions of currents in the coils. This is known as dot convention.

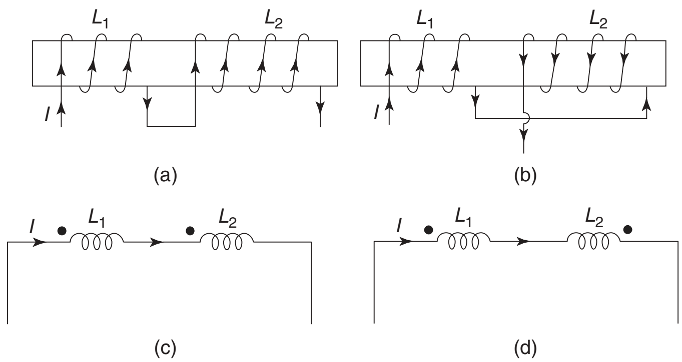


Fig. 4.7 *Dot convention*

Figure 4.7 shows the schematic connection of the two coils. It is not possible to state from Fig. 4.7(a) and Fig. 4.7(b) whether the mutual fluxes are additive or in opposition. However dot convention removes this confusion.

If the current enters from both the dotted ends of Coil 1 and Coil 2, the mutual fluxes of the two coils aid each other as shown in Fig. 4.7(c). If the current enters from the dotted end of Coil 1 and leaves from the dotted end of Coil 2, the mutual fluxes of the two coils oppose each other as shown in Fig. 4.7(d).

When two mutual fluxes aid each other, the mutual inductance is positive and polarity of the mutually induced emf is same as that of the self-induced emf. When two mutual fluxes oppose each other, the mutual inductance is negative and polarity of the mutually induced emf is opposite to that of the self-induced emf.

Example 4.5 *Obtain the dotted equivalent circuit for Fig. 4.8 shown below.*

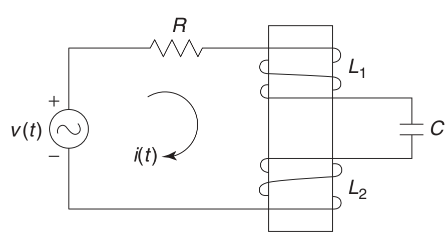


Fig. 4.8

Solution The current in the two coils is shown in Fig. 4.9. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

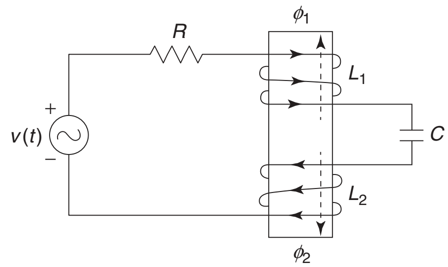


Fig. 4.9

From Fig. 4.9, it is seen that, the flux ϕ_1 is in upward direction in Coil 1, and flux ϕ_2 is in downward direction in Coil 2. Hence, fluxes are opposing each other. The mutual inductances are negative and mutually induced emfs have opposite polarities as that of self-induced emf. The dots are placed in two coils to illustrate these conditions. Hence, current $i(t)$ enters from the dotted end in Coil 1 and leaves from the dotted end in Coil 2.

The dotted equivalent circuit is shown in Fig. 4.10.

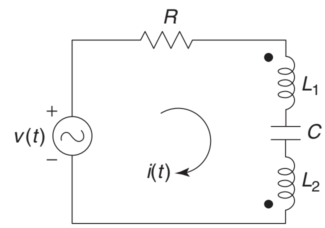


Fig. 4.10

Example 4.6 *Obtain the dotted equivalent circuit for the circuit of Fig. 4.11.*

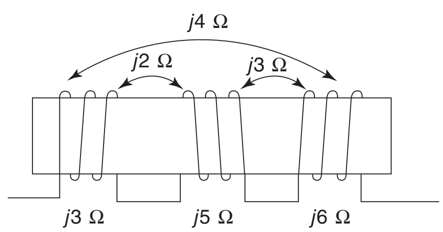


Fig. 4.11

Solution The current in the three coils is shown in Fig. 4.12. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

From Fig. 4.12, it is seen that the flux is towards the left in Coil 1, towards the right in Coil 2 and towards the left in Coil 3. Hence, fluxes ϕ_1 and ϕ_2 oppose each other in coils 1 and 2, fluxes ϕ_2 and ϕ_3 oppose each other in coils 2 and 3, and fluxes ϕ_1 and ϕ_3 aid each other in coils 1 and 3. The dots are placed in three coils to illustrate these conditions. Hence, current enters from the dotted end in Coil 1, leaves from the dotted end in Coil 2 and enters from the dotted end in Coil 3.

The dotted equivalent circuit is shown in Fig. 4.13.

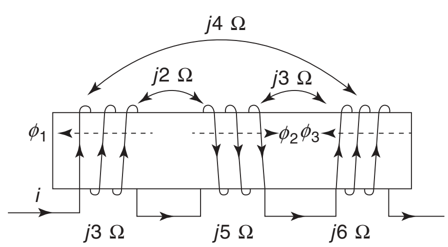


Fig. 4.12

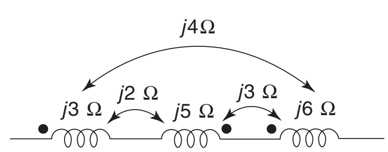


Fig. 4.13

Example 4.7 *Obtain the dotted equivalent circuit for the circuit shown in Fig. 4.14.*

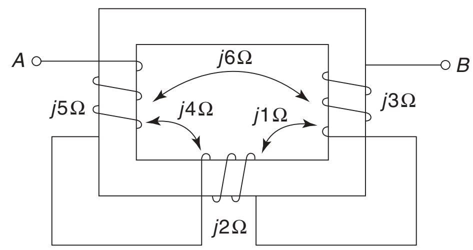


Fig. 4.14

Solution The current in the three coils is shown in Fig. 4.15. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

From Fig. 4.15, it is seen that all the three fluxes ϕ_1, ϕ_2, ϕ_3 aid each other. Hence, all the mutual reactances are positive and mutually induced emfs have same polarities as that of self-induced emfs. The dots are placed in three coils to illustrate these conditions. Hence, currents enter from the dotted end in each of the three coils. The dotted equivalent circuit is shown in Fig. 4.16.

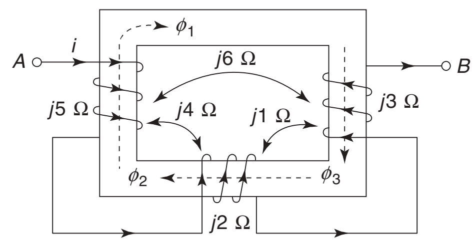


Fig. 4.15

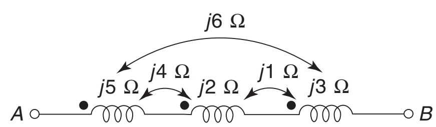


Fig. 4.16

Example 4.8 *Obtain the dotted equivalent circuit for the coupled circuit of Fig. 4.17.*

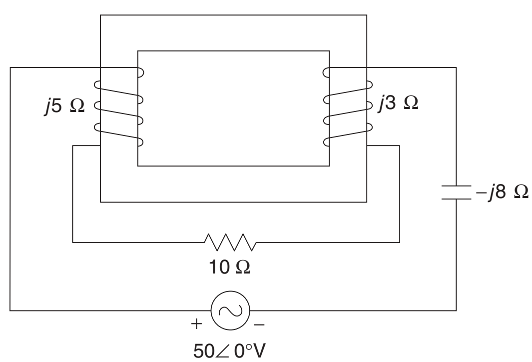


Fig. 4.17

Solution The current in the two coils is shown in Fig. 4.18 . The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

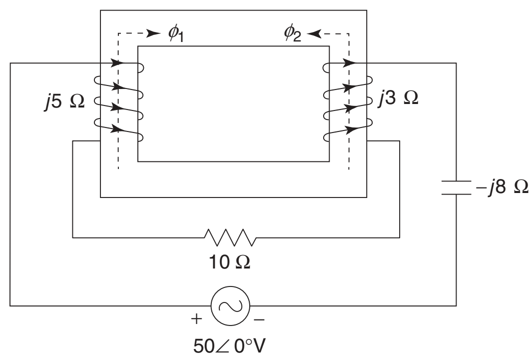


Fig. 4.18

From Fig. 4.18, it is seen that the flux ϕ_1 is in clockwise direction in Coil 1 and in anti-clockwise direction in Coil 2. Hence, fluxes are opposing each other. The dots are placed in two coils to illustrate these conditions. Hence, current enters from the dotted end in Coil 1 and leaves from the dotted end in Coil 2. The dotted equivalent circuit is shown in Fig. 4.19.

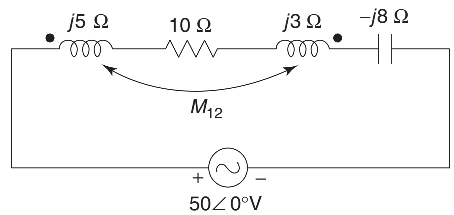


Fig. 4.19

Example 4.9 *Find the equivalent inductance of the network shown in Fig. 4.20.*

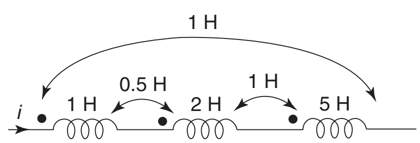


Fig. 4.20

Solution

$$\begin{aligned} L &= (L_1 + M_{12} + M_{13}) + (L_2 + M_{23} + M_{21}) + (L_3 + M_{31} + M_{32}) \\ &= (1 + 0.5 + 1) + (2 + 1 + 0.5) + (5 + 1 + 1) \\ &= 13\text{ H} \end{aligned}$$

Example 4.10 *Find the equivalent inductance of the network shown in Fig. 4.21.*

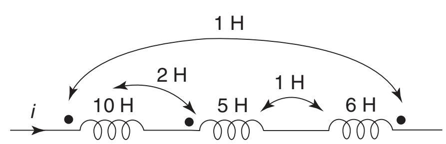


Fig. 4.21

Solution

$$\begin{aligned} L &= (L_1 + M_{12} - M_{13}) + (L_2 - M_{23} + M_{21}) + (L_3 - M_{31} - M_{23}) \\ &= (10 + 2 - 1) + (5 - 1 + 2) + (6 - 1 - 1) = 21\text{ H} \end{aligned}$$

Example 4.11 *Find the equivalent inductance of the network shown in Fig. 4.22.*

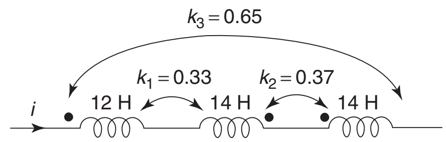


Fig. 4.22

Solution

$$M_{12} = M_{21} = k_1\sqrt{L_1 L_2} = 0.33\sqrt{(12)(14)} = 4.28 \text{ H}$$
$$M_{23} = M_{32} = k_2\sqrt{L_2 L_3} = 0.37\sqrt{(14)(14)} = 5.18 \text{ H}$$
$$M_{31} = M_{13} = k_3\sqrt{L_3 L_1} = 0.65\sqrt{(12)(14)} = 8.42 \text{ H}$$
$$\begin{aligned} L &= (L_1 - M_{12} + M_{13}) + (L_2 - M_{23} - M_{21}) + (L_3 + M_{31} - M_{32}) \\ &= (12 - 4.28 + 8.42) + (14 - 5.18 - 4.28) + (14 + 8.42 - 5.18) \\ &= 37.92 \text{ H} \end{aligned}$$

Example 4.12 *Find the equivalent inductance of the network shown in Fig. 4.23.*

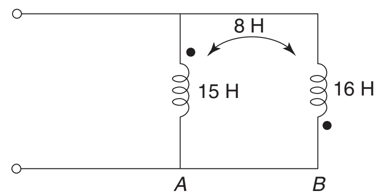


Fig. 4.23

Solution For Coil A,

$$L_A = L_1 - M_{12} = 15 - 8 = 7 \text{ H}$$

For Coil B,

$$L_B = L_2 - M_{12} = 16 - 8 = 8 \text{ H}$$
$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} = \frac{1}{7} + \frac{1}{8} = \frac{15}{56}$$
$$L = \frac{56}{15} = 3.73 \text{ H}$$

Example 4.13 *Find the equivalent inductance of the network shown in Fig. 4.24.*

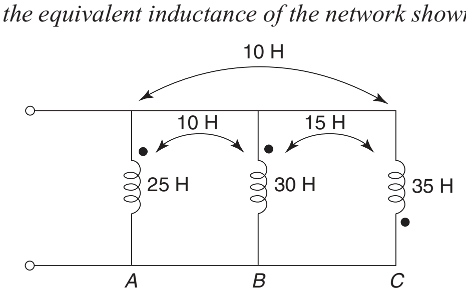


Fig. 4.24

Solution For Coil A,

$$L_A = L_1 + M_{12} - M_{13} = 25 + 10 - 10 = 25 \text{ H}$$

For Coil B,

$$L_B = L_2 - M_{23} + M_{21} = 35 - 15 + 10 = 25 \text{ H}$$

For Coil C,

$$L_C = L_3 - M_{32} - M_{31} = 35 - 15 - 10 = 10 \text{ H}$$

$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} + \frac{1}{L_C} = \frac{1}{25} + \frac{1}{25} + \frac{1}{10} = \frac{9}{50}$$

$$L = \frac{50}{9} = 5.55 \text{ H}$$

Example 4.14 *Find the equivalent impedance across the terminals A and B in Fig. 4.25.*

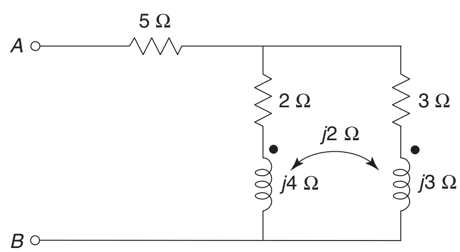


Fig. 4.25

Solution

$$\mathbf{Z}_1 = 5\,\Omega, \quad \mathbf{Z}_2 = (2 + j4)\,\Omega, \quad \mathbf{Z}_3 = (3 + j3)\,\Omega, \quad \mathbf{Z}_M = j2\,\Omega$$

$$\mathbf{Z} = \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3 - \mathbf{Z}_M^2}{\mathbf{Z}_2 + \mathbf{Z}_3 - 2\mathbf{Z}_M} = 5 + \frac{(2 + j4)(3 + j3) - (j2)^2}{2 + j4 + 3 + j3 - 2(j2)} = 6.9 \angle 24.16° \; \Omega$$

4.8 COUPLED CIRCUITS

Consider two coils located physically close to one another as shown in Fig. 4.26.

When current i_1 flows in the first coil and $i_2 = 0$ in the second coil, flux ϕ_1 is produced in the coil. A fraction of this flux also links the second coil and induces a voltage in this coil. The voltage v_1 induced in the first coil is

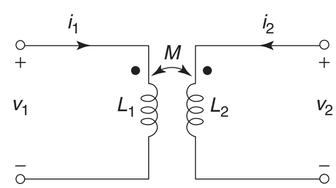


Fig. 4.26 *Coupled circuit*

$$v_1 = L_1 \left.\frac{di_1}{dt}\right|_{i_2 = 0}$$

The voltage v_2 induced in the second coil is

$$v_2 = M \left.\frac{di_1}{dt}\right|_{i_2 = 0}$$

The polarity of the voltage induced in the second coil depends on the way the coils are wound and it is usually indicated by dots. The dots signify that the induced voltages in the two coils (due to single current) have the same polarities at the dotted ends of the coils. Thus, due to i_1, the induced voltage v_1 must be positive at the dotted end of Coil 1. The voltage v_2 is also positive at the dotted end in Coil 2.

The same reasoning applies if a current i_2 flows in Coil 2 and $i_1 = 0$ in Coil 1. The induced voltages v_2 and v_1 are

$$v_2 = L_2 \frac{di_2}{dt}\bigg|_{i_1 = 0}$$

and

$$v_1 = M \frac{di_2}{dt}\bigg|_{i_1 = 0}$$

The polarities of v_1 and v_2 follow the dot convention. The voltage polarity is positive at the doted end of inductor L_2 when the current direction for i_2 is as shown in Fig. 4.26. Therefore, the voltage induced in Coil 1 must be positive at the dotted end also.

Now if both currents i_1 and i_2 are present, by using superposition principle, we can write

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

This can be represented in terms of dependent sources, as shown in Fig. 4.27.

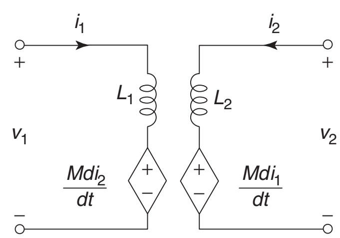


Fig. 4.27 *Equivalent circuit*

Now consider the case when the dots are placed at the opposite ends in the two coils, as shown in Fig. 4.28.

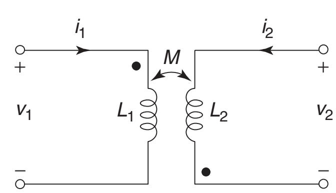


Fig. 4.28 *Coupled circuit*

Due to i_1, with $i_2 = 0$, the dotted end in Coil 1 is positive, so the induced voltage in Coil 2 is positive at the dot, which is the reverse of the designated polarity for v_2. Similarly, due to i_2, with $i_1 = 0$, the dotted ends have negative polarities for the induced voltages. The mutually induced voltages in both cases have polarities that are the reverse of terminal voltages and the equations are

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

This can be repressed in terms of dependent sources as shown in Fig. 4.29.

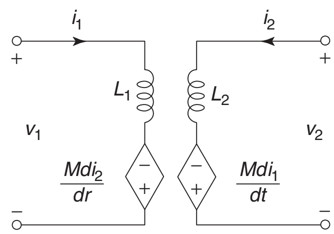


Fig. 4.29 *Equivalent circuit*

The various cases are summarised in the table shown in Fig. 4.30.

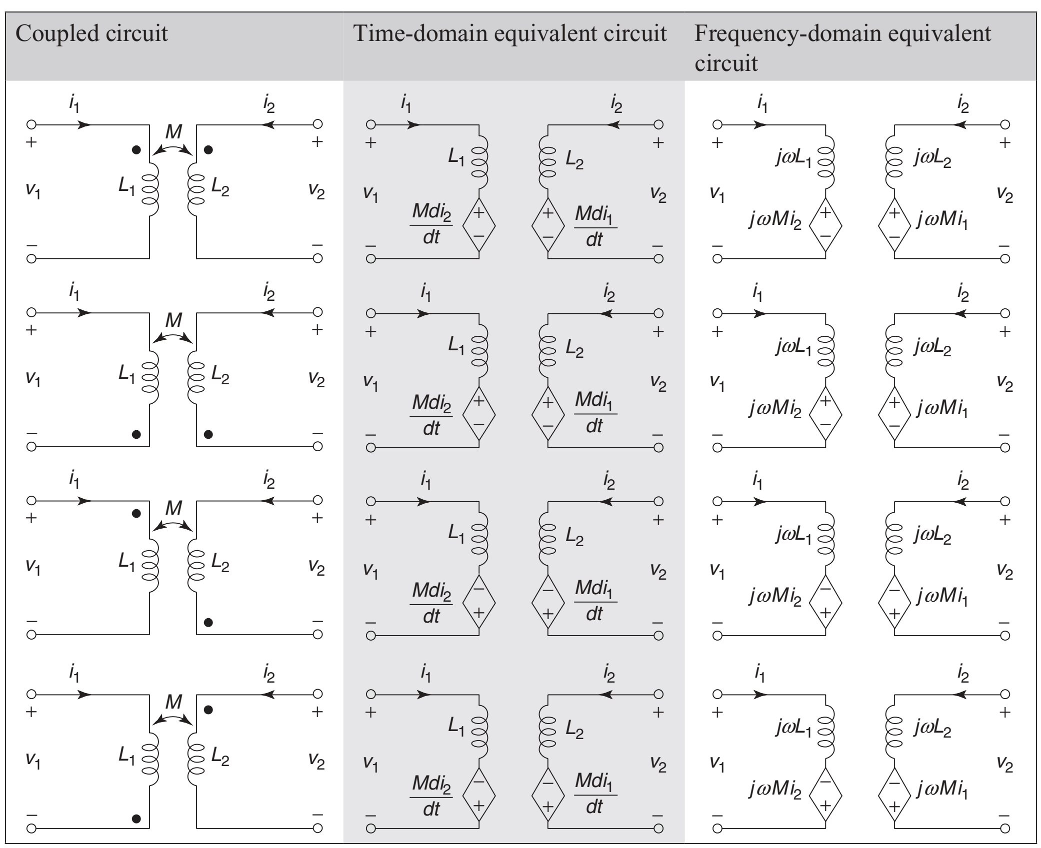


Fig. 4.30 *Coupled circuits for various cases*

Example 4.15 *Write mesh equations for the network shown in Fig. 4.31.*

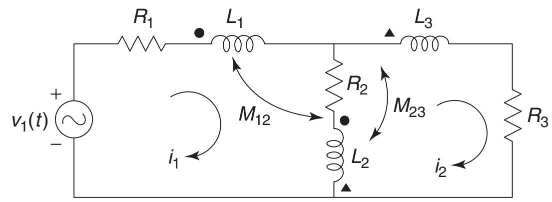


Fig. 4.31

Solution Coil 1 is magnetically coupled to Coil 2. Similarly, Coil 2 is magnetically coupled with Coil 1 and Coil 3. By applying dot convention, the equivalent circuit is drawn with the dependent sources.

The equivalent circuit in terms of dependent sources is shown in Fig. 4.32.

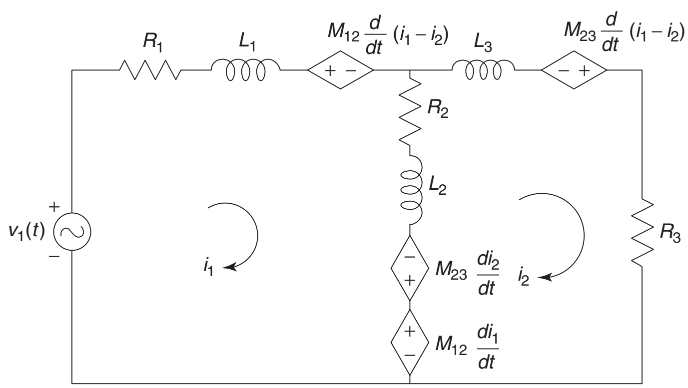


Fig. 4.32

(a) In Coil 1, there is a mutually induced emf due to current $(i_1 - i_2)$ in Coil 2. The polarity of the mutually induced emf is same as that of self-induced emf because currents i_1 and $(i_1 - i_2)$ enter in respective coils from the dotted ends.

(b) In Coil 2, there are two mutually induced emfs, one due to current i_1 in Coil 1 and the other due to current i_2 in Coil 3. The polarity of the mutually induced emf in Coil 2 due to the current i_1 is same as that of the self-induced emf because currents i_1 and $(i_1 - i_2)$ enter in respective coils from dotted ends. The polarity of the mutually induced emf in Coil 2 due to the current i_2 is opposite to that of the self-induced emf because current $(i_1 - i_2)$ leaves from the dotted end in Coil 2 and the current i_2 enters from the dotted end in Coil 3.

(c) In Coil 3, there is a mutually induced emf due to the current $(i_1 - i_2)$ in Coil 2. The polarity of the mutually induced emf is opposite to that of self-induced emf because the current $(i_1 - i_2)$ leaves from the dotted end in Coil 2 and the current i_2 enters from the dotted end in Coil 3.

Applying KVL to Mesh 1,

$$v_1(t) - R_1 i_1 - L_1 \frac{di_1}{dt} - M_{12}\frac{d}{dt}(i_1 - i_2) - R_2(i_1 - i_2) - L_2\frac{d}{dt}(i_1 - i_2) + M_{23}\frac{di_2}{dt} - M_{12}\frac{di_1}{dt} = 0$$

$$(R_1 + R_2)\, i_1 + (L_1 + L_2 + 2M_{12})\frac{di_1}{dt} - R_2 i_2 - (L_2 + M_{12} + M_{23})\frac{di_2}{dt} = v_1(t) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$M_{12}\frac{di_1}{dt} - M_{23}\frac{di_2}{dt} - L_2\frac{d}{dt}(i_2 - i_1) - R_2(i_2 - i_1) - L_3\frac{di_2}{dt} + M_{23}\frac{d}{dt}(i_1 - i_2) - R_3 i_2 = 0$$

$$-R_2\, i_1 - (L_2 + M_{12} + M_{23})\frac{di_1}{dt} + (R_2 + R_3)\, i_2 + (L_2 + L_3 + 2M_{23})\frac{di_2}{dt} = 0 \qquad \ldots\text{(ii)}$$

Example 4.16 *Write KVL equations for the circuit shown in Fig. 4.33.*

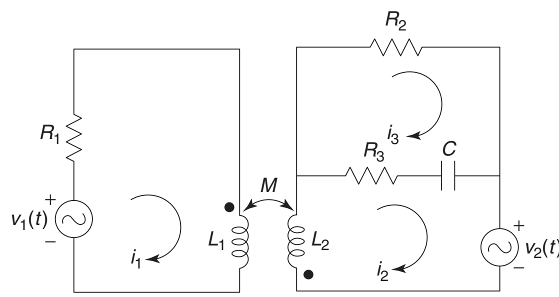


Fig. 4.33

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.34.

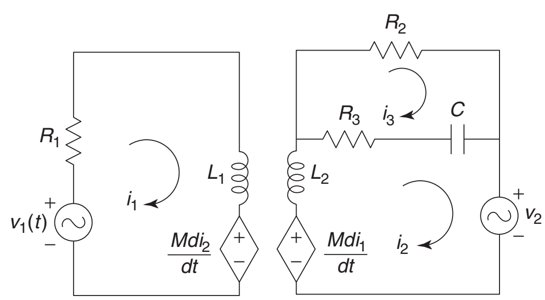


Fig. 4.34

Applying KVL to Mesh 1,

$$v_1(t) - R_1\, i_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

$$R_1\, i_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = v_1(t) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$M\frac{di_1}{dt} - L_2\frac{di_2}{dt} - R_3(i_2 - i_3) - \frac{1}{C}\int_0^t (i_2 - i_3)\, dt - v_2(t) = 0$$

$$M\frac{di_1}{dt} - L_2\frac{di_2}{dt} - R_3(i_2 - i_3) - \frac{1}{C}\int_0^t (i_2 - i_3)\,dt = v_2(t) \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-R_2 i_3 - \frac{1}{C}\int_0^t (i_3 - i_2)dt - R_3(i_3 - i_2) = 0 \qquad \text{...(iii)}$$

Example 4.17 *Write down the mesh equations for the network shown in Fig. 4.35.*

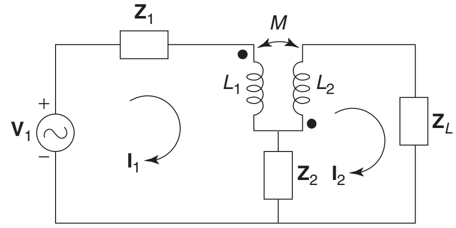


Fig. 4.35

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.36.

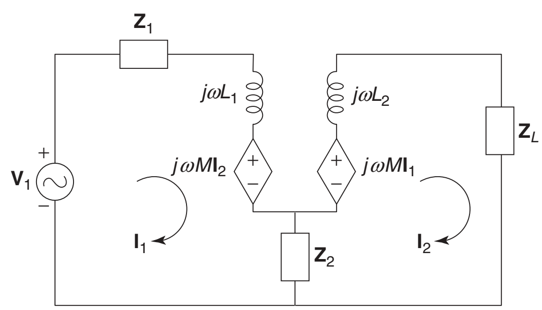


Fig. 4.36

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - \mathbf{Z}_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(\mathbf{Z}_1 + j\omega L_1 + \mathbf{Z}_2)\,\mathbf{I}_1 - (\mathbf{Z}_2 - j\omega M)\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) + j\omega M\mathbf{I}_1 - j\omega L_2\mathbf{I}_2 - \mathbf{Z}_L\mathbf{I}_2 = 0$$

$$-(\mathbf{Z}_2 - j\omega M)\,\mathbf{I}_1 + (\mathbf{Z}_2 + j\omega L_2 + \mathbf{Z}_L)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Example 4.18 *Write mesh equations for the network shown in Fig. 4.37.*

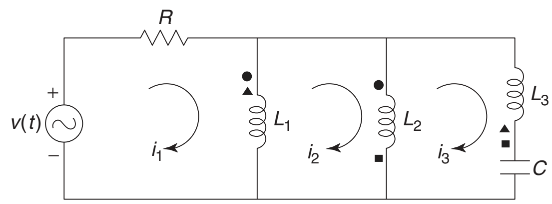


Fig. 4.37

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.38.

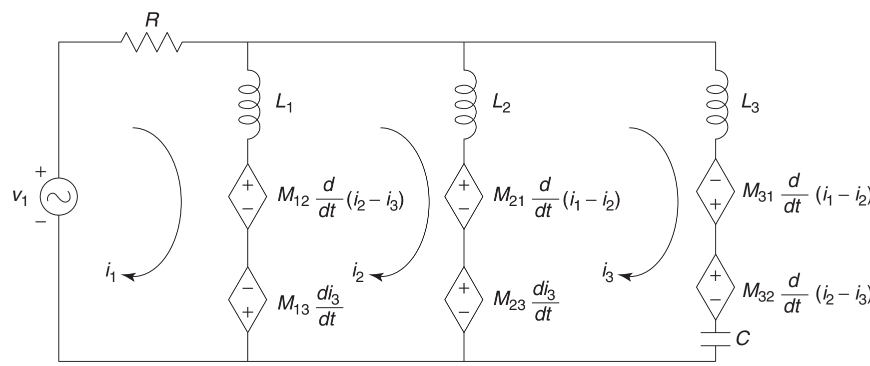


Fig. 4.38

Applying KVL to Mesh 1,

$$v(t) - Ri_1 - L_1 \frac{d}{dt}(i_1 - i_2) - M_{12}\frac{d}{dt}(i_2 - i_3) + M_{13}\frac{di_3}{dt} = 0$$

$$Ri_1 + L_1 \frac{d}{dt}(i_1 - i_2) + M_{12}\frac{d}{dt}(i_2 - i_3) - M_{13}\frac{di_3}{dt} = v(t) \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-M_{13}\frac{di_3}{dt} + M_{12}\frac{d}{dt}(i_2 - i_3) - L_1\frac{d}{dt}(i_2 - i_1) - L_2\frac{d}{dt}(i_2 - i_3) - M_{21}\frac{d}{dt}(i_1 - i_2) - M_{23}\frac{di_3}{dt} = 0$$

$$M_{13}\frac{di_3}{dt} - M_{12}\frac{d}{dt}(i_2 - i_3) + L_1\frac{d}{dt}(i_2 - i_1) + L_2\frac{d}{dt}(i_2 - i_3) + M_{21}\frac{d}{dt}(i_1 - i_2) + M_{23}\frac{di_3}{dt} = 0 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$M_{23}\frac{di_3}{dt} + M_{21}\frac{d}{dt}(i_1 - i_2) - L_2\frac{d}{dt}(i_3 - i_2) - L_3\frac{di_3}{dt} + M_{31}\frac{d}{dt}(i_1 - i_2) - M_{32}\frac{d}{dt}(i_2 - i_3) - \frac{1}{C}\int i_3\, dt = 0$$

$$-M_{23}\frac{di_3}{dt} - M_{21}\frac{d}{dt}(i_1 - i_2) + L_2\frac{d}{dt}(i_3 - i_2) + L_3\frac{di_3}{dt} - M_{31}\frac{d}{dt}(i_1 - i_2) + M_{32}\frac{d}{dt}(i_2 - i_3) + \frac{1}{C}\int i_3\, dt = 0 \quad \ldots\text{(iii)}$$

Example 4.19 *Write KVL equations for the network shown in Fig. 4.39.*

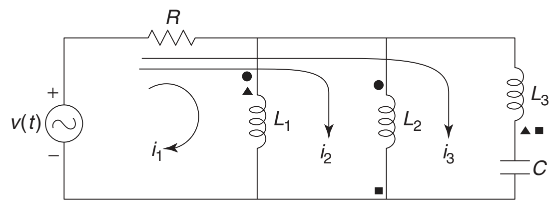


Fig. 4.39

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.40.

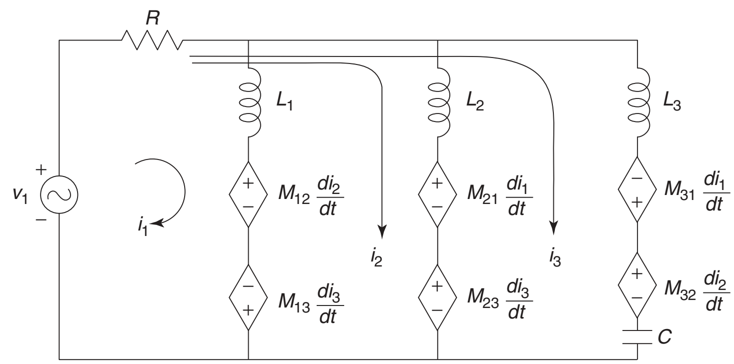


Fig. 4.40

Applying KVL to Loop 1,

$$v(t) - R(i_1 + i_2 + i_3) - L_1 \frac{di_1}{dt} - M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt} = 0$$

$$R(i_1 + i_2 + i_3) + L_1 \frac{di_1}{dt} + M_{12}\frac{di_2}{dt} - M_{13}\frac{di_3}{dt} = v(t) \qquad \text{...(i)}$$

Applying KVL to Loop 2,

$$v(t) - R(i_1 + i_2 + i_3) - L_2 \frac{di_2}{dt} - M_{21}\frac{di_1}{dt} - M_{23}\frac{di_3}{dt} = 0$$

$$R(i_1 + i_2 + i_3) + L_2 \frac{di_2}{dt} + M_{21}\frac{di_1}{dt} + M_{23}\frac{di_3}{dt} = v(t) \qquad \text{...(ii)}$$

Applying KVL to Loop 3,

$$v(t) - R(i_1 + i_2 + i_3) - L_3 \frac{di_3}{dt} + M_{31}\frac{di_1}{dt} - M_{32}\frac{di_2}{dt} - \frac{1}{C}\int i_3\, dt = 0$$

$$R(i_1 + i_2 + i_3) + L_3 \frac{di_3}{dt} - M_{31}\frac{di_1}{dt} + M_{32}\frac{di_2}{dt} + \frac{1}{C}\int i_3\, dt = v(t) \qquad \text{...(iii)}$$

Example 4.20 *In the network shown in Fig. 4.41, find the voltages V_1 and V_2.*

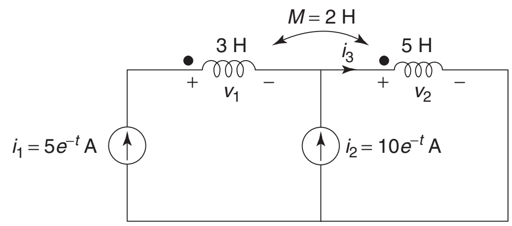


Fig. 4.41

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.42.

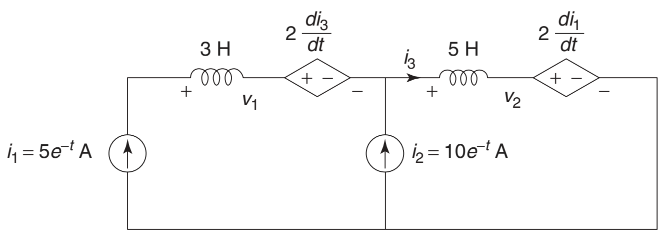


Fig. 4.42

From Fig. 4.42,

$$i_3 = i_1 + i_2 = 5\,e^{-t} + 10\,e^{-t} = 15\,e^{-t}\text{ A}$$

$$v_1 = 3\frac{di_1}{dt} + 2\frac{di_3}{dt} = 3\frac{d}{dt}(5\,e^{-t}) + 2\frac{d}{dt}(15\,e^{-t}) = -15\,e^{-t} - 30\,e^{-t} = -45\,e^{-t}\text{V}$$

$$v_2 = 5\frac{di_3}{dt} + 2\frac{di_1}{dt} = 5\frac{d}{dt}(15\,e^{-t}) + 2\frac{d}{dt}(5\,e^{-t}) = -75\,e^{-t} - 10\,e^{-t} = -85\,e^{-t}\text{V}$$

Example 4.21 *In the network shown in Fig. 4.43, find the voltages V_1 and V_2.*

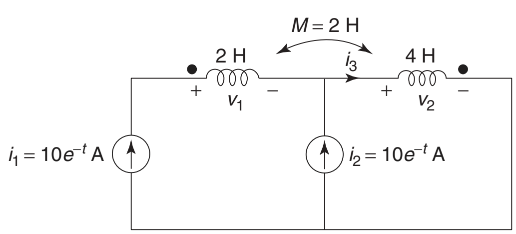


Fig. 4.43

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.44.

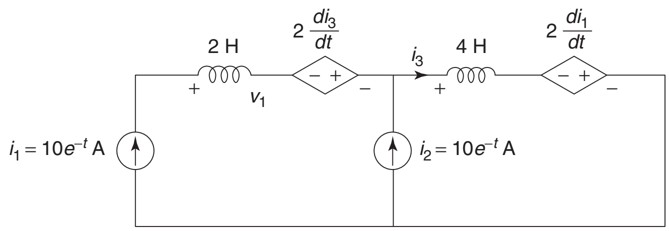


Fig. 4.44

From Fig. 4.44,

$$i_3 = i_1 + i_2 = 10\,e^{-t} + 10\,e^{-t} = 20\,e^{-t}\,\text{A}$$

$$v_1 = 2\frac{di_1}{dt} - 2\frac{di_3}{dt} = 2\frac{d}{dt}(10\,e^{-t}) - 2\frac{d}{dt}(20\,e^{-t}) = -20\,e^{-t} + 40\,e^{-t} = 20\,e^{-t}\,\text{A}$$

$$v_2 = 4\frac{di_3}{dt} - 2\frac{di_1}{dt} = 4\frac{d}{dt}(20\,e^{-t}) - 2\frac{d}{dt}(10\,e^{-t}) = -80\,e^{-t} + 20\,e^{-t} = -60\,e^{-t}\,\text{A}$$

Example 4.22 *Calculate the current $i_2(t)$ in the coupled circuit of Fig. 4.45.*

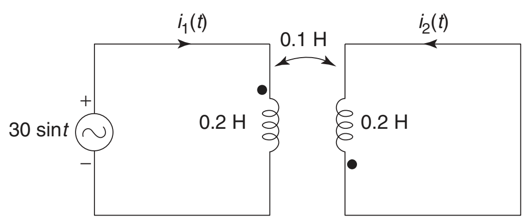


Fig. 4.45

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.46.

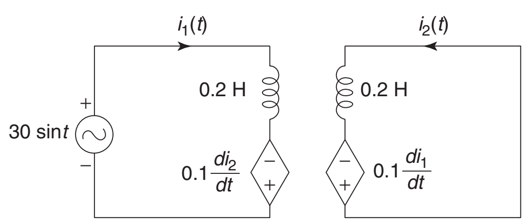


Fig. 4.46

Applying KVL to Mesh 1,

$$30\sin t - 0.2\frac{di_1}{dt} + 0.1\frac{di_2}{dt} = 0 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-0.2\frac{di_2}{dt}+0.1\frac{di_1}{dt}=0$$

$$\frac{di_1}{dt}=2\frac{di_2}{dt} \qquad \text{...(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$30\sin t-0.2\left(2\frac{di_2}{dt}\right)+0.1\frac{di_2}{dt}=0$$

$$0.3\frac{di_2}{dt}=30\sin t$$

$$\frac{di_2}{dt}=100\sin t$$

$$di_2=100\sin t\,dt$$

Integrating both the sides,

$$i_2(t)=100\int_0^t \sin t\,dt$$

$$=100\left[-\cos t\right]_0^t$$

$$=100(1-\cos t)$$

Example 4.23 *Find the voltage V_2 in the circuit shown in Fig. 4.47 such that the current in the left-hand loop (Loop 1) is zero.*

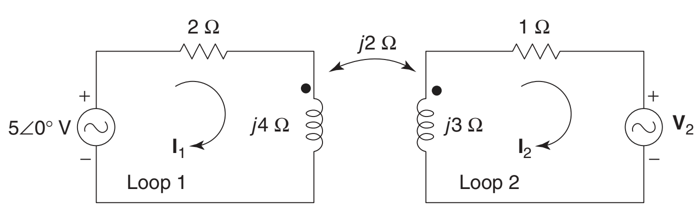


Fig. 4.47

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.48.

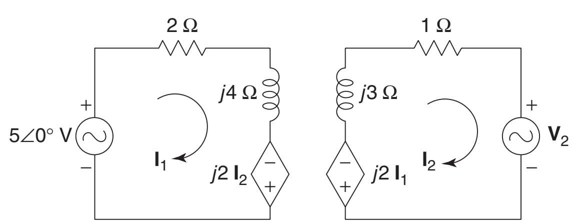


Fig. 4.48

Applying KVL to Loop 1,

$$5\angle 0° - 2\,\mathbf{I}_1 - j4\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$(2 + j4)\,\mathbf{I}_1 - j2\mathbf{I}_2 = 5\angle 0° \qquad \ldots\text{(i)}$$

Applying KVL to Loop 2,

$$-j2\mathbf{I}_1 - j3\mathbf{I}_2 - 1\mathbf{I}_2 - \mathbf{V}_2 = 0$$

$$-j2\mathbf{I}_1 - (1 + j3)\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 2+j4 & -j2 \\ -j2 & -(1+j3) \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5\angle 0° \\ \mathbf{V}_2 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 5\angle 0° & -j2 \\ \mathbf{V}_2 & -(1+j3) \end{vmatrix}}{\begin{vmatrix} 2+j4 & -j2 \\ -j2 & -(1+j3) \end{vmatrix}}$$

But $\mathbf{I}_1 = 0$.

$$\therefore -(5\angle 0°)(1 + j3) + j2\,\mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = \frac{(5\angle 0°)(1 + j3)}{j2} = 7.91\angle -18.43° \text{ V}$$

Example 4.24 *Determine the ratio* $\dfrac{\mathbf{V}_2}{\mathbf{V}_1}$ *in the circuit of Fig. 4.49, if* $\mathbf{I}_1 = 0$.

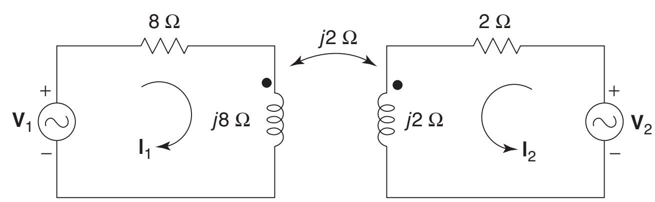


Fig. 4.49

Solution The equivalent circuit in terms of dependent sources is as shown in Fig. 4.50.

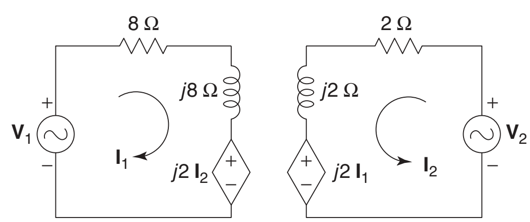


Fig. 4.50

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - 8\mathbf{I}_1 - j8\mathbf{I}_1 - j2\mathbf{I}_2 = 0$$

$$(8 + j8)\,\mathbf{I}_1 + j2\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots\text{(i)}$$

Putting $\mathbf{I}_1 = 0$ in Eq (i),

$$j2\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$\mathbf{V}_2 - 2\mathbf{I}_2 - j2\mathbf{I}_2 - j2\mathbf{I}_1 = 0$$

$$j2\mathbf{I}_1 + (2 + j2)\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \text{...(iii)}$$

Putting $\mathbf{I}_1 = 0$ in Eq (iii),

$$(2 + j2)\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \text{...(iv)}$$

From Eqs (ii) and (iv),

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{(2 + j2)\mathbf{I}_2}{j2\mathbf{I}_2} = \frac{2 + j2}{j2} = 1.41\angle -45°\text{ V}$$

Example 4.25 *For the coupled circuit shown in Fig. 4.51, find input impedance at terminals A and B.*

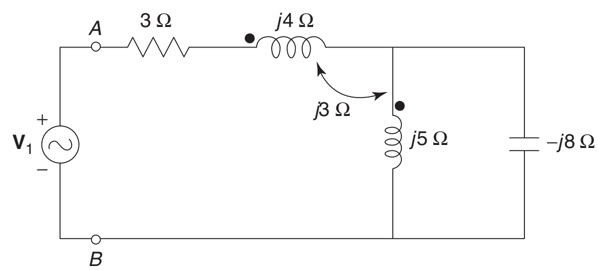


Fig. 4.51

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.52.

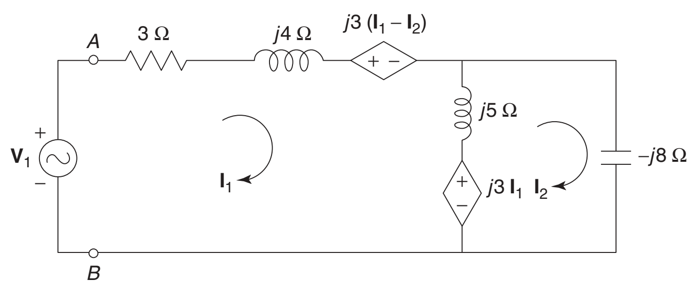


Fig. 4.52

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - 3\mathbf{I}_1 - j4\mathbf{I}_1 - j3\,(\mathbf{I}_1 - \mathbf{I}_2) - j5\,(\mathbf{I}_1 - \mathbf{I}_2) - j3\mathbf{I}_1 = 0$$

$$(3 + j15)\,\mathbf{I}_1 - j8\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$j3\,\mathbf{I}_1 - j5\,(\mathbf{I}_2 - \mathbf{I}_1) + j8\,\mathbf{I}_2 = 0$$

$$j8\,\mathbf{I}_1 + j3\,\mathbf{I}_2 = 0$$

$$\mathbf{I}_2 = -\frac{j8}{j3}\mathbf{I}_1 = -2.67\,\mathbf{I}_1 \qquad \text{...(ii)}$$

Substituting Eq (ii) in Eq (i),

$$(3+j15)\mathbf{I}_1 - j8\,(-2.67\,\mathbf{I}_1) = \mathbf{V}_1$$

$$(3+j\,36.36)\,\mathbf{I}_1 = \mathbf{V}_1$$

$$\mathbf{Z}_i = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (3+j\,36.36)\,\Omega = 36.48\angle 85.28°\,\Omega$$

Example 4.26 *Find equivalent impedance of the network shown in Fig. 4.53.*

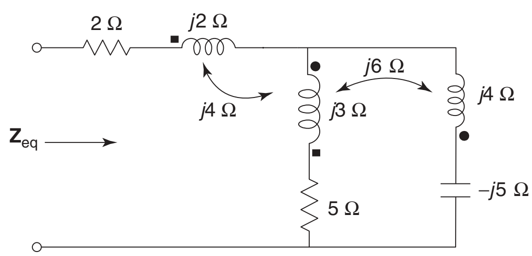


Fig. 4.53

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.54.

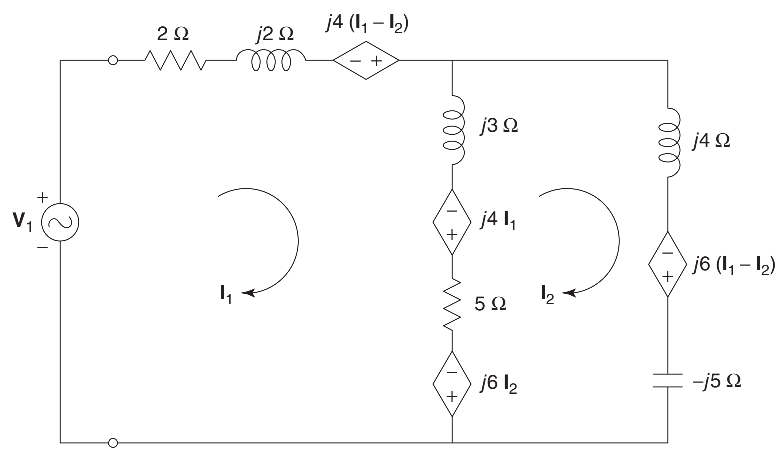


Fig. 4.54

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - 2\mathbf{I}_1 - j2\mathbf{I}_1 + j4\,(\mathbf{I}_1 - \mathbf{I}_2) - j3\,(\mathbf{I}_1 - \mathbf{I}_2) + j4\,\mathbf{I}_1 - 5(\mathbf{I}_1 - \mathbf{I}_2) + j6\mathbf{I}_2 = 0$$

$$(7-j3)\,\mathbf{I}_1 - (5+j5)\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j6\mathbf{I}_2 - 5(\mathbf{I}_2 - \mathbf{I}_1) - j4\mathbf{I}_1 - j3(\mathbf{I}_2 - \mathbf{I}_1) - j4\mathbf{I}_2 + j6(\mathbf{I}_1 - \mathbf{I}_2) + j5\mathbf{I}_2 = 0$$

$$(5 + j5)\mathbf{I}_1 = (5 + j4)\,\mathbf{I}_2$$

$$\mathbf{I}_2 = \left(\frac{5 + j5}{5 + j4}\right)\mathbf{I}_1 \qquad \text{...(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$(7 - j3)\mathbf{I}_1 - (5 + j5)\left(\frac{5 + j5}{5 + j4}\right)\mathbf{I}_1 = \mathbf{V}_1$$

$$\mathbf{Z}_i = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 7 - j3 - \frac{(5 + j5)\,(5 + j5)}{5 + j4} = 5.63 \angle -47.15° \ \Omega$$

Example 4.27 *Find the voltage across the 5 Ω resistor in Fig. 4.55 using mesh analysis.*

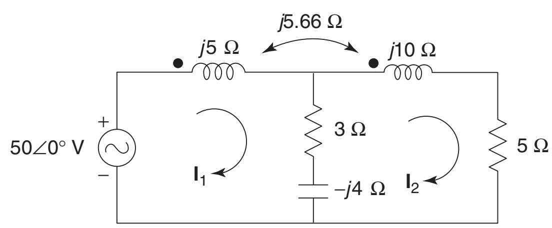


Fig. 4.55

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.56.

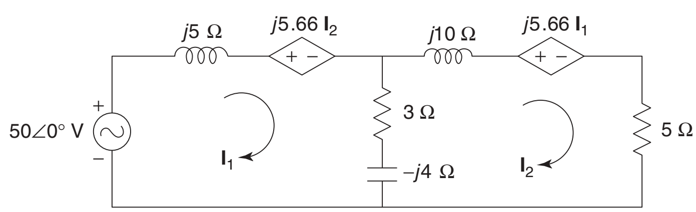


Fig. 4.56

Applying KVL to Mesh 1,

$$50 \angle 0° - j5\,\mathbf{I}_1 - j\,5.66\,\mathbf{I}_2 - (3 - j4)\,(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(3 + j\,1)\,\mathbf{I}_1 - (3 - j\,9.66)\,\mathbf{I}_2 = 50 \angle 0° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(3 - j4)\,(\mathbf{I}_2 - \mathbf{I}_1) - j\,10\mathbf{I}_2 - j\,5.66\,\mathbf{I}_1 - 5\mathbf{I}_2 = 0$$

$$-(3 - j\,9.66)\,\mathbf{I}_1 + (8 + j6)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 3+j1 & -(3-j9.66) \\ -(3-j9.66) & 8+j6 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0° \\ -(3-j9.66) & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -(3-j9.66) \\ -(3-j9.66) & 8+j6 \end{vmatrix}} = 3.82\angle -112.14° \text{ A}$$

$$\mathbf{V}_{5\,\Omega} = 5\,\mathbf{I}_2 = 5\,(3.82\angle -112.14°) = 19.1\angle -112.14° \text{ V}$$

Example 4.28 *Find the voltage across the 5 Ω resistor in Fig. 4.57 using mesh analysis.*

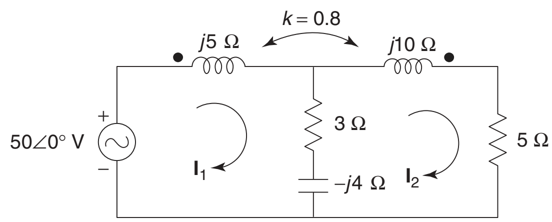


Fig. 4.57

Solution For a magnetically coupled circuit,

$$\begin{aligned} X_M &= k\sqrt{X_{L_1} X_{L_2}} \\ &= 0.8\sqrt{(5)(10)} \\ &= 5.66\ \Omega \end{aligned}$$

The equivalent circuit in terms of dependent sources is shown in Fig. 4.58.

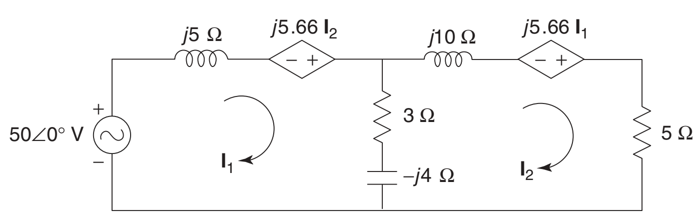


Fig. 4.58

Applying KVL to Mesh 1,

$$50\angle 0° - j5\,\mathbf{I}_1 + j\,5.66\,\mathbf{I}_2 - (3-j4)\,(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(3+j1)\,\mathbf{I}_1 - (3+j1.66)\,\mathbf{I}_2 = 50\angle 0° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(3-j4)(\mathbf{I}_2-\mathbf{I}_1)-j\,10\mathbf{I}_2+j\,5.66\,\mathbf{I}_1-5\mathbf{I}_2=0$$

$$-(3+j\,1.66)\,\mathbf{I}_1+(8+j6)\,\mathbf{I}_2=0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 3+j\,1 & -(3+j\,1.66) \\ -(3+j\,1.66) & 8+j\,6 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}=\begin{bmatrix} 50\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2=\frac{\begin{vmatrix} 3+j\,1 & 50\angle 0^\circ \\ -(3+j\,1.66) & 0 \end{vmatrix}}{\begin{vmatrix} 3+j\,1 & -(3+j\,1.66) \\ -(3+j\,1.66) & 8+j6 \end{vmatrix}}=8.62\angle -24.79^\circ \text{ A}$$

$$\mathbf{V}_{5\,\Omega}=5\,\mathbf{I}_2=5\,(8.62\angle -24.79^\circ)=43.1\angle -24.79^\circ \text{ A}$$

Example 4.29 *Find the current through the capacitor in Fig. 4.59 using mesh analysis.*

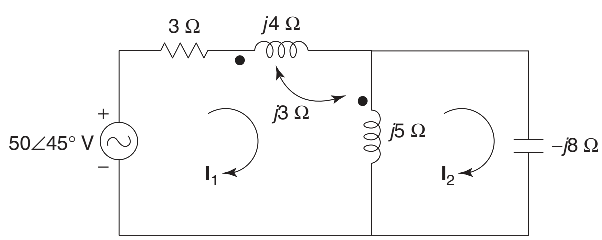


Fig. 4.59

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.60.

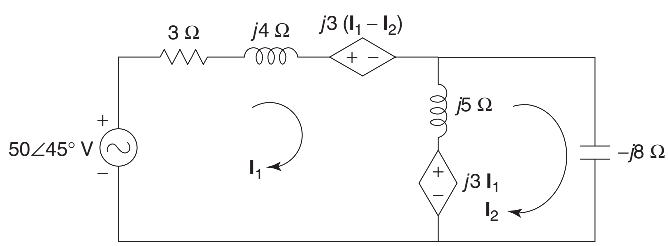


Fig. 4.60

Applying KVL to Mesh 1,

$$50\angle 45^\circ-(3+j4)\,\mathbf{I}_1-j3(\mathbf{I}_1-\mathbf{I}_2)-j5\,(\mathbf{I}_1-\mathbf{I}_2)-j3\,\mathbf{I}_1=0$$

$$(3+j\,15)\,\mathbf{I}_1-j\,8\,\mathbf{I}_2=50\angle 45^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$j3\,\mathbf{I}_1 - j5\,(\mathbf{I}_2 - \mathbf{I}_1) + j8\,\mathbf{I}_2 = 0$$

$$-j8\,\mathbf{I}_1 - j3\,\mathbf{I}_2 = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 3+j\,15 & -j8 \\ -j8 & -j3 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 45^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 3+j\,15 & 50\angle 45^\circ \\ -j8 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j\,15 & -j8 \\ -j8 & -j3 \end{vmatrix}} = 3.66\angle 139.72^\circ\ \text{A}$$

$$\mathbf{I}_C = \mathbf{I}_2 = 3.66\angle 139.72^\circ\text{A}$$

Example 4.30 *Find the voltage across the 15 Ω resistor in Fig. 4.61 using mesh analysis.*

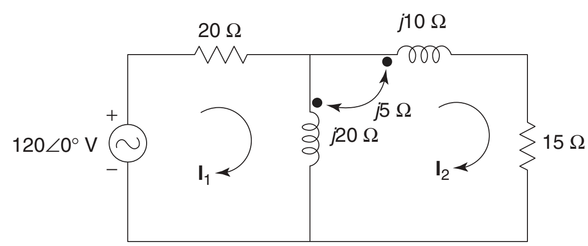


Fig. 4.61

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.62.

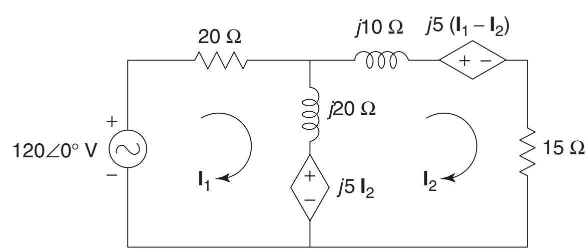


Fig. 4.62

Applying KVL to Mesh 1,

$$120\angle 0^\circ - 20\,\mathbf{I}_1 - j20(\mathbf{I}_1 - \mathbf{I}_2) - j5\,\mathbf{I}_2 = 0$$

$$(20 + j\,20)\,\mathbf{I}_1 - j\,15\,\mathbf{I}_2 = 120\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$j5\,\mathbf{I}_2 - j\,20\,(\mathbf{I}_2 - \mathbf{I}_1) - j10\,\mathbf{I}_2 - j5\,(\mathbf{I}_1 - \mathbf{I}_2) - 15\,\mathbf{I}_2 = 0$$

$$-\,j15\,\mathbf{I}_1 + (15 + j20)\,\mathbf{I}_2 = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 20 + j\,20 & -j15 \\ -\,j15 & 15 + j\,20 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120 \angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 20 + j\,20 & 120 \angle 0° \\ -\,j15 & 0 \end{vmatrix}}{\begin{vmatrix} 20 + j\,20 & -j15 \\ -\,j15 & 15 + j\,20 \end{vmatrix}} = 2.53 \angle 10.12°\ \text{A}$$

$$\mathbf{V}_{15\,\Omega} = 15\,\mathbf{I}_2 = 15\,(2.53 \angle 10.12°) = 37.95 \angle 10.12°\ \text{V}$$

Example 4.31 *Find the current through the 6 Ω resistor in Fig. 4.63 using mesh analysis.*

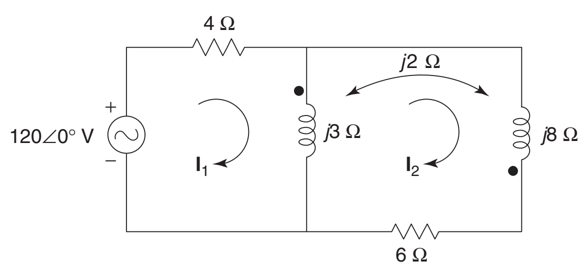


Fig. 4.63

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.64.

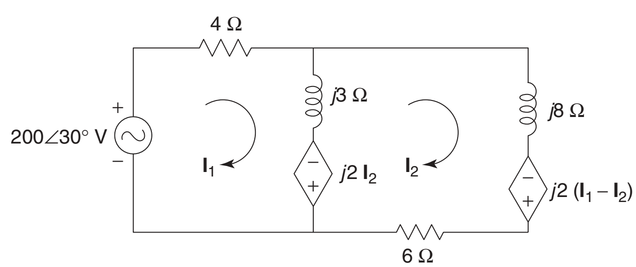


Fig. 4.64

Applying KVL to Mesh 1,

$$120 \angle 0° - 4\,\mathbf{I}_1 - j3(\mathbf{I}_1 - \mathbf{I}_2) + j2\,\mathbf{I}_2 = 0$$

$$(4 + j\,3)\,\mathbf{I}_1 - j\,5\,\mathbf{I}_2 = 120 \angle 0° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j\,2\,\mathbf{I}_2 - j\,3\,(\mathbf{I}_2 - \mathbf{I}_1) - j8\,\mathbf{I}_2 + j2\,(\mathbf{I}_1 - \mathbf{I}_2) - 6\,\mathbf{I}_2 = 0$$

$$-j5\,\mathbf{I}_1 + (6 + j15)\,\mathbf{I}_2 = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 4 + j\,3 & -j5 \\ -j5 & 6 + j\,15 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120 \angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 4 + j\,3 & 120 \angle 0° \\ -j5 & 0 \end{vmatrix}}{\begin{vmatrix} 4 + j\,3 & -j5 \\ -j5 & 6 + j\,15 \end{vmatrix}} = 7.68 \angle 2.94°\text{A}$$

Example 4.32 *Determine the mesh current I_3 in the network of Fig. 4.65.*

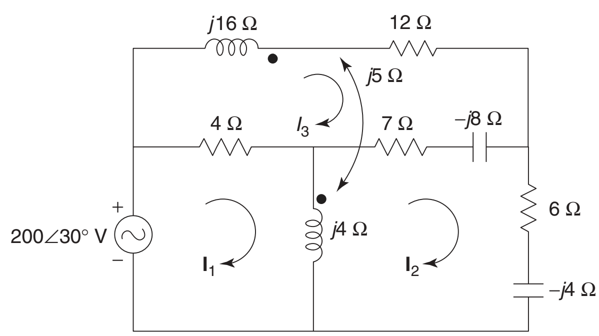


Fig. 4.65

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.66.

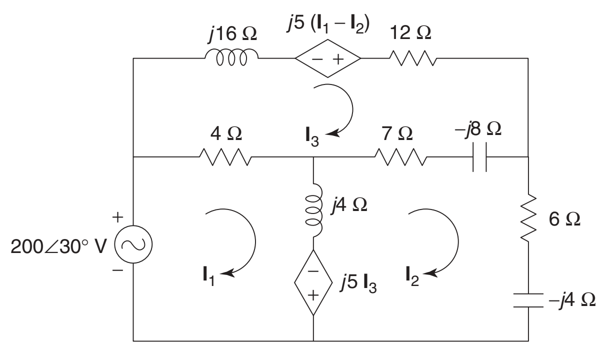


Fig. 4.66

Applying KVL to Mesh 1,

$$200\angle 30° - 4(\mathbf{I}_1 - \mathbf{I}_3) - j4(\mathbf{I}_1 - \mathbf{I}_2) + j5\,\mathbf{I}_3 = 0$$

$$(4 + j4)\,\mathbf{I}_1 - j4\,\mathbf{I}_2 - (4 + j5)\,\mathbf{I}_3 = 200\angle 30° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j5\,\mathbf{I}_3 - j4(\mathbf{I}_2 - \mathbf{I}_1) - (7 - j8)(\mathbf{I}_2 - \mathbf{I}_3) - (6 - j4)\,\mathbf{I}_2 = 0$$

$$-j4\,\mathbf{I}_1 + (13 - j8)\,\mathbf{I}_2 - (7 - j13)\,\mathbf{I}_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-j16\,\mathbf{I}_3 + j5(\mathbf{I}_1 - \mathbf{I}_2) - 12\,\mathbf{I}_3 - (7 - j8)(\mathbf{I}_3 - \mathbf{I}_2) - 4(\mathbf{I}_3 - \mathbf{I}_1) = 0$$

$$-(4 + j5)\,\mathbf{I}_1 - (7 - j13)\,\mathbf{I}_2 + (23 + j8)\,\mathbf{I}_3 = 0 \qquad \text{...(iii)}$$

Writing Eqs. (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4 + j4 & -j4 & -(4 + j5) \\ -j4 & 13 - j8 & -(7 - j13) \\ -(4 + j5) & -(7 - j13) & 23 + j8 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 200\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 4 + j4 & -j4 & 200\angle 30° \\ -j4 & 13 - j8 & 0 \\ -(4 + j5) & -(7 - j13) & 0 \end{vmatrix}}{\begin{vmatrix} 4 + j4 & -j4 & -(4 + j5) \\ -j4 & 13 - j8 & -(7 - j13) \\ -(4 + j5) & -(7 - j13) & 23 + j8 \end{vmatrix}} = 16.28\angle 16.87°\text{A}$$

Example 4.33 *Obtain the dotted equivalent circuit for the coupled circuit shown in Fig. 4.67 and find mesh currents. Also find the voltage across the capacitor.*

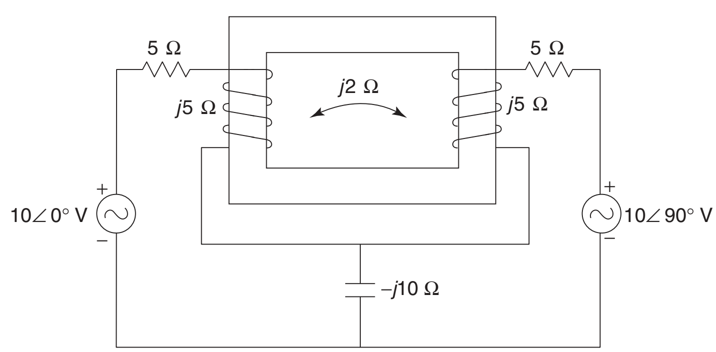


Fig. 4.67

Solution The currents in the coils are as shown in Fig. 4.68. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

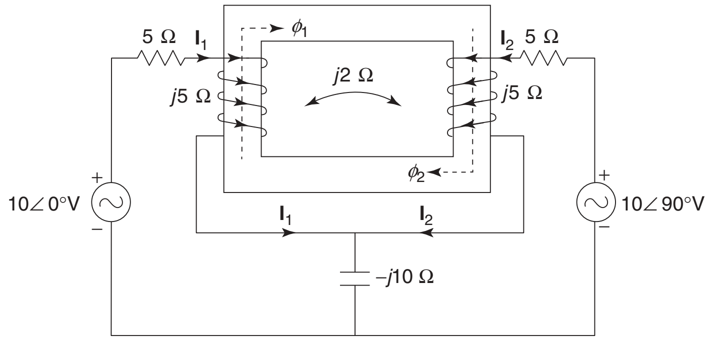


Fig. 4.68

From Fig. 4.68, it is seen that two fluxes ϕ_1 and ϕ_2 aid each other. Hence, dots are placed at the two coils as shown in Fig. 4.69.

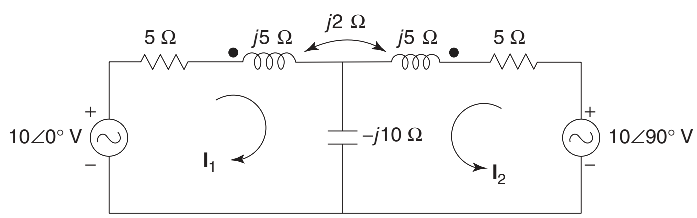


Fig. 4.69

The equivalent circuit in terms of dependent sources is shown in Fig. 4.70.

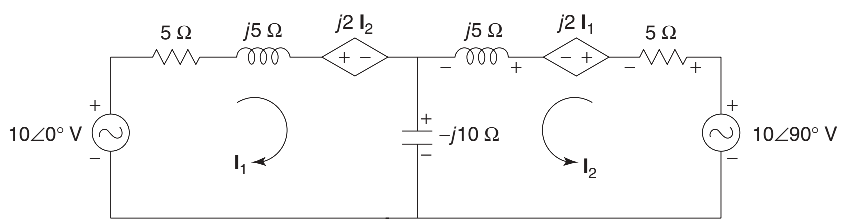


Fig. 4.70

Applying KVL to Mesh 1,

$$10 \angle 0° - (5 + j5)\,\mathbf{I}_1 - j2\,\mathbf{I}_2 + j10\,(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$(5 - j\,5)\,\mathbf{I}_1 - j\,8\,\mathbf{I}_2 = 10 \angle 0° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j10\,(\mathbf{I}_2 + \mathbf{I}_1) + j5\,\mathbf{I}_2 - j2\,\mathbf{I}_1 + 5\,\mathbf{I}_2 - 10 \angle 90° = 0$$

$$-j\,8\,\mathbf{I}_1 + (5 - j5)\,\mathbf{I}_2 = 10 \angle 90° \qquad \ldots\text{(ii)}$$

Writing Eqs. (i) and (ii) in matrix form,

$$\begin{bmatrix} 5 - j\,5 & -j8 \\ -j8 & 5 - j\,5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10 \angle 0° \\ 10 \angle 90° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10 \angle 0° & -j8 \\ 10 \angle 90° & 5-j5 \end{vmatrix}}{\begin{vmatrix} 5-j5 & -j8 \\ -j8 & 5-j5 \end{vmatrix}} = 0.72 \angle -82.97° \text{A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5-j5 & 10 \angle 0° \\ -j8 & 10 \angle 90° \end{vmatrix}}{\begin{vmatrix} 5-j5 & -j8 \\ -j8 & 5-j5 \end{vmatrix}} = 1.71 \angle 106.96° \text{A}$$

$$\mathbf{V}_C = -j\,10\,(\mathbf{I}_1 + \mathbf{I}_2) = (-j\,10)\,(0.72 \angle -82.97° + 1.71 \angle 106.96°\text{ A})$$
$$= 10.08 \angle 24.03° \text{V}$$

4.9 CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS

For simplifying circuit analysis, it is desirable to replace a magnetically coupled circuit with an equivalent circuit called conductively coupled circuit. In this circuit, no magnetic coupling is involved. The dot convention is also not needed in the conductively coupled circuit.

Consider a coupled circuit as shown in Fig. 4.71.

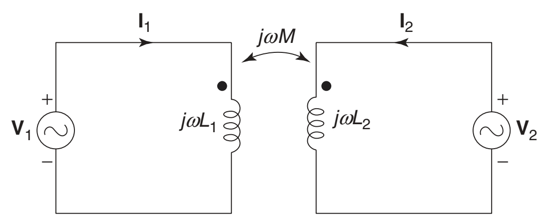


Fig. 4.71 *Coupled circuit*

The equivalent circuit in terms of dependent sources is shown in Fig. 4.72.

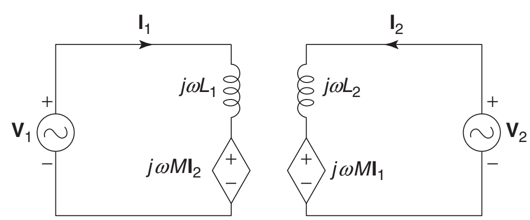


Fig. 4.72 *Equivalent circuit*

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - j\omega L_1\,\mathbf{I}_1 - j\omega M\,\mathbf{I}_2$$
$$j\omega L_1\,\mathbf{I}_1 + j\omega M\,\mathbf{I}_2 \qquad \ldots(4.11)$$

Applying KVL to Mesh 2,

$$\mathbf{V}_2 - j\omega L_2\,\mathbf{I}_2 - j\omega M\,\mathbf{I}_1 = 0$$
$$j\omega M\,\mathbf{I}_1 + j\omega L_2\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \ldots(4.12)$$

Writing Eqs (4.11) and (4.12) in matrix form,

$$\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad \text{...(4.13)}$$

Consider a T-network as shown in Fig. 4.73.

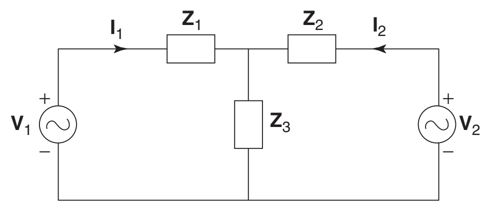


Fig. 4.73 *T-network*

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - \mathbf{Z}_1\,\mathbf{I}_1 - \mathbf{Z}_3(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$(\mathbf{Z}_1 + \mathbf{Z}_3)\,\mathbf{I}_1 + \mathbf{Z}_3\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{... (4.14)}$$

Applying KVL to Mesh 2,

$$R_{\text{Th}} = [(2\,||\,12) + 1]\,||\,3 = 1.43\ \Omega \qquad \text{... (4.15)}$$

Writing Eqs (4.14) and (4.15) in matrix form,

$$\begin{bmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & \mathbf{Z}_3 \\ \mathbf{Z}_3 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Comparing matrix equations,

$$\mathbf{Z}_1 + \mathbf{Z}_3 = j\,\omega\,L_1$$

$$\mathbf{Z}_3 = j\,\omega\,M$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = j\,\omega\,L_2$$

Solving these equations,

$$\mathbf{Z}_1 = j\,\omega\,L_1 - j\,\omega\,M = j\,\omega(L_1 - M)$$

$$\mathbf{Z}_2 = j\,\omega\,L_2 - j\,\omega\,M = j\,\omega(L_2 - M)$$

$$\mathbf{Z}_3 = j\,\omega\,M$$

Hence, the conductively coupled circuit of a magnetically coupled circuit is shown in Fig. 4.74.

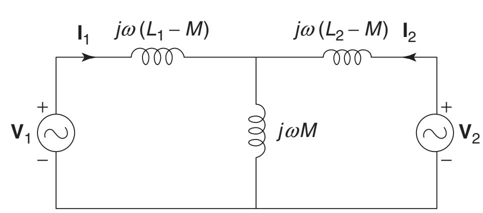


Fig. 4.74 *Conductively coupled equivalent circuit*

Example 4.34 *Find the conductively coupled equivalent circuit for the network shown in Fig. 4.75.*

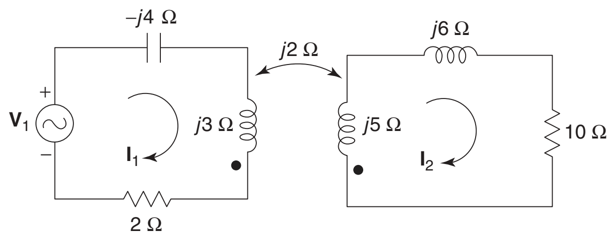


Fig. 4.75

Solution The current $\mathbf{I}_1$ leaves from the dotted end and $\mathbf{I}_2$ enters from the dotted end. Hence, mutual inductance M is negative.

In the conductively coupled equivalent circuit,

$$\mathbf{Z}_1 = j\omega(L_1 - M) = j\omega L_1 - j\omega M = j3 - j2 = j1\ \Omega$$

$$\mathbf{Z}_2 = j\omega(L_2 - M) = j\omega L_2 - j\omega M = j5 - j2 = j3\ \Omega$$

$$\mathbf{Z}_3 = j\omega M = j2\ \Omega$$

The conductively coupled equivalent circuit is shown in Fig. 4.76.

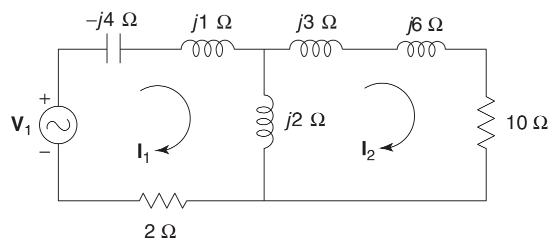


Fig. 4.76

Example 4.35 *Draw the conductively coupled equivalent circuit of Fig. 4.77.*

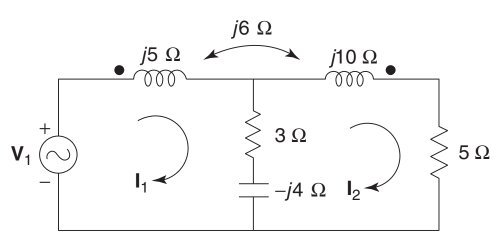


Fig. 4.77

Solution The current $\mathbf{I}_1$ enters from the dotted end and $\mathbf{I}_2$ leaves from the dotted end. Hence, the mutual inductance M is negative.

In the conductively coupled equivalent circuit,

$$\mathbf{Z}_1 = j\omega(L_1 - M) = j\omega L_1 - j\omega M = j5 - j6 = -j1\,\Omega$$
$$\mathbf{Z}_2 = j\omega(L_2 - M) = j\omega L_2 - j\omega M = j10 - j6 = j4\,\Omega$$
$$\mathbf{Z}_3 = j\omega M = j6\,\Omega$$

The conductively coupled equivalent circuit is shown in Fig. 4.78.

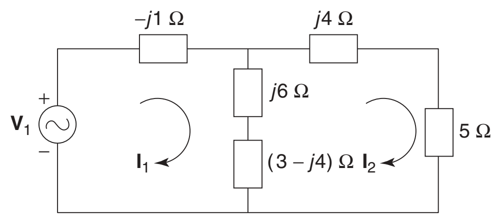


Fig. 4.78

Example 4.36 *Find the conductively coupled equivalent circuit of the network in Fig. 4.79.*

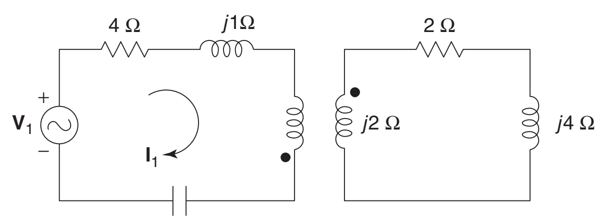


Fig. 4.79

Solution The currents $\mathbf{I}_1$ and $\mathbf{I}_2$ leave from the dotted terminals. Hence, mutual inductance is positive. In the conductively coupled equivalent circuit,

$$\mathbf{Z}_1 = j\omega(L_1 + M) = j\omega L_1 + j\omega M = j4 + j2 = j6\,\Omega$$
$$\mathbf{Z}_2 = j\omega(L_2 + M) = j\omega L_2 + j\omega M = j2 + j2 = j4\,\Omega$$
$$\mathbf{Z}_3 = -j\omega M = -j2\,\Omega$$

The conductively coupled equivalent circuit is shown in Fig. 4.80.

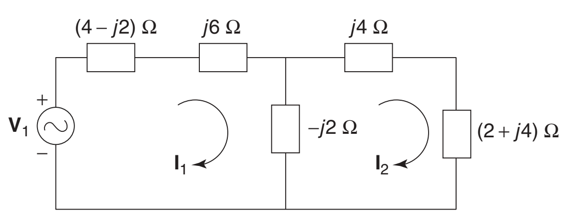


Fig. 4.80

Exercises

4.1 Two coupled coils have inductances of 0.8 H and 0.2 H. The coefficient of coupling is 0.90. Find the mutual inductance and the turns ratio $\frac{N_1}{N_2}$.

[0.36 H, 2]

4.2 Two coils with coefficient of coupling 0.5 are connected in such a way that they magnetise (i) in the same direction, and (ii) in opposite directions. The corresponding equivalent inductances are 1.9 H and 0.7 H. Find self-inductances of the two coils and the mutual inductance between them.

[0.4 H, 0.9 H, 0.3 H]

4.3 Two coils having 3000 and 2000 turns are wound on a magnetic ring. 60% of the flux produced in the first coil links with the second coil. A current of 3 A produce a flux of 0.5 mwb in the first coil and 0.3 mwb in the second coil. Determine the mutual inductance and coefficient of coupling.

[0.2 H, 0.63]

4.4 Find the equivalent inductance of the network shown in Fig. 4.81.

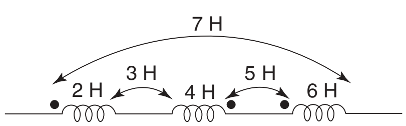


Fig. 4.81

[10 H]

4.5 Find the effective inductance of the network shown in Fig. 4.82.

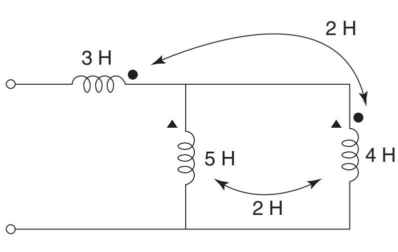


Fig. 4.82

[4.8 H]

4.6 Write mesh equations of the network shown in Fig. 4.83.

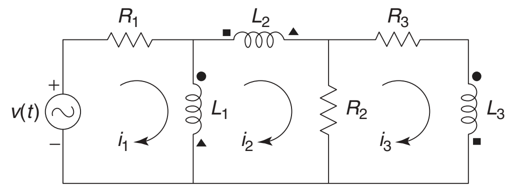


Fig. 4.83

$$\left[\begin{array}{l} v = i_1R_1 + L_1\frac{d}{dt}(i_1 - i_2) + M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt} \\ R_2(i_3 - i_2) + R_3 i_3 + L_3\frac{di_3}{dt} + M_{13}\frac{d}{dt}(i_1 - i_2) \\ -M_{23}\frac{di_2}{dt} = 0 \end{array}\right]$$

4.7 Find the input impedance at terminals *AB* of the coupled circuits shown in Fig. 4.84 to 4.85.

(i)

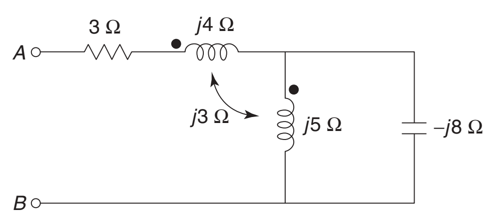


Fig. 4.84

(ii)

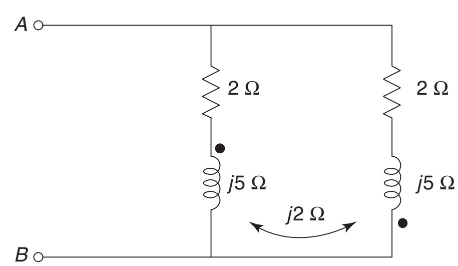


Fig. 4.85

(iii)

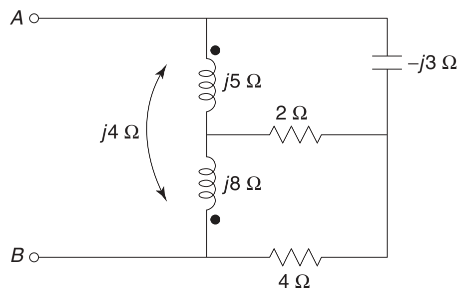


Fig. 4.86

$$\begin{bmatrix} \text{(a)}(3+j36.3)\Omega \quad \text{(b)}(1+j1.5)\Omega \\ \text{(c)}(6.22+j4.65)\Omega \end{bmatrix}$$

4.8 In the coupled circuit shown in Fig. 4.87, find $\mathbf{V}_2$ for which $\mathbf{I}_1 = 0$. What voltage appears at the 8 Ω inductive reactance under this condition?

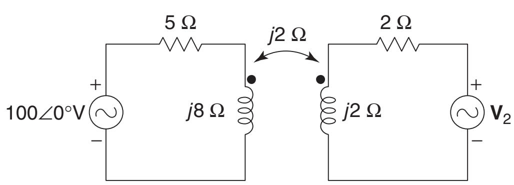


Fig. 4.87

[141.5∠−45°V, 100∠0°V]

4.9 For the coupled circuit shown in Fig. 4.88, find the components of the current $\mathbf{I}_2$ resulting from each source $\mathbf{V}_1$ and $\mathbf{V}_2$.

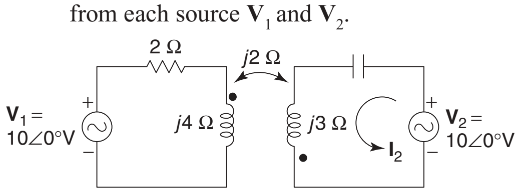


Fig. 4.88

[0.77∠112.6°A, 1.72∠86.05°A]

4.10 Find the voltage across the 5 Ω resistor in the network shown in Fig. 4.89.

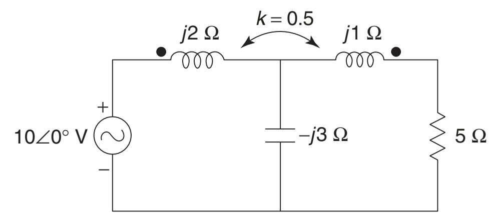


Fig. 4.89

[19.2∠−33.02°V]

4.11 Find the power dissipated in the 5 Ω resistor in the network of Fig. 4.90.

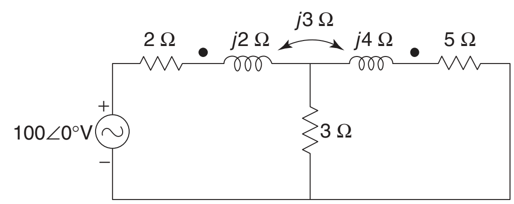


Fig. 4.90

[668.16 W]

4.12 Find the current **I** in the circuit of Fig. 4.91.

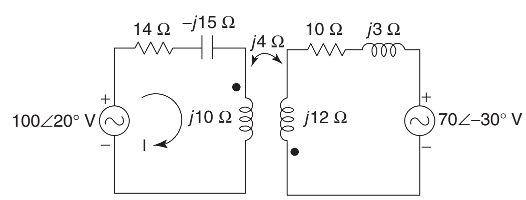


Fig. 4.91

[7.07∠45° V, 1.

4.13 Obtain a conductively coupled circuit for the circuit shown in Fig. 4.92.

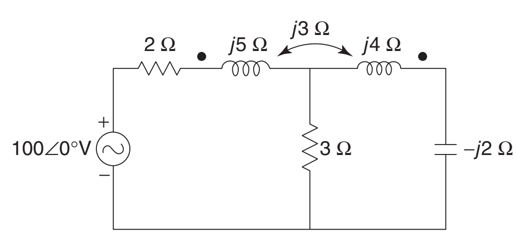


Fig. 4.92

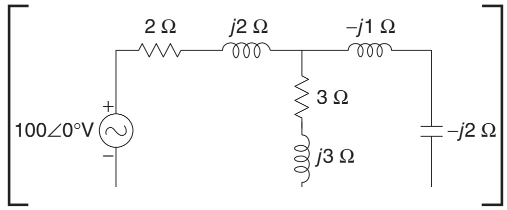


Fig. 4.93

Objective-Type Questions

4.1 Two coils are wound on a common magnetic core. The sign of mutual inductance M for finding out effective inductance of each coil is positive if the

(a) two coils are wound in the same sense.
(b) fluxes produced by the two coils are equal
(c) fluxes produced by the coils act in the same direction
(d) fluxes produced by the two coils act in opposition

4.2 When two coils having self-inductances of L_1 and L_2 are coupled through a mutual inductance M, the coefficient of coupling k is given by

(a) $k = \dfrac{M}{\sqrt{2L_1L_2}}$ (b) $k = \dfrac{M}{\sqrt{L_1L_2}}$

(c) $k = \dfrac{2M}{\sqrt{L_1L_2}}$ (d) $k = \dfrac{L_1L_2}{M}$

4.3 The overall inductance of two coils connected in series, with mutual inductance aiding self-inductance is L_1; with mutual inductance opposing self-inductance, the overall inductance is L_2. The mutual inductance M is given by

(a) $L_1 + L_2$ (b) $L_1 - L_2$

(c) $\dfrac{1}{4}(L_1 - L_2)$ (d) $\dfrac{1}{2}(L_1 + L_2)$

4.4 Consider the following statements:
The coefficient of coupling between two oils depends upon
1. Orientation of the coils
2. Core material
3. Number of turns on the two coils
4. Self-inductance of the two coils
of these statements,
(a) 1, 2 and 3 are correct
(b) 1 and 2 are correct
(c) 3 and 4 are correct
(d) 1, 2 and 4 are correct

4.5 Two coupled coils connected in series have an equivalent inductance of 16 mH or 8 mH depending on the inter connection.
Then the mutual inductance M between the coils is
(a) 12 mH (b) $8\sqrt{2}$ mH
(c) 4 mH (d) 2 mH

4.6 Two coupled coils with $L_1 = L_2 = 0.6$ H have a coupling coefficient of $k = 0.8$. The turns ratio $\dfrac{N_1}{N_2}$ is
(a) 4 (b) 2
(c) 1 (d) 0.5

4.7 The coupling between two magnetically coupled coils is said to be ideal if the coefficient of coupling is
(a) zero (b) 0.5
(c) 1 (d) 2

4.8 The mutual inductance between two coupled coils is 10 mH . If the turns in one coil are doubled and that in the other are halved then the mutual inductance will be
(a) 5 mH (b) 10 mH
(c) 14 mH (d) 20 mH

4.9 Two perfectly coupled coils each of 1 H self-inductance are connected in parallel so as to aid each other. The overall inductance in henrys is

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) Zero

4.10 The impedance Z as shown in Fig. 4.94 is

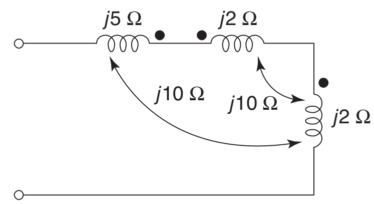


Fig. 4.94

(a) j 29 Ω (b) j9 Ω

(c) j19 Ω (d) j39 Ω

Answers to Objective-Type Questions

4.1 (c)	4.2 (b)	4.3 (c)	4.4 (d)	4.5 (d)	4.6 (c)
4.7 (c)	4.8 (b)	4.9 (b)	4.10 (b)		

Solution Nodes 1 and 2 will form a supernode.
Writing the voltage equation for the supernode,

$$\mathbf{V}_1 - \mathbf{V}_2 = 12\angle 30° \qquad \text{...(i)}$$

Applying KCL to the supernode,

$$\frac{\mathbf{V}_1}{j1} + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2}{-j2} = 2\angle 60°$$

$$(-j1)\mathbf{V}_1 + (0.5 + j0.5)\mathbf{V}_2 = 2\angle 60° \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 1 & -1 \\ -j1 & 0.5 + j0.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 12\angle 30° \\ 2\angle 60° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 1 & 12\angle 30° \\ -j1 & 2\angle 60° \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -j1 & 0.5 + j0.5 \end{vmatrix}} = 18.55\angle 157.42° \text{ V}$$

$$\mathbf{V}_c = \mathbf{V}_2 = 18.55\angle 157.42° \text{ V}$$

3.4 SUPERPOSITION THEOREM

The superposition theorem can be used to analyse an ac network containing more than one source. The superposition theorem states that *in a network containing more than one voltage source or current source, the total current or voltage in any branch of the network is the phasor sum of currents or voltages produced in that branch by each source acting separately*. As each source is considered, all of the other sources are replaced by their internal impedances. This theorem is valid only for linear systems.

Example 3.17 *Find the current through the 3 + j4 ohm impedance.*

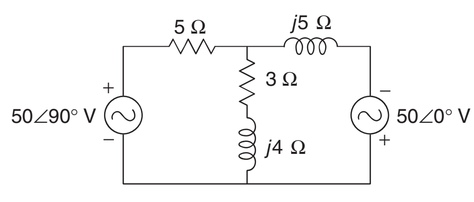


Fig. 3.17

Solution

Step I When the 50 ∠90° V source is acting alone (Fig. 3.18)

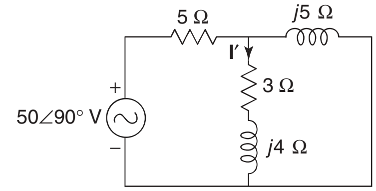


Fig. 3.18

$$\mathbf{Z}_T = 5 + \frac{(3 + j4)(j5)}{3 + j9} = 6.35\angle 23.2° \ \Omega$$

$$\mathbf{I}_T = \frac{50\angle 90°}{6.35\angle 23.2°} = 7.87\angle 66.8° \text{ A}$$

By current division rule,

$$\mathbf{I}' = (7.87\angle 66.8°)\left(\frac{j5}{3+j9}\right) = 4.15\angle 85.3° \text{ A}(\downarrow)$$

Step II When the 50∠0° V source is acting alone (Fig. 3.19)

$$\mathbf{Z}_T = j5 + \frac{5(3+j4)}{8+j4} = 6.74\angle 68.2° \ \Omega$$

$$\mathbf{I}_T = \frac{50\angle 0°}{6.74\angle 68.2°} = 7.42\angle -68.2° \text{ A}$$

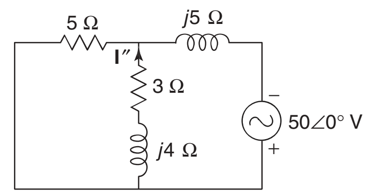


Fig. 3.19

By current division rule,

$$\mathbf{I}'' = (7.42\angle -68.2°)\left(\frac{5}{8+j4}\right) = 4.15\angle -94.77° \text{ A}(\uparrow) = 4.15\angle 85.3° \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 4.15\ \angle 85.3° + 4.15\ \angle 85.3° = 8.31\ \angle 85.3° \text{A}(\downarrow)$$

Example 3.18 *Determine the voltage across the (2 + j5) ohm impedance for the network shown in Fig. 3.20.*

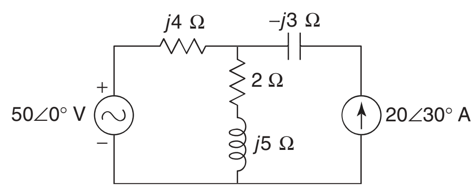


Fig. 3.20

Solution

Step I When the 50∠0° V source is acting alone (Fig. 3.21)

$$\mathbf{I} = \frac{50\angle 0°}{2+j4+j5} = 5.42\angle -77.47° \text{ A}$$

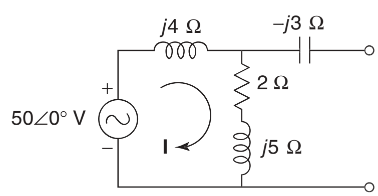


Fig. 3.21

Voltage cross (2 + *j*5) Ω impedance

$$\mathbf{V}' = (2+j5)(5.42\ \angle -77.47°) = 29.16\ \angle -9.28° \text{ V}$$

Step II When the 20∠30° A source is acting alone (Fig. 3.22)
By current division rule,

$$\mathbf{I} = (20\angle 30°)\left(\frac{j4}{2+j9}\right) = 8.68\angle 42.53° \text{ A}$$

Voltage across (2 + *j*5) Ω impedance

$$\mathbf{V}'' = (2+j5)(8.68\ \angle 42.53°) = 46.69\ \angle 110.72° \text{ V}$$

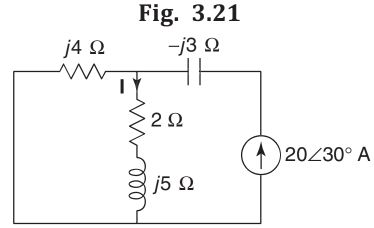


Fig. 3.22

Step III By superposition theorem,

$$\mathbf{V} = \mathbf{V}' + \mathbf{V}'' = 29.16 \angle -9.28° + 46.69 \angle 110.72° = 40.85 \angle 72.53° \text{ V}$$

Example 3.19 *Determine the voltage $\mathbf{V}_{AB}$ for the network shown in Fig. 3.23.*

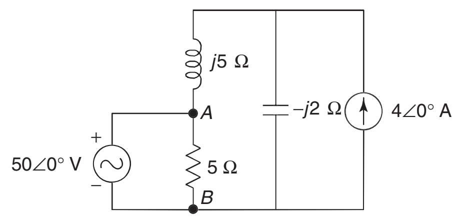


Fig. 3.23

Solution

Step I When the 50∠0° V source is acting alone (Fig. 3.24)

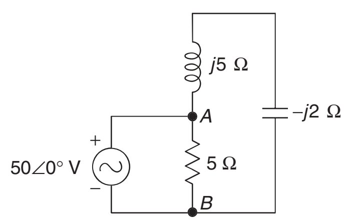


Fig. 3.24

$$\mathbf{V}'_{AB} = 50\angle 0° \text{ V}$$

Step II When the 4∠0° A source is acting alone (Fig. 3.25)

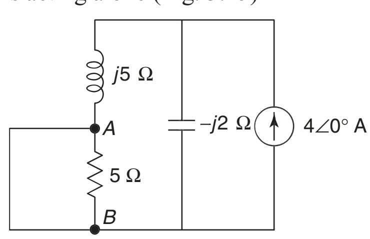


Fig. 3.25

$$\mathbf{V}''_{AB} = 0$$

Step III By superposition theorem,

$$\mathbf{V}_{AB} = \mathbf{V}'_{AB} + \mathbf{V}''_{AB} = 50\angle 0° + 0 = 50\angle 0° \text{ V}$$

Example 3.20 *Find the current **I** in the network shown in Fig. 3.26.*

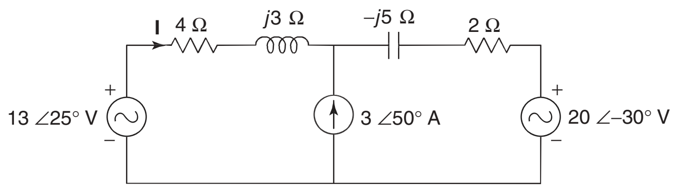


Fig. 3.26

Solution

Step I When the 13∠25° V source is acting alone (Fig. 3.27)

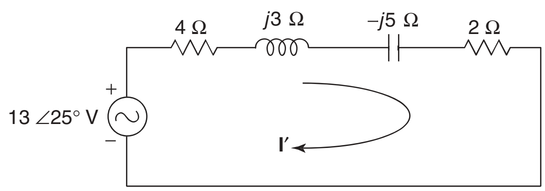


Fig. 3.27

$$\mathbf{I}' = \frac{13\angle 25^\circ}{6 - j2} = 2.057\angle 43.43^\circ \text{ A} \quad (\rightarrow)$$

Step II When the 20∠−30° V source is acting alone (Fig. 3.28)

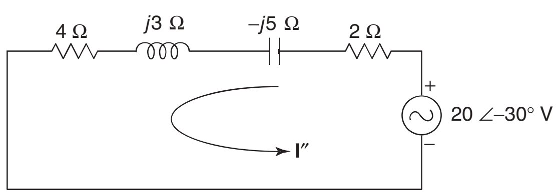


Fig. 3.28

$$\mathbf{I}'' = \frac{20\angle -30^\circ \text{ V}}{6 - j2} = 3.16\angle -11.57^\circ \text{ A}(\leftarrow) = 3.16\angle 168.43^\circ \text{ A}(\rightarrow)$$

Step III When the 3∠50° A source is acting alone (Fig. 3.29)

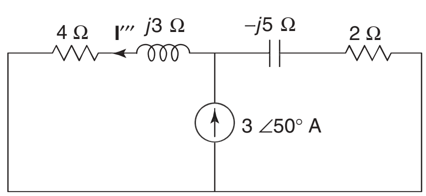


Fig. 3.29

By current division rule,

$$\mathbf{I'''} = 3\angle 50° \times \frac{2 - j5}{6 - j2} = 2.56\angle 0.23° \text{ A}(\leftarrow) = 2.56\angle -179.77° \text{ A}(\rightarrow)$$

Step IV By superposition theorem,

$$\mathbf{I} = \mathbf{I'} + \mathbf{I''} + \mathbf{I'''} = 2.057\ \angle 43.13° + 3.16\ \angle 168.43° + 2.56\ \angle -179.77° \text{ A} = 4.62\ \angle 153.99° \text{ A } (\rightarrow)$$

Example 3.21 *Find the current through the j3 Ω reactance in the network of Fig. 3.30.*

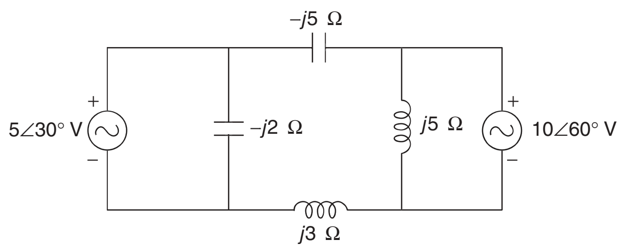


Fig. 3.30

Solution

Step I When the 5∠30° V source is acting alone (Fig. 3.31)

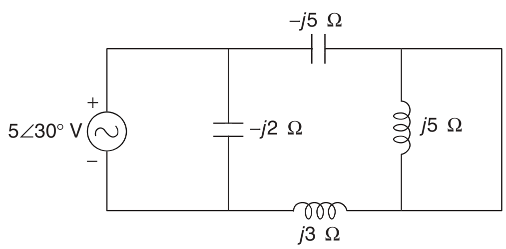


Fig. 3.31

When a short circuit is placed across *j*15 Ω reactance, it gets shorted as shown in Fig. 3.32.

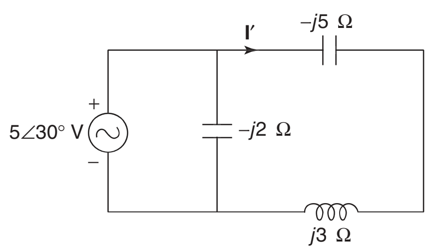


Fig. 3.32

$$\mathbf{I'} = \frac{5\angle 30°}{-j5 + j3} = 2.5\angle 120° \text{ A}(\leftarrow)$$

Step II When the $10\angle 60°$ V source is acting alone (Fig. 3.33)

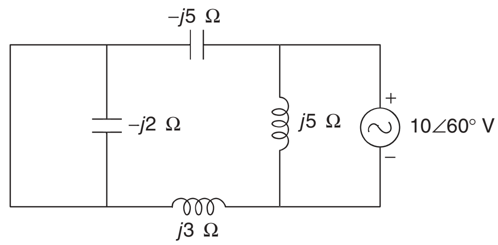


Fig. 3.33

When a short circuit is placed across the $-j2\ \Omega$ reactance, it gets shorted as shown in Fig. 3.34

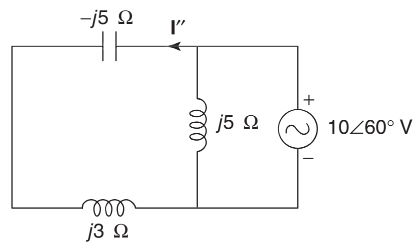


Fig. 3.34

$$\mathbf{I}'' = \frac{10\angle 60°}{-j5 + j3} = 5\angle 150°\ \text{A}\ (\rightarrow) = 5\angle -30°\ \text{A}\ (\leftarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 2.5\angle 120° + 5\angle -30° = 3.1\angle -6.21°\ \text{A} \quad (\leftarrow)$$

Example 3.22 *Find the current $\mathbf{I}_0$ in the network of Fig. 3.35.*

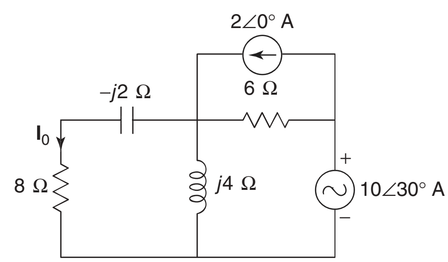


Fig. 3.35

Solution

Step I When the $10\angle 30°$ V source is acting alone (Fig. 3.36)

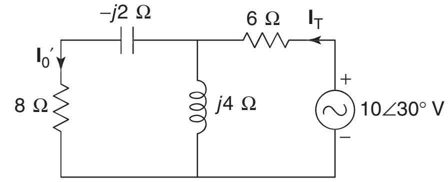


Fig. 3.36

$$\mathbf{Z}_T = 6 + \frac{j4(8 - j2)}{j4 + 8 - j2} = 8.64\angle 24.12°\ \Omega$$

$$\mathbf{I}_T = \frac{10\angle 30°}{8.64\angle 24.12°} = 1.16\angle 5.88°\ \text{A}$$

By current division rule,

$$\mathbf{I}_0' = 1.16\angle 5.88° \times \frac{j4}{8 - j2 + j4} = 0.56\angle 81.84° \text{ A} \quad (\downarrow)$$

Step II When the 2∠0° A source is acting alone (Fig. 3.37)

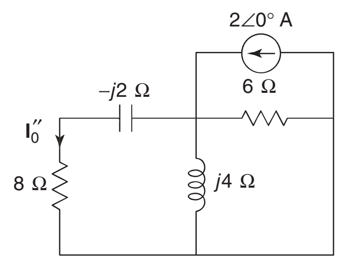


Fig. 3.37

The network can be redrawn as shown in Fig. 3.38.

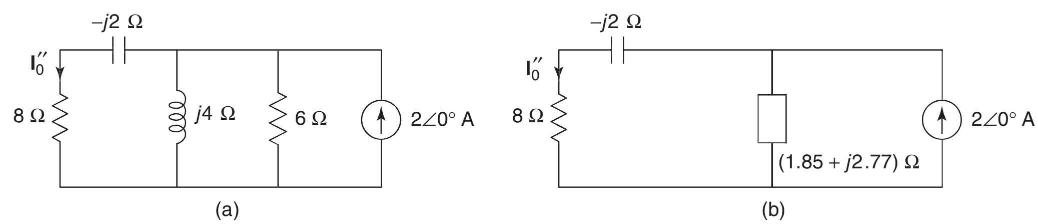


Fig. 3.38

By current division rule,

$$\mathbf{I}_0'' = 2\angle 0° \times \frac{1.85 + j2.77}{1.85 + j2.77 + 8 - j2} = 0.67\angle 51.83° \text{ A } (\downarrow)$$

Step III By superposition theorem,

$$\mathbf{I}_0 = \mathbf{I}_0' + \mathbf{I}_0'' = 0.56\angle 81.84° + 0.67\angle 51.83° = 1.19\angle 65.46° \text{ A } (\downarrow)$$

Example 3.23 *Find the current through the j5 Ω branch for the network shown in Fig. 3.39.*

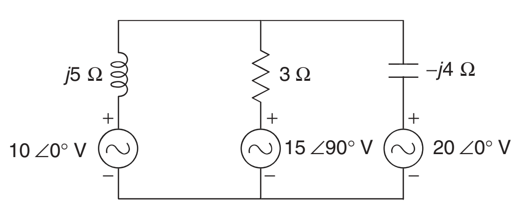


Fig. 3.39

Solution

Step I When the 10∠0° V source is acting alone (Fig. 3.40)

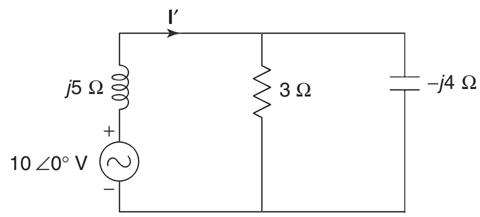


Fig. 3.40

$$\mathbf{Z}_T = j5 + \frac{3(-j4)}{3-j4} = 4.04\angle 61.66°\ \Omega$$

$$\mathbf{I}' = \frac{10\angle 0°}{4.04\angle 61.66°} = 2.48\angle -61.66°\text{A} \quad (\rightarrow)$$

Step II When the 15∠90° V source is acting alone (Fig. 3.41)

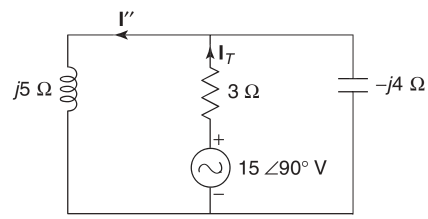


Fig. 3.41

$$\mathbf{Z}_T = 3 + \frac{(j5)(-j4)}{j5-j4} = 20.22\angle -81.47°\ \Omega$$

$$\mathbf{I}_T = \frac{15\angle 90°}{20.22\angle -81.47°} = 0.74\angle 171.47°\ \text{A}$$

By current division rule,

$$\mathbf{I}'' = 0.74\angle 171.47° \times \frac{-j4}{-j4+j5} = 2.96\angle -8.53°\ \text{A}\ (\leftarrow) = 2.96\angle 171.47°\ \text{A} \quad (\rightarrow)$$

Step III When the 20 ∠0° V source is acting along (Fig. 3.42)

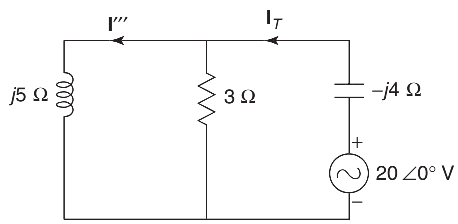


Fig. 3.42

$$\mathbf{Z}_T = -j4 + \frac{3(j5)}{3+j5} = 3.47\angle -50.51°\ \Omega$$

$$\mathbf{I}_T = \frac{20\angle 0°}{3.47\angle -50.51°} = 5.76\angle 50.51°\ \text{A}$$

By current division rule,

$$\mathbf{I}''' = 5.76\angle 50.51° \times \frac{3}{3+j5} = 2.96\angle -8.53°\ \text{A}\ (\leftarrow) = 2.96\angle 171.47°\ \text{A}\ (\rightarrow)$$

Step IV By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 2.48\angle -61.66° + 2.96\angle 171.47° + 2.96\angle 171.47° = 4.86\angle -164.41° \text{ A}$$

Example 3.24 *Find the voltage drop across the capacitor for the network shown in Fig. 3.43.*

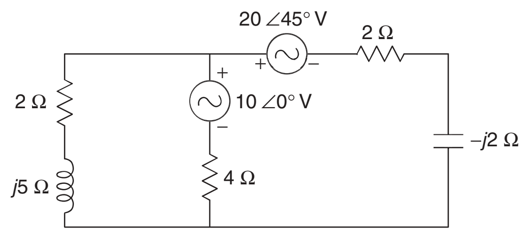


Fig. 3.43

Solution

Step I When the 10∠0° V source is acting alone (Fig. 3.44)

$$\mathbf{Z}_T = 4 + \frac{(2+j5)(2-j2)}{2+j5+2-j2}$$

$$= 7\angle -5.91° \ \Omega$$

$$\mathbf{I}_T = \frac{10\angle 0°}{7\angle -5.91°} = 1.43\angle 5.91° \text{ A}$$

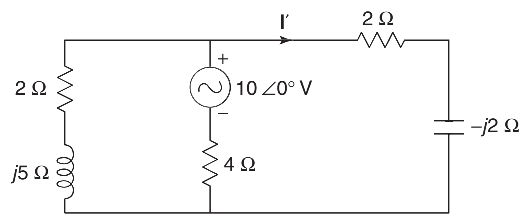


Fig. 3.44

By current division rule,

$$\mathbf{I}' = (1.43\angle 5.91°)\left(\frac{2+j5}{2+j5+2-j2}\right) = 1.54\angle 37.24° \text{ A} \quad (\rightarrow)$$

Step II When the 20∠45° V source is acting alone (Fig. 3.45)

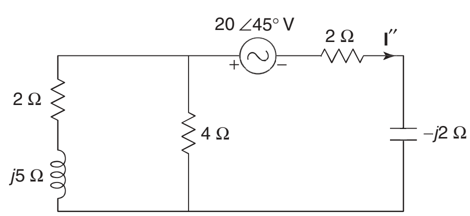


Fig. 3.45

$$\mathbf{Z}_T = (2-j2) + \frac{4(2+j5)}{4+2+j5} = 4.48\angle -8.84° \ \Omega$$

$$\mathbf{I}'' = \frac{20\angle 45°}{4.48\angle -8.84°} = 4.46\angle 53.84° \text{ A} \quad (\leftarrow) = -4.46\angle 53.84° \text{ A} \quad (\rightarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 1.54\angle 37.24 - 4.46\angle 53.84° = 3.01\angle -117.78° \text{ A}$$

$$\mathbf{V}_c = (-j2)\mathbf{I} = (-j2)\ (3.01\angle -117.78°) = 6.02\angle 152.22° \text{ V}$$

Example 3.25 *Find the node voltage $\boldsymbol{V}_2$ in the network of Fig. 3.46.*

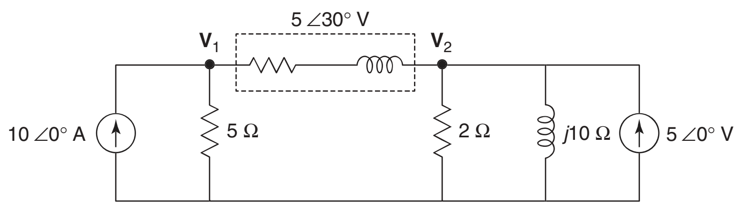


Fig. 3.46

Solution

Step I When the $10\angle 0°$ A source is acting alone (Fig. 3.47)

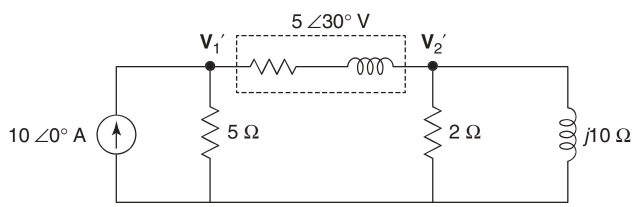


Fig. 3.47

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1'}{5} + \frac{\mathbf{V}_1' - \mathbf{V}_2'}{5\angle 30°} = 10\angle 0°$$

$$\left(\frac{1}{5} + \frac{1}{5\angle 30°}\right)\mathbf{V}_1' - \frac{1}{5\angle 30°}\mathbf{V}_2' = 10\angle 0°$$

$$(0.37 - j0.1)\mathbf{V}_1' - (0.17 - j0.1)\mathbf{V}_2' = 10\angle 0° \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2' - \mathbf{V}_1'}{5\angle 30°} + \frac{\mathbf{V}_2'}{2} + \frac{\mathbf{V}_2'}{j10} = 0$$

$$-\frac{1}{5\angle 30°}\mathbf{V}_1' + \left(\frac{1}{5\angle 30°} + \frac{1}{2} + \frac{1}{j10}\right)\mathbf{V}_2' = 0$$

$$-(0.17 - j0.1)\mathbf{V}_1' + (0.67 - j0.2)\mathbf{V}_2' = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.37 - j0.1 & -(0.17 - j0.1) \\ -(0.17 - j0.1) & 0.67 - j0.2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1' \\ \mathbf{V}_2' \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_2' = \frac{\begin{vmatrix} 0.37 - j0.1 & 10\angle 0^\circ \\ -(0.17 - j0.1) & 0 \end{vmatrix}}{\begin{vmatrix} 0.37 - j0.1 & -(0.17 - j0.1) \\ -(0.17 - j0.1) & 0.67 - j0.2 \end{vmatrix}} = 8.57\angle -3.36^\circ \text{ V}$$

Step II When the $5\angle 0^\circ$ A source is acting alone (Fig. 3.48)

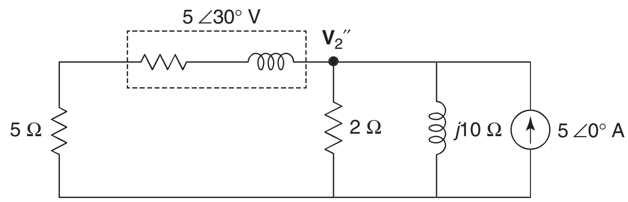


Fig. 3.48

$$\frac{\mathbf{V}_2''}{5\angle 30^\circ + 5} + \frac{\mathbf{V}_2''}{2} + \frac{\mathbf{V}_2''}{j10} = 5\angle 0^\circ$$

$$(0.61\angle -11.93^\circ)\mathbf{V}_2'' = 5\angle 0^\circ$$

$$\mathbf{V}_2'' = 8.2\angle 11.93^\circ \text{ V}$$

Step III By superposition theorem,

$$\mathbf{V}_2 = \mathbf{V}_2' + \mathbf{V}_2'' = 8.57\angle -3.36^\circ + 8.2\angle 11.93^\circ = 16.62\angle 4.12^\circ \text{ V}$$

Example 3.26 *Find current through inductor in the network of Fig. 3.49.*

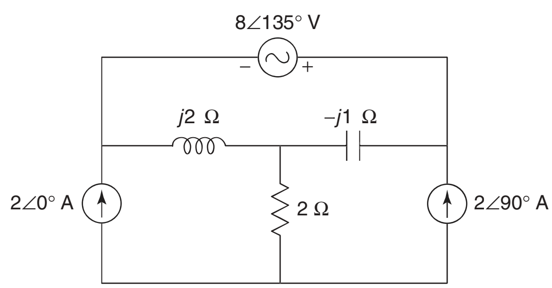


Fig. 3.49

Solution

Step I When the $8\angle 135^\circ$ V source is acting alone (Fig. 3.50)
Applying KVL to the mesh,

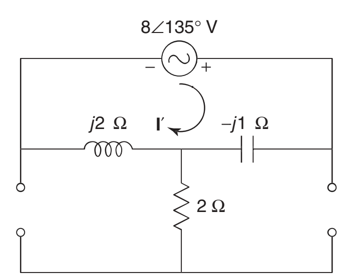


Fig. 3.50

$$8\angle 135^\circ - (-j1)\mathbf{I}' - j2\mathbf{I}' = 0$$

$$\mathbf{I}' = \frac{8\angle 135^\circ}{j1} = 8\angle 45^\circ \text{ A } (\leftarrow) = 8\angle -135^\circ \text{ A } (\rightarrow)$$

Step II When the $2\angle 0°$ A source is acting alone (Fig. 3.51)

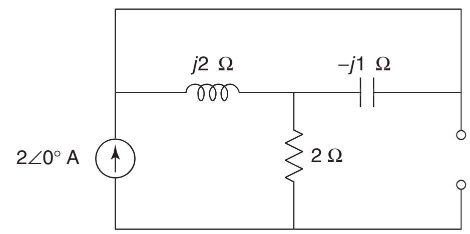


Fig. 3.51

The network can be redrawn as shown in Fig. 3.52.

By current division rule,

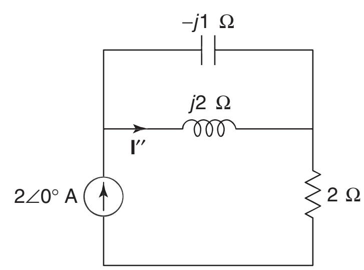


Fig. 3.52

$$\mathbf{I}'' = 2\angle 0°\left(\frac{-j1}{-j1+j2}\right) = 2\angle 0°\left(\frac{-j1}{j1}\right) = 2\angle 180° \text{ A}(\rightarrow)$$

Step III When the $2\angle 90°$ A source is acting alone (Fig. 3.53)

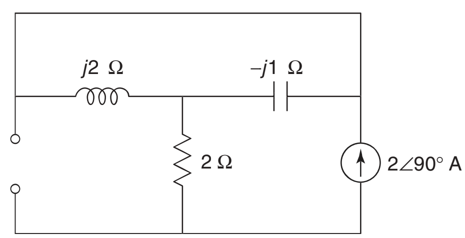


Fig. 3.53

The network can be redrawn as shown in Fig. 3.54.

By current division rule,

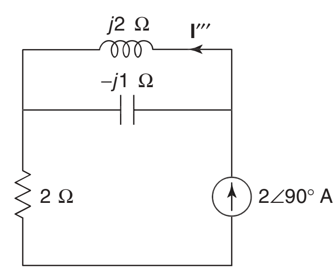


Fig. 3.54

$$\mathbf{I}''' = 2\angle 90°\left(\frac{-j1}{-j1+j2}\right) = 2\angle -90° \text{ A } (\leftarrow) = 2\angle 90° \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 8\angle -135° + 2\angle 180° + 2\angle 90° = 8.49\angle -154.47° \text{A}$$

Example 3.27 *Determine the source voltage $\mathbf{V}_S$ so that the current through 2 Ω resistor is zero in the network of Fig. 3.55.*

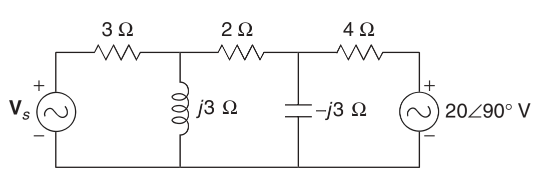


Fig. 3.55

Solution

Step I When the voltage source $\mathbf{V}_s$ is acting alone (Fig. 3.56)

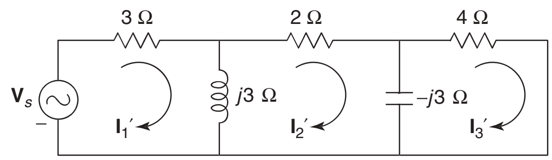


Fig. 3.56

Appling KVL to Mesh 1,

$$\mathbf{V}_s - 3\mathbf{I}_1' - j3(\mathbf{I}_1' - \mathbf{I}_2') = 0$$

$$(3 + j3)\mathbf{I}_1' - j3\mathbf{I}_2' = \mathbf{V}_s \qquad \ldots\text{(i)}$$

Appling KVL to Mesh 2,

$$-j3(\mathbf{I}_2' - \mathbf{I}_1') - 2\mathbf{I}_2' + j3(\mathbf{I}_2' - \mathbf{I}_3') = 0$$

$$-j3\mathbf{I}_1' + 2\mathbf{I}_2' + j3\mathbf{I}_3' = 0 \qquad \ldots\text{(ii)}$$

Appling KVL to Mesh 3,

$$-j3(\mathbf{I}_3' - \mathbf{I}_2') - 4\mathbf{I}_3' = 0$$

$$j3\mathbf{I}_2' + (4 - j3)\mathbf{I}_3' = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1' \\ \mathbf{I}_2' \\ \mathbf{I}_3' \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2' = \frac{\begin{vmatrix} 3+j3 & \mathbf{V}_s & 0 \\ -j3 & 0 & j3 \\ 0 & 0 & 4-j3 \end{vmatrix}}{\begin{vmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{vmatrix}} = \frac{(9 + j12)\,\mathbf{V}_s}{\Delta}$$

Step II When the 20 ∠90° V source is acting alone (Fig. 3.57)

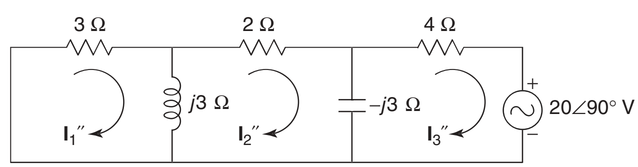


Fig. 3.57

Applying KVL to Mesh 1,

$$-3\mathbf{I}_1'' - j3(\mathbf{I}_1'' - \mathbf{I}_2'') = 0$$

$$(3 + j3)\,\mathbf{I}_1'' - j3\mathbf{I}_2'' = 0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j3(\mathbf{I}_2'' - \mathbf{I}_1'') - 2\mathbf{I}_2'' + j3(\mathbf{I}_2'' - \mathbf{I}_3'') = 0$$

$$-j3\mathbf{I}_1'' + 2\mathbf{I}_2'' + j3\mathbf{I}_3'' = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$j3(\mathbf{I}_3'' - \mathbf{I}_2'') - 4\mathbf{I}_3'' - 20\angle 90° = 0$$

$$j3\mathbf{I}_2'' + (4 - j3)\mathbf{I}_3'' = -20\angle 90° \quad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1'' \\ \mathbf{I}_2'' \\ \mathbf{I}_3'' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -20\angle 90° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2'' = \frac{\begin{vmatrix} 3+j3 & 0 & 0 \\ -j3 & 0 & j3 \\ 0 & -20\angle 90° & 4-j3 \end{vmatrix}}{\begin{vmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{vmatrix}} = \frac{-180 - j180}{\Delta}$$

Step III By superposition theorem,

$$\mathbf{I}_2 = \mathbf{I}_2' + \mathbf{I}_2'' = \frac{(9 + j12)\mathbf{V}_s + (-180 - j180)}{\Delta} = 0$$

$$(9 + j12)\mathbf{V}_s + (-180 - j180) = 0$$

$$(9 + j12)\mathbf{V}_s = 180 + j180$$

$$\mathbf{V}_s = 16.97\angle -8.13° \text{ V}$$

3.5 THEVENIN'S THEOREM

Thevenin's theorem gives us a method for simplifying a network. In Thevenin's theorem, *any linear network can be replaced by a voltage source* $\mathbf{V}_{\text{Th}}$ *in series with an impedance* $\mathbf{Z}_{\text{Th}}$.

Example 3.28 *Obtain Thevenin's equivalent network for the terminals A and B in Fig. 3.58.*

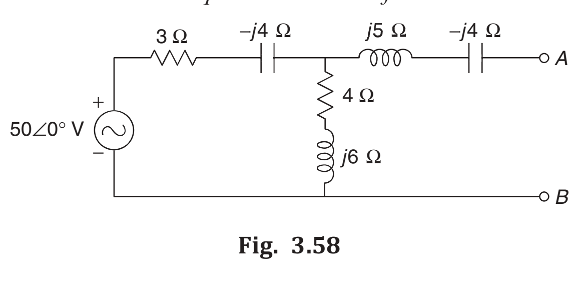


Fig. 3.58

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.59)

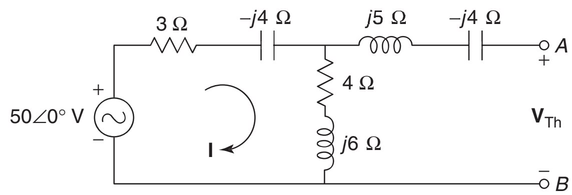


Fig. 3.59

Applying KVL to the mesh,

$$50\angle 0° - (3 - j4)\,\mathbf{I} - (4 + j6)\,\mathbf{I} = 0$$

$$\mathbf{I} = \frac{50\angle 0°}{(3 - j4) + (4 + j6)} = 6.87\angle -15.95°\ \text{A}$$

$$\mathbf{V}_{Th} = (4 + j6)\,\mathbf{I} = (4 + j6)\,(6.87\angle -15.95°) = 49.5\angle 40.35°\ \text{V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.60)

$$\mathbf{Z}_{Th} = (j5 - j4) + \frac{(3 - j4)(4 + j6)}{(3 - j4) + (4 + j6)} = 4.83\angle -1.13°\ \Omega$$

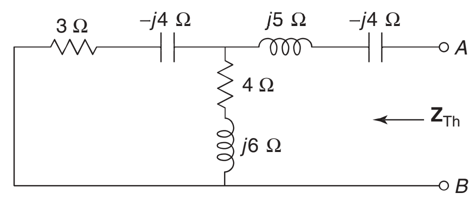


Fig. 3.60

Step III Thevenin's Equivalent Network (Fig. 3.61)

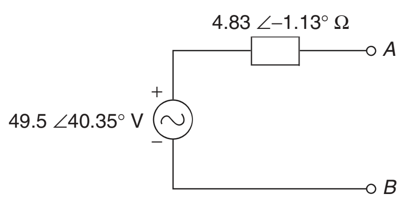


Fig. 3.61

Example 3.29 *Find Thevenin's equivalent network for Fig. 3.62.*

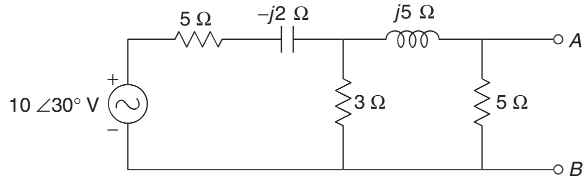


Fig. 3.62

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.63)

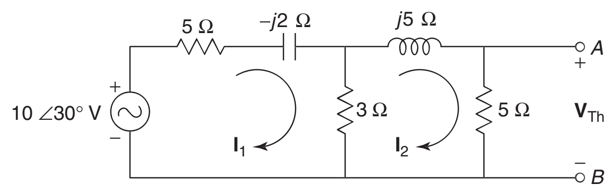


Fig. 3.63

Applying KVL to Mesh 1,

$$10\angle 30^\circ - (5 - j2)\,\mathbf{I}_1 - 3(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(8 - j2)\,\mathbf{I}_1 - 3\mathbf{I}_2 = 10\angle 30^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(\mathbf{I}_2 - \mathbf{I}_1) - j5\,\mathbf{I}_2 - 5\,\mathbf{I}_2 = 0$$

$$-3\mathbf{I}_1 + (8 + j5)\,\mathbf{I}_2 = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form;

$$\begin{bmatrix} 8-j2 & -3 \\ -3 & 8+j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 30^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 8-j2 & 10\angle 30^\circ \\ -3 & 0 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 \\ -3 & 8+j5 \end{vmatrix}} = 0.433\angle 9.7^\circ \text{ A}$$

$$\mathbf{V}_{Th} = 5\mathbf{I}_2 = 5(0.433\angle 9.7^\circ) = 2.16\angle 9.7^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.64)

$$\mathbf{Z}_{Th} = \left[\left\{\frac{(5-j2)3}{5-j2+3}\right\} + j5\right] \parallel 5$$

$$= [1.94 - j0.265 + j5] \parallel 5 = (1.94 + j4.735) \parallel 5$$

$$= \frac{(1.94 + j4.735)5}{6.94 + j4.735} = 3.04\angle 33.4^\circ\ \Omega$$

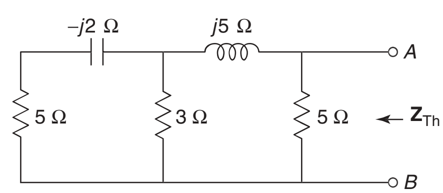


Fig. 3.64

Step III Thevenin's equivalent Network (Fig. 3.65)

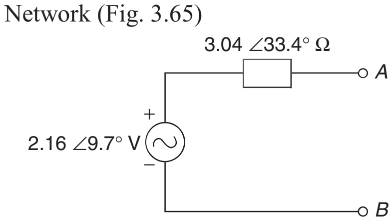


Fig. 3.65

Example 3.30 *Obtain Thevenin's equivalent network for Fig. 3.66.*

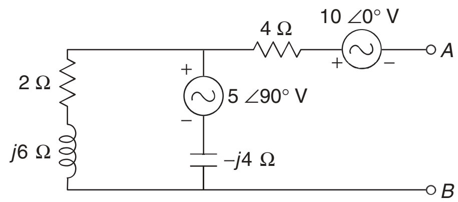


Fig. 3.66

Solution

Step I Calculation of $\mathbf{V}_{Th}$

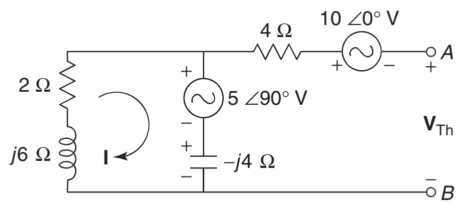


Fig. 3.67

Applying KVL to the mesh,

$$(2+j6-j4)\mathbf{I}-5\angle 90^\circ = 0$$

$$\mathbf{I} = \frac{5\angle 90^\circ}{2+j2} = 1.77\angle 45^\circ \text{ A}$$

$$\mathbf{V}_{Th} = (-j4)\,\mathbf{I} + 5\angle 90^\circ - 10\angle 0^\circ = (4\angle -90^\circ)(1.77\angle 45^\circ) + 5\angle 90^\circ - 10\angle 0^\circ = 18\angle 146.31^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.67)

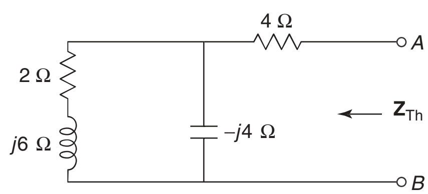


Fig. 3.68

$$\mathbf{Z}_{Th} = 4 + \frac{(2+j6)(-j4)}{2+j2} = 11.3\angle -44.93^\circ\ \Omega$$

Step III Thevenin's Equivalent Network

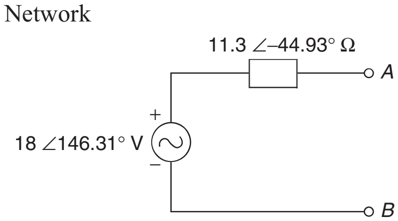


Fig. 3.69

Example 3.31 *Obtain Thevenin's equivalent network for Fig. 3.70.*

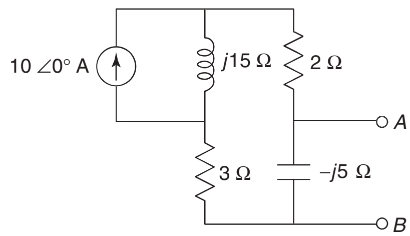


Fig. 3.70

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.71)
By current division rule,

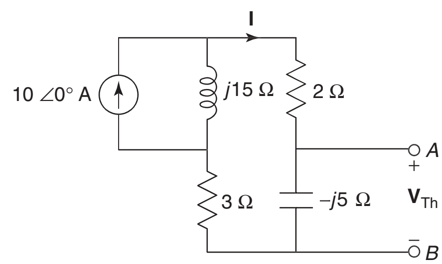


Fig. 3.71

$$\mathbf{I} = \frac{(10\angle 0°)(j15)}{5 - j5 + j15} = 13.42\angle 26.57° \text{ A}$$

$$\mathbf{V}_{Th} = (-j5)\,\mathbf{I}$$
$$= (5\angle -90°)(13.42\angle 26.57°) = 67.08\angle -63.43° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.72)

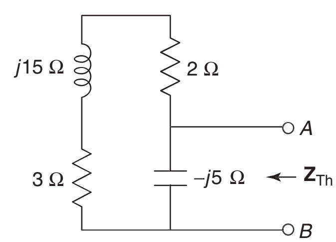


Fig. 3.72

$$\mathbf{Z}_{Th} = \frac{(-j5)(5 + j15)}{-j5 + 5 + j15} = 7.07\angle -81.86° \ \Omega$$

Step III Thevenin's Equivalent Network

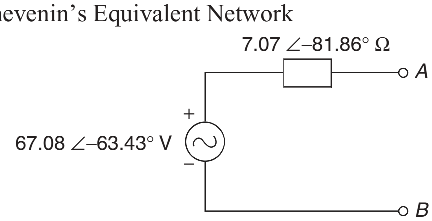


Fig. 3.73

Example 3.32 *Obtain Thevenin's equivalent network for Fig. 3.74.*

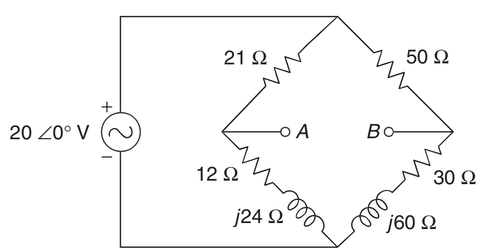


Fig. 3.74

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.75)

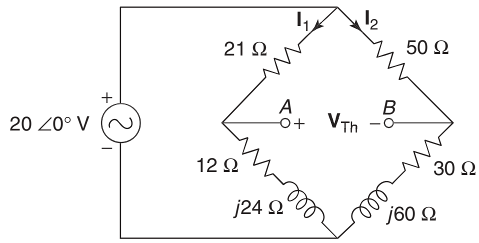


Fig. 3.75

$$\mathbf{I}_1 = \frac{20\angle 0^\circ}{21+12+j24} = 0.49\angle -36.02^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{20\angle 0^\circ}{80+j60} = 0.2\angle -36.86^\circ \text{ A}$$

$$\begin{aligned}\mathbf{V}_{Th} &= (12+j24)\,\mathbf{I}_1 - (30+j60)\,\mathbf{I}_2\\ &= (26.83\angle 63.43^\circ)(0.49\angle -36.02^\circ) - (67.08\angle 63.43^\circ)(0.2\angle -36.86^\circ)\\ &= 0.33\angle 171.12^\circ \text{ V}\end{aligned}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.76)

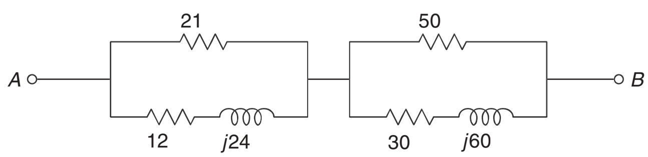


Fig. 3.76

$$\mathbf{Z}_{Th} = \frac{21(12+j24)}{33+j24} + \frac{50(30+j60)}{80+j60} = 47.4\angle 26.8^\circ \ \Omega$$

Step III Thevenin's Equivalent Network

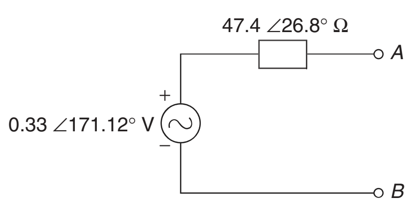


Fig. 3.77

Example 3.33 *Find Thevenin's equivalent network across terminals A and B for Fig. 3.78.*

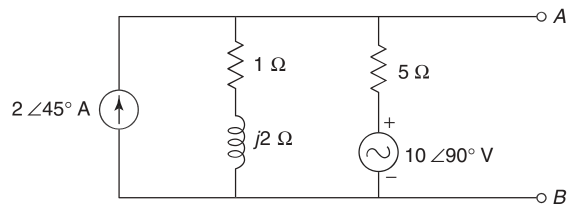


Fig. 3.78

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.79)

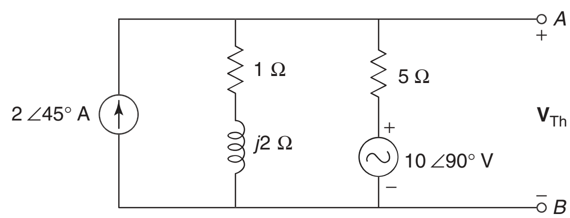


Fig. 3.79

Applying KCL at the node,

$$\frac{\mathbf{V}_{Th}}{1+j2}+\frac{\mathbf{V}_{Th}-10\angle 90^\circ}{5}=2\angle 45^\circ$$

$$\left(\frac{1}{1+j2}+\frac{1}{5}\right)\mathbf{V}_{Th}=2\angle 45^\circ+2\angle 90^\circ$$

$$(0.57\angle -45^\circ)\mathbf{V}_{Th}=3.7\angle 67.5^\circ$$

$$\mathbf{V}_{Th}=6.49\angle 112.5^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.80)

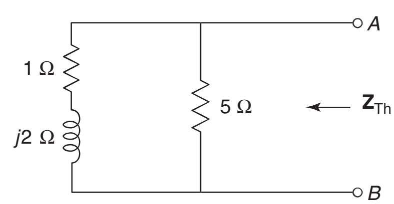


Fig. 3.80

$$\mathbf{Z}_{Th}=\frac{5(1+j2)}{5+1+j2}=1.77\angle 45^\circ\ \Omega$$

Step III Thevenin's Equivalent Network (Fig. 3.81)

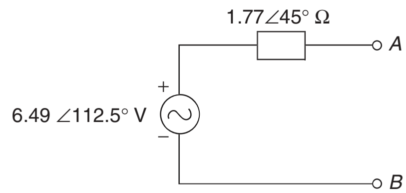


Fig. 3.81

Example 3.34 *Find the current through the* $(5 + j2)\ \Omega$ *impedance in the network of Fig. 3.82.*

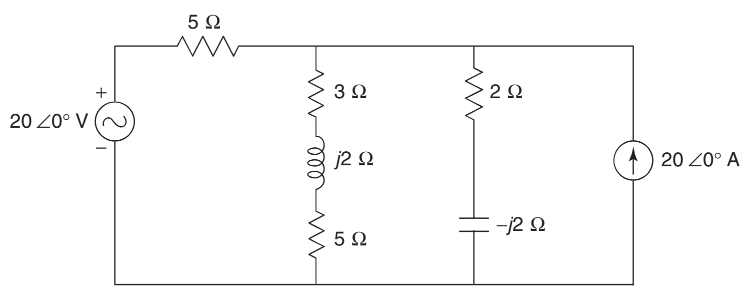


Fig. 3.82

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.83)

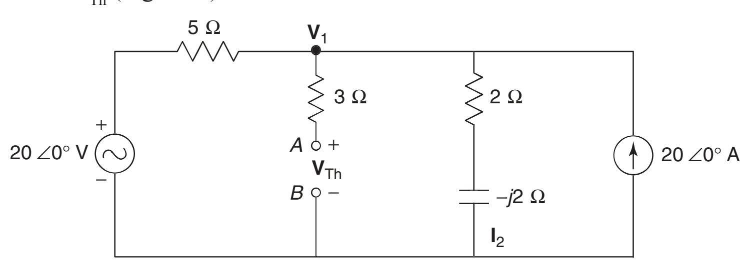


Fig. 3.83

Applying KCL at the node,

$$\frac{\mathbf{V}_1 - 20\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{2 - j2} = 20\angle 0^\circ$$

$$\left(\frac{1}{5} + \frac{1}{2 - j2}\right)\mathbf{V}_1 = 20\angle 0^\circ + 4\angle 0^\circ$$

$$0.51\angle 29.05^\circ\ \mathbf{V}_1 = 24\angle 0^\circ$$

$$\mathbf{V}_1 = 47.06\angle -29.05^\circ \text{ V}$$

$$\mathbf{V}_{Th} = \mathbf{V}_1 = 47.06\angle -29.05^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.84)

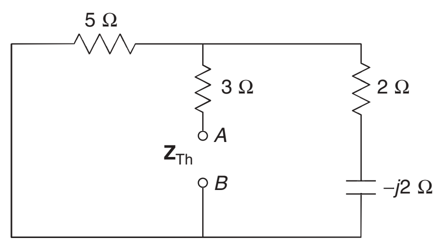


Fig. 3.84

$$\mathbf{Z}_{Th} = 3 + \frac{5(2 - j2)}{5 + 2 - j2} = 4.79\angle{-11.35°}\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 3.85)

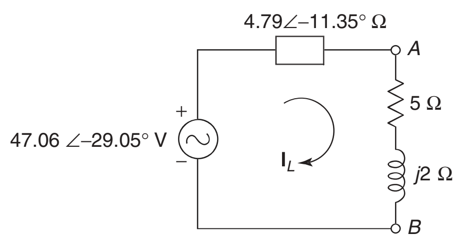


Fig. 3.85

$$\mathbf{I}_L = \frac{47.06\angle{-29.05°}}{4.79\angle{-11.35°} + 5 + j2} = 4.73\angle{-39.96°}\ \text{A}$$

Example 3.35 *Find the current through the 5 Ω resistor in the network of Fig. 3.86.*

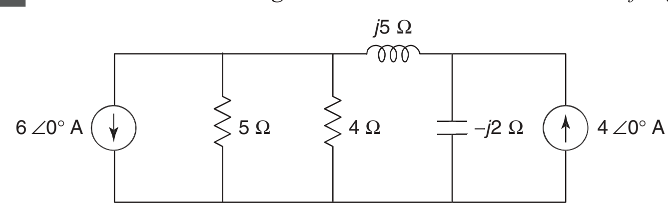


Fig. 3.86

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.87)

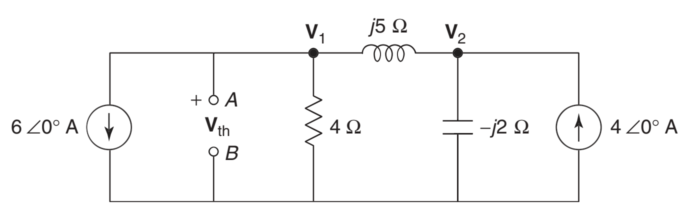


Fig. 3.87

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1}{4}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j5}+6\angle 0^\circ=0$$

$$\left(\frac{1}{4}+\frac{1}{j5}\right)\mathbf{V}_1-\frac{1}{j5}\mathbf{V}_2=-6\angle 0^\circ$$

$$(0.25-j0.2)\mathbf{V}_1+j0.2\mathbf{V}_2=-6\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{j5}+\frac{\mathbf{V}_2}{-j2}=4\angle 0^\circ$$

$$\left(-\frac{1}{j5}\right)\mathbf{V}_1+\left(\frac{1}{j5}-\frac{1}{j2}\right)\mathbf{V}_2=4\angle 0^\circ$$

$$j0.2\,\mathbf{V}_1+j0.3\,\mathbf{V}_2=4\angle 0^\circ \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix}0.25-j0.2 & j0.2\\ j0.2 & j0.3\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}=\begin{bmatrix}-6\angle 0^\circ\\ 4\angle 0^\circ\end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1=\frac{\begin{vmatrix}-6\angle 0^\circ & j0.2\\ 4\angle 0^\circ & j0.3\end{vmatrix}}{\begin{vmatrix}0.25-j0.2 & j0.2\\ j0.2 & j0.3\end{vmatrix}}=20.8\angle -126.87^\circ \text{ V}$$

$$\mathbf{V}_{\text{Th}}=\mathbf{V}_1=20.8\angle -126.87^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 3.88)

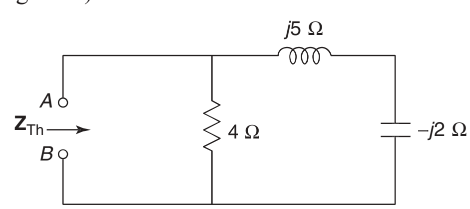


Fig. 3.88

$$\mathbf{Z}_{\text{Th}}=\frac{4(-j2+j5)}{4-j2+j5)}=2.4\angle 53.13^\circ\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 3.89)

$$\mathbf{I}_L=\frac{20.8\angle -126.87^\circ}{2.4\angle 53.13^\circ+5}=3.1\angle -143.47^\circ \text{ A}$$

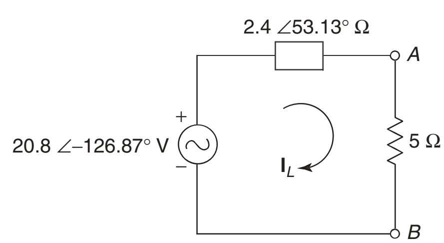


Fig. 3.89

Example 3.36 *In the network of Fig. 3.90, find the current through the 10 Ω resistor.*

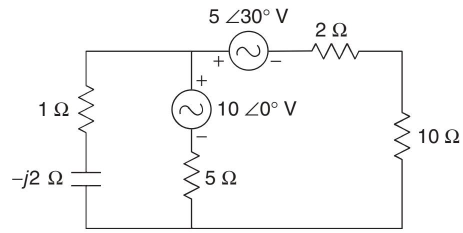


Fig. 3.90

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.91)

Applying KVL to the mesh,

$$j2\mathbf{I} - 1\mathbf{I} - 10\angle 0° - 5\mathbf{I} = 0$$

$$(j2-6)\mathbf{I} = 10\angle 0°$$

$$\mathbf{I} = 1.58\angle -161.57° \text{ A}$$

Writing $\mathbf{V}_{Th}$ equation,

$$5\mathbf{I} + 10\angle 0° - 5\angle 30° - 0 - \mathbf{V}_{Th} = 0$$

$$5(1.58\angle -161.57°) - 10\angle 0° - 5\angle 30° - \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Th} = 5.32\angle -110.06° \text{ V}$$

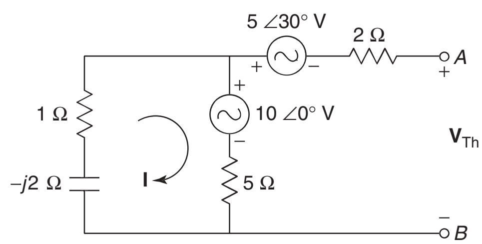


Fig. 3.91

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.92)

$$\mathbf{Z}_{Th} = 2 + \frac{5(1-j2)}{5+1-j2} = 3.48\angle -21.04° \ \Omega$$

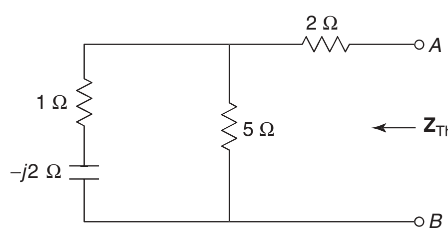


Fig. 3.92

Step III Calculation of $\mathbf{I}_L$ (Fig. 3.93)

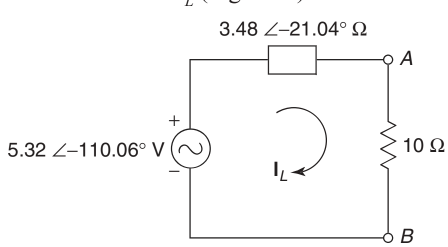


Fig. 3.93

$$\mathbf{I}_L = \frac{5.32\angle -110.06°}{3.48\angle -21.04° + 10} = 0.4\angle -104.67° \text{ A}$$

Example 3.37 *Find the current through $(4+j6)\ \Omega$ impedance in the network of Fig. 3.94.*

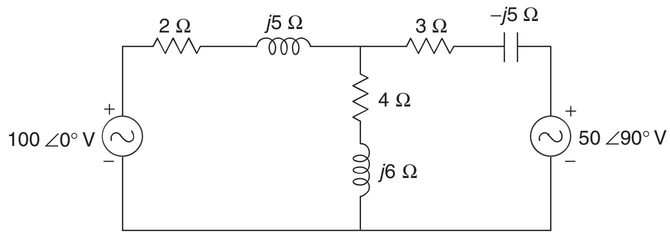


Fig. 3.94

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.95)

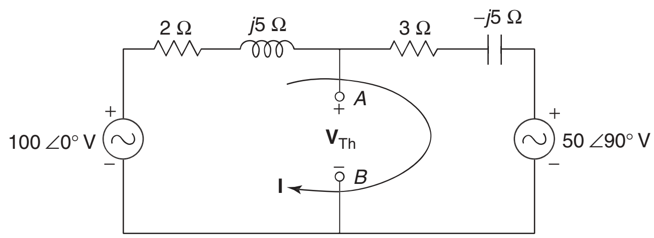


Fig. 3.95

Applying KVL to the mesh,

$$100\angle 0° - 2\mathbf{I} - j5\mathbf{I} - 3\mathbf{I} + j5\mathbf{I} - 50\angle 90° = 0$$

$$\mathbf{I} = 22.36\angle -26.57°\ \text{A}$$

Writing $\mathbf{V}_{Th}$ equation,

$$\mathbf{V}_{Th} - 3\mathbf{I} + j5\mathbf{I} - 50\angle 90° = 0$$

$$\mathbf{V}_{Th} - (3 - j5)(22.36\angle -26.57°) - 50\angle 90° = 0$$

$$\mathbf{V}_{Th} = 80.61\angle -82.88°\ \text{V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.96)

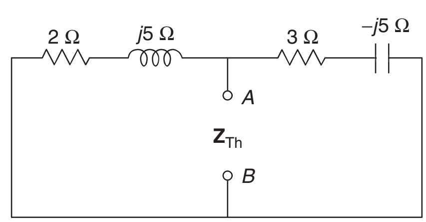


Fig. 3.96

$$\mathbf{Z}_{Th} = \frac{(2+j5)(3-j5)}{2+j5+3-j5} = 6.28\angle 9.16°\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 3.97)

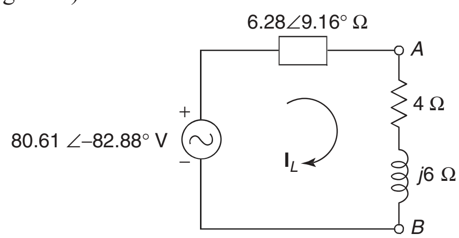


Fig. 3.97

$$\mathbf{I}_L = \frac{80.61\angle -82.88^\circ}{6.28\angle 9.16^\circ + 4 + j6} = 6.52\angle -117.34^\circ \text{ A}$$

Example 3.38 *Obtain Thevenin's equivalent network across terminals A and B in Fig. 3.98.*

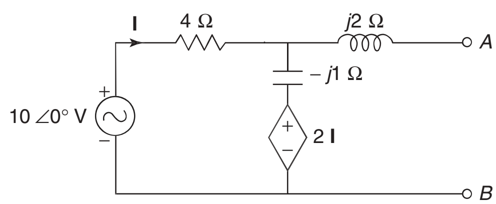


Fig. 3.98

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 3.99)

Applying KVL to the mesh,

$$10\angle 0^\circ - 4\mathbf{I} + j1\mathbf{I} - 2\mathbf{I} = 0$$

$$\mathbf{I} = 1.64\angle 9.46^\circ \text{ A}$$

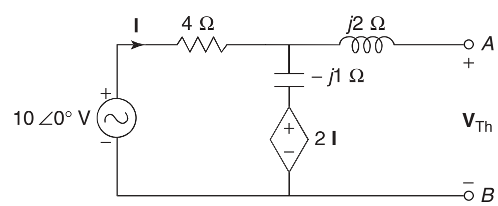


Fig. 3.99

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0^\circ - 4\mathbf{I} - 0 - \mathbf{V}_{\text{Th}} = 0$$

$$10\angle 0^\circ - 4(1.64\angle 9.46^\circ) - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 3.69\angle -17^\circ \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 3.100)

From Fig. 3.100,

$$\mathbf{I} = \mathbf{I}_1$$

Applying KVL to Mesh 1,

$$10\angle 0^\circ - 4\mathbf{I}_1 + j1(\mathbf{I}_1 - \mathbf{I}_2) - 2\mathbf{I} = 0$$

$$10\angle 0^\circ - 4\mathbf{I}_1 + j1\mathbf{I}_1 - j1\mathbf{I}_2 - 2\mathbf{I}_1 = 0$$

$$(6 - j1)\mathbf{I}_1 + j1\mathbf{I}_2 = 10\angle 0^\circ \quad \ldots\text{(i)}$$

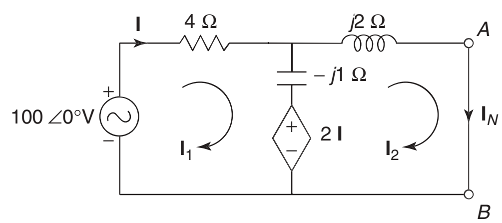


Fig. 3.100

Applying KVL to Mesh 2,

$$2\mathbf{I} + j1(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 = 0$$

$$2\mathbf{I}_1 + j1\mathbf{I}_2 - j1\mathbf{I}_1 - j_2\mathbf{I}_2 = 0$$

$$(2 - j1)\mathbf{I}_1 - j1\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 6-j1 & j1 \\ 2-j1 & -j1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 6-j1 & 10\angle 0° \\ 2-j1 & 0 \end{vmatrix}}{\begin{vmatrix} 6-j1 & j1 \\ 2-j1 & -j1 \end{vmatrix}} = 2.71\angle{-102.53°}\text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 2.71\angle{-102.53°}\text{ A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{3.69\angle -17°}{2.71\angle -102.53°} = 1.36\angle 85.53°\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.101)

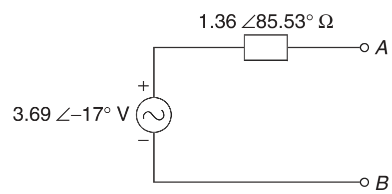


Fig. 3.101

Example 3.39 *Find Thevenin's equivalent network across terminals A and B for Fig. 3.102.*

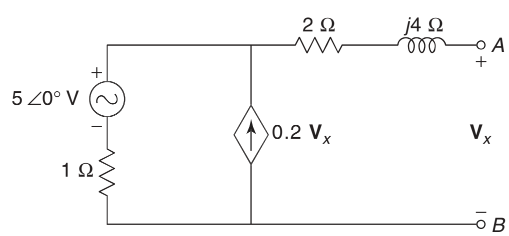


Fig. 3.102

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 3.103)

From Fig. 3.103,

$$\mathbf{I} = -0.2\mathbf{V}_x \qquad \text{... (i)}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$-\mathbf{I} + 5\angle 0° - 0 - \mathbf{V}_x = 0$$

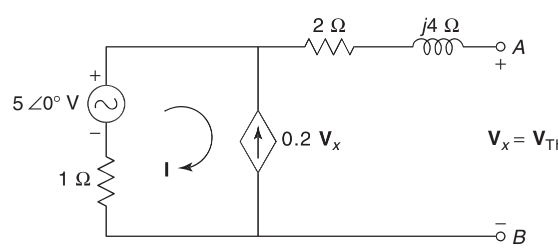


Fig. 3.103

$$0.2\mathbf{V}_x + 5\angle 0° - \mathbf{V}_x = 0$$
$$\mathbf{V}_x = 6.25\angle 0° \text{ V}$$
$$\mathbf{V}_{\text{Th}} = \mathbf{V}_x = 6.25\angle 0° \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 3.104)

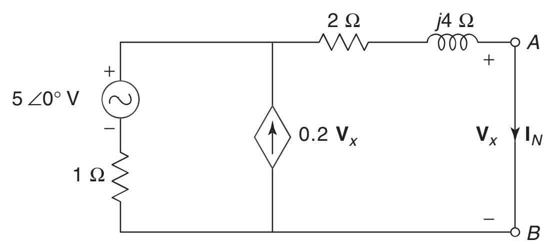


Fig. 3.104

From Fig. 3.104,

$$\mathbf{V}_x = 0$$

The dependent source depends on the controlling variable $\mathbf{V}_x$. When $\mathbf{V}_x = 0$, the dependent source vanishes, i.e. $0.2\,\mathbf{V}_x = 0$ as shown in Fig. 3.105.

$$\mathbf{I}_N = \frac{5\angle 0°}{1+2+j4} = 1\angle -53.13° \text{ A}$$

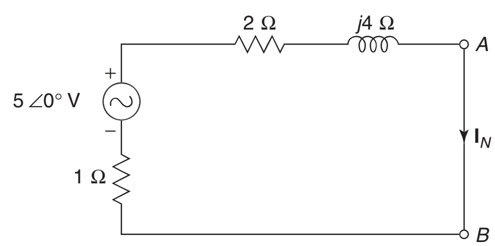


Fig. 3.105

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{6.25\angle 0°}{1\angle -53.13°} = 6.25\angle 53.13° \ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.106)

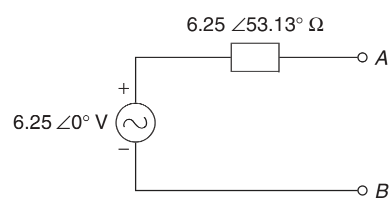


Fig. 3.106

3.6 NORTON'S THEOREM

Norton's theorem states that *any linear network can be replaced by a current source* $\mathbf{I}_N$ *parallel with an impedance* $\mathbf{Z}_N$ *where* $\mathbf{I}_N$ *is the current flowing through the short-circuited path placed across the terminals*.

Example 3.40 *Obtain Norton's equivalent network between terminals A and B as shown in Fig. 3.107.*

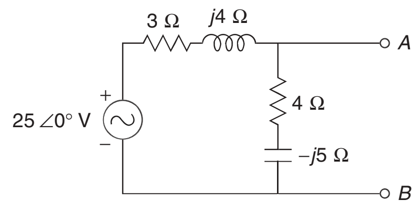


Fig. 3.107

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 3.108)

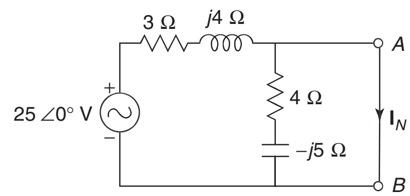


Fig. 3.108

When a short circuit is placed across $(4 - j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 3.109.

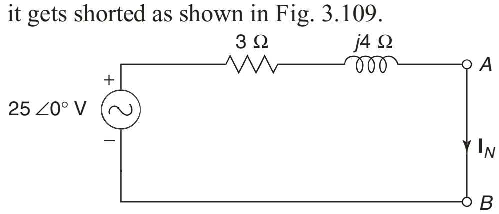


Fig. 3.109

$$\mathbf{I}_N = \frac{25\angle 0^\circ}{3 + j4} = 5\angle -53.13^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 3.110)

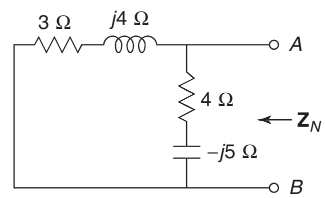


Fig. 3.110

$$\mathbf{Z}_N = \frac{(3 + j4)(4 - j5)}{3 + j4 + 4 - j5} = 4.53\angle 9.92^\circ\ \Omega$$

Step III Norton's Equivalent Network

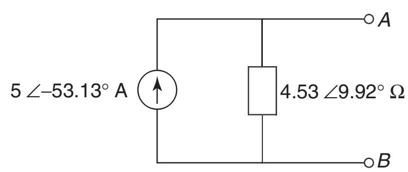


Fig. 3.111

Example 3.41 *Obtain Norton's equivalent network at the terminals A and B in Fig. 3.112.*

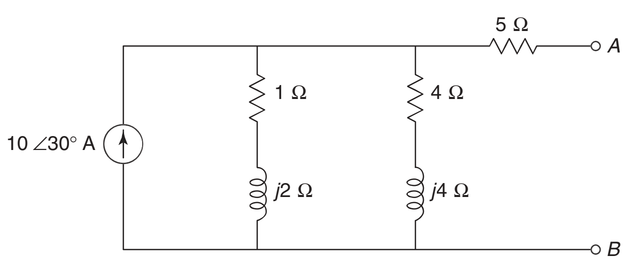


Fig. 3.112

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 3.113)

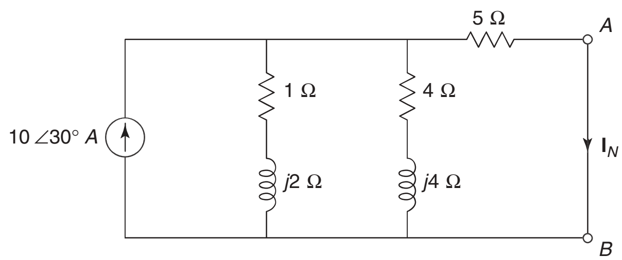


Fig. 3.113

By series-parallel reduction technique (Fig. 3.114)

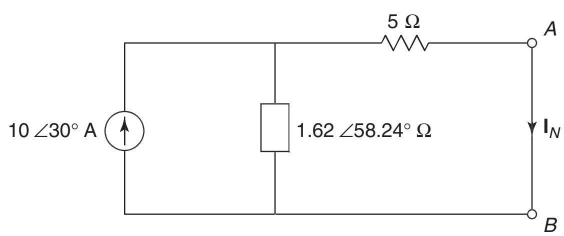


Fig. 3.114

$$\mathbf{I}_N = (10\angle 30°)\left(\frac{1.62\angle 58.24°}{1.62\angle 58.24° + 5}\right) = 2.69\angle 75° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 3.115)

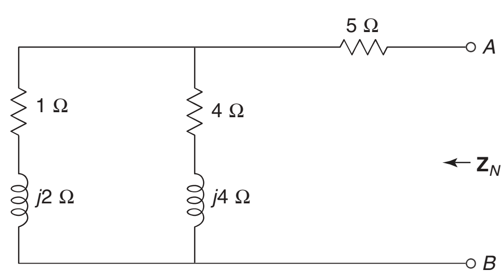


Fig. 3.115

$$\mathbf{Z}_N = 5 + \frac{(1+j2)(4+j4)}{1+j2+4+j4} = 6.01\angle 13.24° \ \Omega$$

Step III Norton's Equivalent Network (Fig. 3.116)

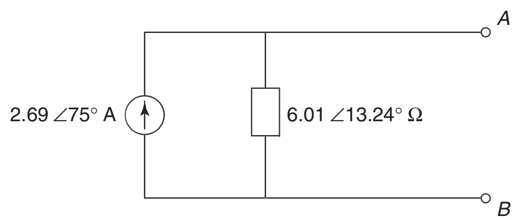


Fig. 3.116

Example 3.42 *Find Norton's equivalent network across terminals A and B in Fig. 3.117.*

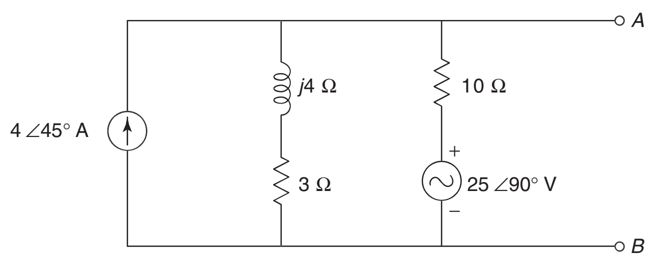


Fig. 3.117

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 3.118)

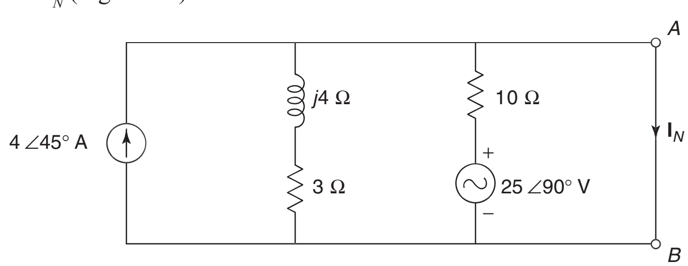


Fig. 3.118

When a short circuit is placed across the $(3 + j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 3.119.

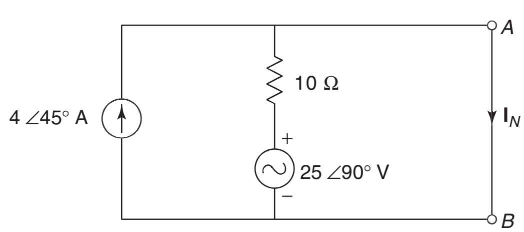


Fig. 3.119

By source transformation, the network is redrawn as shown in Fig. 3.120.

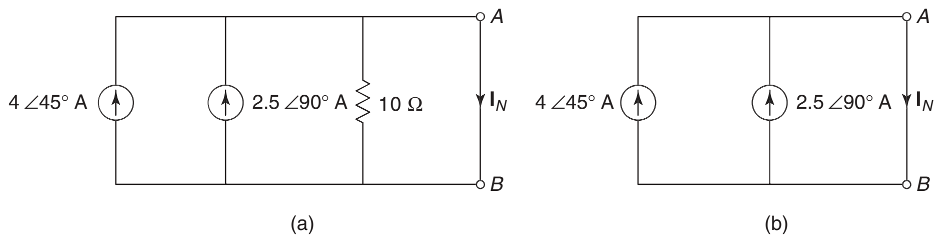


Fig. 3.120

$$I_N = 4\angle 45° + 2.5\angle 90° = 6.03\angle 62.04° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 3.121)

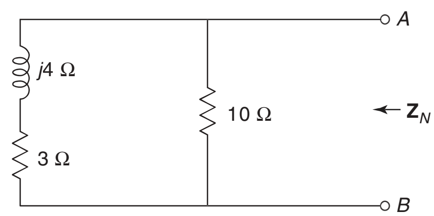


Fig. 3.121

$$\mathbf{Z}_N = \frac{10(3+j4)}{10+3+j4} = 3.68\angle 36.03° \ \Omega$$

Step III Norton's Equivalent Network (Fig. 3.122)

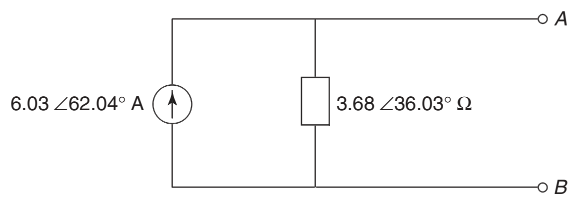


Fig. 3.122

Example 3.43 *Obtain the Norton's equivalent network for Fig. 3.123.*

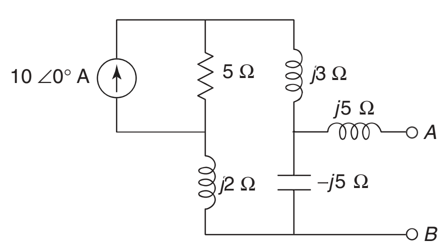


Fig. 3.123

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 3.124)

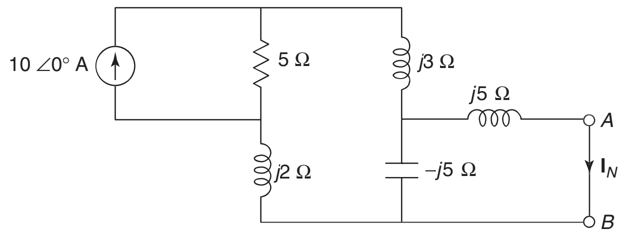


Fig. 3.124

By source transformation, the network can be redrawn as shown in Fig. 3.125.

Writing KVL equations in matrix form,

$$\begin{bmatrix} 5 & j5 \\ j5 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5 & 50\angle 0° \\ j5 & 0 \end{vmatrix}}{\begin{vmatrix} 5 & j5 \\ j5 & 0 \end{vmatrix}} = 10\angle -90° \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 10\angle -90° \text{ A}$$

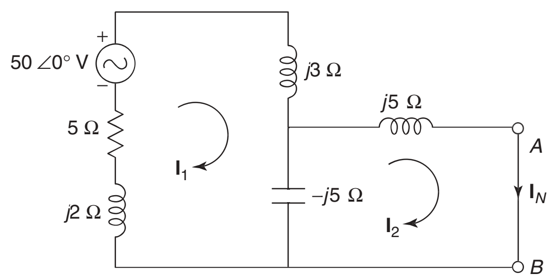


Fig. 3.125

Step II Calculation of $\mathbf{Z}_N$ (Fig. 3.126)

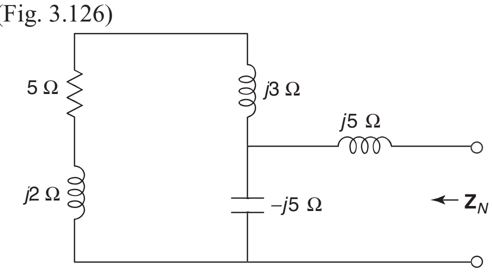


Fig. 3.126

$$\mathbf{Z}_N = j5 + \frac{(5+j5)(-j5)}{5+j5-j5} = 5\ \Omega$$

Step III Norton's Equivalent Network (Fig. 3.127)

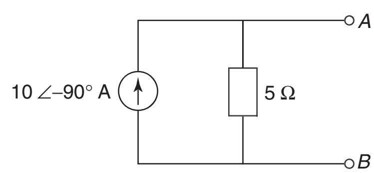


Fig. 3.127

Example 3.44 *Obtain the Norton's equivalent network for Fig. 3.128.*

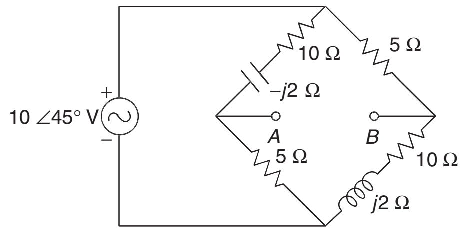


Fig. 3.128

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 3.129)

Writing KVL equations in matrix form,

$$\begin{bmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \\ 0 \end{bmatrix}$$

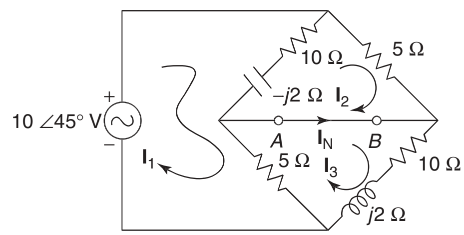


Fig. 3.129

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 15-j2 & 10\angle 45^\circ & -5 \\ -10+j2 & 0 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}}{\begin{vmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}} = 1\angle 41.28^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 15-j2 & -10+j2 & 10\angle 45^\circ \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}} = 0.49\angle 37.41^\circ \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_3 - \mathbf{I}_2 = 0.49\angle 37.41 - 1\angle 41.28^\circ \quad 0.51 \quad -135^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 3.130)

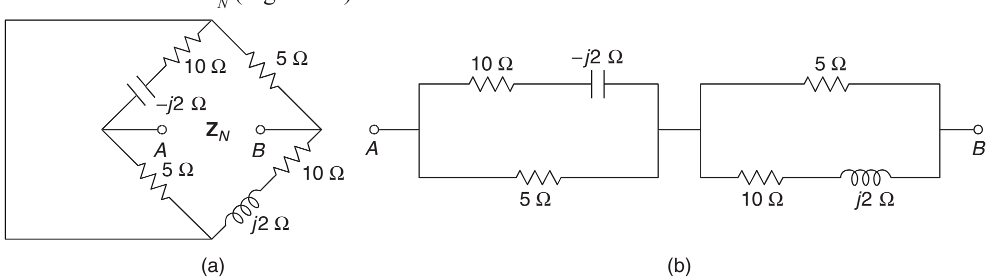


Fig. 3.130

$$\mathbf{Z}_N = \frac{5(10 - j2)}{5 + 10 - j2} + \frac{5(10 + j2)}{5 + 10 + j2} = 6.72\ \Omega$$

Step III Norton's Equivalent Network (Fig. 3.131)

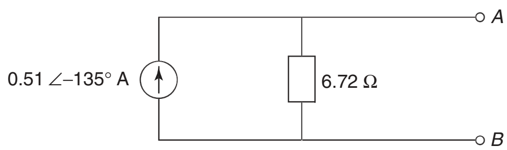


Fig. 3.131

Example 3.45 *Find the current through the 8 Ω resistor in the Network of Fig. 3.132.*

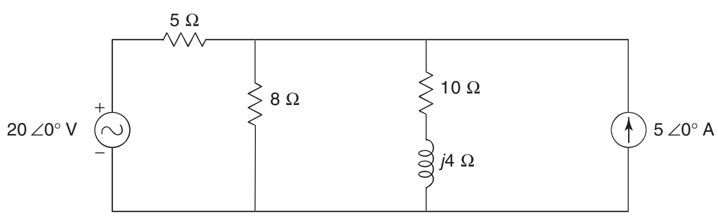


Fig. 3.132

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 3.133)

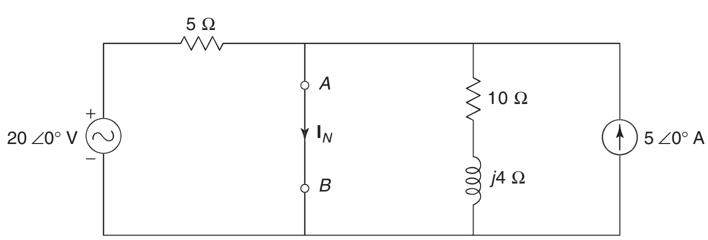


Fig. 3.133

When a short circuit is placed across the $(10 + j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 3.134.

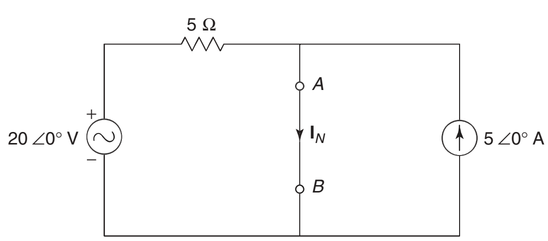


Fig. 3.134

By source transformation, the network is redrawn as shown in Fig. 3.135.

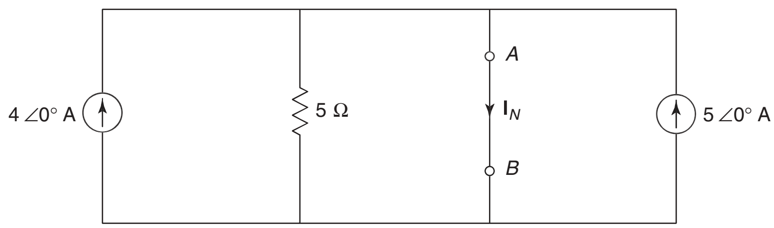


Fig. 3.135

$$\mathbf{I}_N = 4\angle 0° + 5\angle 0° = 9\angle 0° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 3.136)

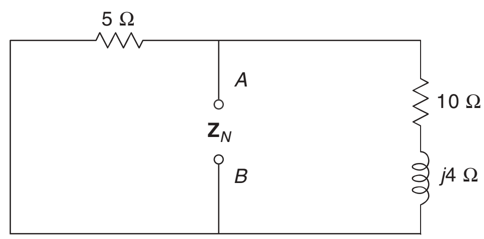


Fig. 3.136

$$\mathbf{Z}_N = \frac{5(10 + j4)}{5 + 10 + j4} = 3.47\angle 6.87° \ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 3.137)

$$\mathbf{I}_L = \frac{9\angle 0°}{3.47\angle 6.87° + 8} = 0.79\angle -2.08° \text{ A}$$

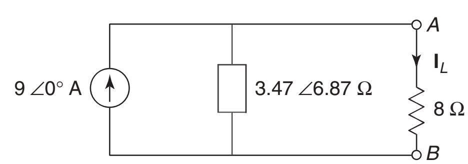


Fig. 3.137

Example 3.46 *Obtain Norton's equivalent network across the terminals A and B in Fig. 3.138.*

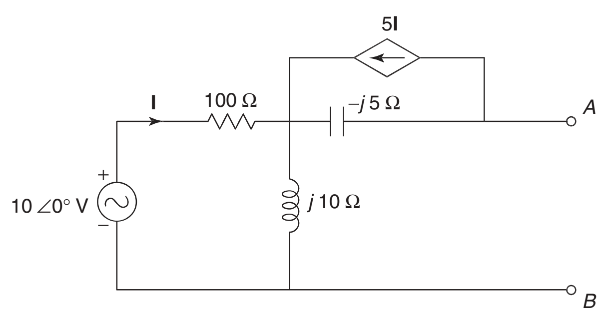


Fig. 3.138

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.139)

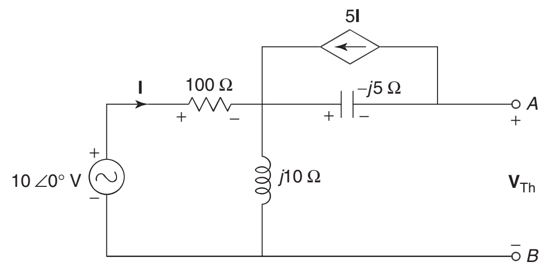


Fig. 3.139

$$\mathbf{I} = \frac{10\angle 0^\circ}{100 + j10} = 0.1\angle -5.71^\circ \text{ A}$$

Writing $\mathbf{V}_{Th}$ equation,

$$10\angle 0^\circ - 100\,\mathbf{I} - (-j5)(5\mathbf{I}) - \mathbf{V}_{Th} = 0$$

$$10\angle 0^\circ - 100(0.1\angle -5.71^\circ) + (j5)(5)(0.1\angle -5.71^\circ) - \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Th} = 3.5\angle 85.1^\circ \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 3.140)

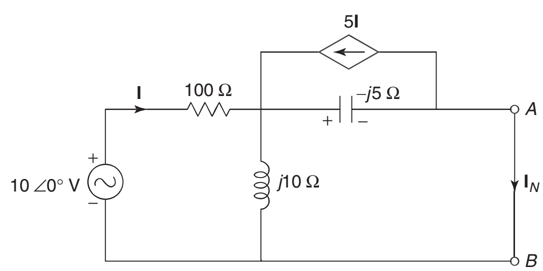


Fig. 3.140

By source transformation, the network is redrawn as shown in Fig. 3.141.

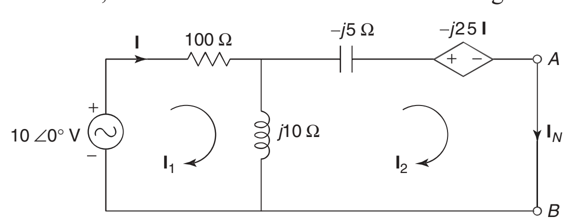


Fig. 3.141

From Fig. 3.141,

$$\mathbf{I} = \mathbf{I}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$10\angle 0° - 100\,\mathbf{I}_1 - j10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$(100 + j10)\,\mathbf{I}_1 - j10\mathbf{I}_2 = 10\angle 0° \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-j10(\mathbf{I}_2 - \mathbf{I}_1) + j5\mathbf{I}_2 + j25\mathbf{I} = 0$$
$$-j10\mathbf{I}_2 + j10\mathbf{I}_1 + j5\mathbf{I}_2 + j25\mathbf{I}_1 = 0$$
$$j35\mathbf{I}_1 - j5\mathbf{I}_2 = 0 \qquad \text{...(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 100 + j10 & -j10 \\ j35 & -j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 100 + j10 & 10\angle 0° \\ j35 & 0 \end{vmatrix}}{\begin{vmatrix} 100 + j10 & -j10 \\ j35 & -j5 \end{vmatrix}} = 0.6\angle 30.96° \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 0.6\angle 30.96° \text{ A}$$

Step III Calculation of $\mathbf{Z}_N$

$$\mathbf{Z}_N = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{3.5\angle 85.1°}{0.6\angle 30.96°} = 5.83\angle 54.14°\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 3.142)

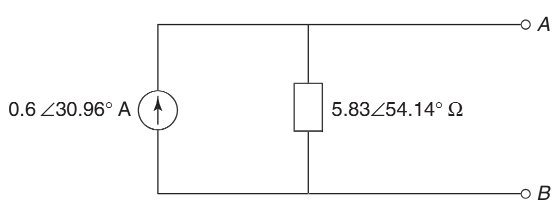


Fig. 3.142

3.7 MAXIMUM POWER TRANSFER THEOREM

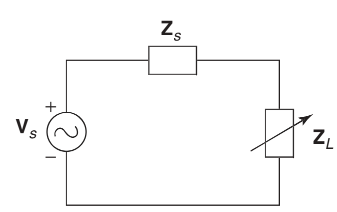


Fig. 3.143

This theorem is used to determine the value of load impedance for which the source will transfer maximum power.

Consider a simple network as shown in Fig. 3.143. There are three possible cases for load impedance $\mathbf{Z}_L$.

Case (i) When the load impedance is variable resistance (Fig. 3.144)

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L} = \frac{\mathbf{V}_s}{R_s + jX_s + R_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

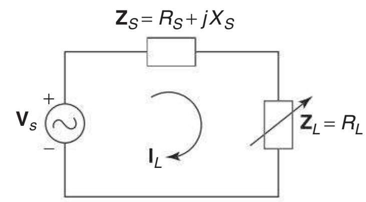


Fig. 3.144 *Purely resistive load*

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_s|^2 R_L}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

For power to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{\{(R_s + R_L)^2 + X_s^2\} - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + X_s^2]^2}\right] = 0$$

$$(R_s + R_L)^2 + X_s^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + X_s^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + X_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + X_s^2$$

$$R_L = \sqrt{R_s^2 + X_s^2} = |Z_s|$$

Hence, load resistance R_L should be equal to the magnitude of the source impedance for maximum power transfer.

Case (ii) When the load impedance is a complex impedance with variable resistance and variable reactance (Fig. 3.145)

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

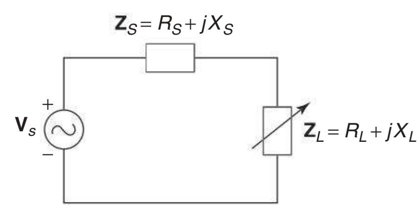


Fig. 3.145 *Complex impedance load*

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_S|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

For maximum value of P_L, denominator of the equation should be small, ie. $X_L = -X_s$.

$$P_L = \frac{|\mathbf{V}_S|^2 R_L}{(R_s + R_L)^2}$$

Differentiating the above equation w.r.t. R_L and equating to zero,

$$\frac{dP_L}{dR_L} = |\mathbf{V}_s|^2 \left[\frac{(R_s + R_L)^2 - 2\, R_L\, (R_s + R_L)}{(R_s + R_L)^2} \right] = 0$$

$$(R_s + R_L)^2 - 2\, R_L\, (R_s + R_L) = 0$$

$$R_s^2 + 2\, R_s R_L + R_L^2 - 2\, R_L R_s - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2$$

$$R_L = R_s$$

Hence, load resistance R_L should be equal to source resistance R_L and load reactance X_L should be equal to negative value of source reactance for maximum power transfer.

$$\mathbf{Z}_L = \mathbf{Z}_s^* = R_s - jX_s$$

i.e. load impedance should be a complex conjugate of the source impedance.

Case (iii) When the load impedance is a complex impedance with variable resistance and fixed reactance (Fig. 3.146)

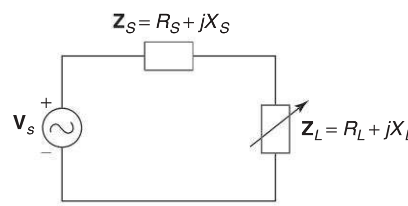


Fig. 3.146 *Complex impedance load*

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2\, R_L = \frac{|\mathbf{V}_s|^2\, R_L}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{(R_s + R_L)^2 + (X_s + X_L)^2 - 2\, R_L (R_s + R_L)}{\{(R_s + R_L)^2 + (X_s + X_L)^2\}^2} \right] = 0$$

$$(R_s + R_L)^2 + (X_s + X_L)^2 - 2\, R_L (R_s + R_L) = 0$$

$$R_s^2 + 2\, R_s\, R_L + R_L^2 + (X_s + X_L)^2 - 2\, R_L R_s - 2\, R_L^2 = 0$$

$$R_s^2 + (X_s + X_L)^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + (X_s + X_L)^2$$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$= |R_s + j(X_s + X_L)|$$

$$= |R_s + jX_s + jX_L|$$

$$= |Z_s + jX_L|$$

Hence, load resistance R_L should be equal to the magnitude of the impedance $\mathbf{Z}_s + jX_L$, i.e. $|\mathbf{Z}_s + jX_L|$ for maximum power transfer.

Example 3.47 *For maximum power transfer, find the value of $\mathbf{Z}_L$ in the network of Fig. 3.147 if (i) $\mathbf{Z}_L$ is an impedance, and (ii) $\mathbf{Z}_L$ is pure resistance.*

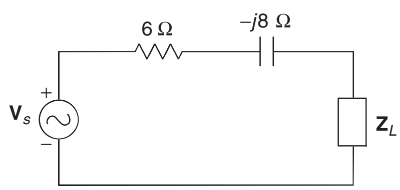


Fig. 3.147

Solution $\mathbf{Z}_s = (6 - j8)\ \Omega$

(i) If $\mathbf{Z}_L$ is an impedance

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_s^* = (6 + j8)\ \Omega$

(ii) If $\mathbf{Z}_L$ is a resistance

For maximum power transfer, $\mathbf{Z}_L = |\mathbf{Z}_S| = |6 + j8| = 10\ \Omega$

Example 3.48 *For the maximum power transfer, find the value of $\mathbf{Z}_L$ in the network of Fig. 3.148 for the following cases:*

(i) $\mathbf{Z}_L$ is variable resistance, (ii) $\mathbf{Z}_L$ is complex impedance, with variable resistance and variable reactance, and (iii) $\mathbf{Z}_L$ is complex impedance with variable resistance and fixed reactance of j5 Ω.

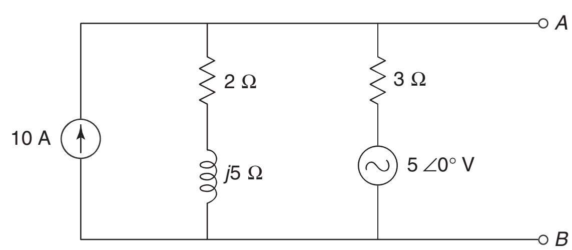


Fig. 3.148

Solution Thevenin's impedance can be calculated by replacing voltage source by a short circuit and current source by an open circuit.

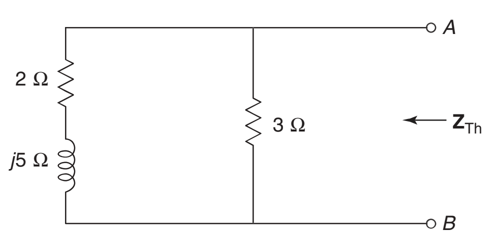


Fig. 3.149

$$\mathbf{Z}_{\text{Th}} = \frac{3(2+j5)}{3+2+j5} = (2.1+j0.9)\ \Omega$$

For maximum power transfer, value of $\mathbf{Z}_L$ will be,

(i) $\mathbf{Z}_L$ is variable resistance

$$\mathbf{Z}_L = |\mathbf{Z}_{\text{Th}}| = |2.1+j0.9| = 2.28\ \Omega$$

(ii) $\mathbf{Z}_L$ is complex impedance with variable resistance and variable reactance

$$\mathbf{Z}_L = \mathbf{Z}_{\text{Th}}^* = (2.1-j0.9)\ \Omega$$

(iii) $\mathbf{Z}_L$ is complex impedance with variable resistance and fixed reactance of $j5\ \Omega$

$$\mathbf{Z}_L = |\mathbf{Z}_{\text{Th}} + j5| = |2.1+j\,0.9+j5| = 6.26\ \Omega$$

Example 3.49 *Find the impedance $\mathbf{Z}_L$ so that maximum power can be transferred to it in the network of Fig. 3.150. Find maximum power.*

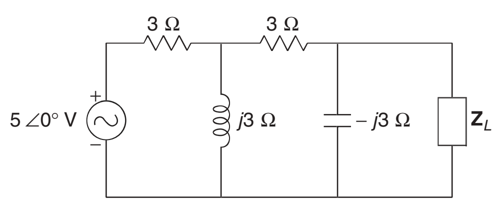


Fig. 3.150

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 3.151)

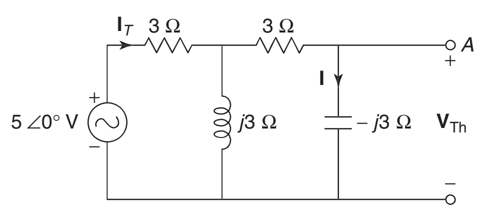


Fig. 3.151

$$\mathbf{Z}_T = 3 + \frac{j3(3-j3)}{3+j3-j3} = 6.71\angle 26.57^\circ\ \Omega$$

$$\mathbf{I}_T = \frac{5\angle 0^\circ}{6.71\angle 26.57^\circ} = 0.75\angle -26.57^\circ\ \text{A}$$

By current division rule,

$$\mathbf{I} = 0.75\angle -26.57^\circ \times \frac{j3}{3+j3-j3} = 0.75\angle 63.43^\circ\ \text{A}$$

$$\mathbf{V}_{\text{Th}} = (-j3)(0.75\angle 63.43^\circ) = 2.24\angle -26.57^\circ\ \text{V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.152)

$$\mathbf{Z}_{Th} = [(3 \parallel j3) + 3] \parallel (-j3)$$
$$= 3\angle -53.12°\ \Omega$$
$$= (1.8 - j2.4)\ \Omega$$

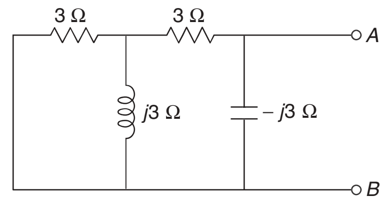


Fig. 3.152

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be a complex conjugate of the source impedance.

$$\mathbf{Z}_L = (1.8 + j2.4)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.153)

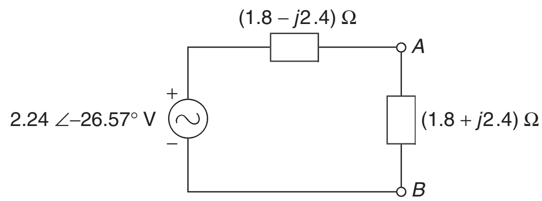


Fig. 3.153

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{|2.24|^2}{4 \times 1.8} = 0.7\ \text{W}$$

Example 3.50 *Find the value of $\mathbf{Z}_L$ for maximum power transfer in the network shown and find maximum power.*

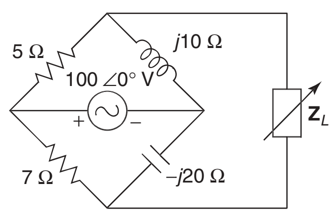


Fig. 3.154

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.155)

$$\mathbf{I}_1 = \frac{100\angle 0°}{5 + j10} = 8.94\angle -63.43°\ \text{A}$$

$$\mathbf{I}_2 = \frac{100\angle 0°}{7 - j20} = 4.72\angle 70.7°\ \text{A}$$

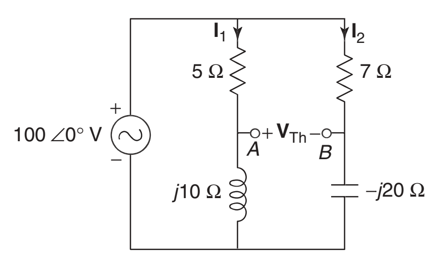


Fig. 3.155

$$\mathbf{V}_{Th} = \mathbf{V}_A - \mathbf{V}_B = (8.94\angle -63.43°)(j10) - (4.72\angle 70.7°)(-j20) = 71.76\angle 97.3° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.156)

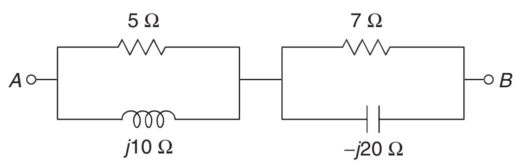


Fig. 3.156

$$\mathbf{Z}_{Th} = \frac{5(j10)}{5+j10} + \frac{7(-j20)}{7-j20} = \frac{50\angle 90°}{11.18\angle 63.43°} + \frac{140\angle -90°}{21.19\angle -70.7°} = (10.23 - j0.18)\ \Omega$$

Step III For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\mathbf{Z}_L = (10.23 + j0.18)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.157)

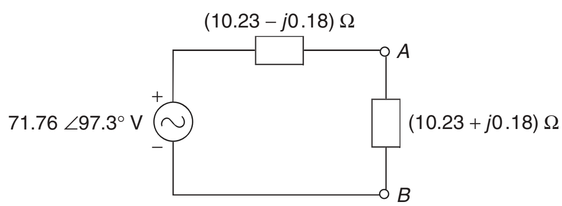


Fig. 3.157

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{|71.76|^2}{4 \times 10.23} = 125.84 \text{ W}$$

Example 3.51 *Find the value of load impedance $\mathbf{Z}_L$ so that maximum power can be transferred to it in the network of Fig. 3.158. Find maximum power.*

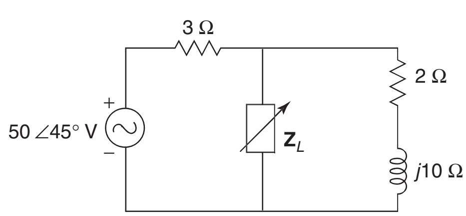


Fig. 3.158

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.159)

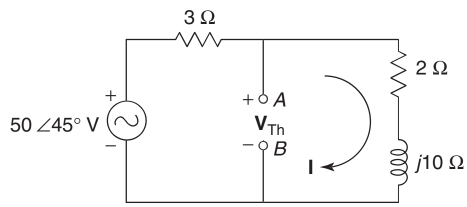


Fig. 3.159

$$\mathbf{I} = \frac{50\angle 45^\circ}{3+2+j10} = 4.47\angle -18.43^\circ \text{ A}$$

$$\mathbf{V}_{Th} = (2+j10)\,\mathbf{I} = (2+j10)(4.47\angle -18.43^\circ) = 45.6\angle 60.26^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.160)

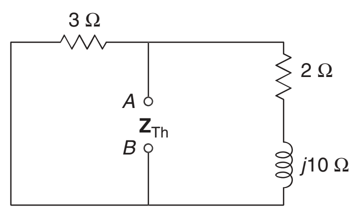


Fig. 3.160

$$\mathbf{Z}_{Th} = \frac{3(2+j10)}{3+2+j10} = (2.64+j0.72)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\mathbf{Z}_L = (2.64 - j0.72)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.161)

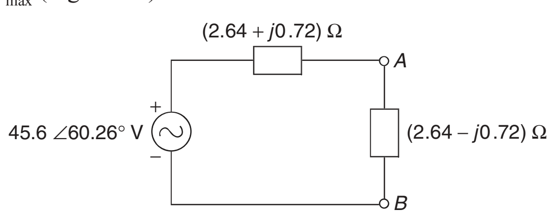


Fig. 3.161

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{|45.6|^2}{4\times 2.64} = 196.91 \text{ W}$$

Example 3.52 *Determine the load $\mathbf{Z}_L$ required to be connected in the network of Fig. 3.162 for maximum power transfer. Determine the maximum power drawn.*

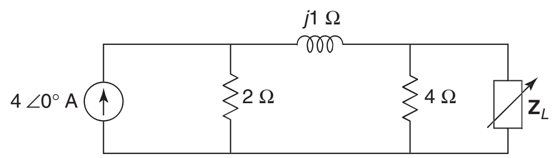


Fig. 3.162

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.163)

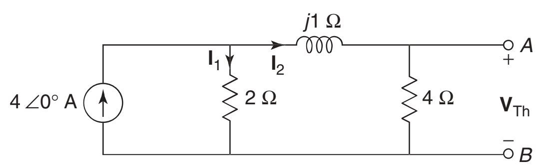


Fig. 3.163

$$\mathbf{I}_2 = 4\angle 0° \times \frac{2}{6+j1} = 1.315\angle -9.46° \text{ A}$$

$$\mathbf{V}_{Th} = 4\mathbf{I}_2 = 4(1.315\angle -9.46°) = 5.26\angle -9.46° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.164)

$$\mathbf{Z}_{Th} = \frac{4(2+j1)}{4+2+j1} = 1.47\angle 17.1° = (1.41 + j0.43)\ \Omega$$

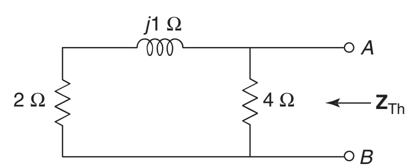


Fig. 3.164

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$\mathbf{Z}_L = (1.41 - j0.43)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.165)

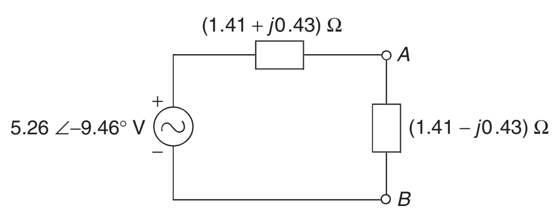


Fig. 3.165

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{|5.26|^2}{4 \times 1.41} = 4.91 \text{ W}$$

Example 3.53 *In the network shown in Fig. 3.166, find the value of $\mathbf{Z}_L$ for which the power transferred will be maximum. Also find maximum power.*

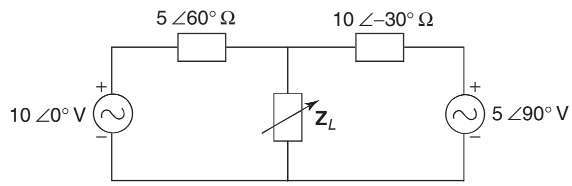


Fig. 3.166

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 3.167)

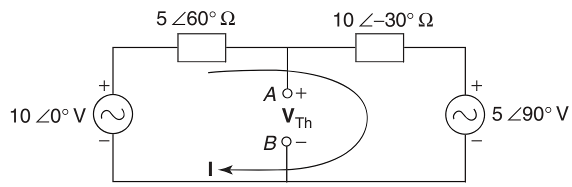


Fig. 3.167

Applying KVL to the mesh,

$$10\angle 0° - (5\angle 60°)\,\mathbf{I} - (10\angle -30°)\,\mathbf{I} - 5\angle 90° = 0$$
$$11.18\angle -26.57° - (11.18\angle -3.43°)\,\mathbf{I} = 0$$
$$\mathbf{I} = 1\angle -23.14° \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0° - (5\angle 60°)\mathbf{I} - \mathbf{V}_{\text{Th}} = 0$$
$$10\angle 0° - (5\angle 60°)(1\angle -23.14°) - \mathbf{V}_{\text{Th}} = 0$$
$$\mathbf{V}_{\text{Th}} = 6.71\angle -26.56° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 3.168)

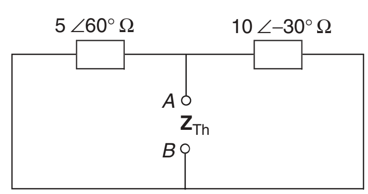


Fig. 3.168

$$\mathbf{Z}_{\text{Th}} = \frac{(5\angle 60°)(10\angle -30°)}{5\angle 60° + 10\angle -30°} = 4.47\angle 33.43° \ \Omega = (3.73 + j2.46)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$Z_L = Z_{Th}^* = (3.73 - j2.46)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.169)

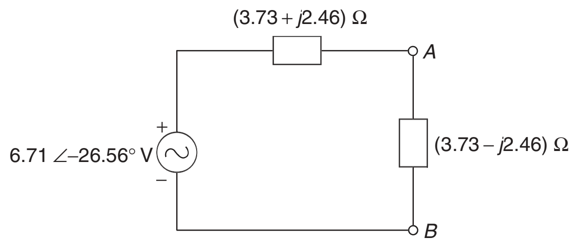


Fig. 3.169

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{(6.71)^2}{4 \times 3.73} = 3.02\ \text{W}$$

Example 3.54 *In the network shown in Fig. 3.170, find the value of $\mathbf{Z}_L$ so that power transfer from the source is maximum. Also find maximum power.*

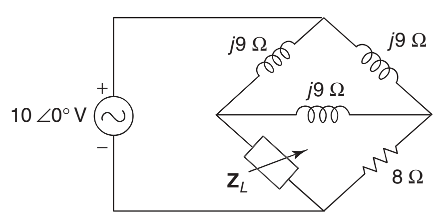


Fig. 3.170

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 3.171)

Applying Star-delta transformation (Fig. 3.172)

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \frac{(j9)(j9)}{j9 + j9 + j9} = j3\ \Omega$$

$\mathbf{V}_{Th}$ = Voltage drop across $(8 + j3)\Omega$ impedence

$$= (8 + j3)\frac{10\angle 0^\circ}{8 + j3 + j3} = 8.54\angle -16.31^\circ\ \text{V}$$

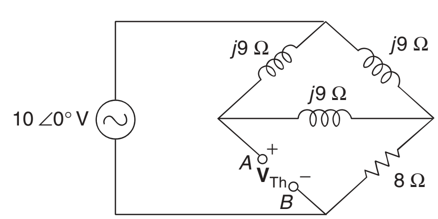


Fig. 3.171

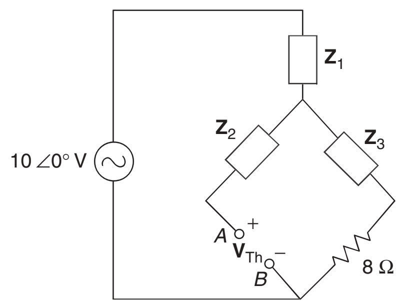


Fig. 3.172

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 3.173)

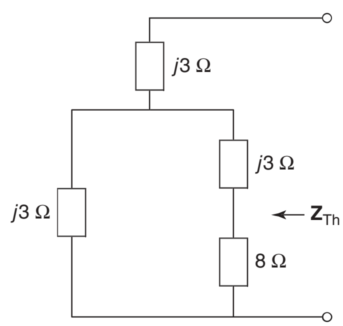


Fig. 3.173

$$\mathbf{Z}_{Th} = j3 + \frac{j3(8 + j3)}{j3 + 8 + j3)} = 5.51\angle 82.49^\circ\ \Omega = (0.72 + j5.46)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = (0.72 - j5.46)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.174)

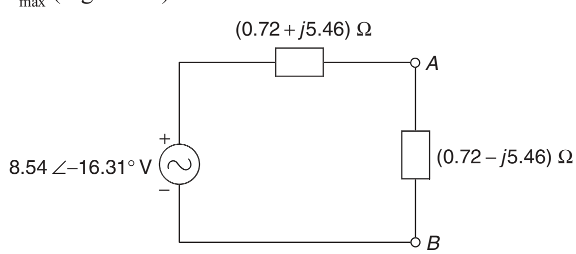


Fig. 3.174

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{(8.54)^2}{4 \times 0.72} = 25.32\ \text{W}$$

Example 3.55 *For the network shown in Fig. 3.175, find the value of $\mathbf{Z}_L$ that will transfer maximum power from the source. Also find maximum power.*

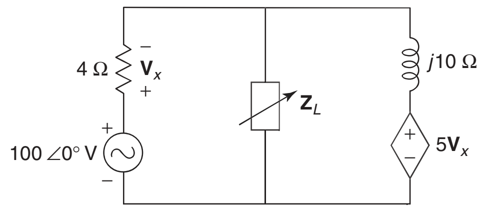


Fig. 3.175

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 3.176)

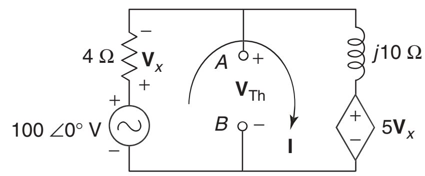


Fig. 3.176

From Fig. 3.176,

$$\mathbf{V}_x = 4\mathbf{I}$$

Applying KVL to the mesh,

$$100\angle 0° - 4\mathbf{I} - j\,10\,\mathbf{I} - 5\mathbf{V}_x = 0$$
$$100\angle 0° - (4 + j10)\,\mathbf{I} - 5(4\mathbf{I}) = 0$$
$$\mathbf{I} = \frac{100\ \angle 0°}{24 + j10} = 3.85\ \angle -22.62°\ \text{A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$100\ \angle\ 0° - 4\mathbf{I} - \mathbf{V}_{\text{Th}} = 0$$
$$100\angle 0° - 4(3.85\ \angle -22.62°) - \mathbf{V}_{\text{Th}} = 0$$
$$\mathbf{V}_{\text{Th}} = 86\angle 3.95°\ \text{V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 3.177)

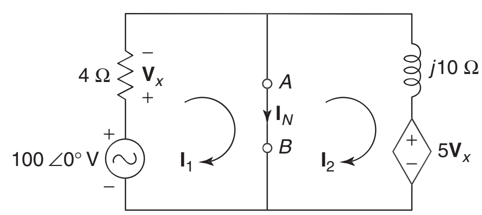


Fig. 3.177

From Fig. 3.177,

$$\mathbf{V}_x = 4\mathbf{I}_1$$

Applying KVL to Mesh 1,

$$100\ \angle 0° - 4\mathbf{I}_1 = 0$$
$$\mathbf{I}_1 = 25\ \text{A}$$

Applying KVL to Mesh 2,

$$-j10\mathbf{I}_2 - 5\mathbf{V}_x = 0$$
$$-j10\mathbf{I}_2 - 5(4\mathbf{I}_1) = 0$$
$$-j10\mathbf{I}_2 - 5(100) = 0$$
$$\mathbf{I}_2 = 50\angle 90°\text{A}$$
$$\mathbf{I}_N = \mathbf{I}_1 - \mathbf{I}_2 = 25 - 50\angle 90° = 55.9\angle -63.43°\text{A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{86\angle 3.95°}{55.9\angle -63.43°} = 1.54\angle 67.38°\ \Omega = (0.59 + j1.42)\ \Omega$$

Step IV Calculation of $\mathbf{Z}_L$

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = (0.59 - j1.42)\ \Omega$

Step V Calculation of P_{max} (Fig. 3.178)

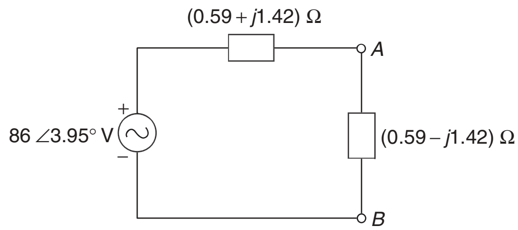


Fig. 3.178

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{4R_L} = \frac{(86)^2}{4 \times 0.59} = 3133.9\ \text{W}$$

3.8 RECIPROCITY THEOREM

The Reciprocity theorem states that '*In a linear, bilateral, active, single-source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged*.'

Example 3.56 *Find the current through the 6 Ω resistor and verify the reciprocity theorem.*

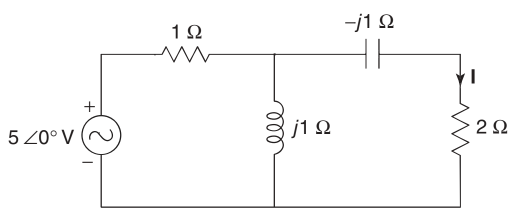


Fig. 3.179

Solution

Case I Calculation of current **I** when excitation and response are not interchanged (Fig. 3.180)

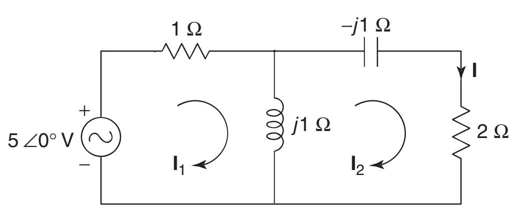


Fig. 3.180

Applying KVL to Mesh 1,

$$5\angle 0° - 1\mathbf{I}_1 - j1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(1 + j1)\mathbf{I}_1 - j1\mathbf{I}_2 = 5\angle 0° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j1(\mathbf{I}_2 - \mathbf{I}_1) + j1\mathbf{I}_2 - 2\mathbf{I}_2 = 0$$

$$-j1\mathbf{I}_1 + 2\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 1+j1 & -j1 \\ -j1 & 2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 1+j1 & 5\angle 0^\circ \\ -j1 & 0 \end{vmatrix}}{\begin{vmatrix} 1+j1 & -j1 \\ -j1 & 2 \end{vmatrix}} = 1.39\angle 56.31^\circ \text{ A}$$

$$\mathbf{I} = \mathbf{I}_2 = 1.39\angle 56.31^\circ \text{ A}$$

Case II Calculation of current **I** when excitation and response are interchanged (Fig. 3.181)

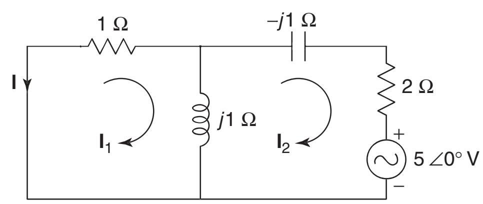


Fig. 3.181

Applying KVL to Mesh 1,

$$-1\mathbf{I}_1 - j1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(1+j1)\mathbf{I}_1 - j1\mathbf{I}_2 = 0 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j1(\mathbf{I}_2 - \mathbf{I}_1) + j1\mathbf{I}_2 - 2\mathbf{I}_2 - 5\angle 0^\circ = 0$$

$$-j1\mathbf{I}_1 + 2\mathbf{I}_2 = -5\angle 0^\circ \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 1+j1 & -j1 \\ -j1 & 2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -5\angle 0^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 0 & -j1 \\ -5\angle 0^\circ & 2 \end{vmatrix}}{\begin{vmatrix} 1+j1 & -j1 \\ -j1 & 2 \end{vmatrix}} = 1.38\angle -123.69^\circ \text{ A}$$

$$\mathbf{I} = -\mathbf{I}_1 = 1.39\angle 56.31^\circ \text{ A}$$

Since the current **I** is same in both the cases, the reciprocity theorem is verified.

Example 3.57 *In the network of Fig. 3.182, find the voltage $\mathbf{V}_x$ and verify the reciprocity theorem.*

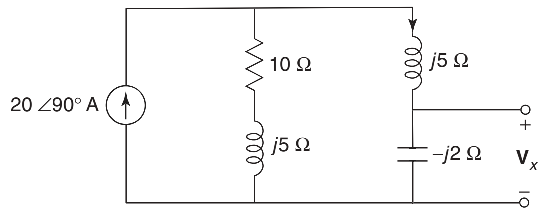


Fig. 3.182

Solution

Case I Calculation of voltage $\mathbf{V}_x$ when excitation and response are interchanged. (Fig. 3.183)

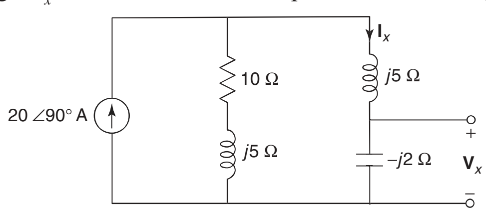


Fig. 3.183

By current division rule,

$$\mathbf{I}_x = (20\angle 90^\circ)\frac{(10+j5)}{(10+j5)+(j5-j2)} = 17.46\angle 77.91^\circ \text{ A}$$

$$\mathbf{V}_x = (-j2)\mathbf{I}_x = (-j2)(17.46\angle 77.91^\circ) = 34.92\ \angle -12.09^\circ \text{ V}$$

Case II Calculation of voltage $\mathbf{V}_x$ when excitation and response are interchanged (Fig. 3.184)

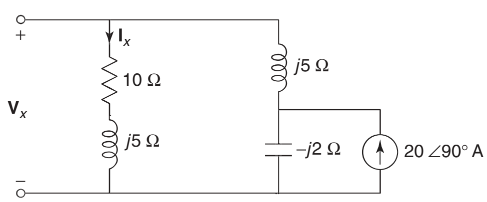


Fig. 3.184

$$\mathbf{I}_x = (20\angle 90^\circ)\frac{(-j2)}{(-j2)+(10+j5+j5)} = 3.12\angle -38.66^\circ \text{A}$$

$$\mathbf{V}_x = (10+j5)\mathbf{I}_x = (10+j5)(3.12\angle -38.66^\circ) = 34.88\angle -12.09^\circ \text{V}$$

Since the voltage $\mathbf{V}_x$ is same in both the cases, the reciprocity theorem is verified.

Example 3.58 *Find **I** and verify the reciprocity theorem for the network shown in Fig. 3.185.*

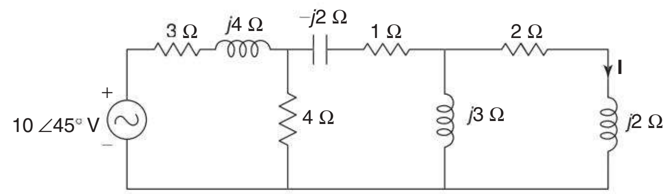


Fig. 3.185

Solution

Case I Calculation of **I** when excitation and response are not interchanged (Fig. 3.186)

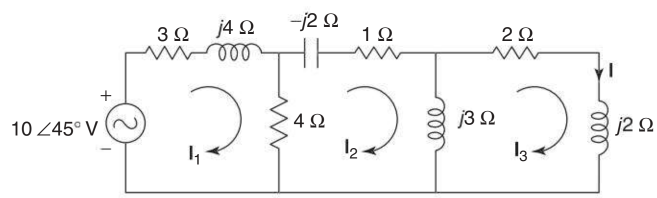


Fig. 3.186

Applying KVL to Mesh 1,

$$10\angle 45° - (3 + j4)\mathbf{I}_1 - 4(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(7 + j4)\mathbf{I}_1 - 4\mathbf{I}_2 = 10\angle 45° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-4(\mathbf{I}_2 - \mathbf{I}_1) - (1 - j2)\,\mathbf{I}_2 - j3(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-4\mathbf{I}_1 + (5 + j1)\,\mathbf{I}_2 - j3\mathbf{I}_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-j3(\mathbf{I}_3 - \mathbf{I}_2) - 2\mathbf{I}_3 - j2\mathbf{I}_3 = 0$$

$$-j3\mathbf{I}_2 + (2 + j5)\mathbf{I}_3 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 7 + j4 & -4 & 0 \\ -4 & 5 + j1 & -j3 \\ 0 & -j3 & 2 + j5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 45° \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 7 + j4 & -4 & 10\angle 45° \\ -4 & 5 + j1 & 0 \\ 0 & -j3 & 0 \end{vmatrix}}{\begin{vmatrix} 7 + j4 & -4 & 0 \\ -4 & 5 + j1 & -j3 \\ 0 & -j3 & 2 + j5 \end{vmatrix}} = 0.704\angle 30.72° \text{ A}$$

$$\mathbf{I} = \mathbf{I}_3 = 0.704\angle 30.72° \text{A}$$

Case II Calculation of **I** when excitation and response are interchanged (Fig. 3.187)

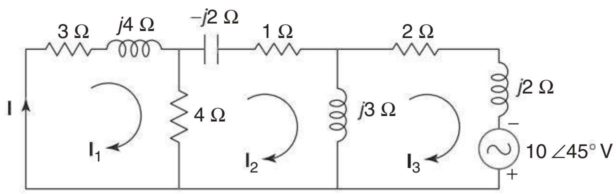


Fig. 3.187

Applying KVL to Mesh 1,

$$-(3+j4)\mathbf{I}_1 - 4(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(7+j4)\mathbf{I}_1 - 4\mathbf{I}_2 = 0 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-4(\mathbf{I}_2 - \mathbf{I}_1) - (1-j2)\,\mathbf{I}_2 - j3(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-4\mathbf{I}_1 + (5+j1)\mathbf{I}_2 - j3\mathbf{I}_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-j3(\mathbf{I}_3 - \mathbf{I}_2) - 2\mathbf{I}_3 - j2\mathbf{I}_3 + 10\angle 45^\circ = 0$$

$$-j3\mathbf{I}_2 + (2+j5)\mathbf{I}_3 = 10\angle 45^\circ \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 7+j4 & -4 & 0 \\ -4 & 5+j1 & -j3 \\ 0 & -j3 & 2+j5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10\angle 45^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 0 & -4 & 0 \\ 0 & 5+j1 & -j3 \\ 10\angle 45^\circ & -j3 & 2+j5 \end{vmatrix}}{\begin{vmatrix} 7+j4 & -4 & 0 \\ -4 & 5+j1 & -j3 \\ 0 & -j3 & 2+j5 \end{vmatrix}} = 0.704\angle 30.72^\circ \text{A}$$

$$\mathbf{I} = \mathbf{I}_1 = 0.704\angle 30.72^\circ \text{A}$$

Since the current **I** is same in both the cases, the reciprocity theorem, is verified.

3.9 MILLMAN'S THEOREM

Millman's theorem states that '*If there are n voltage sources $\boldsymbol{V_1}$, $\boldsymbol{V_2}$, … $\boldsymbol{V_n}$ with internal impedances $\boldsymbol{Z_1}$, $\boldsymbol{Z_2}$, … $\boldsymbol{Z_n}$ respectively connected in parallel then these voltage sources can be replaced by a single voltage source $\boldsymbol{V_m}$ and a single series impedance $\boldsymbol{Z_m}$.*

$$\mathbf{V}_m = \frac{\mathbf{V}_1\,\mathbf{Y}_1 + \mathbf{V}_2\,\mathbf{Y}_2 + \ldots + \mathbf{V}_n\,\mathbf{Y}_n}{\mathbf{Y}_1 + \mathbf{Y}_2 + \ldots + \mathbf{Y}_n}$$

$$\mathbf{Z}_m = \frac{1}{\mathbf{Y}_m} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2 + \ldots + \mathbf{Y}_n}$$

2.15 Find the value of current I.

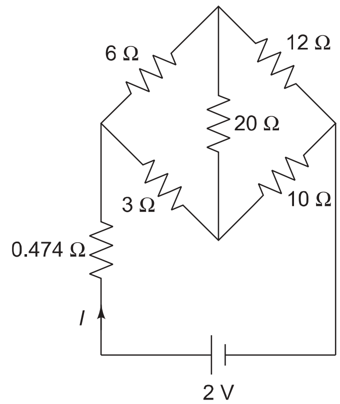


Fig. 2.259

[0.25 A]

2.16 Determine the value of current flowing through the 10 Ω resistor.

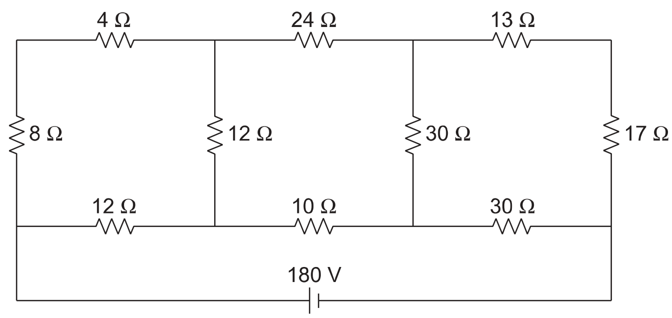


Fig. 2.260

[3.84 A]

2.8 SUPERPOSITION THEOREM

It states that '*In a linear network containing more than one independent sources, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned.

The independent current sources are represented by infinite resistances, i.e., open circuits.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the circuit shown in Fig. 2.261. Suppose we have to find current I_4 flowing through R_4.

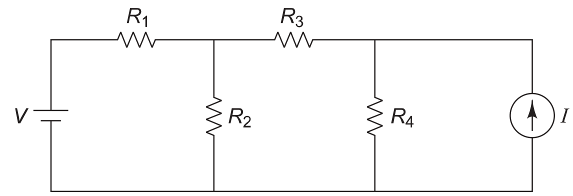


Fig. 2.261 *Superposition theorem*

2.8.1 Steps to be followed in Superposition Theorem

1. Find the current I_4' flowing through R_4 due to independent voltage source 'V', representing independent current source with infinite resistance, i.e., open circuit.

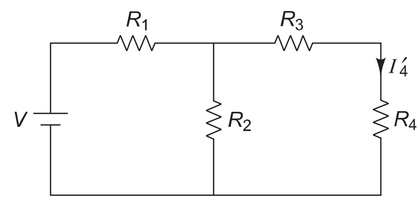


Fig. 2.262 *Step 1*

2. Find the current I_4'' flowing through R_4 due to independent current source 'I', representing the independent voltage source with zero resistance or short circuit.

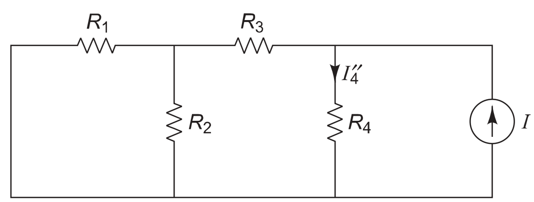


Fig. 2.263 *Step 2*

3. Find the resultant current I_4 through R_4 by the superposition theorem.

$$I_4 = I_4' + I_4''$$

Example 1

Find the value of current flowing through the 2 Ω resistor.

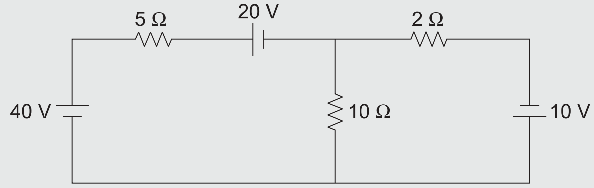


Fig. 2.264

Solution *Step I: When the 40 V source is acting alone*

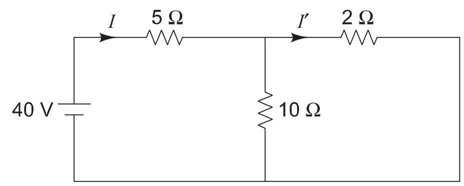


Fig. 2.265

By series–parallel reduction technique,

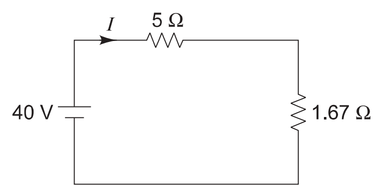


Fig. 2.266

$$I = \frac{40}{5+1.67} = 6\text{ A}$$

From Fig. 2.265, by current-division rule,

$$I' = 6 \times \frac{10}{10+2} = 5\text{ A } (\rightarrow)$$

Step II: When the 20 V source is acting alone

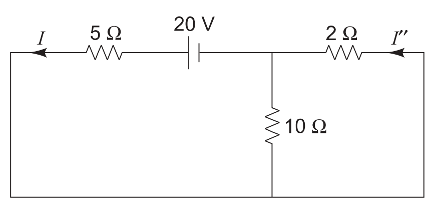


Fig. 2.267

By series–parallel reduction technique,

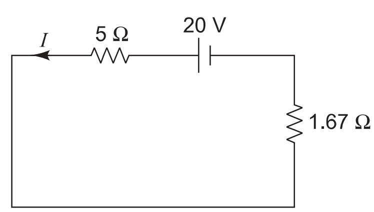


Fig. 2.268

$$I = \frac{20}{5+1.67} = 3\text{ A}$$

From Fig. 2.267, by current-division rule,

$$I'' = 3 \times \frac{10}{10+2} = 2.5 \text{ A} (\leftarrow) = -2.5\text{A} (\rightarrow)$$

Step III: *When the 10 V source is acting alone*

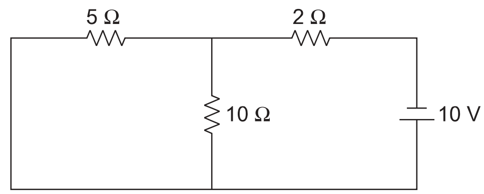


Fig. 2.269

By series–parallel reduction technique,

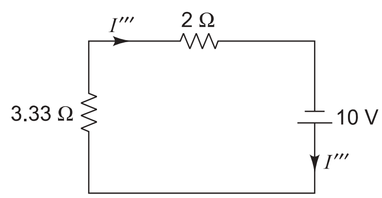


Fig. 2.270

$$I''' = \frac{10}{3.33+2} = 1.88 \text{ A} (\rightarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 5 - 2.5 + 1.88$$
$$= 4.38 \text{ A} (\rightarrow)$$

Example 2

Find the value of current flowing through the 1 Ω resistor.

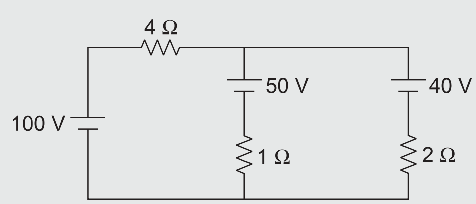


Fig. 2.271

Solution *Step I: When the 100 V source is acting alone*

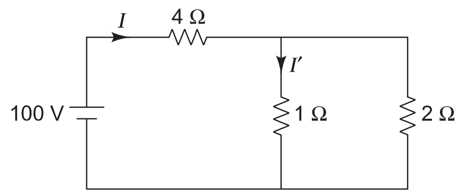


Fig. 2.272

By series–parallel reduction technique

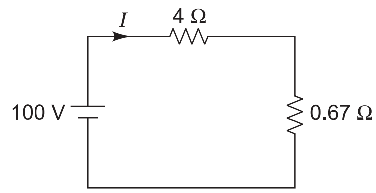


Fig. 2.273

$$I = \frac{100}{4+0.67} = 21.41 \text{ A}$$

From Fig. 2.272, by current-division rule,

$$I' = 21.41 \times \frac{2}{1+2} = 14.27 \text{ A } (\downarrow)$$

Step II: When the 50 V source is acting alone

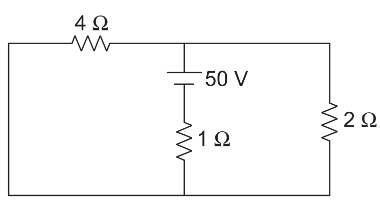


Fig. 2.274

By series–parallel reduction technique,

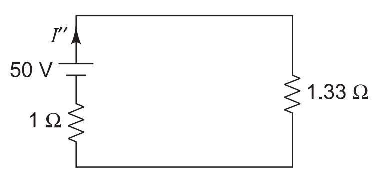


Fig. 2.275

$$I'' = \frac{50}{1+1.33} = 21.46 \text{ A } (\uparrow) = -21.46 \text{ A } (\downarrow)$$

Step III: When the 40 V source is acting alone

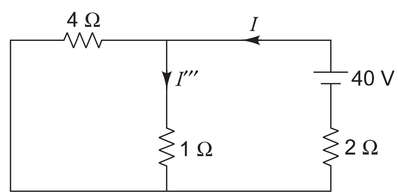


Fig. 2.276

By series–parallel reduction technique,

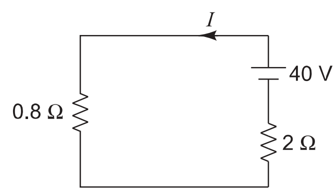


Fig. 2.277

$$I = \frac{40}{0.8 + 2} = 14.29 \text{ A}$$

From Fig. 2.276, by current-division rule,

$$I''' = 14.29 \times \frac{4}{4+1} = 11.43 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$

$$= 14.27 - 21.46 + 11.43$$

$$= 4.24 \text{ A } (\downarrow)$$

Example 3

Find the value of current flowing through the 8 Ω resistor.

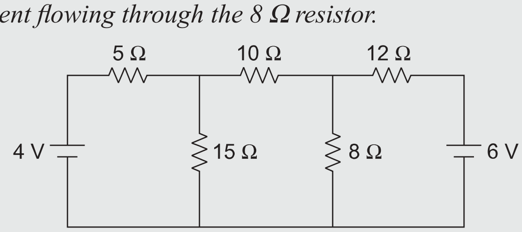


Fig. 2.278

Solution *Step I: When the 4 V source is acting alone*

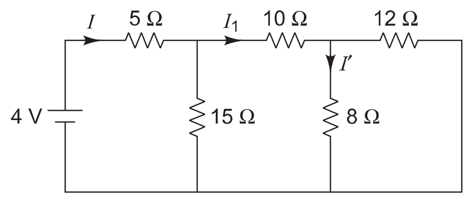


Fig. 2.279

By series–parallel reduction technique,

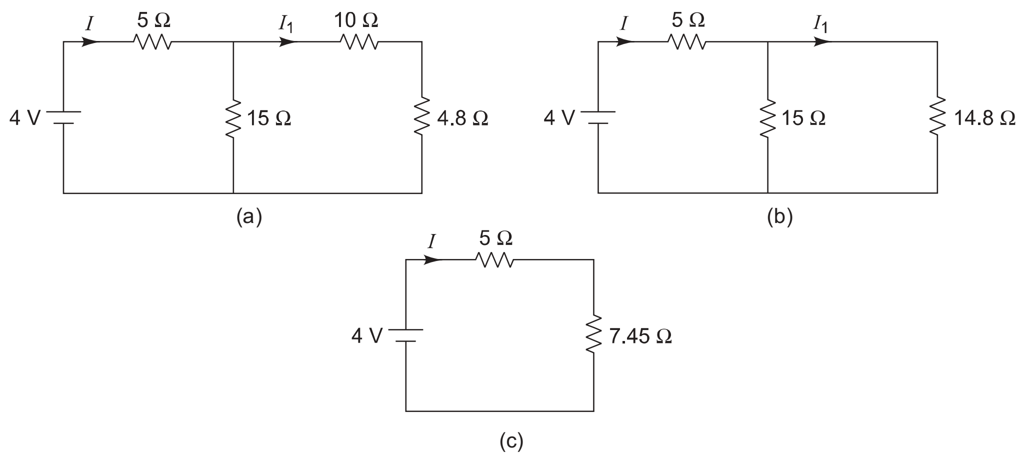


Fig. 2.280

$$I = \frac{4}{5+7.45} = 0.32 \text{ A}$$

From Fig. 2.280(b), by current-division rule,

$$I_1 = 0.32 \times \frac{15}{15+14.8} = 0.16 \text{ A}$$

From Fig. 2.279, by current-division rule,

$$I' = 0.16 \times \frac{12}{12+8} = 0.096 \text{ A} (\downarrow)$$

S*tep II: When the 6 V source is acting alone*

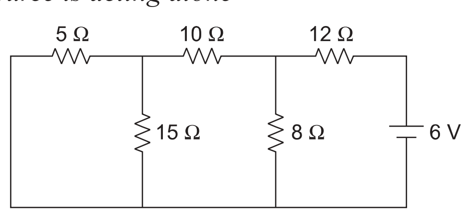
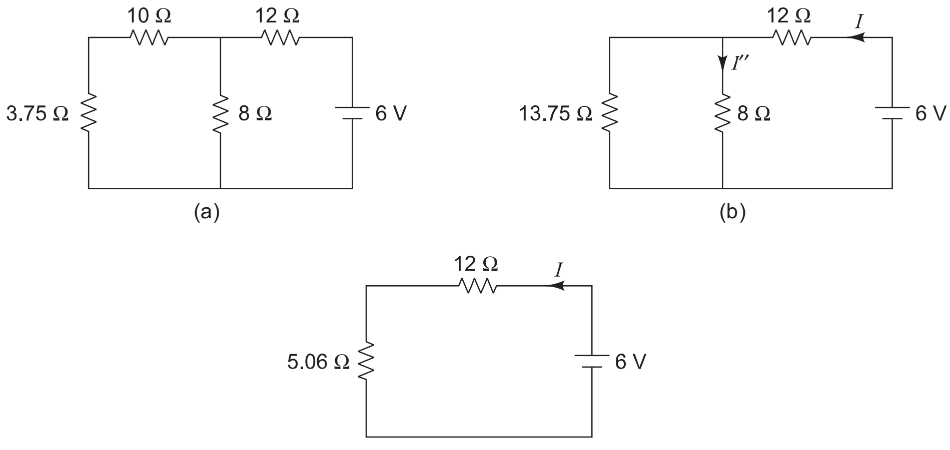


Fig. 2.281

By series–parallel reduction technique,



(c)

Fig. 2.282

$$I = \frac{6}{12+5.06} = 0.35 \text{ A}$$

From Fig. 2.282(b), by current division rule,

$$I'' = 0.35 \times \frac{13.75}{13.75+8} = 0.22 \text{ A} (\downarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 0.096 + 0.22 \\ &= 0.316 \text{ A} (\downarrow) \end{aligned}$$

Example 4

Find the value of current flowing through the 4 Ω resistor.

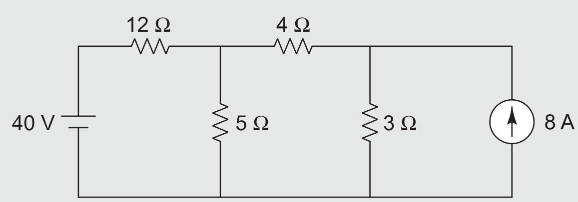


Fig. 2.283

Solution *Step I: When the 40 V source is acting alone*

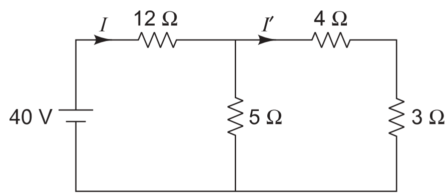


Fig. 2.284

By series–parallel reduction technique,

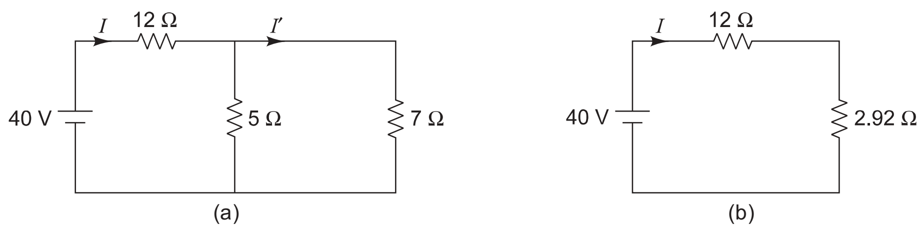


Fig. 2.285

$$I = \frac{40}{12 + 2.92} = 2.68 \text{ A}$$

From Fig. 2.285(a), by current-division rule,

$$I' = 2.68 \times \frac{5}{5+7} = 1.12 \text{ A} (\rightarrow) = -1.12 \text{ A} (\leftarrow)$$

Step II: When the 8 A source is acting alone

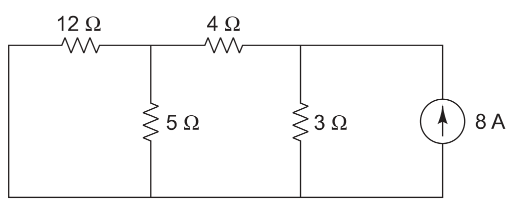


Fig. 2.286

By series–parallel reduction technique,

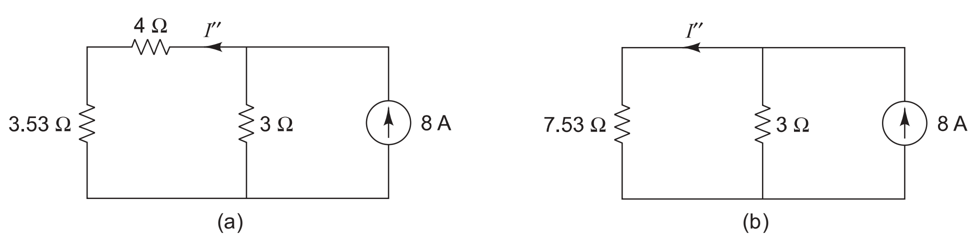


Fig. 2.287

From Fig. 2.287(b), by current-division rule,

$$I'' = 8 \times \frac{3}{7.53+3} = 2.28 \text{ A} (\leftarrow)$$

Step III: By superposition theorem

$$I = I' + I''$$
$$= -1.12 + 2.28$$
$$= 1.16 \text{ A} (\leftarrow)$$

Example 5

Find the value of current flowing in the 10 Ω resistor.

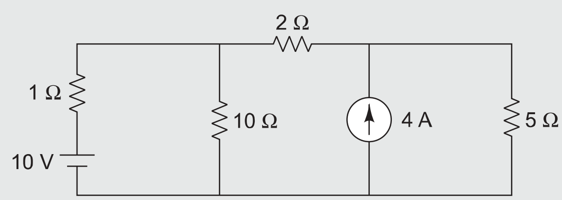


Fig. 2.288

Solution *Step I: When the 10 V source is acting alone*

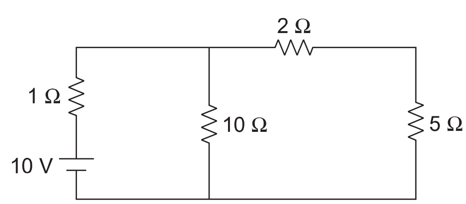


Fig. 2.289

By series–parallel reduction technique,

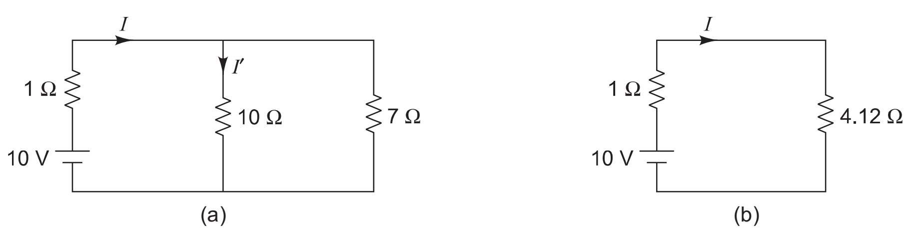


Fig. 2.290

$$I = \frac{10}{1+4.12} = 1.95 \text{ A}$$

From Fig. 2.290(a), by current-division rule,

$$I' = 1.95 \times \frac{7}{7+10} = 0.8 \text{ A } (\downarrow)$$

Step II: When the 4 A source is acting alone

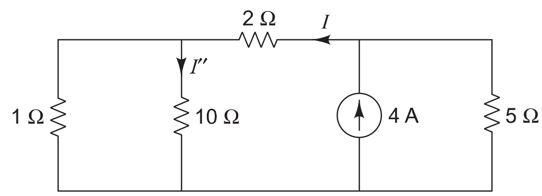


Fig. 2.291

By series–parallel reduction technique,

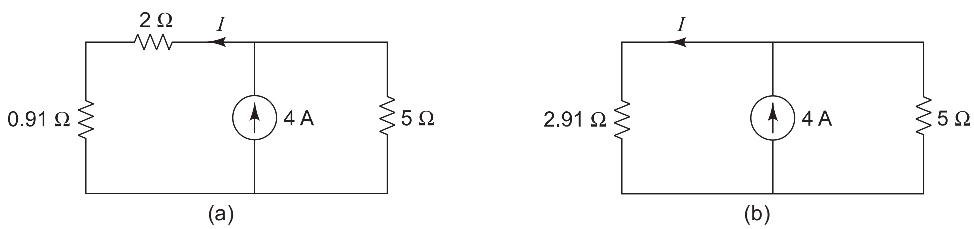


Fig. 2.292

$$I = 4 \times \frac{5}{2.91+5} = 2.53 \text{ A}$$

From Fig. 2.291, by current-division rule,

$$I'' = 2.53 \times \frac{1}{1+10} = 0.23 \text{ A } (\downarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 0.8 + 0.23 \\ &= 1.03 \text{ A } (\downarrow) \end{aligned}$$

Example 6

Find the value of current flowing through the 8 Ω resistor.

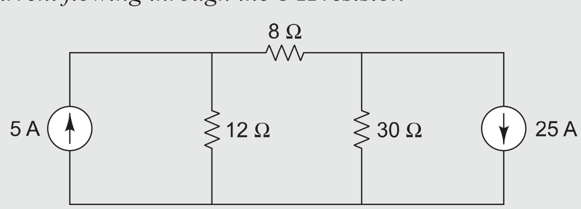


Fig. 2.293

Solution *Step I: When the 5A source is acting alone*

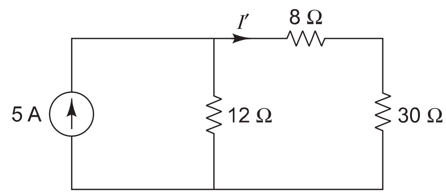


Fig. 2.294

By current-division rule,

$$I' = 5 \times \frac{12}{12+8+30} = 1.2\ \text{A}\ (\rightarrow)$$

Step II: When the 25 A source is acting alone

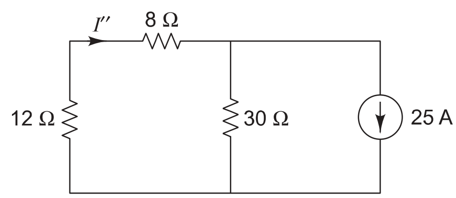


Fig. 2.295

By current-division rule,

$$I'' = 25 \times \frac{30}{30+12+8} = 15\ \text{A}\ (\rightarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 1.2 + 15 \\ &= 16.2\ \text{A}\ (\rightarrow) \end{aligned}$$

Example 7

Find the value of current flowing through the 4 Ω resistor.

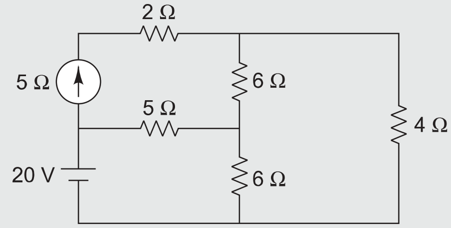


Fig. 2.296

Solution *Step I: When the 5 A source is acting alone*

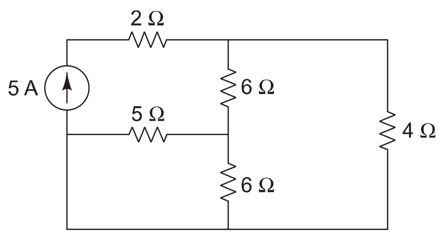


Fig. 2.297

By series–parallel reduction technique,

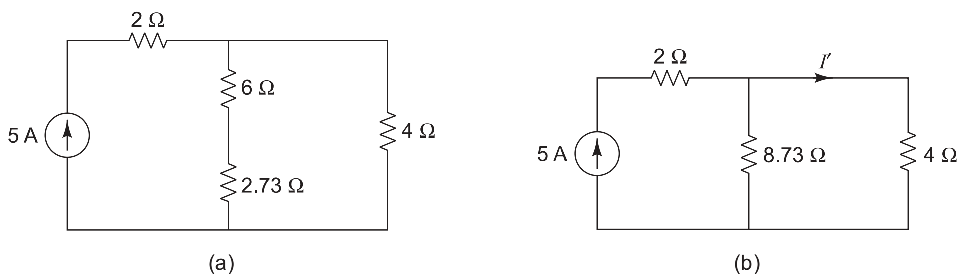


Fig. 2.298

From Fig. 2.298(b), by current-division rule,

$$I' = 5 \times \frac{8.73}{8.73 + 4} = 3.43 \text{ A } (\downarrow)$$

Step II: When the 20 V source is acting alone

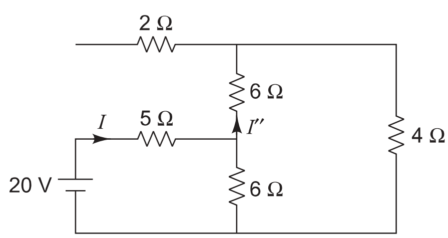


Fig. 2.299

By series–parallel reduction technique.

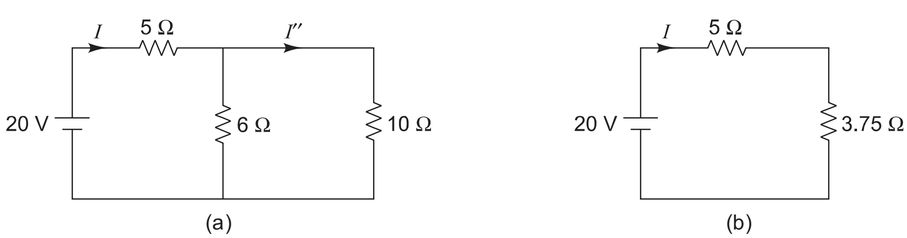


Fig. 2.300

$$I = \frac{20}{5 + 3.75} = 2.29 \text{ A}$$

From Fig. 2.300(a), by current-division rule,

$$I'' = 2.29 \times \frac{6}{6 + 10} = 0.86 \text{ A} (\downarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= 3.43 + 0.86 \\ &= 4.29 \text{ A} (\downarrow) \end{aligned}$$

Example 8

Find the value of current flowing through the 3 Ω resistor.

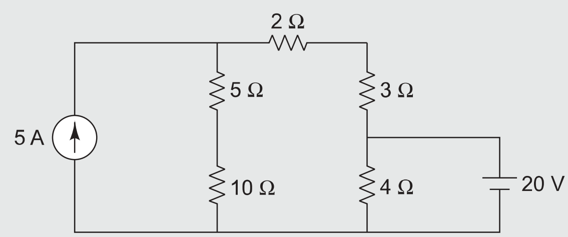


Fig. 2.301

Solution *Step I: When the 5 A source is acting alone*

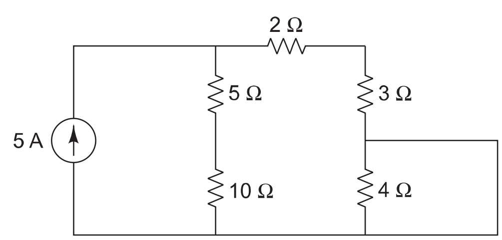


Fig. 2.302

By series–parallel reduction technique,

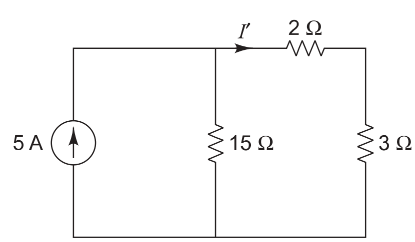


Fig. 2.303

By current-division rule,

$$I' = 5 \times \frac{15}{15+2+3} = 3.75\text{ A }(\downarrow)$$

Step II: When the 20 V source is acting alone

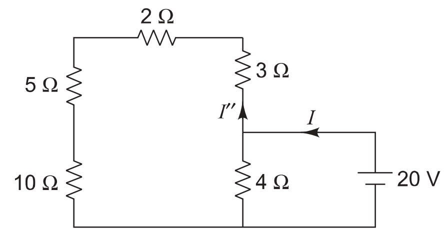


Fig. 2.304

By series–parallel reduction technique,

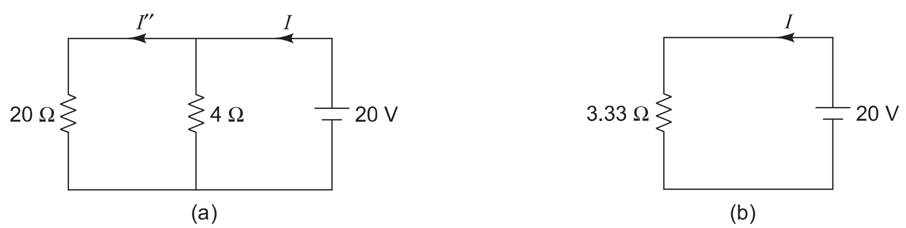


Fig. 2.305

$$I = \frac{20}{3.33} = 6\text{ A}$$

From Fig. 2.305(a), by current-division rule,

$$I'' = 6 \times \frac{4}{20+4} = 1\text{ A }(\uparrow) = -1\text{ A }(\downarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 3.75 - 1 \\ &= 2.75\text{ A }(\downarrow) \end{aligned}$$

Example 9

Find the value of current flowing in the 1 Ω resistor.

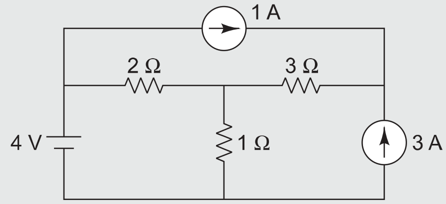


Fig. 2.306

Solution *Step I: When the 4 V source is acting alone*

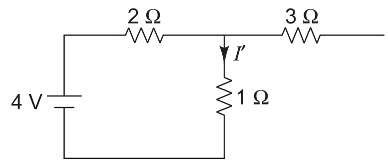


Fig. 2.307

By current-division rule,

$$I' = \frac{4}{2+1} = 1.33 \text{ A } (\downarrow)$$

Step II: When the 3 A source is acting alone

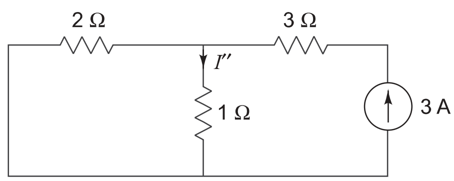


Fig. 2.308

By current-division rule,

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A } (\downarrow)$$

Step III: When the 1 A source is acting alone

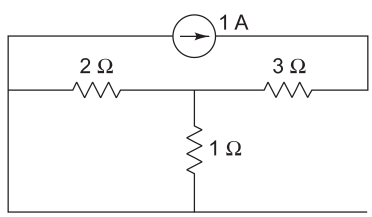


Fig. 2.309

Redrawing the circuit,

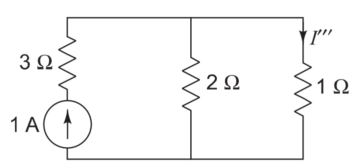


Fig. 2.310

By current-division rule,

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 1.33 + 2 + 0.66$$
$$= 4 \text{ A} (\downarrow)$$

Example 10

Find the voltage V_{AB}.

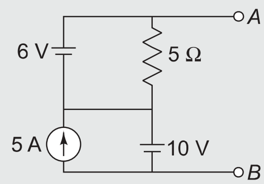


Fig. 2.311 [Dec 2014]

Solution *Step I: When the 6 V source is acting alone*

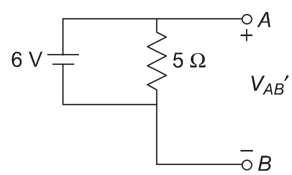


Fig. 2.312

$$V_{AB}' = 6 \text{ V}$$

Step II: When the 10 V source is acting alone

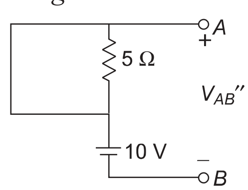


Fig. 2.313

Since the resistor of 5 Ω is shorted, the voltage across it is zero.

$$V_{AB}'' = 10 \text{ V}$$

Step III: When the 5 A source is acting alone

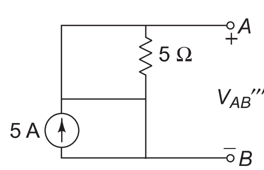


Fig. 2.314

Due to short circuit in both the parts,

$$V_{AB}''' = 0 \text{ V}$$

Step IV: By superposition theorem,

$$\begin{aligned} V_{AB} &= V_{AB}' + V_{AB}'' + V_{AB}''' \\ &= 6 + 10 + 0 \\ &= 16 \text{ V} \end{aligned}$$

Example 11

Find the voltage across 4 kΩ.

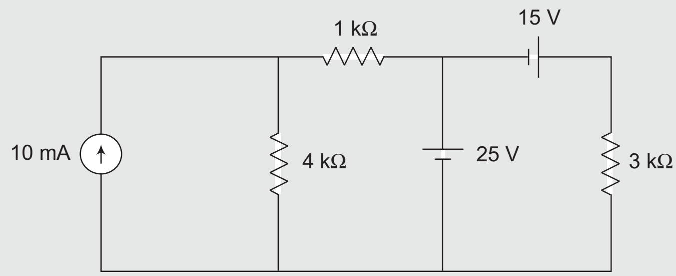


Fig. 2.315 **[May 2016]**

Solution *Step I: When the 10 mA source is acting alone*

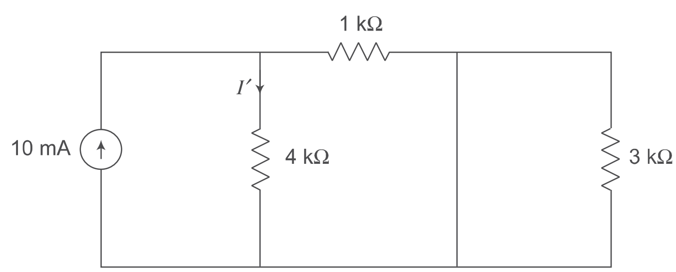


Fig. 2.316

Since 3 kΩ resistor is connected in parallel with short circuit, it gets shorted.

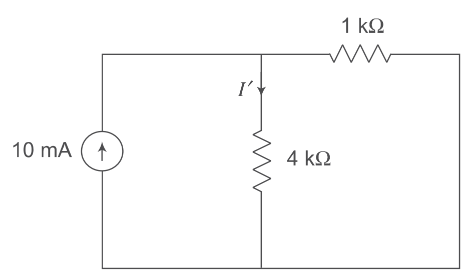


Fig. 2.317

By current division rule,

$$I' = 10\,m \times \frac{1k}{1k + 4k} = 2\text{ mA}(\downarrow)$$

Step II: When the 25 V source is acting alone

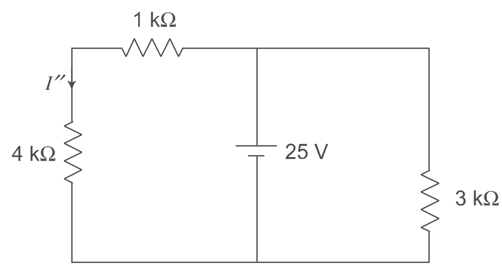


Fig. 2.318

Since 3 kΩ resistor is connected in parallel with 25 V source, it becomes redundant.

$$I'' = \frac{25}{4k + 1k} = 5\,mA(\downarrow)$$

Step III: When the 15 V source is acting alone

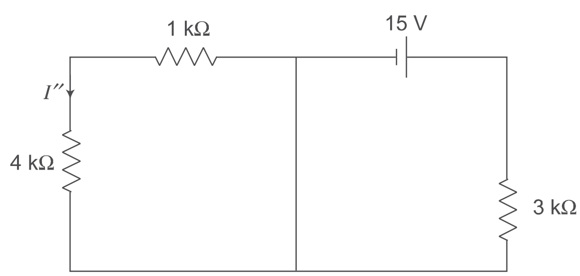


Fig. 2.319

Since series combination of 4 kΩ and 1 kΩ resistor is connected across a short circuit, it gets shorted.

$$I''' = 0$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= 2\,mA + 5\,mA + 0 \\ &= 7\,mA(\downarrow) \end{aligned}$$

Example 12

Find the current through the 5 Ω resistor.

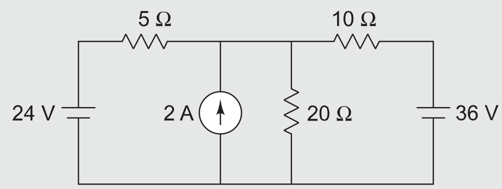


Fig. 2.320

Solution *Step I: When the 24 V source is acting alone*

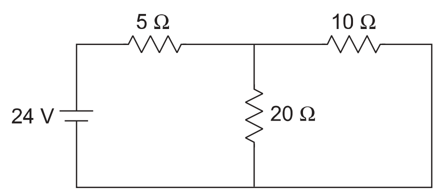


Fig. 2.321

By series–parallel reduction technique,

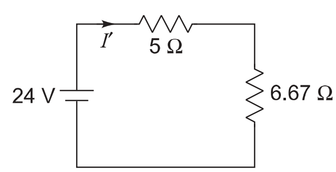


Fig. 2.322

$$I' = \frac{24}{5+6.67} = 2.06 \text{ A } (\rightarrow) = -2.06 \text{ A } (\leftarrow)$$

Step II: When the 2 A source is acting alone

By series-parallel reduction technique,

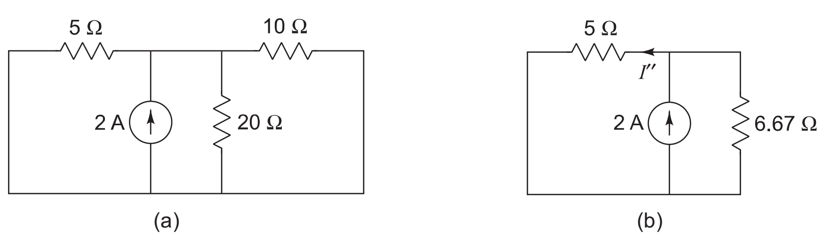


Fig. 2.323

From Fig. 2.323(b), by current-division rule,

$$I'' = 2 \times \frac{6.67}{5 + 6.67} = 1.14 \text{ A } (\leftarrow)$$

Step III: When the 36 V source is acting alone

By series-parallel reduction technique,

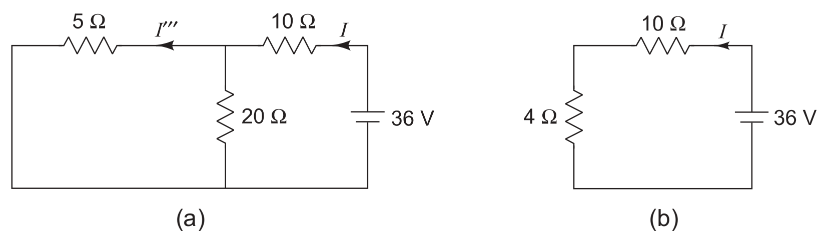


Fig. 2.324

$$I = \frac{36}{10 + 4} = 2.57 \text{ A}$$

From Fig. 2.324(a), by current-division rule,

$$I''' = 2.57 \times \frac{20}{20 + 5} = 2.06 \text{ A } (\leftarrow)$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= -2.06 + 1.14 + 2.06 \\ &= 1.14 \text{ A } (\leftarrow) \end{aligned}$$

Example 13

Find the value of current flowing through 30 Ω resistor.

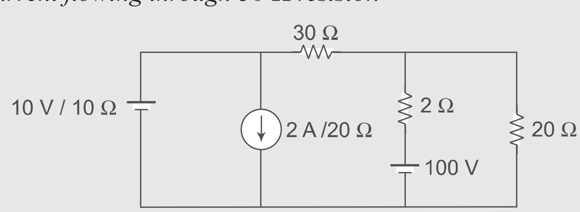


Fig. 2.325 **[Dec 2015]**

Solution *Step I: When the 10 V source is acting alone*

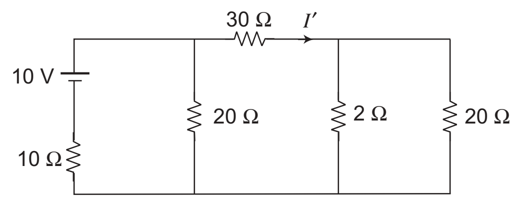


Fig. 2.326

By series-parallel reduction technique,

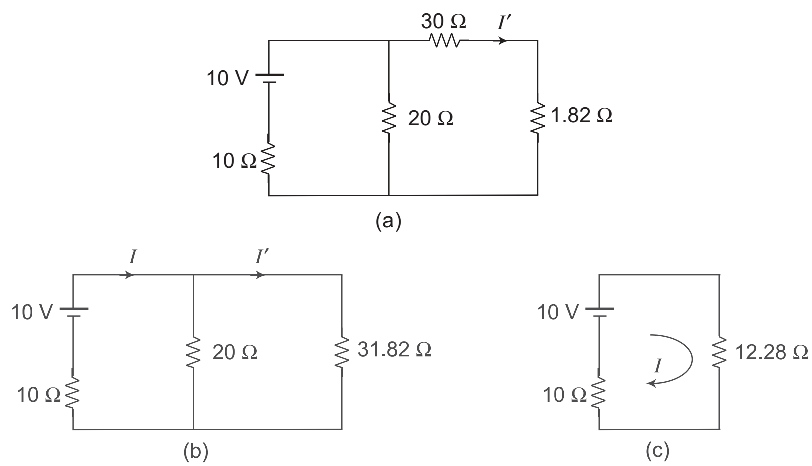


Fig. 2.327

$$I = \frac{10}{10+12.28} = 0.45 \text{ A}$$

From Fig. 2.327(b), by current-division rule,

$$I' = 0.45 \times \frac{20}{20+31.82} = 0.17 \text{ A} (\rightarrow) = -0.17 \text{ A} (\leftarrow)$$

Step II: When the 2A source is acting alone

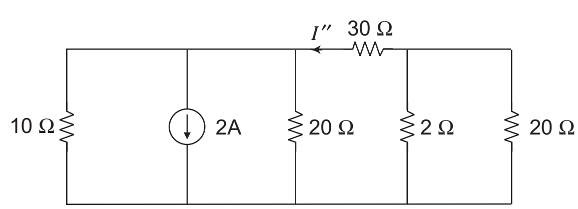


Fig. 2.328

By series-parallel reduction technique,

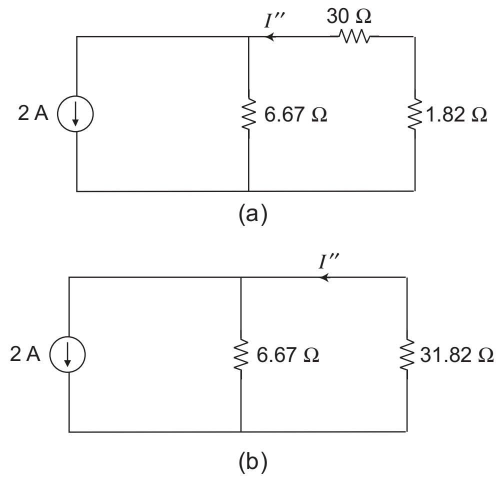


Fig. 2.329

By current-division rule,

$$I'' = 2 \times \frac{6.67}{6.67 + 31.82} = 0.35\,\text{A} \quad (\leftarrow)$$

Step-III: When the 100 V source is acting alone

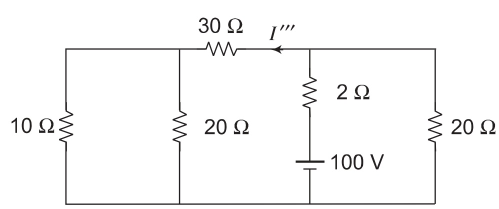
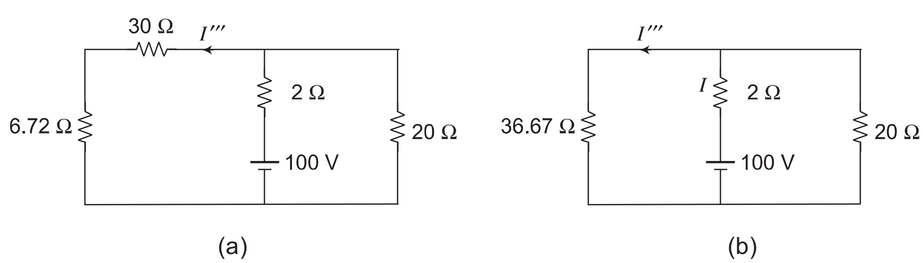
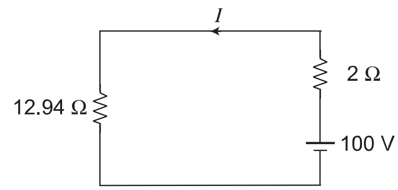


Fig. 2.330

By series-parallel reduction technique,





(c)

Fig. 2.331

$$I = \frac{100}{12.94+2} = 6.69 \text{ A}$$

By current-division rule,

$$I''' = 6.69 \times \frac{20}{20+36.67} = 2.36 \text{ A} (\leftarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= -0.17 + 0.35 + 2.36$$
$$= 2.54 \text{ A} \ (\leftarrow)$$

Example 14

Find the value of current flowing through the 5 Ω resistor.

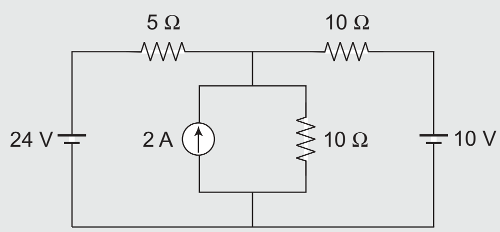
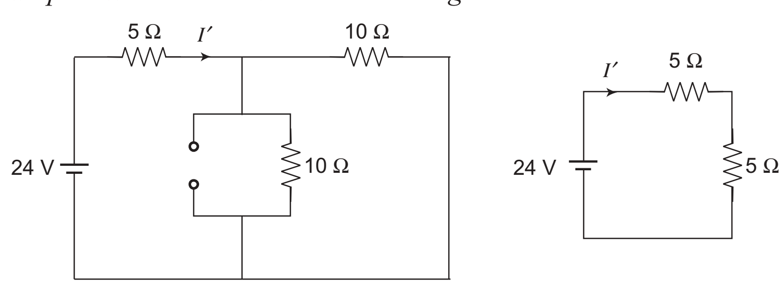


Fig. 2.332 **[May 2015]**

Solution *Step I: When the 24 V source is acting alone*

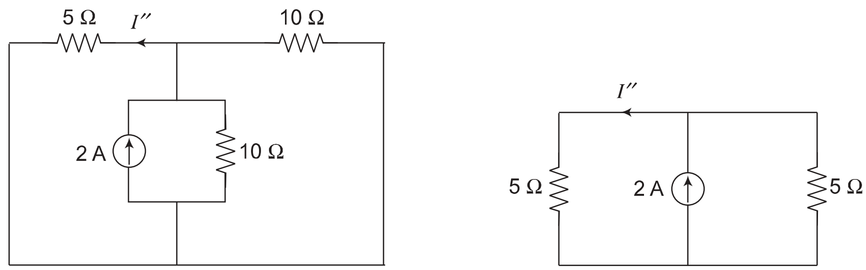


(a) (b)

Fig. 2.333

$$I' = \frac{24}{5+5} = 2.4 \text{ A} (\rightarrow) = -2.4 \text{ A} (\leftarrow)$$

Step II *When the 2 A source is acting alone*

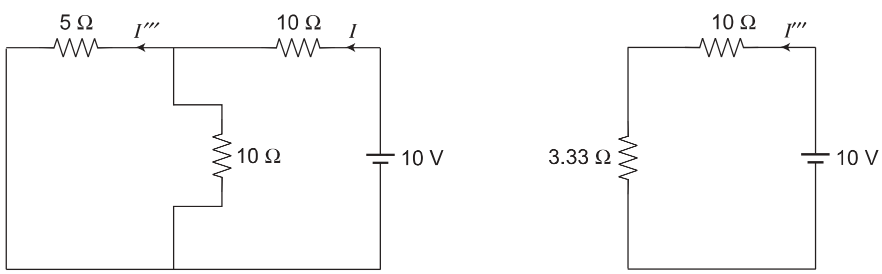


(a) (b)

Fig. 2.334

$$I'' = 2 \times \frac{5}{5+5} = 1 \text{ A}(\leftarrow)$$

Step III When the 10 V source is acting alone



(a) (b)

Fig. 2.335

$$I = \frac{10}{10+3.33} = 0.75 \text{ A}$$

By current-division rule,

$$I''' = 0.75 \times \frac{10}{10+5} = 0.5 \text{ A}(\leftarrow)$$

Step IV *By superposition theorem,*

$$I = I' + I'' + I'''$$
$$= -2.4 + 1 + 0.5$$
$$= -0.9 \text{ A} (\leftarrow)$$
$$I = 0.9 \text{ A} (\rightarrow)$$

Example 15

Find the value of current flowing through the 4 Ω resistor.

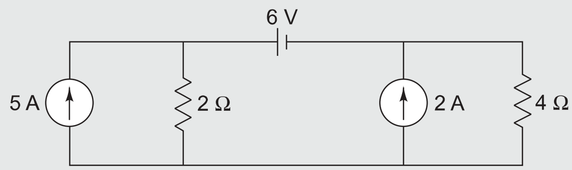


Fig. 2.336

Solution *Step I: When the 5 A source is acting alone*

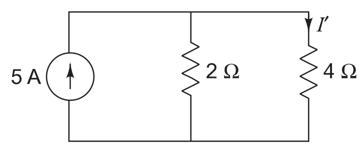


Fig. 2.337

By current-division rule,

$$I' = 5 \times \frac{2}{2+4} = 1.67 \text{ A } (\downarrow)$$

Step II: When the 2 A source is acting alone

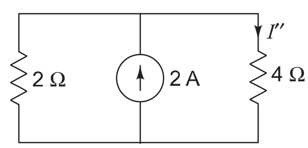


Fig. 2.338

By current-division rule,

$$I'' = 2 \times \frac{2}{2+4} = 0.67 \text{ A } (\downarrow)$$

Step III: When the 6 V source is acting alone

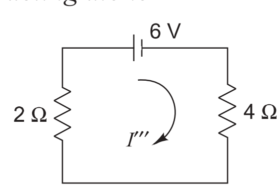


Fig. 2.339

Applying KVL to the mesh,

$$-2I''' - 6 - 4I''' = 0$$

$$I''' = -1 \text{ A} (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$

$$= 1.67 + 0.67 - 1$$

$$= 1.34 \text{ A} (\downarrow)$$

Example 16

Find the value of current flowing through the 5 Ω resistor.

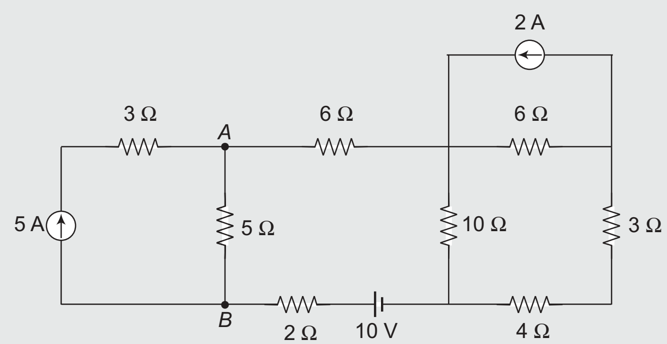


Fig. 2.340 [Dec 2014]

Solution *Step I: When the 5 A source is acting alone*

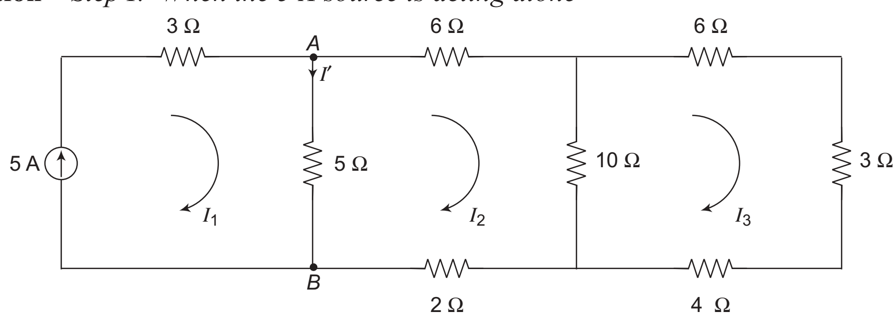


Fig. 2.341

Writing equations in matrix form,

$$\begin{bmatrix} 1 & 0 & 0 \\ -5 & 23 & -10 \\ 0 & -10 & 23 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = 5$$

$$I_2 = 1.34 \text{ A}$$

$$I_3 = 0.58 \text{ A}$$
$$I' = I_1 - I_2 = 5 - 1.34 = 3.66 \text{ A } (\downarrow)$$

Step II: When the 10 V source is acting alone

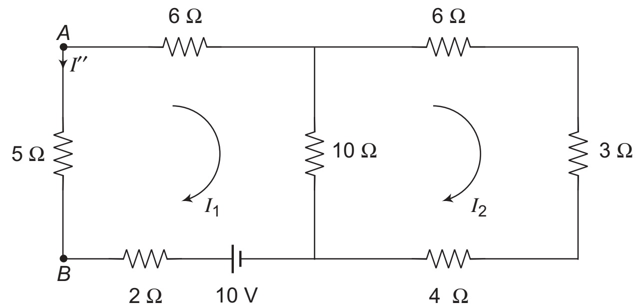


Fig. 2.342

Writing KVL equations in matrix form,

$$\begin{bmatrix} 23 & -10 \\ -10 & 23 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$I_1 = 0.54 \text{ A}$$
$$I_2 = 0.23 \text{ A}$$
$$I'' = -I_1 = -0.54 \text{ A } (\downarrow)$$

Step III: When the 2 A source is acting alone

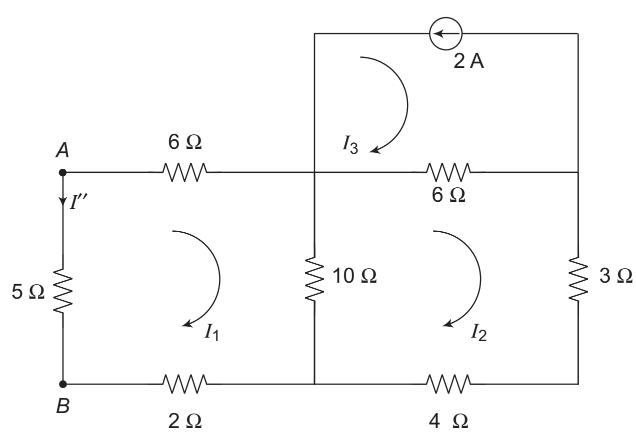


Fig. 2.343

Writing equations in matrix form,

$$\begin{bmatrix} 23 & -10 & 0 \\ -10 & 23 & -6 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

$$I_1 = -0.28 \text{ A}$$
$$I_2 = -0.64 \text{ A}$$
$$I_3 = -2 \text{ A}$$
$$I''' = -I_1 = -0.28 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 3.66 - 0.54 + 0.28$$
$$= 3.4 \text{ A}$$

Example 16

Find the value of current flowing through the 3 Ω resistor.

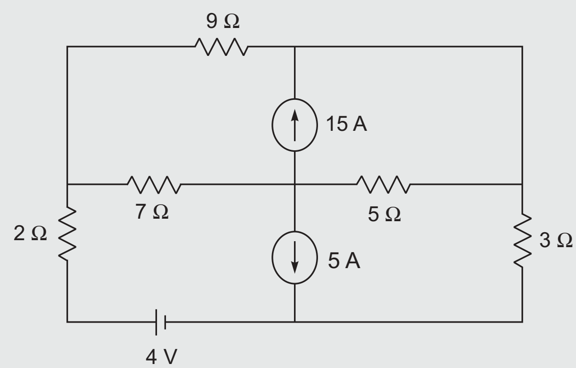


Fig. 2.344 [Dec 2012]

Solution *Step I:* When the 4 V source is acting alone

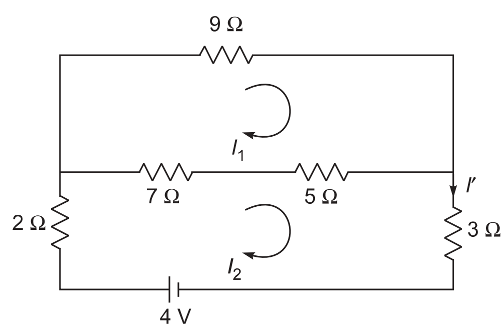


Fig. 2.345

Writing KVL equation in matrix form,

$$\begin{bmatrix} 21 & -12 \\ -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$I' = I_2 = 0.39 \text{ A} \quad (\downarrow)$$

Step II When the 15 A source is acting alone

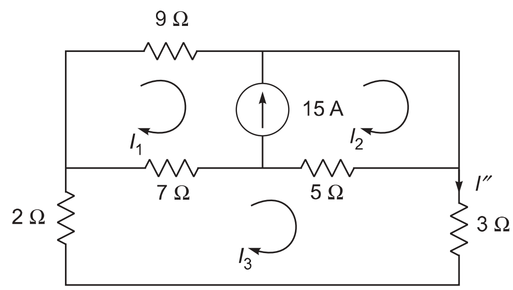


Fig. 2.346

Writing the current equation for the supermesh,

$$I_2 - I_1 = 15 \tag{1}$$

Writing the voltage equation for the supermesh,

$$-9I_1 - 5(I_2 - I_3) - 7(I_1 - I_3) = 0$$
$$-16I_1 - 5I_2 + 12I_3 = 0 \tag{2}$$

Applying KVL to Mesh 3,

$$-2I_3 - 7(I_3 - I_1) - 5(I_3 - I_2) - 3I_3 = 0$$
$$-7I_1 - 5I_2 + 17I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I'' = I_3 = 3.17\text{ A} \quad (\downarrow)$$

Sep III When the 5 A source is acting alone

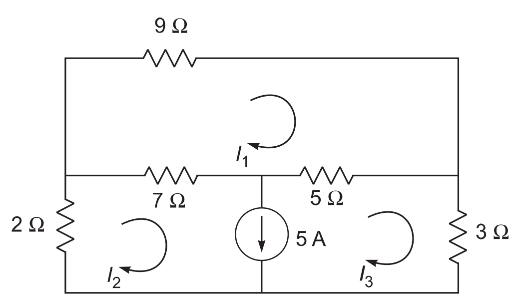


Fig. 2.347

Applying KVL to Mesh 1,

$$-9I_1 - 5(I_1 - I_3) - 7(I_1 - I_2) = 0$$
$$21I_1 - 7I_2 - 5I_3 = 0 \tag{1}$$

Writing the current equation for the supermesh,

$$I_2 - I_3 = 5 \qquad (2)$$

Writing the voltage equation for the supermesh,

$$-2I_2 - 7(I_2 - I_1) - 5(I_3 - I_1) - 3I_3 = 0$$

$$12I_1 - 9I_2 - 8I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I''' = I_3 = -2.46 \text{ A} \quad (\downarrow)$$

Step IV *By superposition theorem,*

$$I = I' + I'' + I''' = 0.39 + 3.17 - 2.46 = 1.1 \text{ A}$$

$$V_{3\,\Omega} = 3I = 3(1.1) = 3.3 \text{ V}$$

Example 17

Determine the value of current flowing through $R_L = 2\ \Omega$ in the circuit shown in Fig. 2.348.

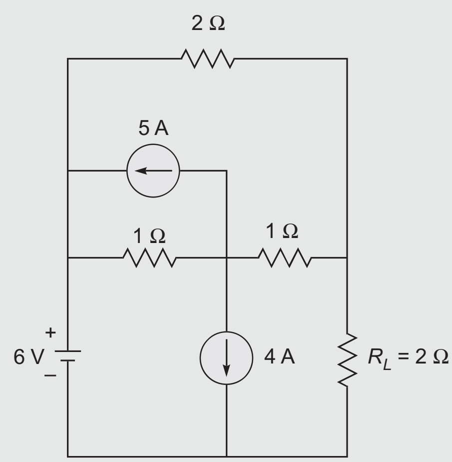


Fig. 2.348 **[May 2013]**

Solution *Step I:* When the 6 V source is acting alone

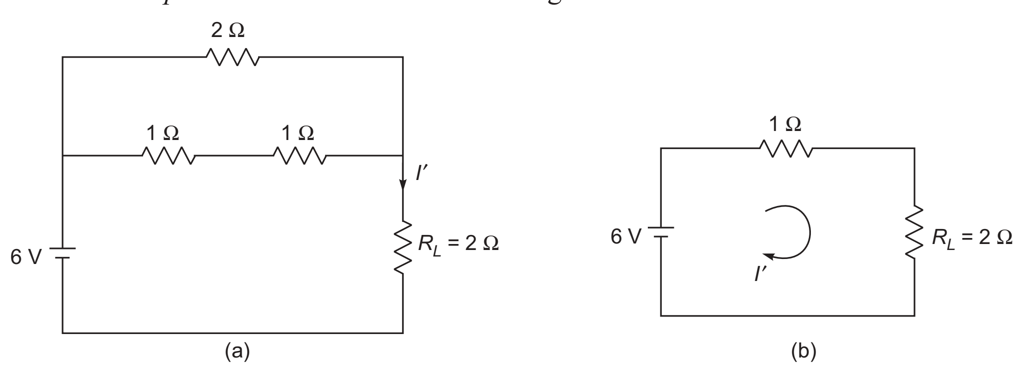


Fig. 2.349

$$I' = \frac{6}{1+2} = 2\text{ A} \quad (\downarrow)$$

Step II When the 4 A source is acting alone

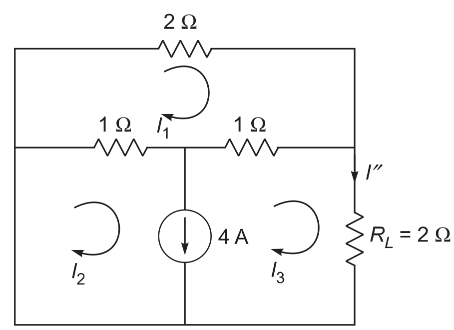


Fig. 2.350

Applying KVL to Mesh 1,

$$4I_1 - I_2 - I_3 = 0 \qquad (1)$$

Writing the current equation for the supermesh,

$$I_2 - I_3 = 4 \qquad (2)$$

Writing the voltage equation for the supermesh,

$$-1(I_2 - I_1) - 1(I_3 - I_1) - 2I_3 = 0$$
$$2I_1 - I_2 - 3I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I'' = I_3 = -0.67\text{ A} \quad (\downarrow)$$

Step III When the 5 A source is acting alone

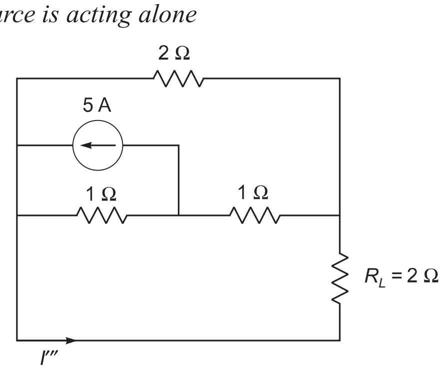


Fig. 2.351

Simplifying the circuit,

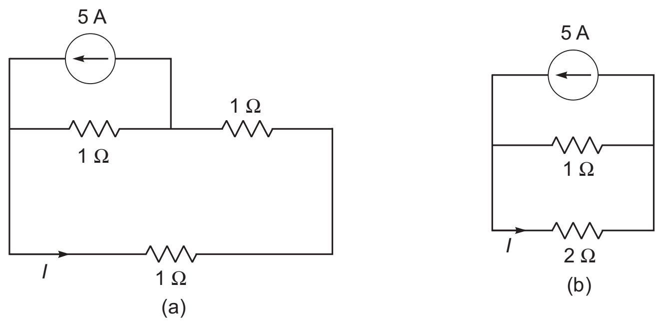


Fig. 2.352

$$I = 5 \times \frac{1}{1+2} = 1.67 \text{ A} \quad (\uparrow)$$

$$I''' = -1.67 \times \frac{1}{2} = -0.84 \text{ A} \quad (\downarrow)$$

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 2 - 0.67 - 0.84 = -0.49 \text{ A} \quad (\downarrow)$$

Example 18

Determine the value of current flowing in the 1 Ω resistor.

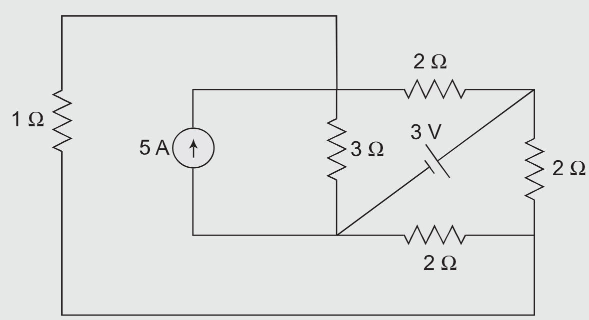


Fig. 2.353 **[Dec 2013]**

Solution *Step I: When the 5 A source is acting alone*

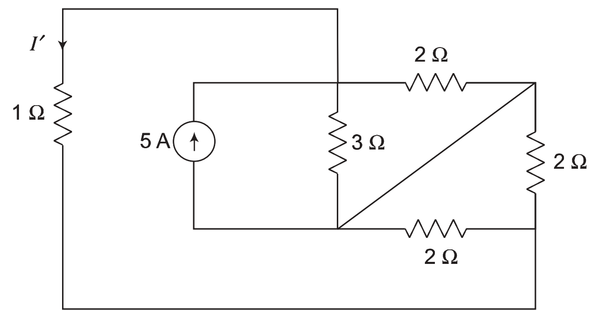


Fig. 2.354

Simplifying the network,

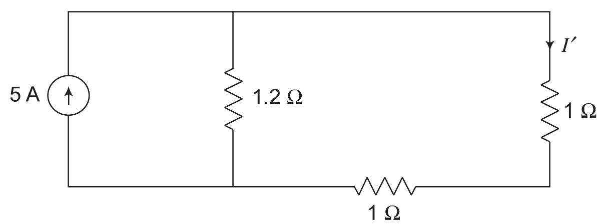


Fig. 2.355

By current-division rule,

$$I' = 5\times\frac{1.2}{1.2+1+1} = 1.875\text{ A }(\downarrow)$$

Step II When the 3 V source is acting alone

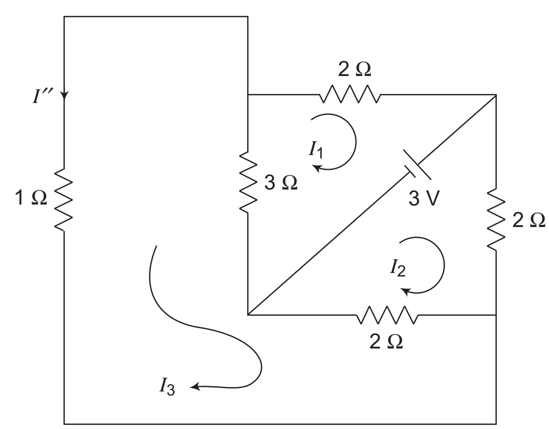


Fig. 2.356

Applying KVL to Mesh 1,

$$-2I_1 - 3 - 3\,(I_1 - I_3) = 0$$

$$5I_1 - 3I_3 = -3 \qquad (1)$$

Applying KVL to Mesh 2,

$$3 - 2I_2 - 2(I_2 - I_3) = 0$$

$$4I_2 - 2\,I_3 = 3 \qquad (2)$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_1) - 2\,(I_3 - I_2) - I_3 = 0$$

$$-3I_1 - 2I_2 + 6I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = -0.66 \text{ A}$$

$$I_2 = 0.7 \text{ A}$$

$$I_3 = -0.09 \text{ A}$$

$$I'' = -I_3 = 0.09 \text{ A}$$

Step III *By superposition theorem*,

$$I = I' + I'' = 1.875 + 0.09 = 1.965 \text{ A } (\downarrow)$$

Example 19

Find the value of current flowing through the 6 Ω resistor.

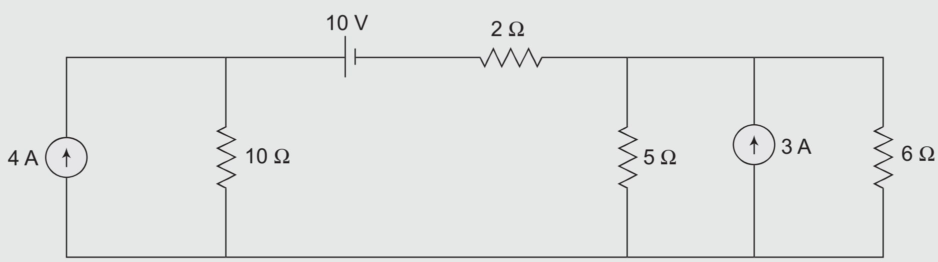


Fig. 2.357 **[May 2014]**

Solution *Step I: When the 4 A source is acting alone*

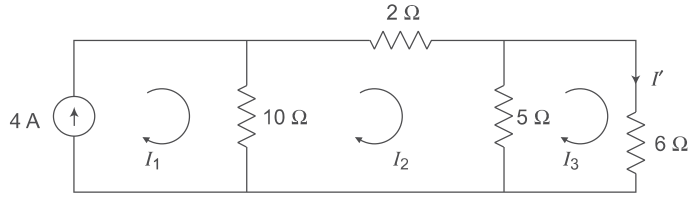


Fig. 2.358

Writing equations in matrix form,

$$\begin{bmatrix} 1 & 0 & 0 \\ -10 & 17 & -5 \\ 0 & -5 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$I' = I_3 = 1.23 \text{ A} \quad (\downarrow)$$

Step II When the 10 V source is acting alone

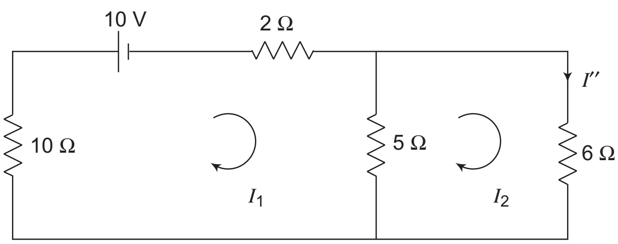


Fig. 2.359

Writing KVL equation in matrix form,

$$\begin{bmatrix} 17 & -5 \\ -5 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

$$I'' = I_2 = -0.31 \text{ A} (\downarrow)$$

Step III When the 3 A source is acting alone

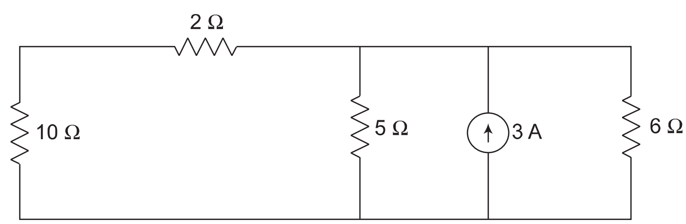


Fig. 2.360

By series-parallel reduction technique,

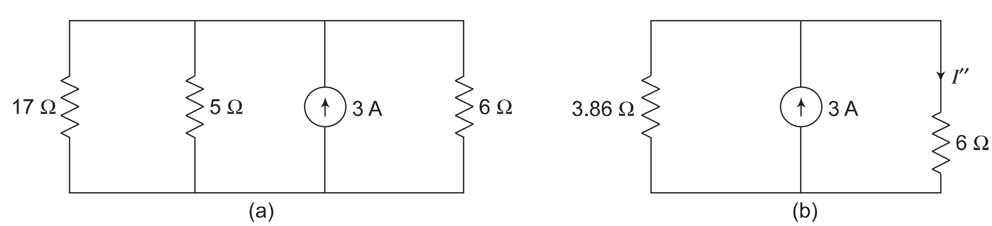


Fig. 2.361

By current-division rule,

$$I''' = 3 \times \frac{3.86}{3.86+6} = 1.17 \text{ A } (\downarrow)$$

Step IV *By superposition theorem,*

$$I = I' + I'' + I''' = 1.23 - 0.31 + 1.17 = 2.09 \text{ A } (\downarrow)$$

Exercise 2.6

2.1 Find the value of current flowing through the 1 Ω resistor.

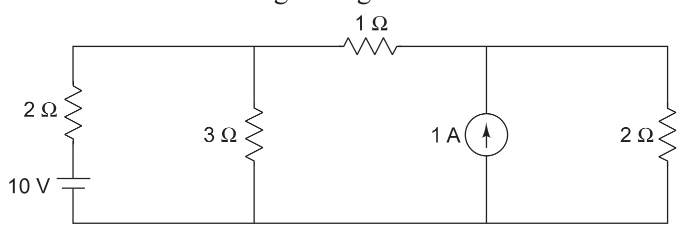


Fig. 2.362

[0.95 A]

2.2 Find the value of current flowing through the 10 Ω resistor.

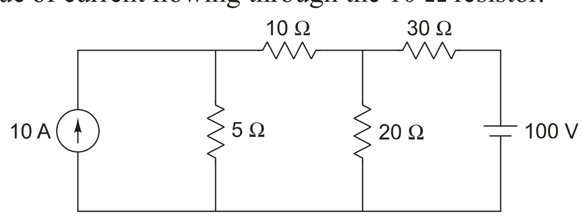


Fig. 2.363

[0.37 A]

2.3 Calculate the value of current flowing through the 10 Ω resistor.

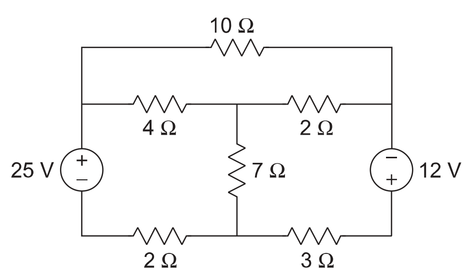


Fig. 2.364

[1.62 A]

2.4 Find the value of current flowing in the 2 Ω resistor. Also, find voltage across the current source.

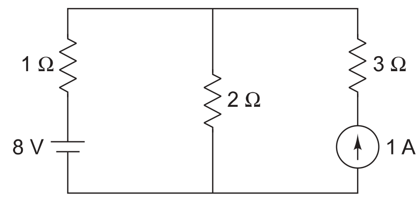


Fig. 2.365

[3 A, 9 V]

2.5 Find the current I_x.

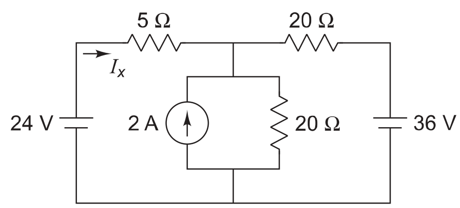


Fig. 2.366

[–0.93 A]

2.9 THEVENIN'S THEOREM

It states that *'Any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.'*

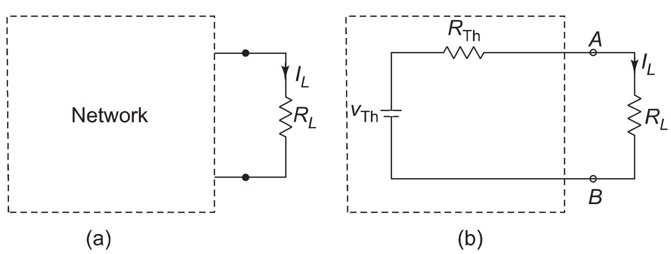


Fig. 2.367 *Thevenin's theorem*

Explanation The above method of determining the load current through a given load resistance can be explained with the help of the following circuit.

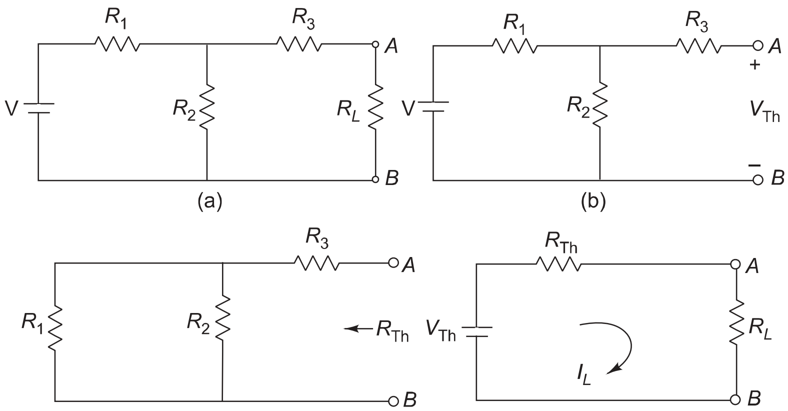


Fig. 2.368 *Steps in Thevenin's theorem*

2.9.1 Steps to be followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with the voltage sources and current sources replaced by internal resistances.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

Example 1

Find the value of current flowing through the 2 Ω resistor.

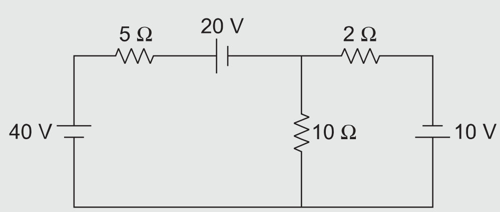


Fig. 2.369

Solution *Step I : Calculation of* V_{Th}

Removing the 2 Ω resistor from the network,

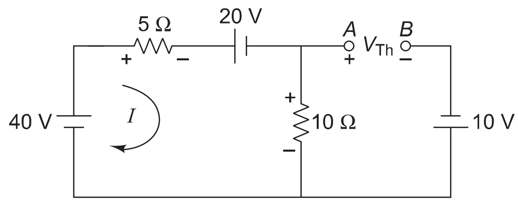


Fig. 2.370

Applying KVL to the mesh,

$$40 - 5I - 20 - 10I = 0$$
$$15I = 20$$
$$I = 1.33 \text{ A}$$

Writing V_{Th} equation,

$$10I - V_{\text{Th}} + 10 = 0$$
$$V_{\text{Th}} = 10I + 10$$
$$= 10\,(1.33) + 10$$
$$= 23.33 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

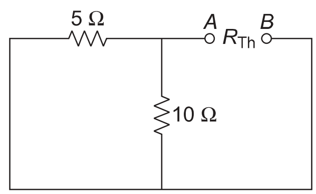


Fig. 2.371

$$R_{\text{Th}} = 5 \parallel 10 = 3.33\ \Omega$$

Step III: Calculation of I_L

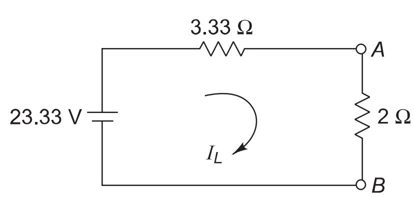


Fig. 2.372

$$I_L = \frac{23.33}{3.33 + 2} = 4.38 \text{ A}$$

Example 2

Find the value of current flowing through the 8 Ω resistor.

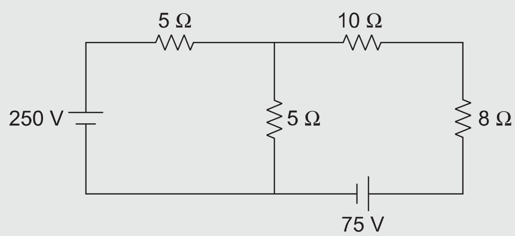


Fig. 2.373

Solution *Step I: Calculation of* V_{Th}

Removing the 8 Ω resistor from the network,

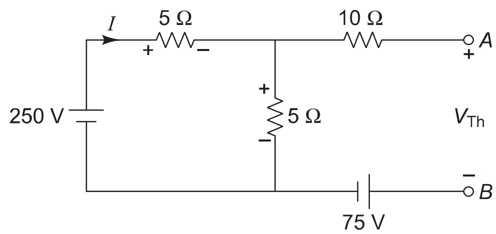


Fig. 2.374

$$I = \frac{250}{5+5} = 25 \text{ A}$$

Writing V_{Th} equation,

$$250 - 5I - V_{\text{Th}} - 75 = 0$$

$$V_{\text{Th}} = 175 - 5I$$

$$= 175 - 5\,(25)$$

$$= 50 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

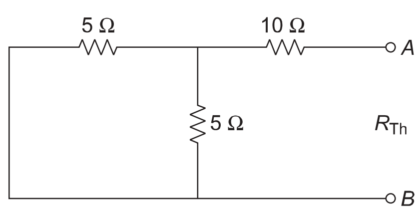


Fig. 2.375

$$R_{\text{Th}} = (5 \parallel 5) + 10 = 12.5\ \Omega$$

Step III: Calculation of I_L

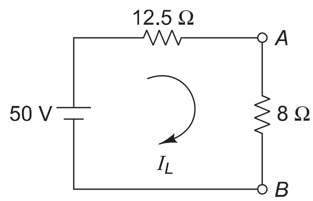


Fig. 2.376

$$I_L = \frac{50}{12.5 + 8} = 2.44 \text{ A}$$

Example 3

Find the value of current flowing through the 2 Ω resistor connected between terminals A and B.

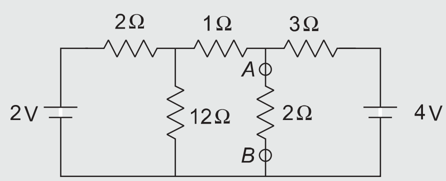


Fig. 2.377

Solution

Step I: Calculation of V_{Th}

Removing the 2 Ω resistor connected between terminals *A* and *B*,

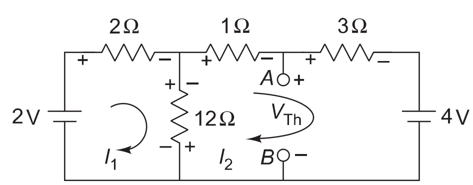


Fig. 2.378

Applying KVL to Mesh 1,

$$2 - 2I_1 - 12\,(I_1 - I_2) = 0$$

$$14I_1 - 12I_2 = 2 \qquad (1)$$

Applying KVL to Mesh 2,

$$-12\,(I_2 - I_1) - 1I_2 - 3I_2 - 4 = 0$$

$$-12I_1 + 16I_2 = -4 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_2 = -0.4 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 3I_2 - 4 = 0$$
$$V_{\text{Th}} = 4 + 3I_2$$
$$= 4 + 3\,(-0.4)$$
$$= 2.8\text{ V}$$

Step II: Calculation of R_{Th}

Replacing all voltage sources by short circuits,

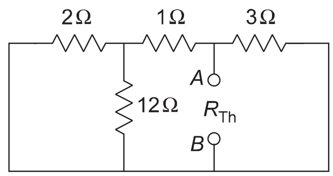


Fig. 2.379

$$R_{\text{Th}} = [|(2||\ 12) + 1]\ ||\ 3 = 1.43\ \Omega$$

Step III: Calculation of I_L

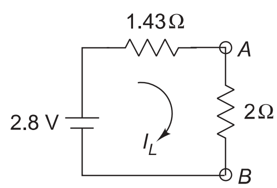


Fig. 2.380

$$I_L = \frac{40}{5+1.67} = 0.82\text{ A}$$

Example 4

Find the value of current flowing through the 8 Ω resistor.

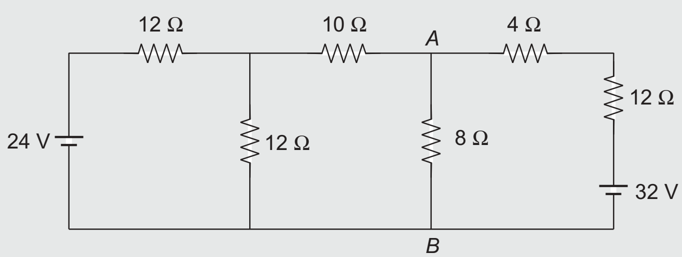


Fig. 2.381 [May 2015]

Solution *Step I: Calculation of V_{Th}*

Removing 8 Ω resistor connected between *A* and *B*,

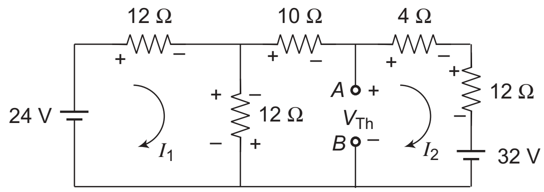


Fig. 2.382

Applying KVL to Mesh 1,

$$24 - 12I_1 - 12\,(I_1 - I_2) = 0$$

$$24I_1 - 12I_2 = 24 \qquad (1)$$

Applying KVL to Mesh 2,

$$-12\,(I_2 - I_1) - 10\,I_2 - 4I_2 - 12I_2 - 32 = 0$$

$$-12\,I_1 + 38\,I_2 = -32 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 0.69 \text{ A}$$

$$I_2 = -0.63 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} - 4I_2 - 12I_2 - 32 = 0$$

$$V_{Th} = 32 + 4\,(-0.63) + 12\,(-0.63)$$

$$= 21.92 \text{ V}$$

Step II: Calculation of R_{TH}

Replacing all voltage sources by short circuits,

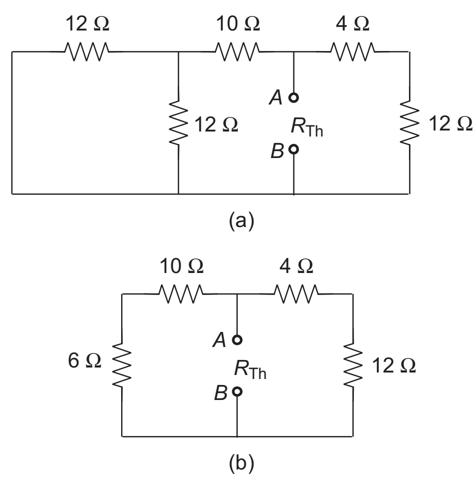


Fig. 2.383

$$R_{TH} = 8\ \Omega$$

Step III: Calculation of I_L

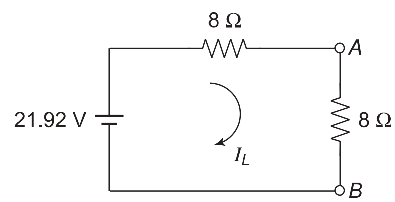


Fig. 2.384

$$I_L = \frac{21.92}{8+8} = 1.37 \text{ A}$$

Example 5

Find the value of current flowing through the 10 Ω resistor.

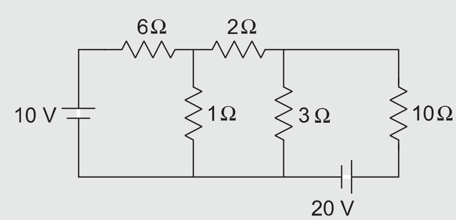


Fig. 2.385

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

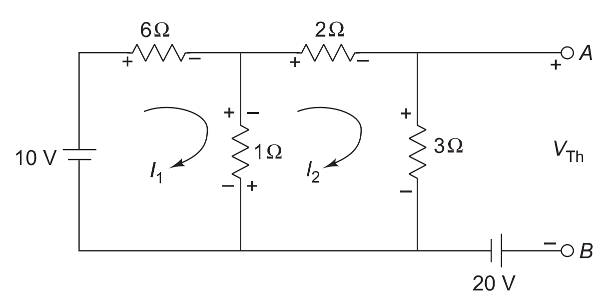


Fig. 2.386

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \quad (1)$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3I_2 = 0$$

$$I_1 - 6I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_2 = 0.24 \text{ A}$$

Writing V_{Th} equation,

$$3I_2 - V_{\text{Th}} - 20 = 0$$

$$V_{\text{Th}} = 3I_2 - 20$$

$$= 3\,(0.24) - 20$$

$$= -\,19.28 \text{ V}$$

$$= 19.28 \text{ V (terminal } B \text{ is positive w.r.t } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

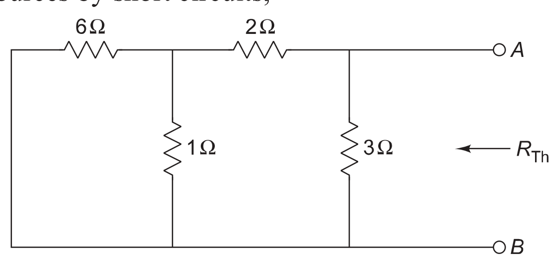


Fig. 2.387

$$R_{\text{Th}} = [(6 \| 1) + 2] \,\|\, 3 = 1.47\ \Omega$$

Step III: Calculation of I_L

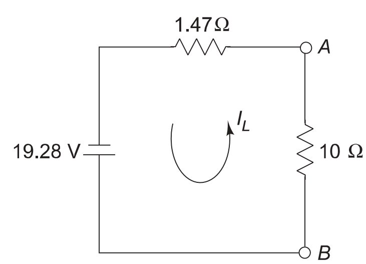


Fig. 2.388

$$I_L = 6 \times \frac{10}{10+2} = 1.68 \text{ A } (\uparrow)$$

Example 6

Find the value of current flowing through the 10 Ω resistor.

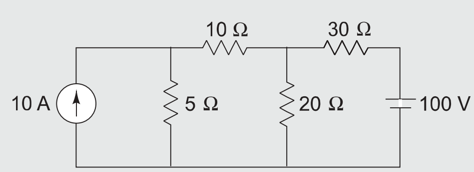


Fig. 2.389

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

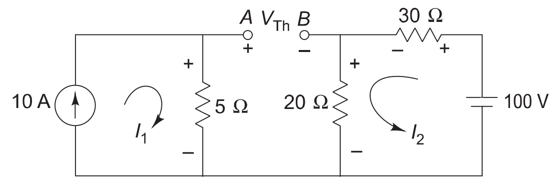


Fig. 2.390

For Mesh 1,

$$I_1 = 10$$

Applying KVL to Mesh 2,

$$100 - 30I_2 - 20I_2 = 0$$

$$I_2 = 2\text{ A}$$

Writing V_{Th} equation,

$$5I_1 - V_{Th} - 20I_2 = 0$$

$$V_{Th} = 5I_1 - 20I_2 = 5(10) - 20(2) = 10\text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source by an open circuit and the voltage source by a short circuit,

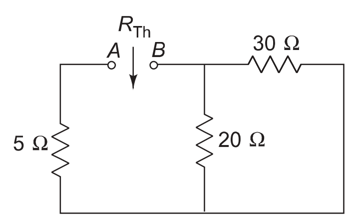


Fig. 2.391

$$R_{Th} = 5 + (20 \,||\, 30) = 17\ \Omega$$

Step III: Calculation of I_L

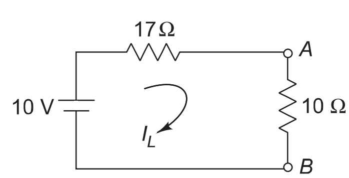


Fig. 2.392

$$I_L = \frac{20}{5 + 1.67} = 0.37\text{ A}$$

Example 7

Find the value of current flowing through the 40 Ω resistor.

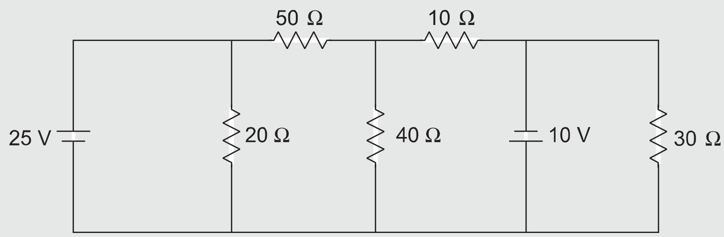


Fig. 2.393

Solution

Step I: Calculation of V_{Th}

Removing the 40 Ω resistor from the network,

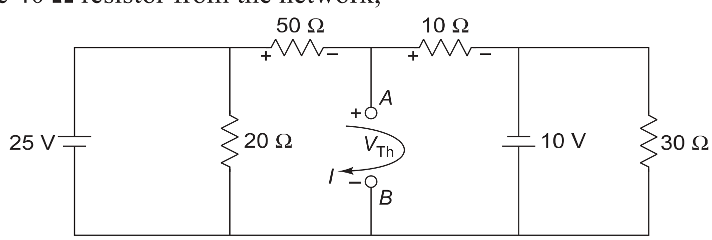


Fig. 2.394

Since the 20 Ω resistor is connected across the 25 V source, the resistor becomes redundant.

$$V_{20\,\Omega} = 25\text{ V}$$

Applying KVL to the mesh,

$$25 - 50I - 10I + 10 = 0$$
$$I = 0.58\text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 10\text{I} + 10 = 0$$
$$V_{\text{Th}} = 10\,(I) - 10$$
$$= 10\,(0.58) - 10$$
$$= -\,4.2\text{ V}$$
$$= 4.2\text{ V (terminal } B \text{ is positive w.r.t. A)}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits,

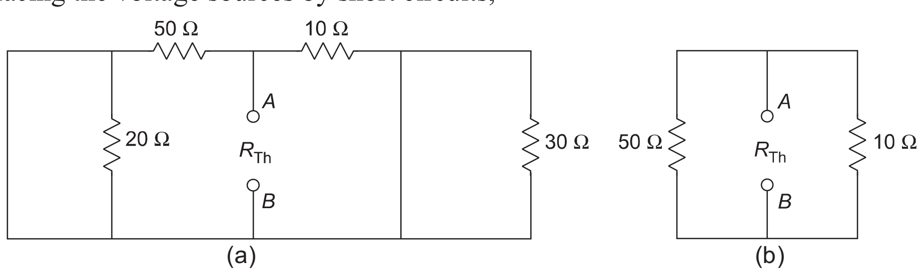


Fig. 2.395

$$R_{Th} = 50 \parallel 10 = 8.33\ \Omega$$

Step III: Calculation of I_L

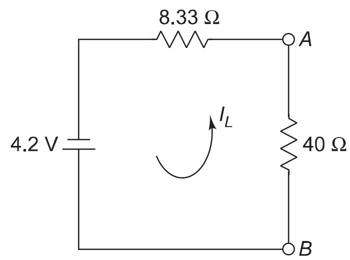


Fig. 2.396

$$I_L = \frac{10}{3.33 + 2} = 0.09\ \text{A}\ (\uparrow)$$

Example 8

Find the values of current flowing through the 10 Ω resistor.

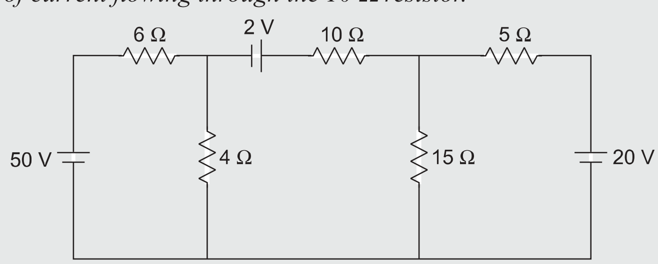


Fig. 2.397

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

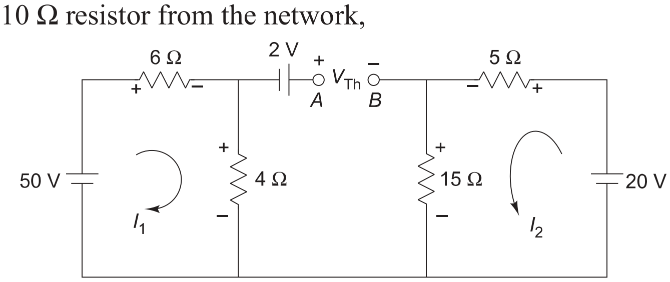


Fig. 2.398

$$I_1 = \frac{50}{10} = 5\ \text{A}$$

$$I_2 = \frac{20}{20} = 1\ \text{A}$$

Writing V_{Th} equation,

$$4I_1 + 2 - V_{\text{Th}} - 15I_2 = 0$$
$$V_{\text{Th}} = 4I_1 + 2 - 15I_2$$
$$= 4(5) + 2 - 15(1)$$
$$= 7\ V$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

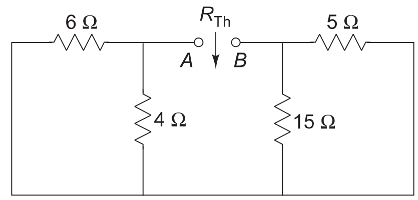


Fig. 2.399

$$R_{\text{Th}} = (6 \parallel 4) + (5 \parallel 15) = 6.15\ \Omega$$

Step III: Calculation of I_L

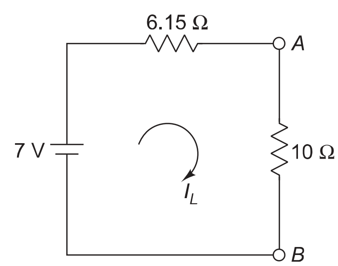


Fig. 2.400

$$I_L = \frac{50}{1+1.33} = 0.43\text{ A}$$

Example 9

Determine the value of current flowing through the 24 Ω resistor.

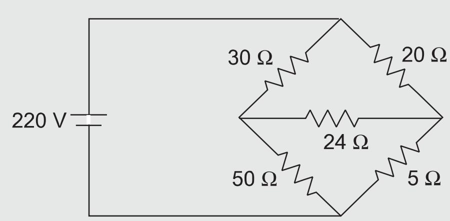


Fig. 2.401

Solution

Step I: Calculation of V_{Th}

Removing the 24 Ω resistor from the network,

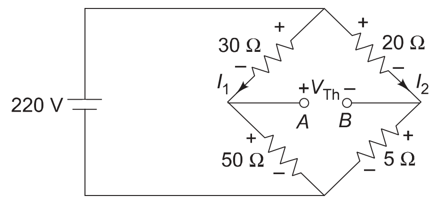


Fig. 2.402

$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} + 30I_1 - 20I_2 = 0$$

$$\begin{aligned} V_{Th} &= 20I_2 - 30I_1 \\ &= 20\,(8.8) - 30\,(2.75) \\ &= 93.5 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

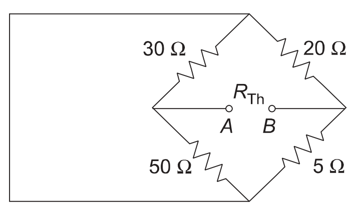


Fig. 2.403

Redrawing the circuit,

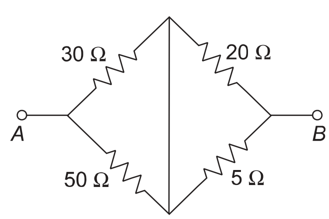


Fig. 2.404

$$R_{Th} = (30\,||\,50) + (20\,||\,5) = 22.75\ \Omega$$

Step III: Calculation of I_L

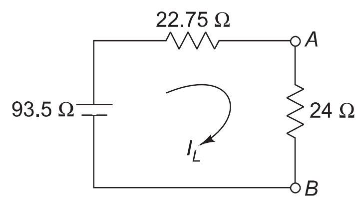


Fig. 2.405

$$I_L = \frac{93.5}{22.75 + 24} = 2 \text{ A}$$

Example 10

Find the value of current flowing through the 3 Ω resistor.

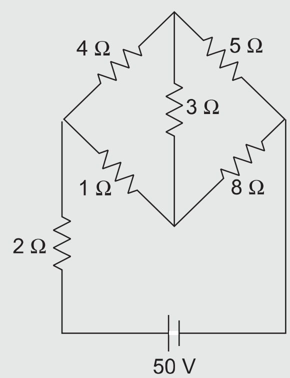


Fig. 2.406

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

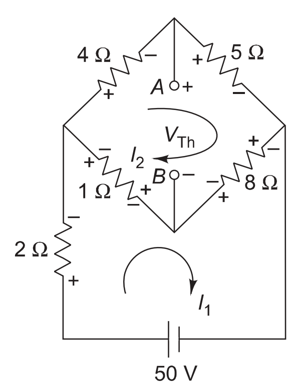


Fig. 2.407

Applying KVL to Mesh 1,

$$50 - 2I_1 - 1(I_1 - I_2) - 8(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 50 \qquad (1)$$

Applying KVL to Mesh 2,

$$-4I_2 - 5I_2 - 8(I_2 - I_1) - 1(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 7.69 \text{ A}$$

$$I_2 = 3.85 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 5I_2 - 8(I_2 - I_1) = 0$$

$$V_{\text{Th}} = 5I_2 + 8\,(I_2 - I_1)$$

$$= 5(3.85) + 8(3.85 - 7.69)$$

$$= -11.47 \text{ V}$$

$$= 11.47 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

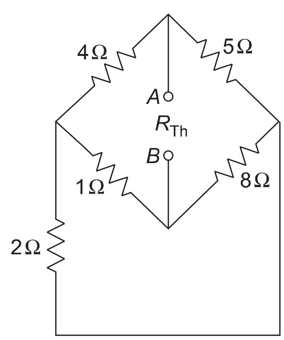


Fig. 2.408

Redrawing the network,

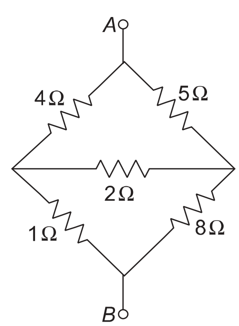


Fig. 2.409

Converting the upper delta into equivalent star network,

$$R_1 = \frac{4 \times 2}{4+2+5} = 0.73\ \Omega$$

$$R_2 = \frac{4 \times 5}{4+2+5} = 1.82\ \Omega$$

$$R_3 = \frac{5 \times 2}{4+2+5} = 0.91\ \Omega$$

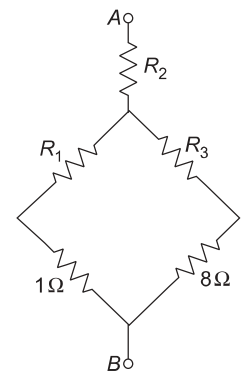


Fig. 2.410

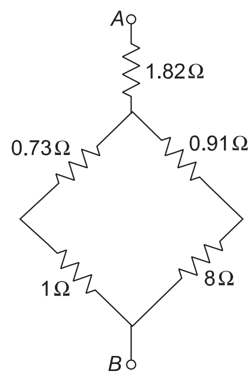


Fig. 2.411

Simplifying the network,

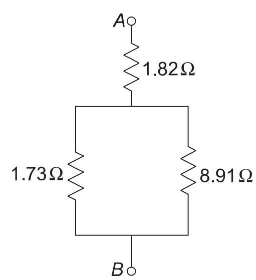


Fig. 2.412

$$R_{Th} = 1.82 + (1.73\ ||\ 8.91) = 3.27\ \Omega$$

Step III: Calculation of I_L

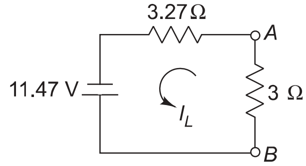


Fig. 2.413

$$I_L = \frac{40}{12 + 2.92} = 1.83 \text{ A } (\uparrow)$$

Example 11

Find the value of current flowing through the 20 Ω resistor.

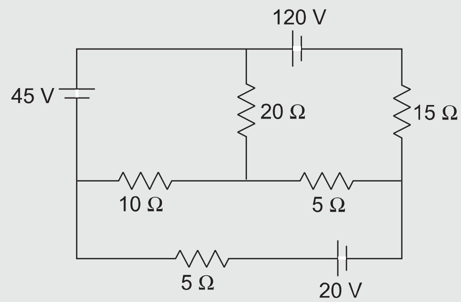


Fig. 2.414

Solution

Step I: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

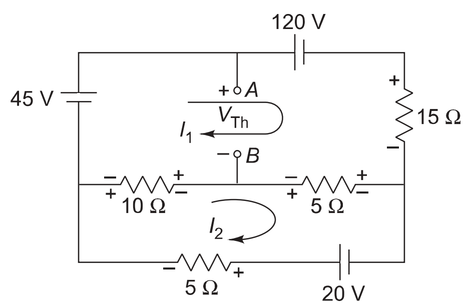


Fig. 2.415

Applying KVL to Mesh 1,

$$45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0$$

$$30I_1 - 15I_2 = -75 \quad (1)$$

Applying KVL to Mesh 2,

$$20 - 5I_2 - 10(I_2 - I_1) - 5(I_2 - I_1) = 0$$

$$-15I_1 + 20I_2 = 20 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -3.2 \text{ A}$$
$$I_2 = -1.4 \text{ A}$$

Writing V_{Th} equation,

$$45 - V_{Th} - 10(I_1 - I_2) = 0$$

$$V_{Th} = 45 - 10(I_1 - I_2)$$

$$= 45 - 10[-3.2 - (-1.4)]$$

$$= 63 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

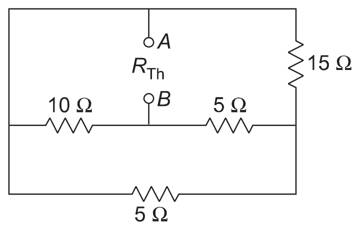


Fig. 2.416

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network,

$$R_1 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_2 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_3 = \frac{5 \times 5}{20} = 1.25\ \Omega$$

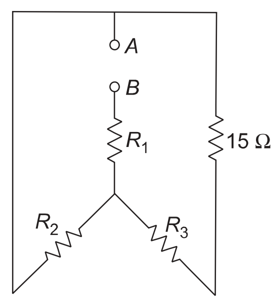


Fig. 2.417

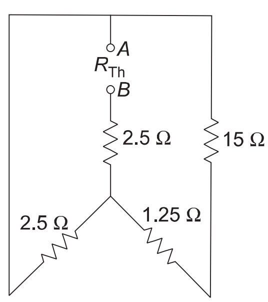


Fig. 2.418

Simplifying the network,

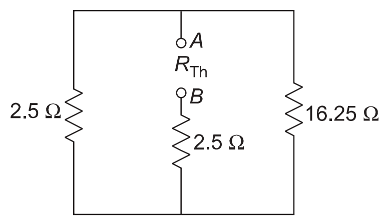


Fig. 2.419

$$R_{Th} = (16.25 \,||\, 2.5) + 2.5 = 4.67\ \Omega$$

Step III: Calculation of I_L

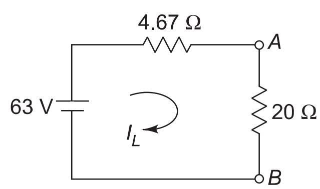


Fig. 2.420

$$I_L = \frac{63}{4.67 + 20} = 2.55\ \text{A}$$

Example 12

Find the value of current flowing through the 3 Ω resistor.

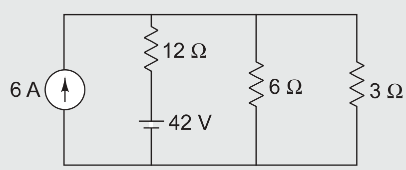


Fig. 2.421

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

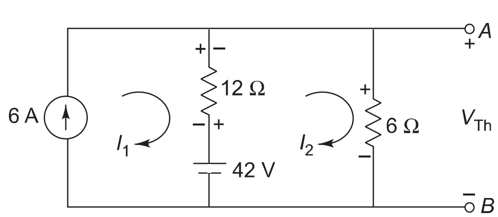


Fig. 2.422

Writing equation for Mesh 1,

$$I_1 = 6 \tag{1}$$

Applying KVL to Mesh 2,

$$42 - 12(I_2 - I_1) - 6\,I_2 = 0$$

$$-12\,I_1 + 18\,I_2 = 42 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = 6.33 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 6\,I_2 = 38 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by short circuit and current source by open circuit,

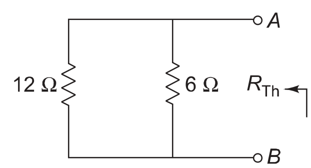


Fig. 2.423

$$R_{\text{Th}} = 6 \,\|\, 12 = 4\ \Omega$$

Step III: Calculation of I_L

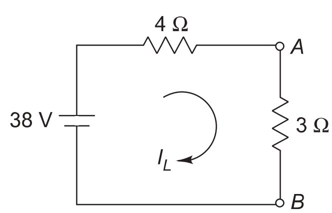


Fig. 2.424

$$I_L = \frac{38}{4+3} = 5.43 \text{ A}$$

Example 13

Find the value of current flowing through the 30 Ω resistor.

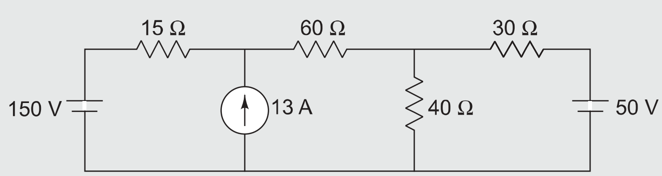


Fig. 2.425 **[May 2016]**

Solution

Step I: Calculation of V_{Th}

Removing the 30 Ω resistor from the network,

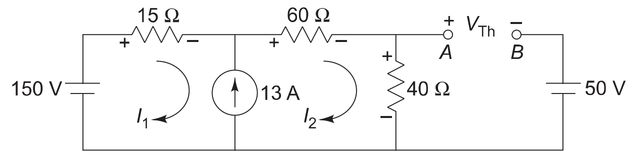


Fig. 2.426

Meshes 1 and 2 form a supermesh.

Writing current equation for supermesh,

$$I_2 - I_1 = 13 \qquad (1)$$

Writing voltage equation for supermesh,

$$150 - 15I_1 - 60I_2 - 40I_2 = 0$$

$$15I_1 + 100I_2 = 150 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -10 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing V_{Th} equation,

$$40I_2 - V_{\text{Th}} - 50 = 0$$

$$V_{\text{Th}} = 40I_2 - 50$$

$$= 40(3) - 50$$

$$= 70 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

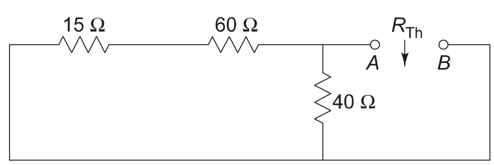


Fig. 2.427

$$R_{\text{Th}} = 75 \parallel 40 = 26.09\ \Omega$$

Step III: Calculation of I_L

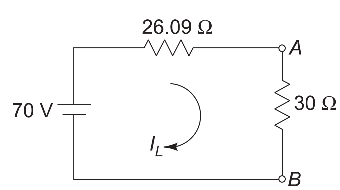


Fig. 2.428

$$I_L = \frac{70}{26.09 + 30} = 1.25 \text{ A}$$

Example 14

Find the value of current flowing through the 20 Ω resistor.

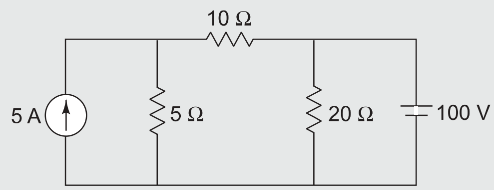


Fig. 2.429

Solution

Step I: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

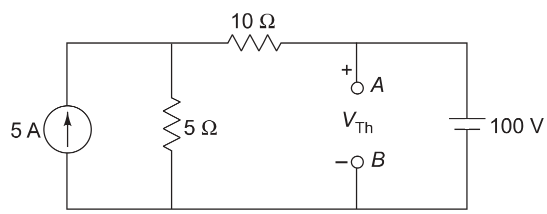


Fig. 2.430

From Fig. 2.430,

$$V_{\text{Th}} = 100 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

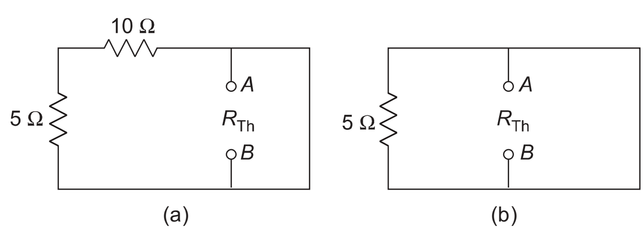


Fig. 2.431

$$R_{\text{Th}} = 0$$

Step III: Calculation of I_L

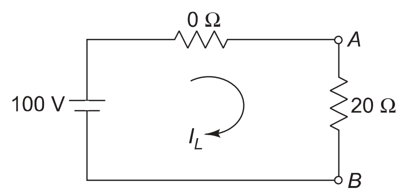


Fig. 2.432

$$I_L = \frac{100}{20} = 5\text{ A}$$

Example 15

Find the value of current flowing through the 20 Ω resistor.

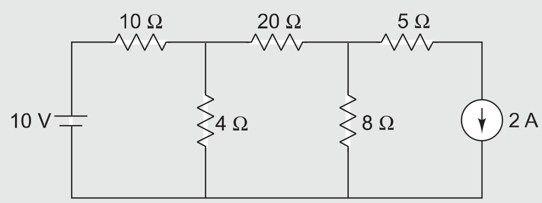


Fig. 2.433

Solution

Step 1: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

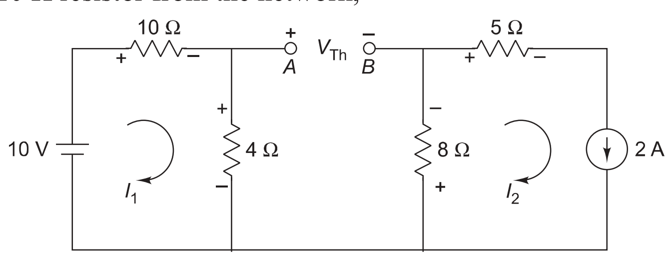


Fig. 2.434

$$I_1 = \frac{10}{10+4} = 0.71\text{ A}$$

$$I_2 = 2\text{ A}$$

Writing the V_{Th} equation,

$$4\,I_1 - V_{\text{Th}} + 8\,I_2 = 0$$

$$V_{\text{Th}} = 4\,(0.71) + 8\,(2)$$

$$= 18.84\text{ V}$$

Step II : Calculation of R_{Th}

Replacing the voltage source by short circuit and current source by an open circuit,

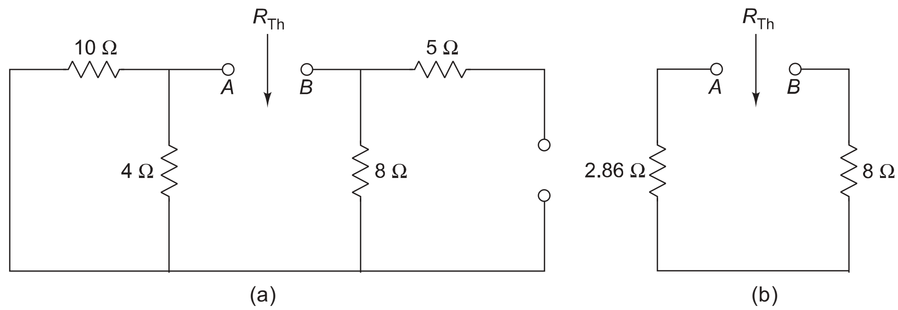


Fig. 2.435

$$R_{\text{Th}} = 10.86\ \Omega$$

Step III : Calculation of I_L

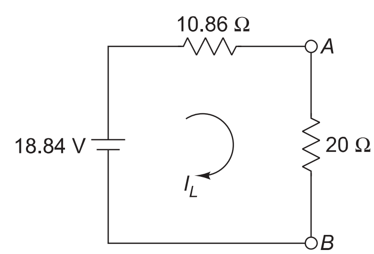


Fig. 2.436

$$I_L = \frac{18.84}{10.86 + 20} = 0.61\ \text{A}$$

Example 16

Find the value of current flowing through the 5 Ω resistor.

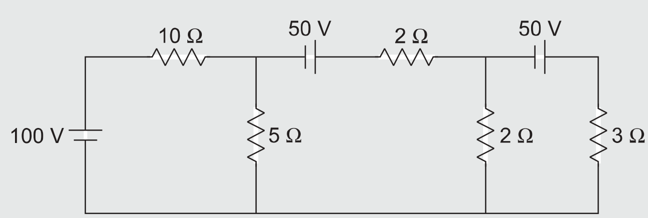


Fig. 2.437

Solution

Step I: Calculation of V_{Th}

Removing the 5 Ω resistor from the network,

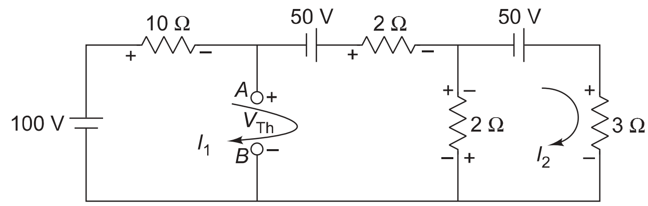


Fig. 2.438

Applying KVL to Mesh 1,

$$100 - 10\, I_1 + 50 - 2\, I_1 - 2\,(I_1 - I_2) = 0$$

$$14\, I_2 - 2\, I_2 = 150 \quad (1)$$

Applying KVL to Mesh 2,

$$-2\,(I_2 - I_1) + 50 - 3\, I_2 = 0$$

$$-2\, I_1 + 5\, I_2 = 50 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 12.88 \text{ A}$$

$$I_2 = 15.15 \text{ A}$$

Writing the V_{Th} equation,

$$100 - 10\, I_1 - V_{Th} = 0$$

$$V_{Th} = 100 - 10\,(12.88)$$

$$= -28.8 \text{ V}$$

$$= 28.8 \text{ V (terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

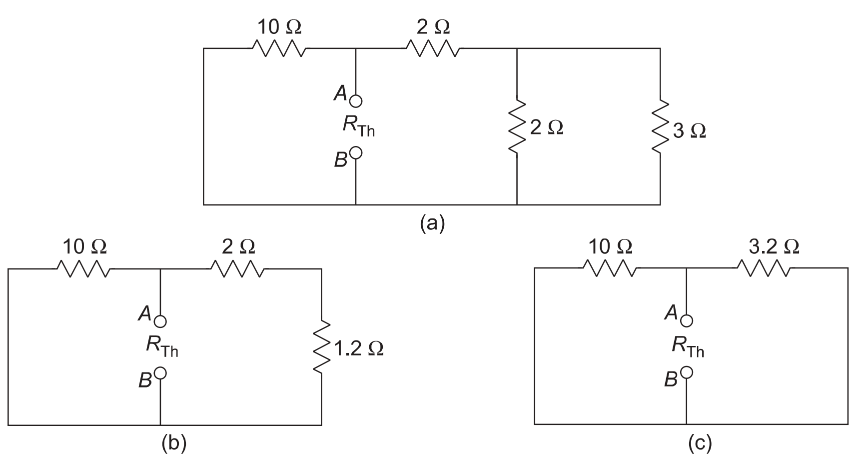


Fig. 2.439

$$R_{Th} = 10 \,||\, 3.2 = 2.42\ \Omega$$

Step III: Calculation of I_L

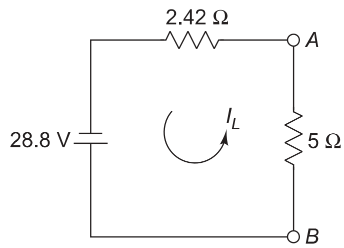


Fig. 2.440

$$I_L = \frac{28.8}{2.42 + 5} = 3.88 \text{ A } (\uparrow)$$

Example 17

Find the value of current flowing through the 10 Ω resistor.

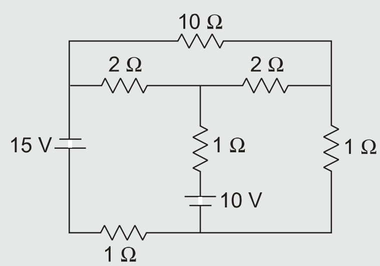


Fig. 2.441

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

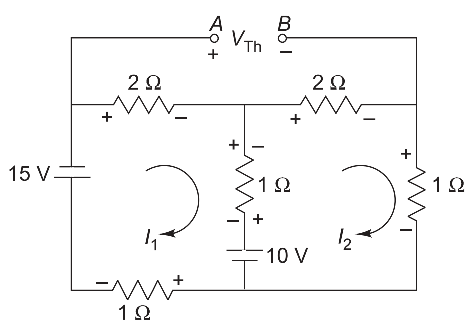


Fig. 2.442

Applying KVL to Mesh 1,

$$-15 - 2I_1 - 1\,(I_1 - I_2) - 10 - 1I_1 = 0$$

$$4I_1 - I_2 = -25 \qquad (1)$$

Applying KVL to Mesh 2,

$$10 - 1(I_2 - I_1) - 2I_2 - I_2 = 0$$

$$-I_1 + 4I_2 = 10 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -6 \text{ A}$$

$$I_2 = 1 \text{ A}$$

Writing V_{Th} equation,

$$-V_{Th} + 2I_2 + 2I_1 = 0$$

$$V_{Th} = 2I_1 + 2I_2$$

$$= 2(-6) + 2\,(1)$$

$$= -10 \text{ V}$$

$$= 10 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

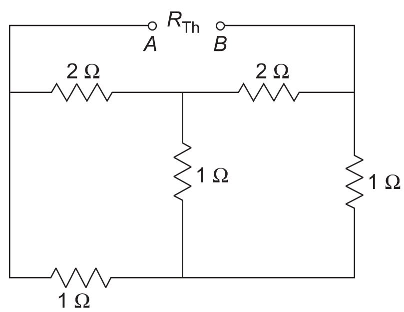


Fig. 2.443

Converting the star network formed by resistors of 2 Ω, 2 Ω and 1 Ω into an equivalent delta network.

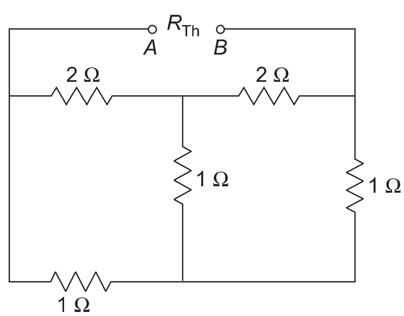


Fig. 2.444

$$R_1 = 2 + 2 + \frac{2 \times 2}{1} = 8\ \Omega$$

$$R_2 = 2 + 1 + \frac{2 \times 1}{2} = 4\ \Omega$$

$$R_3 = 2 + 1 + \frac{2 \times 1}{2} = 4\ \Omega$$

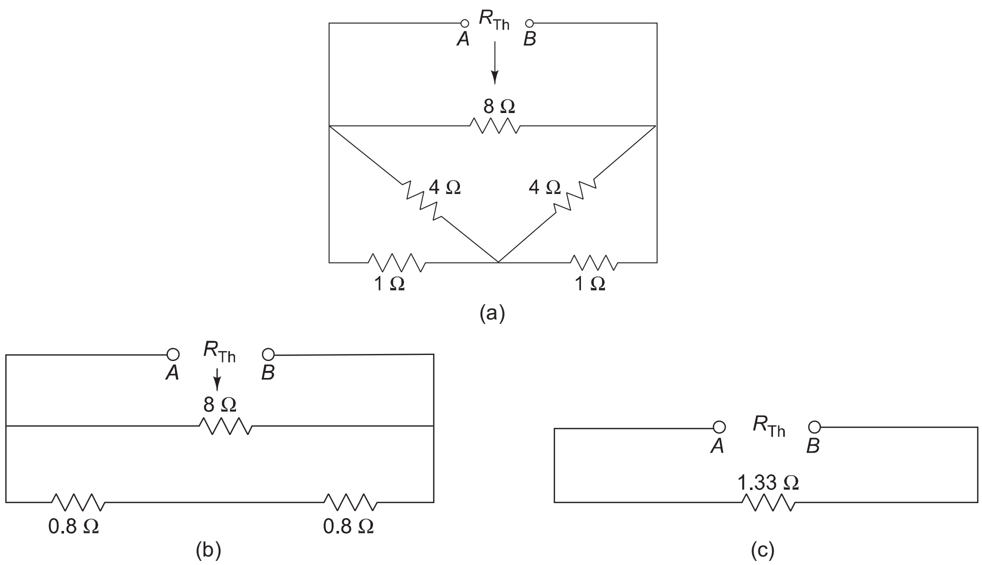


Fig. 2.445

$$R_{Th} = 1.33\ \Omega$$

Step III: Calculation of I_L

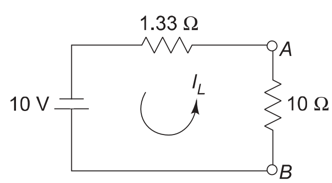


Fig. 2.446

$$I_L = \frac{10}{1.33 + 10} = 0.88\ \text{A}\ (\uparrow)$$

Example 18

Find the value of current flowing through the 1 Ω resistor.

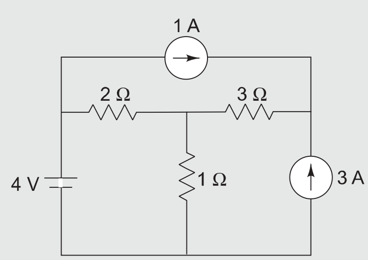


Fig. 2.447

Solution

Step I: Calculation of V_{Th}

Removing the 1 Ω resistor from the network,

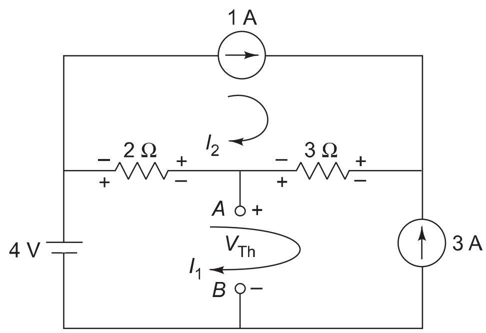


Fig. 2.448

Writing the current equation for meshes 1 and 2,

$$I_1 = -3$$
$$I_2 = 1$$

Writing V_{Th} equation,

$$4 - 2\,(I_1 - I_2) - V_{Th} = 0$$
$$V_{Th} = 4 - 2(-3 - 1)$$
$$= 4 - 2(-4)$$
$$= 12 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

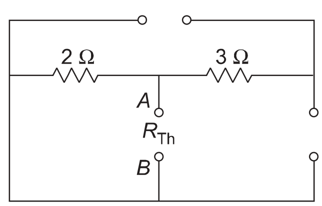


Fig. 2.449

$$R_{Th} = 2\ \Omega$$

Step III: Calculation of I_L

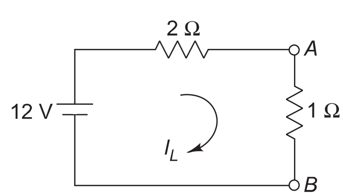


Fig. 2.450

$$I_L = \frac{12}{2+1} = 4\text{ A}$$

Example 19

Find the value of current flowing through the 3 Ω resistor.

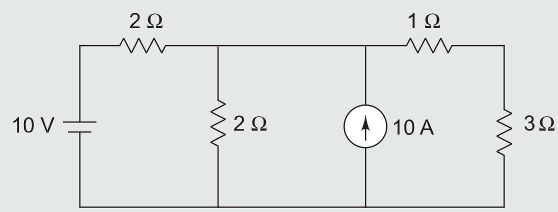


Fig. 2.451

Solution *Step I: Calculation of* V_{Th}

Removing the 3 Ω resistor from the network,

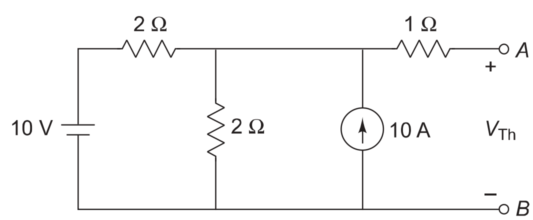


Fig. 2.452

By source transformation,

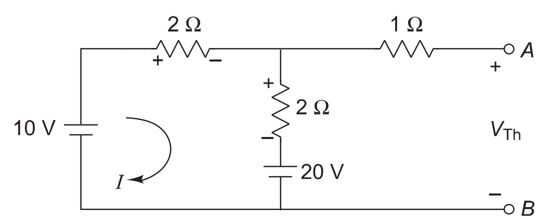


Fig. 2.453

Applying KVL to the mesh,

$$10 - 2I - 2I - 20 = 0$$
$$4I = -10$$
$$I = -2.5\text{ A}$$

Writing V_{Th} equation,

$$10 - 2I - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 10 - 2I$$

$$= 10 - 2(-2.5)$$
$$= 15 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit and current source by an open circuit,

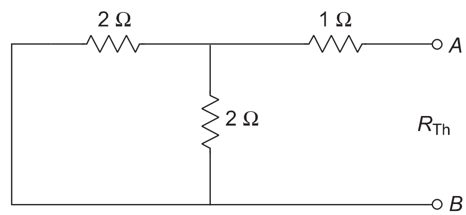


Fig. 2.454

$$R_{Th} = (2 \parallel 2) + 1 = 1 + 1 = 2\ \Omega$$

Step III: Calculation of I_L

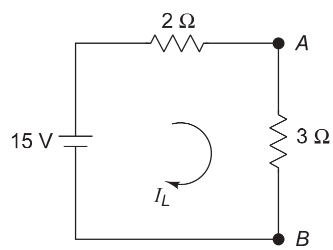


Fig. 2.455

$$I_L = \frac{15}{2+3} = 3 \text{ A}$$

Example 20

Find the value of current flowing through the 60 Ω resistor.

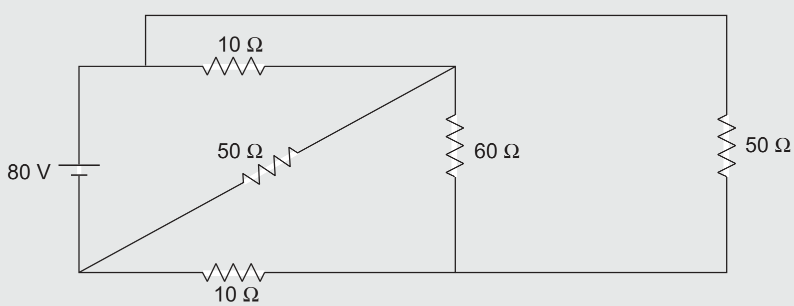


Fig. 2.456 [May 2014]

Solution *Step I: Calculation of V_{Th}*

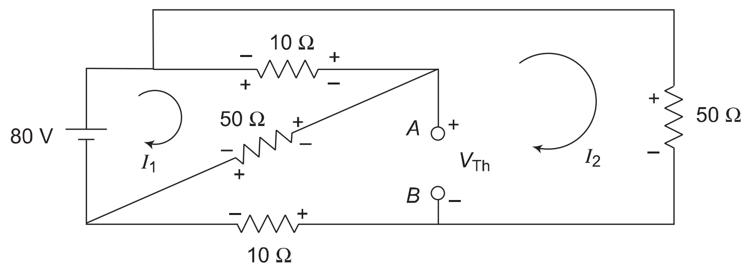


Fig. 2.457

Writing KVL equation in matrix form,

$$\begin{bmatrix} 60 & 0 \\ 0 & 120 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \end{bmatrix}$$

$$I_1 = 2.67 \text{ A}$$
$$I_2 = 1.33 \text{ A}$$

Writing V_{Th} equation,

$$80 - 10\,(I_1 - I_2) - V_{Th} - 10I_2 = 0$$
$$V_{Th} = 80 - 10(I_1 - I_2) - 10I_2$$
$$= 80 - 10(2.67 - 1.33) - 10\,(1.33)$$
$$= 53.3 \text{ V}$$

Step II Calculation of R_{Th}

Replacing voltage source by short circuit,

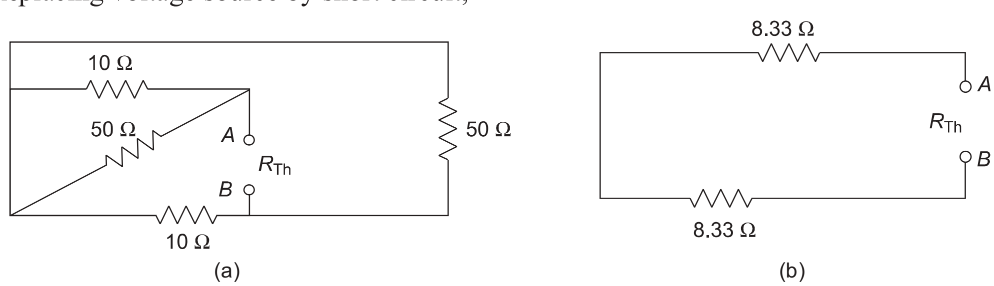


Fig. 2.458

$$R_{Th} = 16.66\ \Omega$$

Step III Calculation of I_L

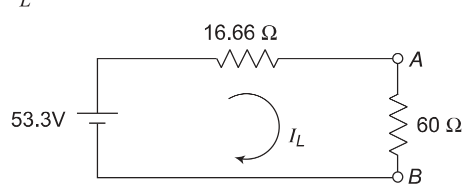


Fig. 2.459

$$I_L = \frac{53.3}{16.66 + 60} = 0.7 \text{ A}$$

Exercise 2.7

2.1 Find the value of current flowing through the 6 Ω resistor.

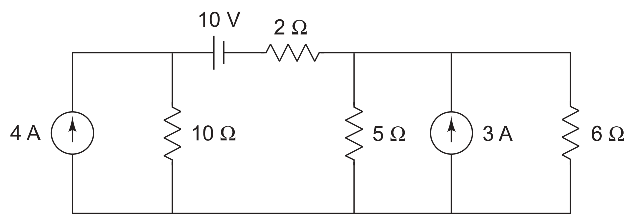


Fig. 2.460

[2.04 A]

2.2 Find the value of current flowing through the 2 Ω resistor connected between terminals *A* and *B*.

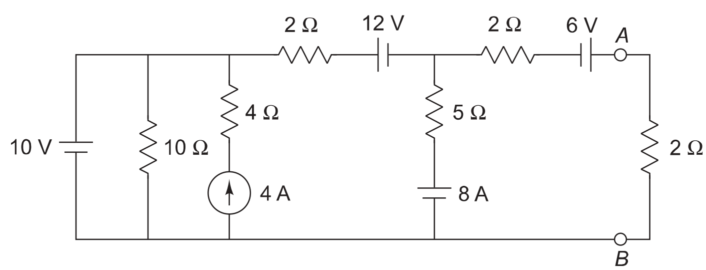


Fig. 2.461

[1.26 A]

2.3 Find the value of current flowing through the 5 Ω resistor.

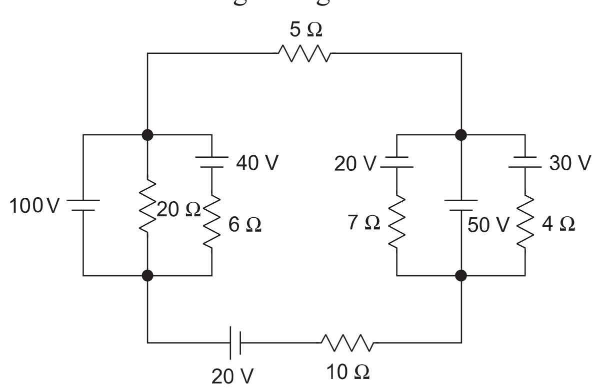


Fig. 2.462

[4.67 A]

2.4 Find the value of current flowing through the 20 Ω resistor.

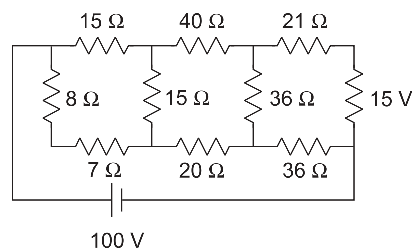


Fig. 2.463

[1.54 A]

2.5 Calculate the value of current flowing through the 10 Ω resistor.

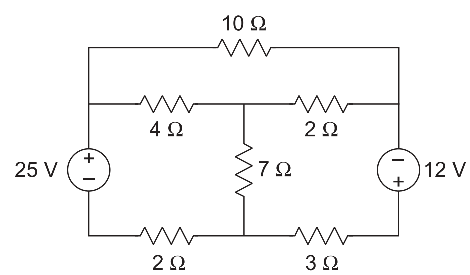


Fig. 2.464

[1.62 A]

2.6 Find the value of current flowing through the 2 Ω resistor.

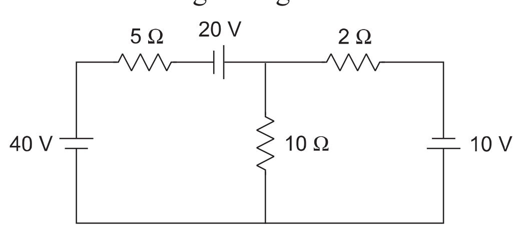


Fig. 2.465

[9.375 A]

2.7 Find the value of current flowing through the 5 Ω resistor.

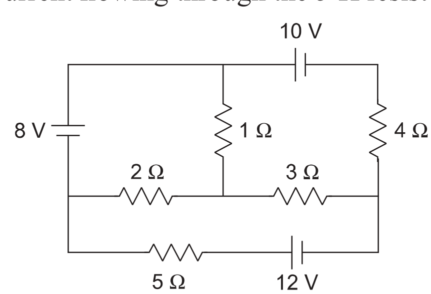


Fig. 2.466

[2 A]

2.10 NORTON'S THEOREM

[Dec 2013]

It states that *'Any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.*

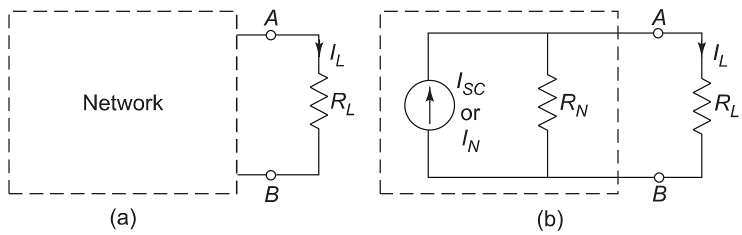


Fig. 2.467 *Norton's theorem*

Explanation The method of determining the load current through a given load resistance can be explained with the help of the following circuit.

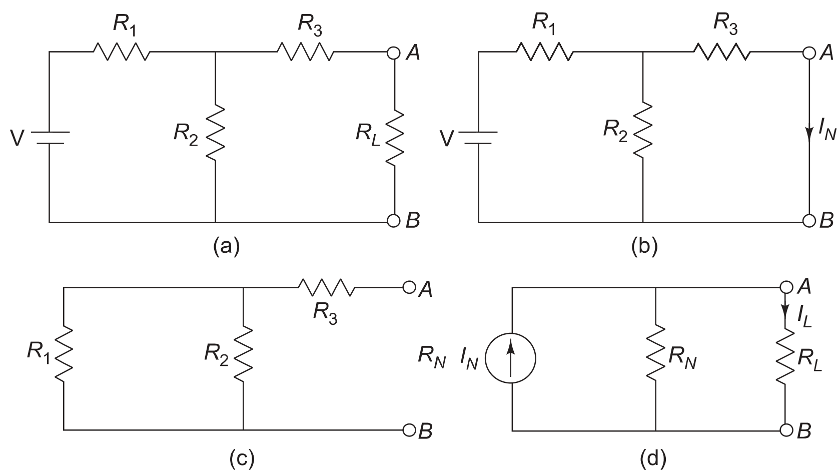


Fig. 2.468 *Steps in Norton's theorem*

2.10.1 Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{sc} or I_N.
3. Find the resistance R_N as seen from points A and B by replacing the voltage sources and current sources by internal resistances.

4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_N by current–division rule,

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 1

For the given circuit in Fig. 2.539, find the Norton equivalent between points A and B.

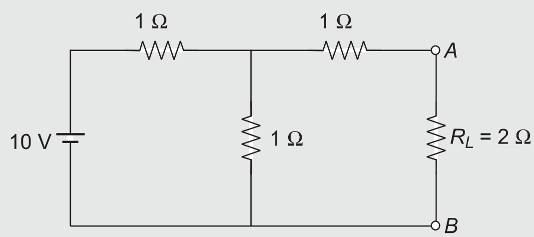


Fig. 2.469 [May 2015]

Solution

Step I: Calculation of I_N

Replacing 2 Ω resistor by short circuit,

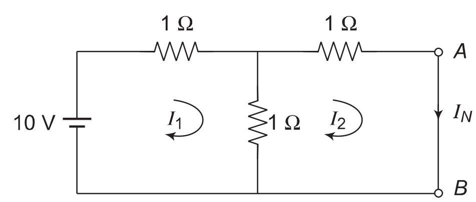


Fig. 2.470

Applying KVL to Mesh 1,

$$10 - 1I_1 - 1(I_1 - I_2) = 0$$

$$2I_1 = I_2 = 10 \quad ...(1)$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 1I_2 = 0$$

$$-I_1 + 2I_2 = 0 \quad ...(2)$$

Solving Eqs (1) and (2),

$$I_1 = 6.67 \text{ A}$$

$$I_2 = I_N = 3.33 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage source by short circuit,

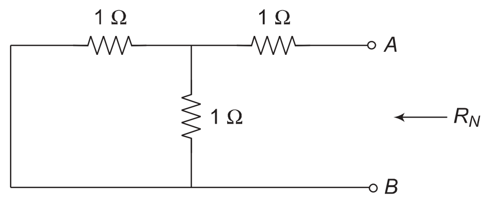


Fig. 2.471

$$R_N = 1.5\ \Omega$$

Step III: Norton's equivalent network

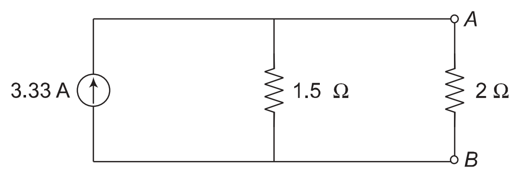


Fig. 2.172

Example 2

Find the value of current through the 10 Ω resistor.

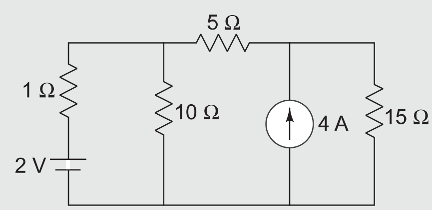


Fig. 2.473

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

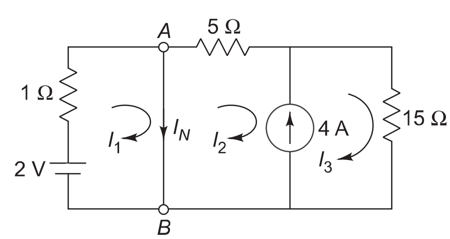


Fig. 2.474

Applying KVL to Mesh 1,

$$2 - 1I_1 = 0 \quad (1)$$

$$I_1 = 2$$

Meshes 2 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_3 - I_2 = 4 \quad (2)$$

Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2 \text{ A}$$

$$I_2 = -3 \text{ A}$$

$$I_3 = 1 \text{ A}$$

$$I_N = I_1 - I_2 = 2 - (-3) = 5 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and current source by an open circuit,

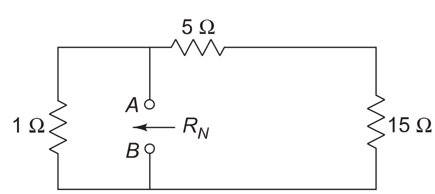


Fig. 2.475

$$R_N = 1 \parallel (5 + 15) = 0.95\ \Omega$$

Step III: Calculation of I_L

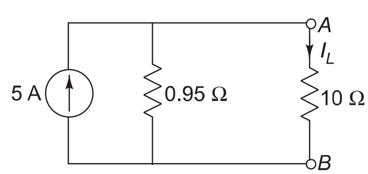


Fig. 2.476

$$I_L = 5 \times \frac{0.95}{10 + 0.95} = 0.43 \text{ A}$$

Example 3

Calculate the value of current flowing through the 15 Ω load resistor in the given circuit.

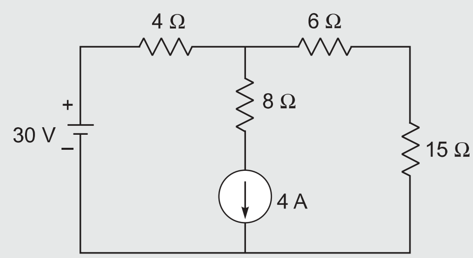


Fig. 2.477 [May 2013]

Solution

Step I: Calculation of I_N

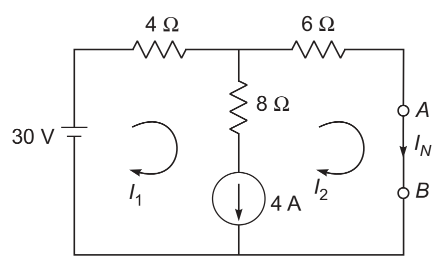


Fig. 2.478

Writing the current equation for the supermesh,

$$I_1 - I_2 = 4 \tag{1}$$

Writing the voltage equation for the supermesh,

$$30 - 4I_1 - 6I_2 = 0$$

$$4I_1 + 6I_2 = 30 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = 5.4 \text{ A}$$

$$I_2 = 1.4 \text{ A}$$

$$I_N = I_2 = 1.4 \text{ A}$$

Step II: Calculation of R_N

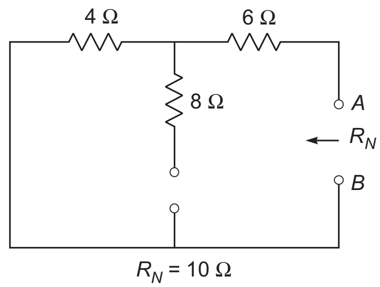


Fig. 2.479

Step III: Calculation of I_L

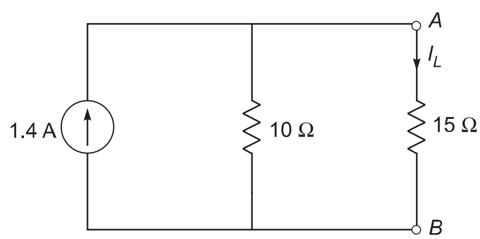


Fig. 2.480

$$I_L = 1.4 \times \frac{10}{10+15} = 0.56 \text{ A}$$

Example 4

Find the value of current flowing through the 10 Ω resistor.

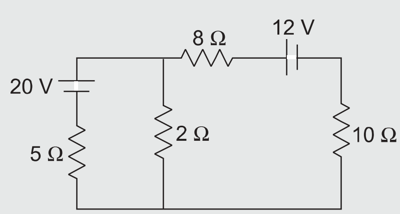


Fig. 2.481

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

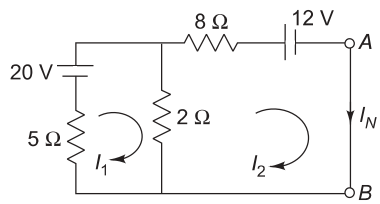


Fig. 2.482

Applying KVL to Mesh 1,

$$-5I_1 + 20 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = 20 \quad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 8I_2 - 12 = 0$$

$$-2I_1 + 10I_2 = -12 \quad (2)$$

Solving Eqs. (1) and (2),

$$I_2 = -0.67 \text{ A}$$

$$I_N = I_2 = -0.67 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

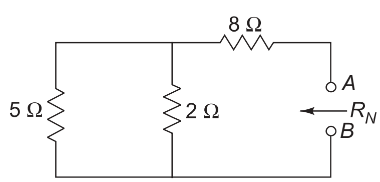


Fig. 2.483

$$R_N = (5 \,||\, 2) + 8 = 9.43 \ \Omega$$

Step III: Calculation of I_L

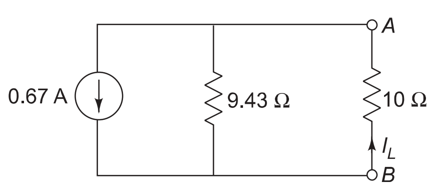


Fig. 2.484

$$I_L = 0.67 \times \frac{9.43}{9.43 + 10} = 0.33 \text{ A} (\uparrow)$$

Example 5

Find the value of current flowing in the 10 Ω resistor.

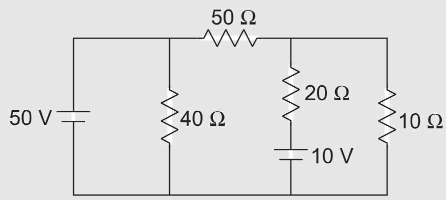


Fig. 2.485

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

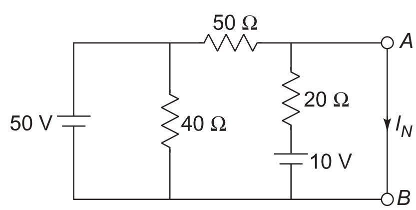


Fig. 2.486

The resistance of 40 Ω becomes redundant as it is connected across the 50 V source.

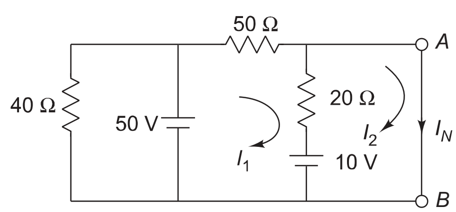


Fig. 2.487

Applying KVL to Mesh 1,

$$50 - 50\,I_1 - 20\,(I_1 - I_2) - 10 = 0$$

$$70\,I_1 - 20\,I_2 = 40 \tag{1}$$

Applying KVL to Mesh 2,

$$10 - 20\,(I_2 - I_1) = 0$$

$$-20\,I_1 + 20\,I_2 = 10 \tag{2}$$

Solving Eqs. (1) and (2),

$$I_1 = 1\text{ A}$$

$$I_2 = 1.5\text{ A}$$

$$I_N = I_2 = 1.5\text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits, resistor of 40 Ω gets shorted.

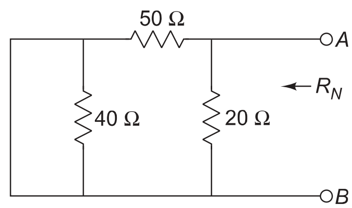


Fig. 2.488

$$R_N = 50 \| 20 = 14.28\ \Omega$$

Step III: Calculation of I_L

$$I_L = 1.5 \times \frac{14.28}{14.28 + 10} = 0.88\text{ A}$$

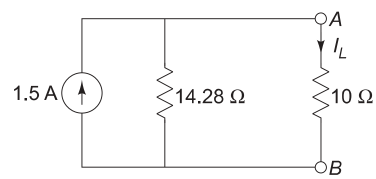


Fig. 2.489

Example 6

Find the value of current flowing through the 10 Ω resistor in Fig. 2.490.

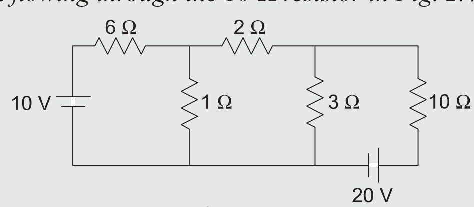


Fig. 2.490

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

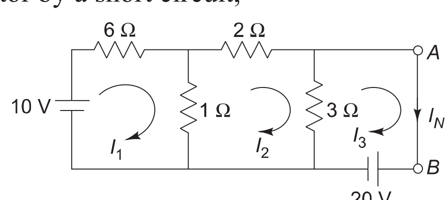


Fig. 2.491

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1\,(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1\,(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \quad (2)$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_2) - 20 = 0$$

$$3I_2 - 3I_3 = 20 \quad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_3 = -13.17 \text{ A}$$

$$I_N = I_3 = -13.17 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

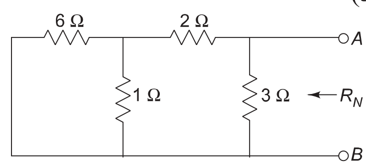


Fig. 2.492

$$R_N = [(6 \parallel 1) + 2] \parallel 3 = 1.46\ \Omega$$

Step III: Calculation of I_L

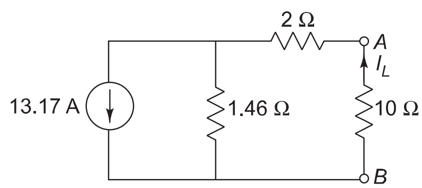


Fig. 2.493

$$I_L = 13.17 \times \frac{1.46}{1.46 + 10} = 1.68 \text{ A } (\uparrow)$$

Example 7

Find the value of current flowing through the 10 Ω resistor.

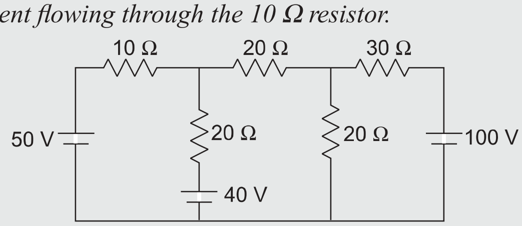


Fig. 2.494

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

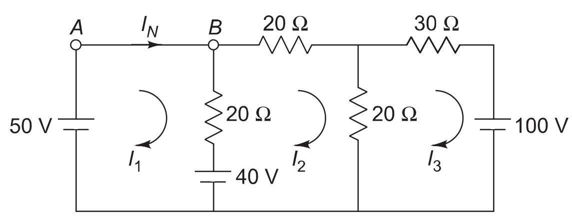


Fig. 2.495

Applying KVL to Mesh 1,

$$50 - 20(I_1 - I_2) - 40 = 0$$

$$20I_1 - 20I_2 = 10 \quad (1)$$

Applying KVL to Mesh 2,

$$40 - 20(I_2 - I_1) - 20I_2 - 20(I_2 - I_3) = 0$$

$$-20I_1 + 60I_2 - 20I_3 = 40 \quad (2)$$

Applying KVL to Mesh 3,

$$-20(I_3 - I_2) - 30I_3 - 100 = 0$$

$$-20I_2 + 50I_3 = -100 \quad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_1 = 0.81 \text{ A}$$

$$I_N = I_1 = 0.81 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

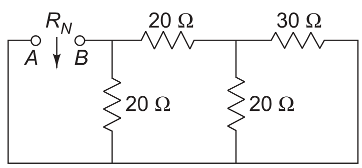


Fig. 2.496

$$R_N = [(20 \,\|\, 30) + 20] \,\|\, 20 = 12.3 \; \Omega$$

Step III: Calculation of I_L

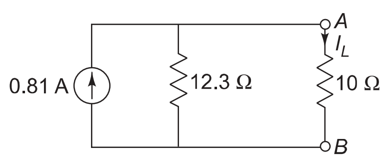


Fig. 2.497

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10} = 0.45 \text{ A}$$

Example 8

Obtain Norton's equivalent network as seen by R_L.

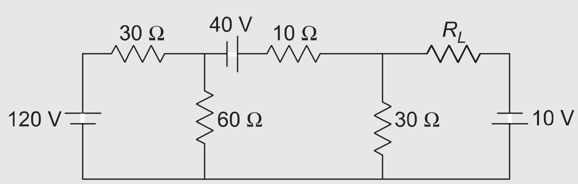


Fig. 2.498

Solution

Step I: Calculation of I_N

Replacing the resistor R_L by a short circuit,

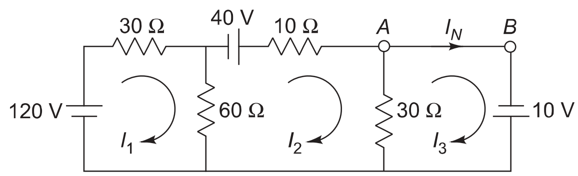


Fig. 2.499

Applying KVL to Mesh 1,

$$120 - 30I_1 - 60\,(I_1 - I_2) = 0$$

$$90I_1 - 60I_2 = 120 \qquad (1)$$

Applying KVL to Mesh 2,

$$-60\,(I_2 - I_1) + 40 - 10I_2 - 30\,(I_2 - I_3) = 0$$

$$-60I_1 + 100I_2 - 30I_3 = 40 \qquad (2)$$

Applying KVL to Mesh 3,

$$-30\,(I_3 - I_2) + 10 = 0$$

$$30I_2 - 30I_3 = -10 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_3 = 4.67 \text{ A}$$

$$I_N = I_3 = 4.67 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

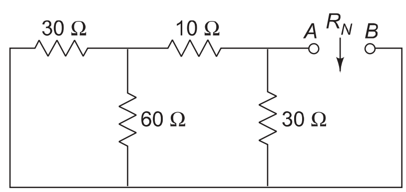


Fig. 2.500

$$R_N = [(30 \,||\, 60) + 10] \,||\, 30 = 15\ \Omega$$

Step III: Norton's equivalent network

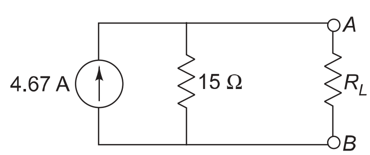


Fig. 2.501

Example 9

Find the value of current flowing through the 8 Ω resistor.

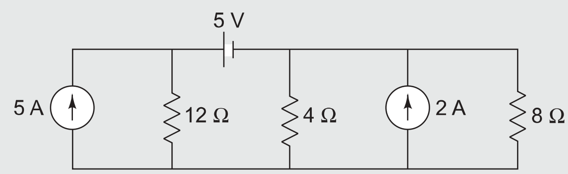


Fig. 2.502

Solution

Step I: Calculation of I_N

Replacing the 8 Ω resistor by a short circuit,

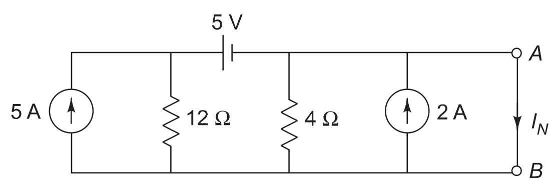


Fig. 2.503

The resistor of the 4 Ω gets shorted as it is in parallel with the short circuit. Simplifying the network by source transformation,

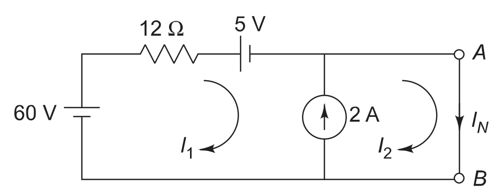


Fig. 2.504

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 2 \quad (1)$$

Applying KVL to the supermesh,

$$60 - 12I_1 - 5 = 0$$

$$12I_1 = 55 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 4.58 \text{ A}$$

$$I_2 = 6.58 \text{ A}$$

$$I_N = I_2 = 6.58 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

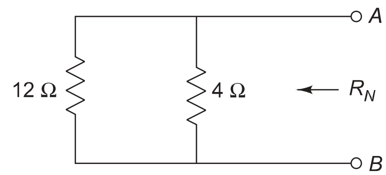


Fig. 2.505

$$R_N = 12 \parallel 4 = 3\ \Omega$$

Step III: Calculation of I_L

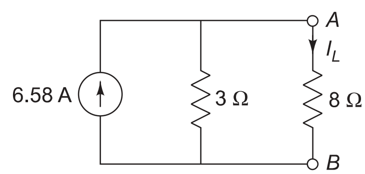


Fig. 2.506

$$I_L = 6.58 \times \frac{15}{2+3} = 1.79\ \text{A}$$

Example 10

Find value of current flowing through the 1 Ω resistor.

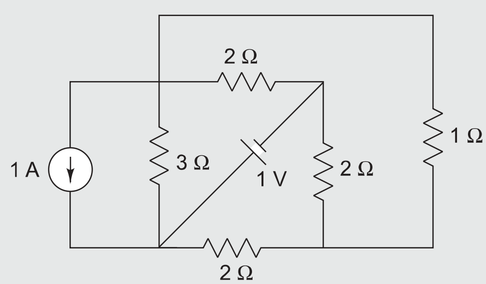


Fig. 2.507

Solution

Step I: Calculation of I_N

Replacing the 1 Ω resistor by a short circuit,

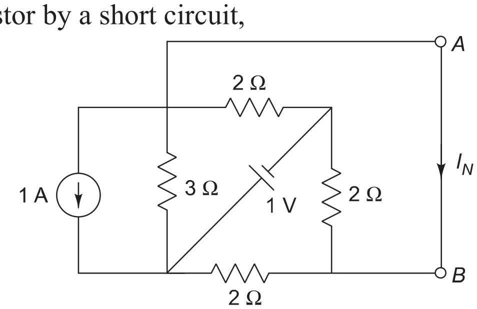


Fig. 2.508

By source transformation,

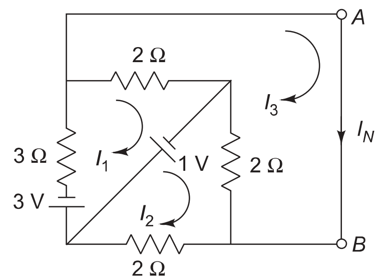


Fig. 2.509

Applying KVL to Mesh 1,

$$-3 - 3I_1 - 2(I_1 - I_3) + 1 = 0$$

$$5I_1 - 2I_3 = -2 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1 - 2(I_2 - I_3) - 2I_2 = 0$$

$$4I_2 - 2I_3 = -1 \qquad (2)$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 2(I_3 - I_2) = 0$$

$$-2I_1 - 2I_2 + 4I_3 = 0 \qquad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_1 = -0.64 \text{ A}$$

$$I_2 = -0.55 \text{ A}$$

$$I_3 = -0.59 \text{ A}$$

$$I_N = I_3 = -0.59 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

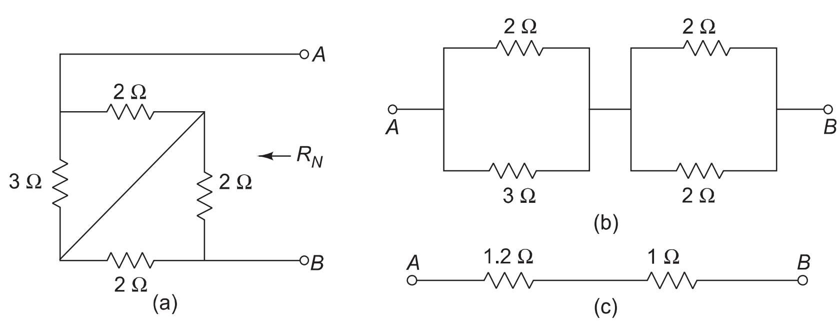


Fig. 2.510

$$R_N = 2.2\ \Omega$$

Step III: Calculation of I_L

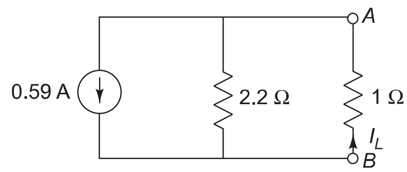


Fig. 2.511

$$I_L = 0.59 \times \frac{2.2}{2.2+1} = 0.41 \text{ A}$$

Exercise 2.8

2.1 Find the value of current flowing through the 10 Ω resistor.

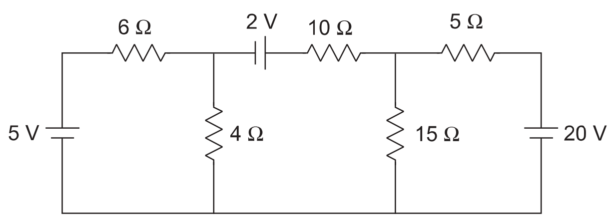


Fig. 2.512

[0.68 A]

2.2 Find the value of current flowing through the 20 Ω resistor.

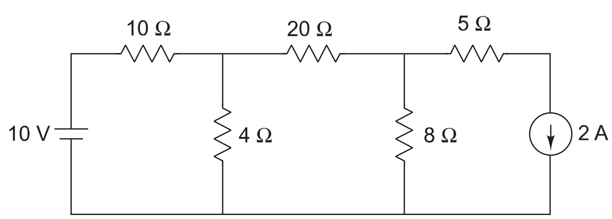


Fig. 2.513

[0.61 A]

2.3 Find the value of current flowing through the 2 Ω resistor.

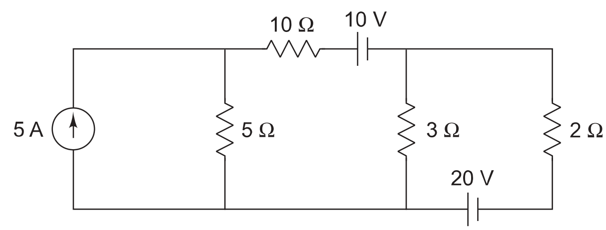


Fig. 2.514

[5 A]

2.4 Find the value of current flowing through the 5 Ω resistor.

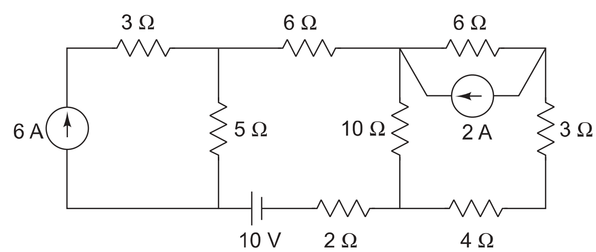


Fig. 2.515

[4.13 A]

2.5 Find the value of current flowing through the 15 Ω resistor.

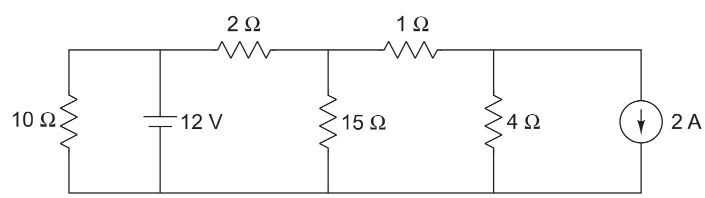


Fig. 2.516

[0.382 A]

2.6 Find Norton's equivalent network.

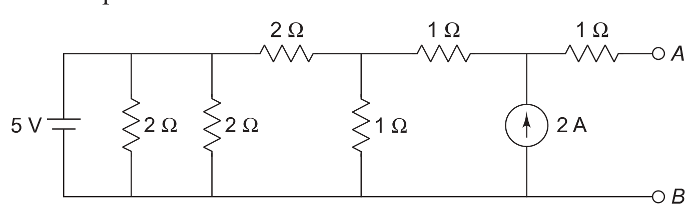


Fig. 2.517

[1.8 A, 1.67 Ω]

2.7 Find Norton's equivalent circuit for the portion of network shown in Fig. 2.518 to the left of *ab*. Hence obtain the current in the 10 Ω resistor.

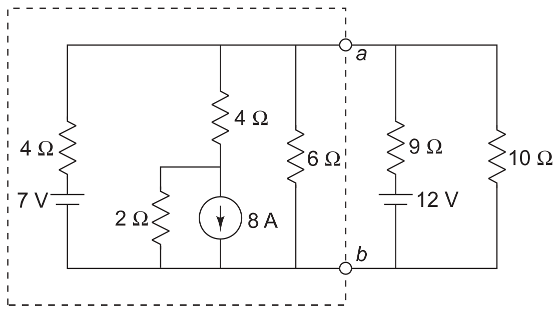


Fig. 2.518

[0.053 A]

2.11 MAXIMUM POWER TRANSFER THEOREM

[Dec 2012, 2015, May 2013, 2014]

It states that *'the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.'*

$$I = \frac{V}{R_S + R_L}$$

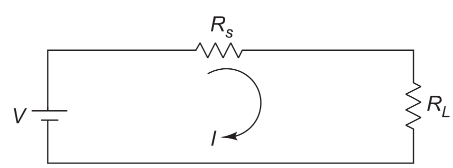


Fig. 2.519 *Maximum power transfer theorem*

Power delivered to the load $R_L = P = I^2 R_L = \dfrac{V^2 R_L}{(R_S + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_S + R_L)^2} R_L$$

$$= \frac{V^2[(R_S + R_L)^2 - (2R_L)(R_S + R_L)]}{(R_S + R_L)^4}$$

$$(R_S + R_L)^2 - 2\,R_L\,(R_S + R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_S R_L - 2R_L R_S - 2R_L^2 = 0$$

$$R_L = R_S$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

2.11.1 Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with voltage sources and current sources replaced by internal resistances.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{\text{Th}}$$

5. Find the maximum power.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{V_{\text{Th}}}{2R_{\text{Th}}}$$

$$P_{\max} = I_L^2\,R_L = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}^2} \times R_{\text{Th}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

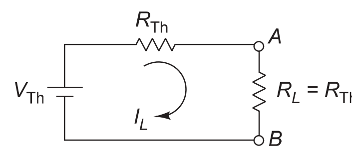


Fig. 2.520 *Equivalent circuit*

Example 1

Find the value of resistance R_L for maximum power transfer calculate maximum power.

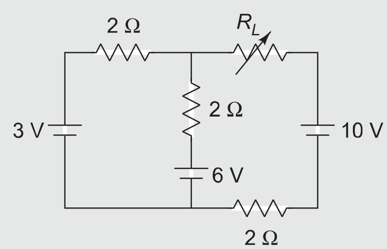


Fig. 2.521

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

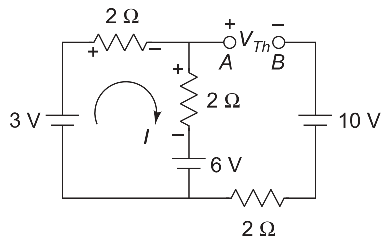


Fig. 2.522

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75 \text{ A}$$

Writing V_{Th} equation,

$$6 + 2I - V_{\text{Th}} - 10 = 0$$

$$V_{\text{Th}} = 6 + 2I - 10$$

$$= 6 + 2(-0.75) - 10$$

$$= -5.5 \text{ V}$$

$$= 5.5 \text{ V (terminal } B \text{ is positive w.r.t } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

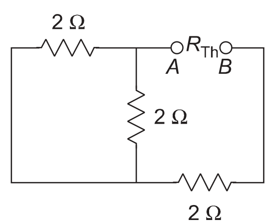


Fig. 2.523

$$R_{\text{Th}} = (2 \parallel 2) + 2 = 3\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 3\ \Omega$$

Step IV: Calculation of P_{max}

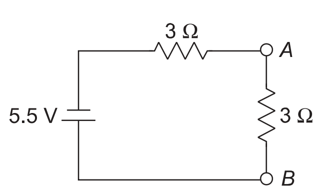


Fig. 2.524

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5.5)^2}{4 \times 3} = 2.52 \text{ W}$$

Example 2

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

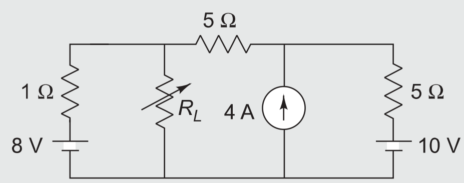


Fig. 2.525

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

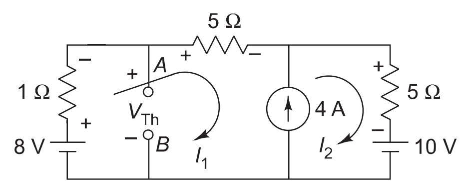


Fig. 2.526

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 4 \quad (1)$$

Applying KVL to the supermesh,

$$8 - 1I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \quad (2)$$

Solving Eqs. (1) and (2),

$$I_1 = -2 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$8 - 1I_1 - V_{Th} = 0$$

$$V_{Th} = 8 - I_1$$

$$= 8 - (-2)$$

$$= 10 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and current source by an open circuit,

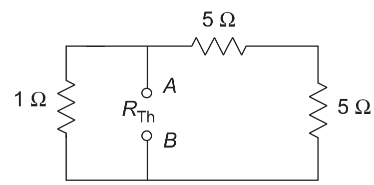


Fig. 2.527

$$R_{Th} = 10 \parallel 1 = 0.91\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 0.91\ \Omega$$

Step IV: Calculation of P_{max}

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(10)^2}{4 \times 0.91} = 27.47\ \text{W}$$

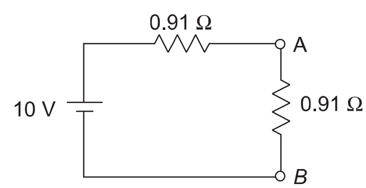


Fig. 2.528

Example 3

Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

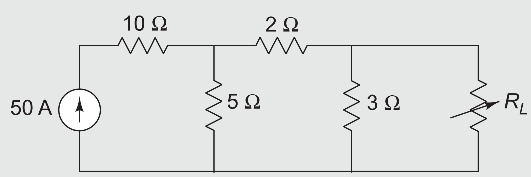


Fig. 2.529

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

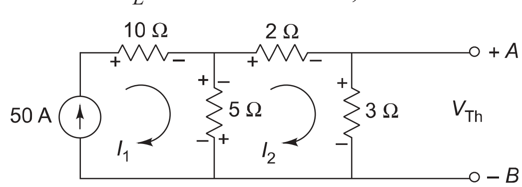


Fig. 2.530

For Mesh 1,

$$I_1 = 50$$

Applying KVL to Mesh 2,

$$-5\,(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$5I_1 - 10I_2 = 0$$
$$I_1 = 2I_2$$
$$I_2 = 25 \text{ A}$$
$$V_{\text{Th}} = 3I_2 = 3(25) = 75 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source by an open circuit,

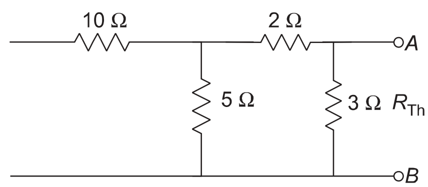


Fig. 2.531

$$R_{\text{Th}} = 7 \,||\, 3 = 2.1\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 2.1\ \Omega$$

Step IV: Calculation of P_{max}

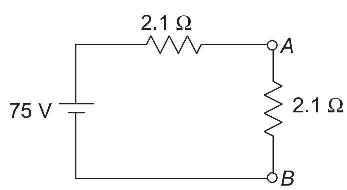


Fig. 2.532

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(75)^2}{4 \times 2.1} = 669.64 \text{ W}$$

Example 4

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

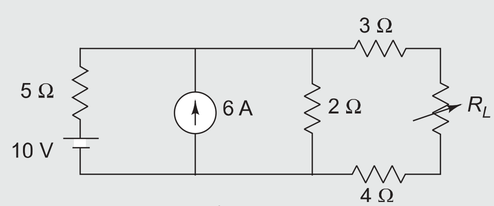


Fig. 2.533

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

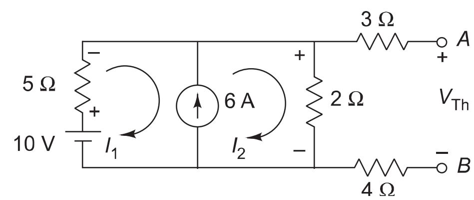


Fig. 2.534

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 6 \tag{1}$$

Applying KVL to the supermesh,

$$10 - 5I_1 - 2I_2 = 0$$

$$5I_1 + 2I_2 = 10 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = -0.29 \text{ A}$$

$$I_2 = 5.71 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 2I_2 = 11.42 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

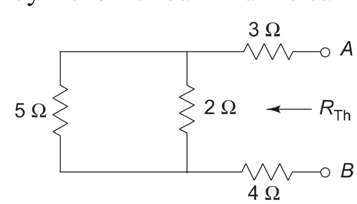


Fig. 2.535

$$R_{\text{Th}} = (5 \parallel 2) + 3 + 4 = 8.43 \ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 8.43 \ \Omega$$

Step IV: Calculation of P_{max}

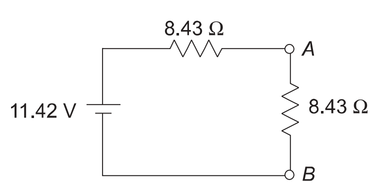


Fig. 2.536

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(11.42)^2}{4 \times 8.43} = 3.87 \text{ W}$$

Example 5

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

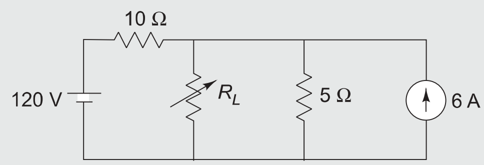


Fig. 2.537

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

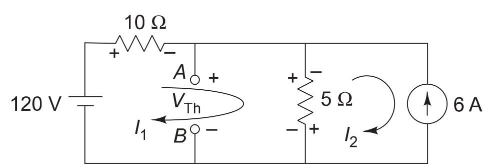


Fig. 2.538

Applying KVL to Mesh 1,

$$120 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15I_1 - 5I_2 = 120 \quad (1)$$

Writing current equation for Mesh 2,

$$I_2 = -6 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 6 \text{ A}$$

Writing V_{Th} equation,

$$120 - 10I_1 - V_{Th} = 0$$

$$V_{Th} = 120 - 10\,(6)$$

$$= 60 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

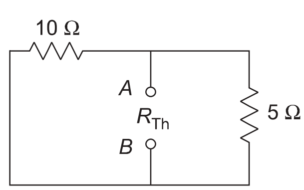


Fig. 2.539

$$R_{Th} = 10 \,||\, 5 = 3.33\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.33\ \Omega$$

Step IV: Calculation of P_{max}

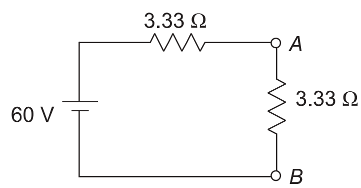


Fig. 2.540

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 3.33} = 270.27\ \text{W}$$

Example 6

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

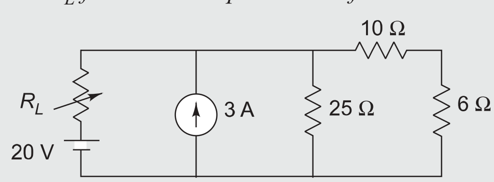


Fig. 2.541

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

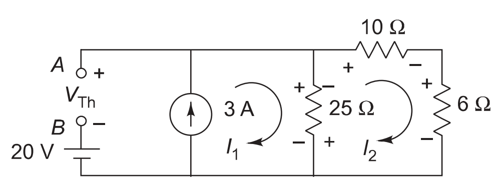


Fig. 2.542

For Mesh 1,

$$I_1 = 3 \tag{1}$$

Applying KVL to Mesh 2,

$$-25(I_2 - I_1) - 10I_2 - 6I_2 = 0$$

$$-25I_1 + 41I_2 = 0 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = 1.83 \text{ A}$$

Writing V_{Th} equation,

$$20 + V_{\text{Th}} - 10I_2 - 6I_2 = 0$$

$$V_{\text{Th}} = -20 + 10\,(1.83) + 6\,(1.83)$$

$$= 9.28 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

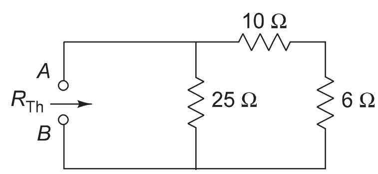


Fig. 2.543

$$R_{\text{Th}} = 25 \parallel 16 = 9.76\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 9.76\ \Omega$$

Step IV: Calculation of P_{max}

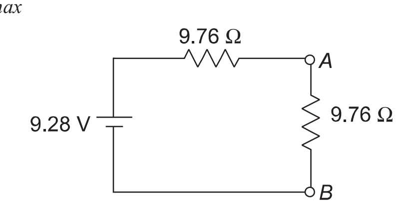


Fig. 2.544

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(9.28)^2}{4 \times 9.76} = 2.21 \text{ W}$$

Example 7

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

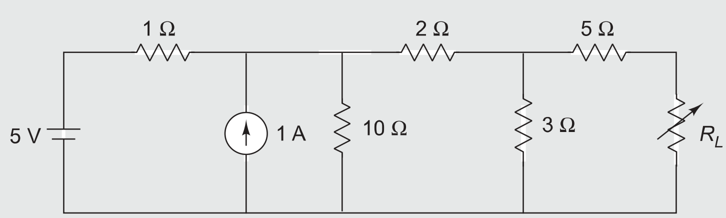


Fig. 2.545

Solution

Step I : Calculation of V_{Th}

Removing the variable resistor R_L from the network,

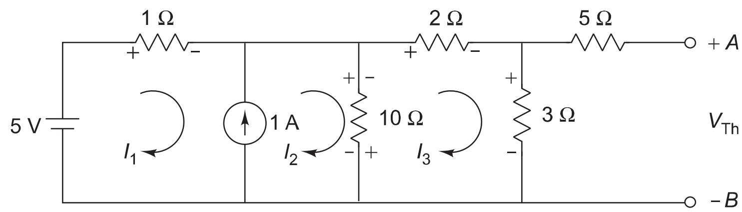


Fig. 2.546

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad (1)$$

Writing the voltage equation for the supermesh,

$$5 - 1I_1 - 10\,(I_2 - I_3) = 0$$

$$I_1 + 10\,I_2 - 10\,I_3 = 5 \qquad (2)$$

Applying KVL to Mesh 3,

$$-\,10\,(I_3 - I_2) - 2\,I_3 - 3\,I_3 = 0$$

$$-\,10\,I_2 + 15\,I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.38\text{ A}$$

$$I_2 = 1.38\text{ A}$$

$$I_3 = 0.92\text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 3\,I_3 = 2.76\text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit and current source by an open circuit,

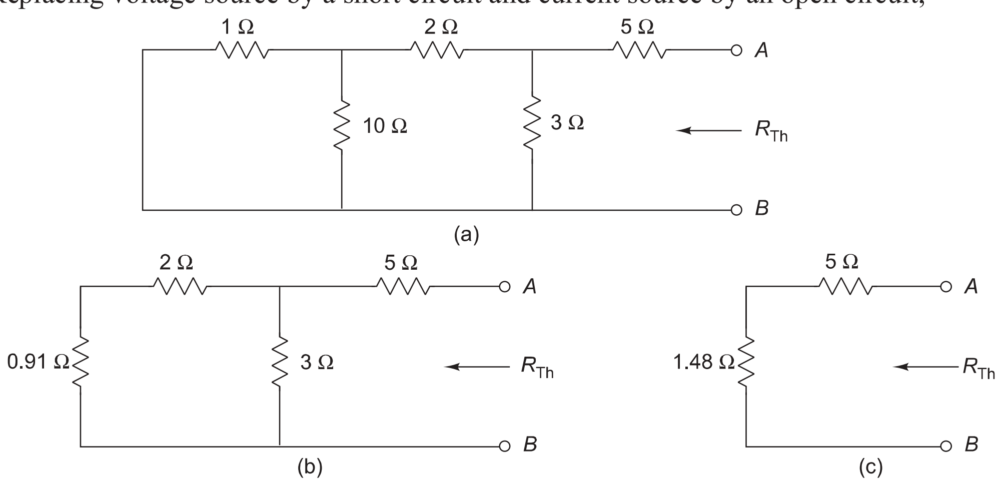


Fig. 2.547

$$R_{Th} = 6.48\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 6.48\ \Omega$$

Step IV: Calculation of P_{max}

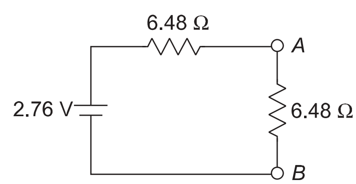


Fig. 2.548

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(2.76)^2}{4 \times 6.48} = 0.29\ \text{W}$$

Example 8

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

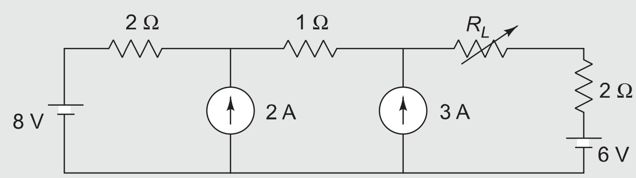


Fig. 2.549

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

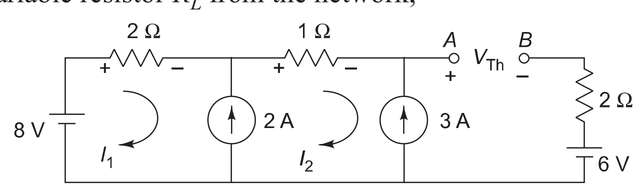


Fig. 2.550

From Fig. 2.550,

$$I_2 - I_1 = 2 \quad (1)$$

$$I_2 = -3\ \text{A} \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -5\ \text{A}$$

Writing V_{Th} equation,

$$8 - 2I_1 - 1I_2 - V_{Th} - 6 = 0$$

$$V_{Th} = 8 - 2(-5) - (-3) - 6$$

$$= 15 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

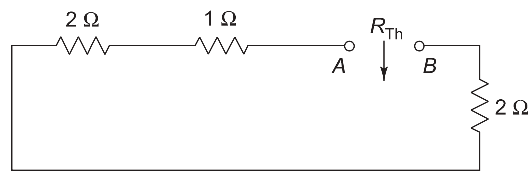


Fig. 2.551

$$R_{Th} = 5 \ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 5 \ \Omega$$

Step IV: Calculation of P_{max}

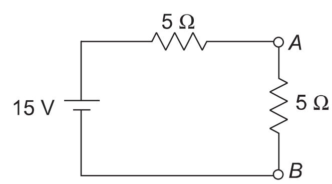


Fig. 2.552

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(15)^2}{4 \times 5} = 11.25 \text{ W}$$

Example 9

Find the value of resistance the R_L for maximum power transfer and calculate the maximum power.

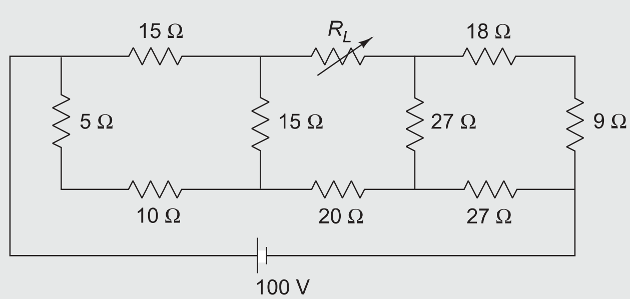


Fig. 2.553

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

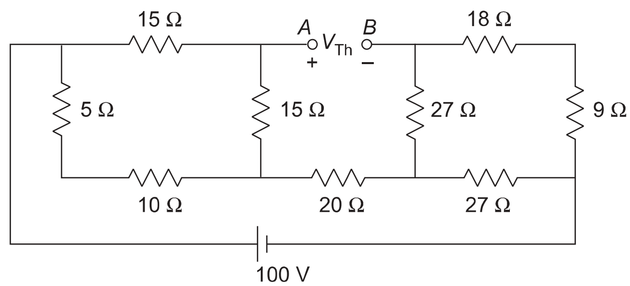


Fig. 2.554

By star-delta transformation,

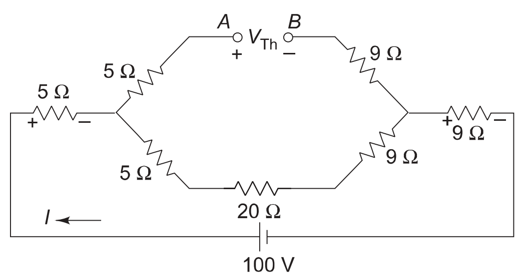


Fig. 2.555

$$I = \frac{100}{5+5+20+9+9} = 2.08 \text{ A}$$

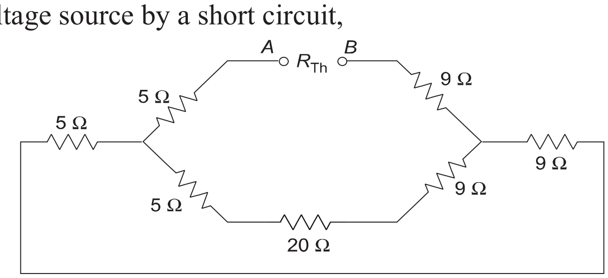
Writing V_{Th} equation,

$$100 - 5I - V_{\text{Th}} - 9I = 0$$

$$\begin{aligned} V_{\text{Th}} &= 100 - 14I \\ &= 100 - 14(2.08) \\ &= 70.88 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,



(a)

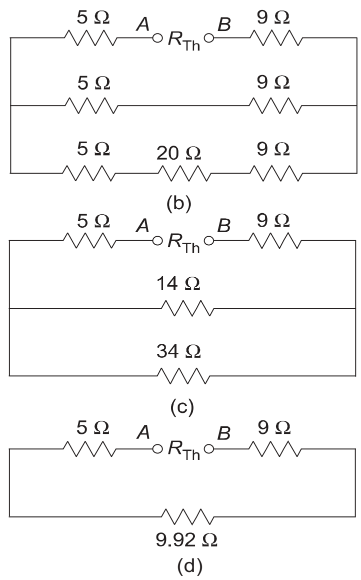


Fig. 2.556

$$R_{\text{Th}} = 23.92\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 23.92\ \Omega$$

Step IV: Calculation of P_{max}

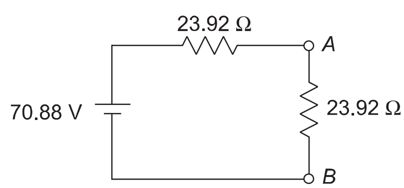


Fig. 2.557

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(70.88)^2}{4 \times 23.92} = 52.51\ \text{W}$$

Example 10

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

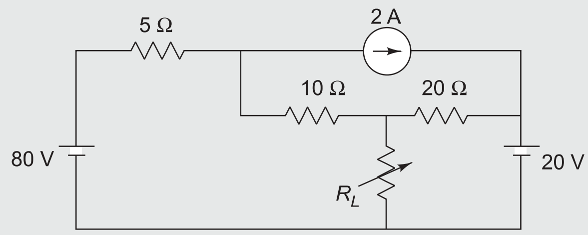


Fig. 2.558

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

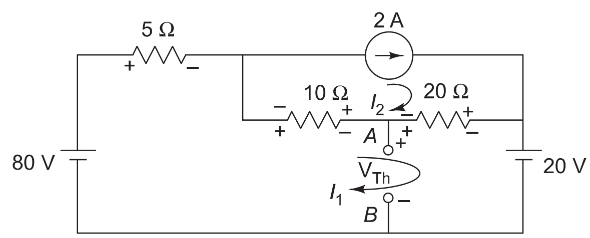


Fig. 2.559

Applying KVL to Mesh 1,

$$80 - 5I_1 - 10(I_1 - I_2) - 20(I_1 - I_2) - 20 = 0$$

$$35I_1 - 30I_2 = 60 \quad (1)$$

Writing the current equation for Mesh 2,

$$I_2 = 2 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 3.43 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 20\,(I_1 - I_2) - 20 = 0$$

$$V_{\text{Th}} = 20(3.43 - 2) + 20$$

$$= 48.6 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

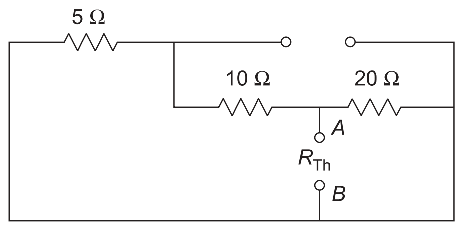


Fig. 2.560

$$R_{Th} = 15 \,||\, 20 = 8.57\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 8.57\ \Omega$$

Step IV: Calculation of P_{max}

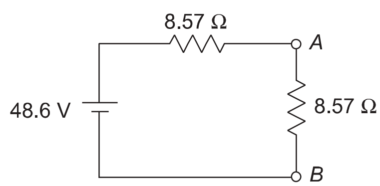


Fig. 2.561

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(48.6)^2}{4 \times 8.57} = 68.9\ \text{W}$$

Example 11

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

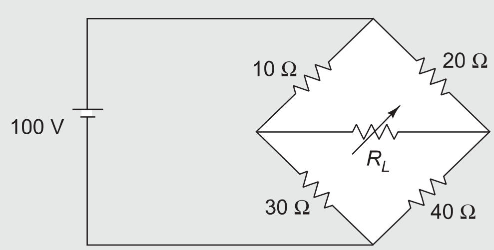


Fig. 2.562

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

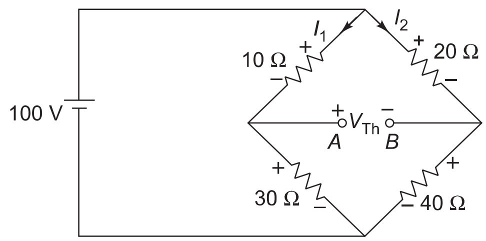


Fig. 2.563

$$I_1 = \frac{100}{10+30} = 2.5 \text{ A}$$

$$I_2 = \frac{100}{20+40} = 1.66 \text{ A}$$

Writing V_{Th} equation,

$V_{Th} + 10I_1 - 20I_2 = 0$

$$\begin{aligned} V_{Th} &= 20I_2 - 10I_1 \\ &= 20(1.66) - 10(2.5) \\ &= 8.2 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

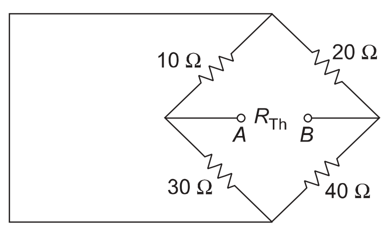


Fig. 2.564

Redrawing the network,

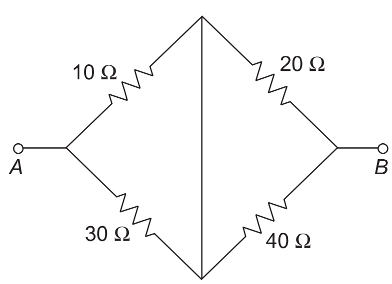


Fig. 2.565

$$R_{Th} = (10 \,||\, 30) + (20 \,||\, 40) = 20.83\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 20.83\ \Omega$$

Step IV: Calculation of P_{max}

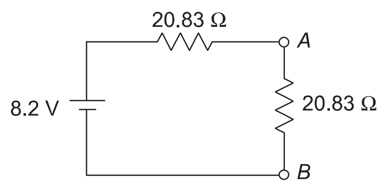


Fig. 2.566

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8.2)^2}{4 \times 20.83} = 0.81\ \text{W}$$

Example 12

For the given circuit find the value of R_L for maximum power transfer and calculate the maximum power absorbed by R_L.

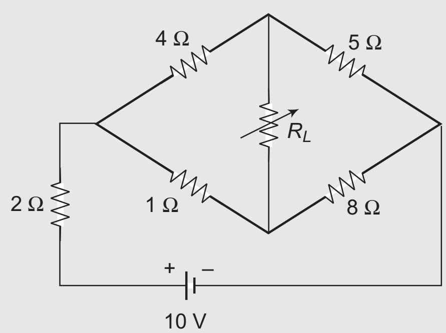


Fig. 2.567 **[Dec 2014]**

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

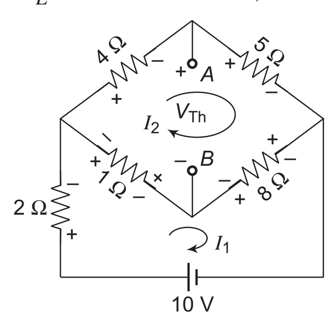


Fig. 2.568

Applying KVL to Mesh 1,

$$10 - 2I_1 - 1(I_1 - I_2) - 8\,(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2

$$-4I_2 - 5I_2 - 8\,(I_2 - I_1) - 1\,(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1.54 \text{ A}$$

$$I_2 = 0.77 \text{ A}$$

Writing V_{Th} equation,

$$-\,1\,(I_2 - I_1) - 4I_2 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -1(I_2 - I_1) - 4I_2$$

$$= -1(0.77 - 1.54) - 4(0.77)$$

$$= -2.31 \text{ V}$$

$$= 2.31 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

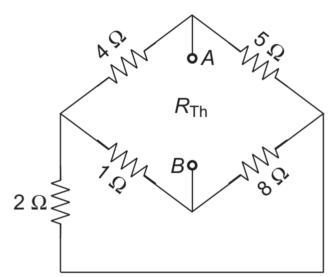


Fig. 2.569

Redrawing the network,

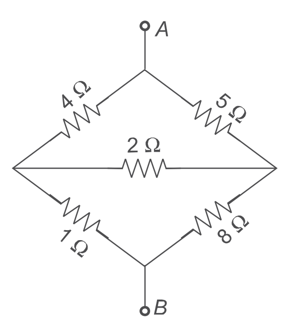


Fig. 2.570

Converting the upper delta into equivalent star network,

$$R_1 = \frac{4 \times 2}{4+2+5} = 0.73\ \Omega$$

$$R_2 = \frac{4 \times 5}{4+2+5} = 1.82\ \Omega$$

$$R_3 = \frac{5 \times 2}{4+2+5} = 0.91\ \Omega$$

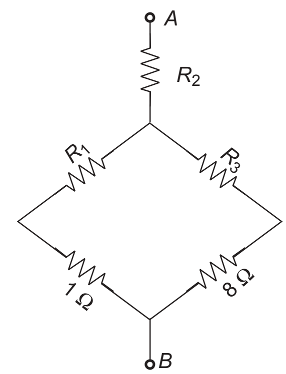


Fig. 2.571

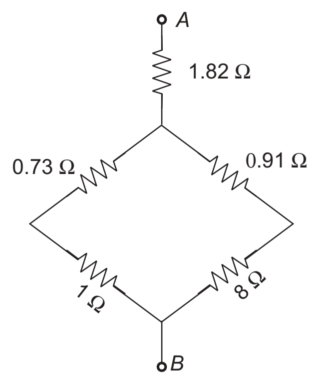


Fig. 2.572

Simplifying the network,

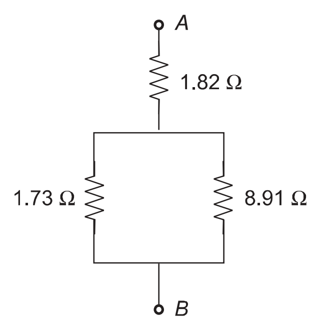


Fig. 2.573

$$R_{Th} = 1.82 + (1.73 \parallel 8.91) = 3.27\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.27\ \Omega$$

Step IV: Calculation of P_{max}

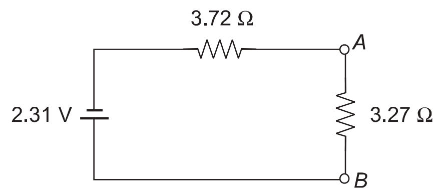


Fig. 2.574

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(2.31)^2}{4\times 3.27} = 0.41 \text{ W}$$

Example 13

Determine the value of R for maximum power transfer. Also find the magnitude of maximum power transferred.

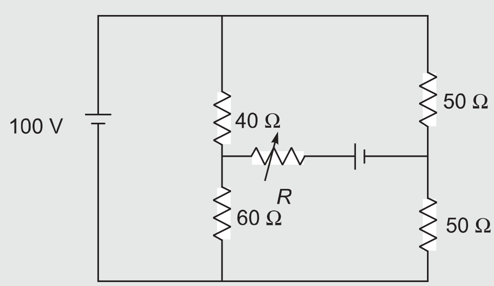


Fig. 2.575 [Dec 2012]

Solution

Step I: Calculation of V_{Th}

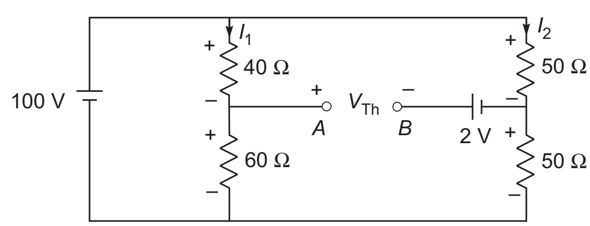


Fig. 2.576

$$I_1 = \frac{100}{40+60} = 1 \text{ A}$$

$$I_2 = \frac{100}{50+50} = 1 \text{ A}$$

Writing V_{Th} equation,

$$-40I_1 + V_{Th} - 2 + 50I_2 = 0$$
$$-40(1) + V_{Th} - 2 + 50(1) = 0$$
$$V_{Th} = -8 \text{ V}$$
$$= 8 \text{ V (terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

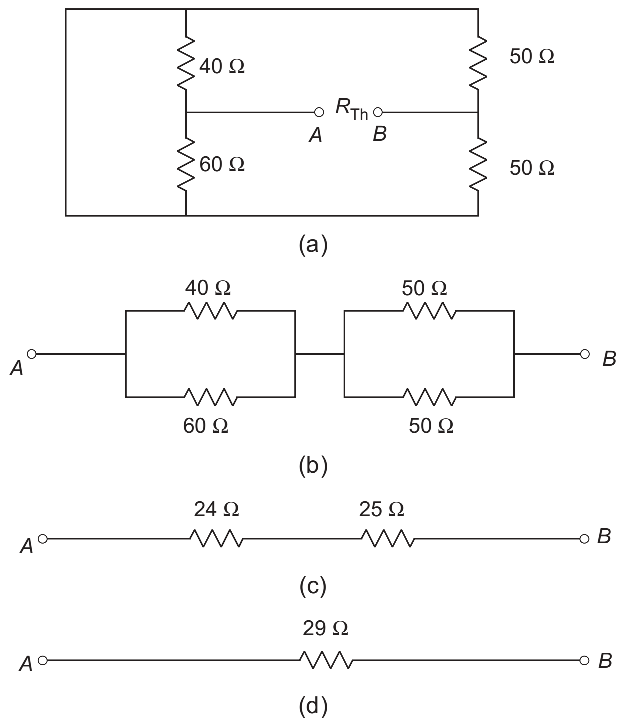


Fig. 2.577

$$R_{Th} = 49\ \Omega$$

Step III: Value of R

For maximum power transfer

$$R = R_{Th} = 49\ \Omega$$

Step IV: Calculation of P_{max}

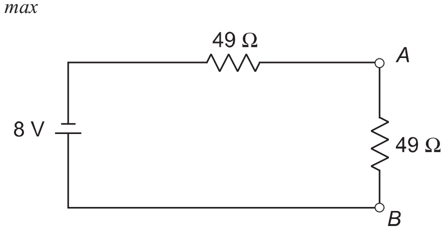


Fig. 2.578

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(8)^2}{4 \times 49} = 0.33 \text{ W}$$

Example 14

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

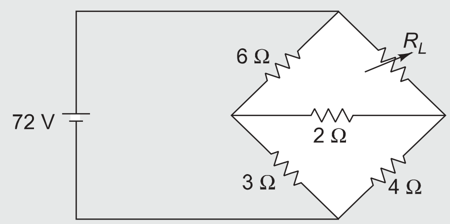


Fig. 2.579

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

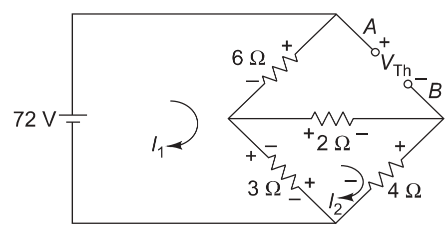


Fig. 2.580

Applying KVL to Mesh 1,

$$72 - 6I_1 - 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 72 \qquad (1)$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 4I_2 = 0$$

$$-3I_1 + 9I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 9 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 6I_1 - 2I_2 = 0$$

$$V_{\text{Th}} = 6I_1 + 2I_2 = 6(9) + 2(3) = 60 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit,

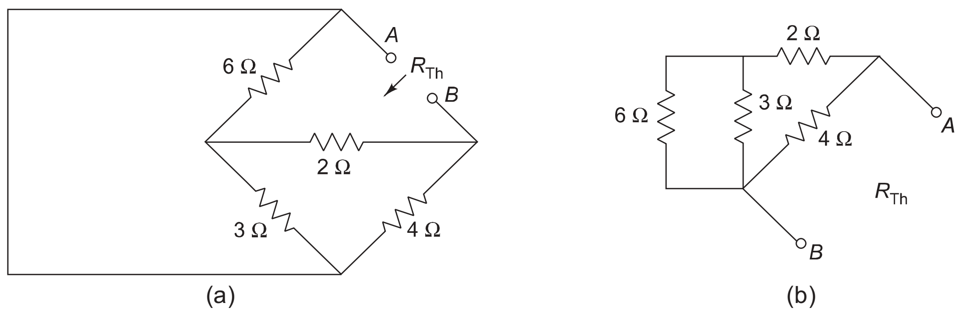


Fig. 2.581

$$R_{Th} = [(6 \,||\, 3) + 2] \,||\, 4 = 2\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 2\ \Omega$$

Step IV: Calculation of P_{max}

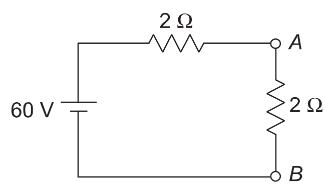


Fig. 2.582

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 2} = 450\ \text{W}$$

Example 15

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate maximum power.

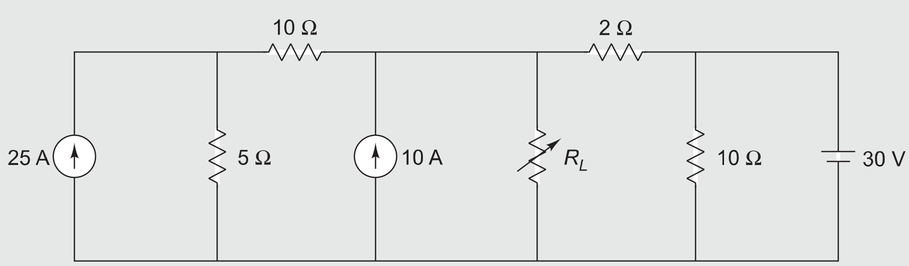


Fig. 2.583

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

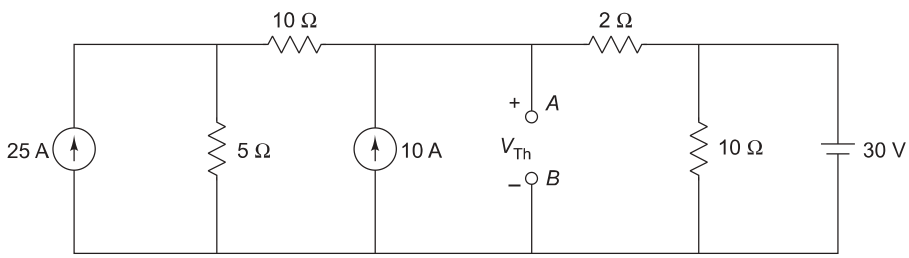


Fig. 2.584

By source transformation, the current source of 25 A and the 5 Ω resistor is converted into an equivalent voltage source of 125 V and a series resistor of 5 Ω. Also the voltage source of 30 V is connected across the 10 Ω resistor. Hence, the 10 Ω resistor becomes redundant.

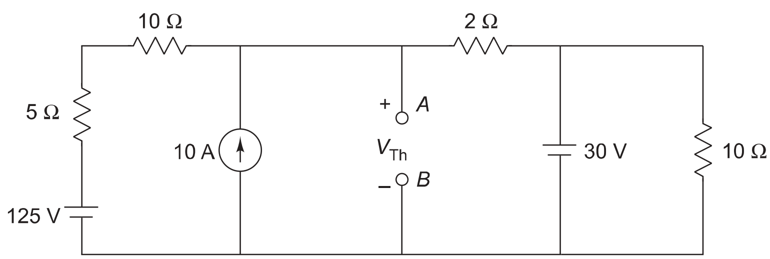


Fig. 2.585

Applying KCL at node,

$$\frac{V_{\text{Th}} - 125}{15} - 10 + \frac{V_{\text{Th}} - 30}{2} = 0$$

$$V_{\text{Th}} = 58.81 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current sources by open circuits,

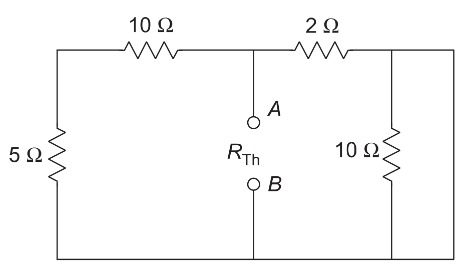


Fig. 2.586

Simplifying the network,

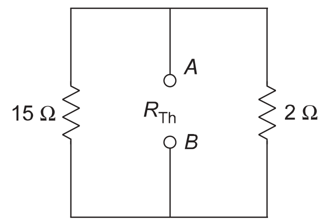


Fig. 2.587

$$R_{Th} = 15 \parallel 2 = 1.76\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 1.76\ \Omega$$

Step IV: Calculation of P_{max}

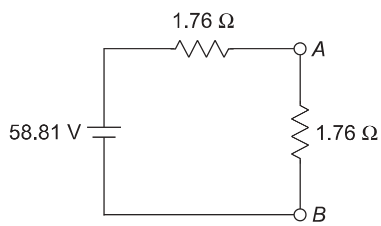


Fig. 2.588

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(58.81)^2}{4 \times 1.76} = 491.28\ \text{W}$$

Example 16

Find the value of R_L for maximum power transfer and calculate maximum power.

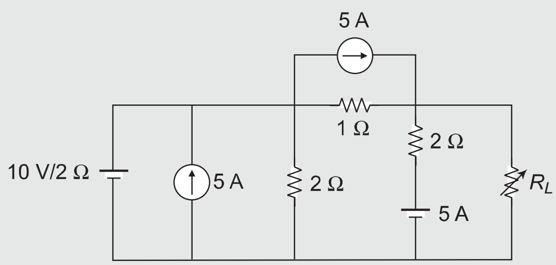


Fig. 2.589 **[Dec 2015]**

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

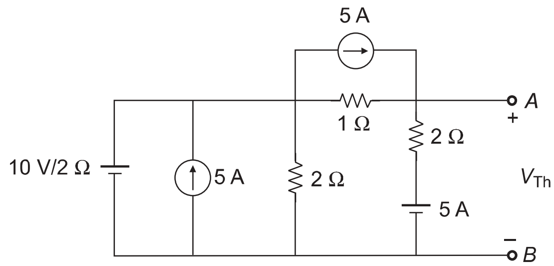


Fig. 2.590

By source transformation, the current source of 5 A and parallel resistor of 2 Ω is converted into an equivalent voltage source of 10 V and series resistor of 2 Ω. Similarly, the other current source of 5 A and parallel resistor of 1 Ω is converted into an equivalent voltage source of 5 V and series resistor of 1 Ω.

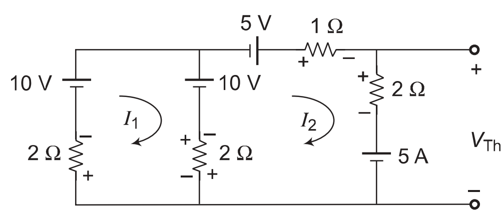


Fig. 2.591

Applying KVL to Mesh 1,

$$-2I_1 + 10 - 10 - 2(I_1 - I_2) = 0$$

$$4I_1 - 2I_2 = 0 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) + 10 + 5 - 1I_2 - 2I_2 - 5 = 0$$

$$-2I_1 + 5I_2 = 10 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1.25 \text{ A}$$

$$I_2 = 2.5 \text{ A}$$

Writing V_{Th} equation,

$$5 + 2I_2 - V_{Th} = 0$$

$$V_{Th} = 5 + 2I_2$$

$$= 5 + 2\,(2.5)$$

$$= 10 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and current sources by open circuits,

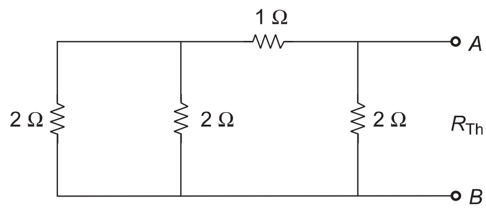


Fig. 2.592

By series-parallel reduction technique,

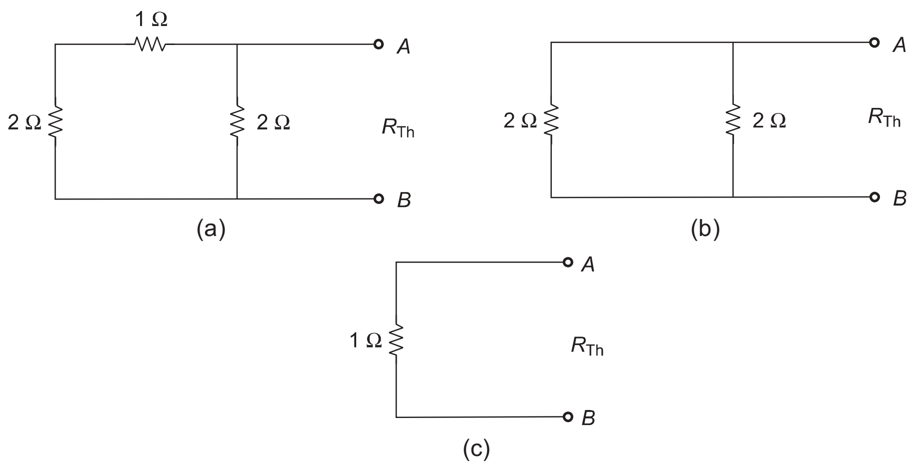


Fig. 2.593

$$R_{Th} = 1\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 1\ \Omega$$

Step IV: Calculation of P_{max}

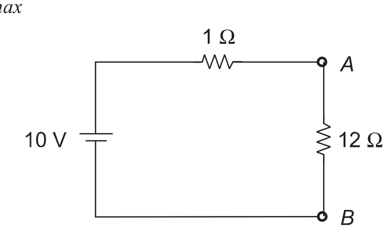


Fig. 2.594

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(10)^2}{4 \times 1} = 25 \text{ W}$$

Example 17

For the given circuit, find the value of 'R_L' so that maximum power is dissipated in it. Also, find P_{max}.

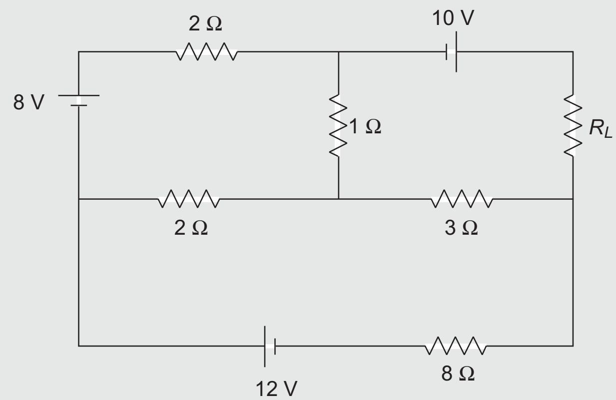


Fig. 2.595 **[Dec 2013]**

Solution

Step I: Calculation of V_{Th}

Removing the resistor R_L from the network,

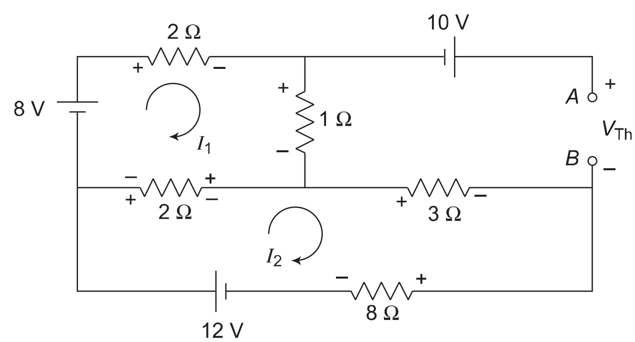


Fig. 2.596

Applying KVL to Mesh 1,

$$8 - 2I_1 - 1I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 8 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 3I_2 - 8I_2 + 12 = 0$$

$$-2I_1 + 13I_2 = 12 \quad (2)$$

Solving Eqs. (1) and (2),

$$I_1 = 2.1 \text{ A}$$
$$I_2 = 1.25 \text{ A}$$

Writing V_{Th} equation,

$$1I_1 + 10 - V_{Th} + 3I_2 = 0$$
$$V_{Th} = 1I_1 + 10 + 3I_2$$
$$= 1(2.1) + 10 + 3(1.25)$$
$$= 15.85 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits,

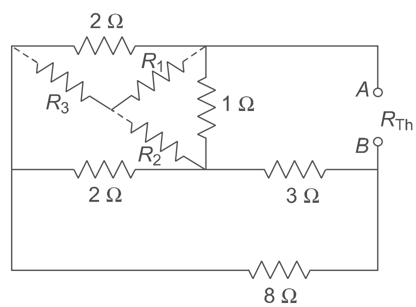


Fig. 2.597

Converting the delta network formed by resistors of 2 Ω, 1 Ω and 2 Ω into equivalent star network,

$$R_1 = \frac{2 \times 1}{2+1+2} = 0.4\ \Omega$$

$$R_2 = \frac{2 \times 1}{2+1+2} = 0.4\ \Omega$$

$$R_3 = \frac{2 \times 2}{2+1+2} = 0.8\ \Omega$$

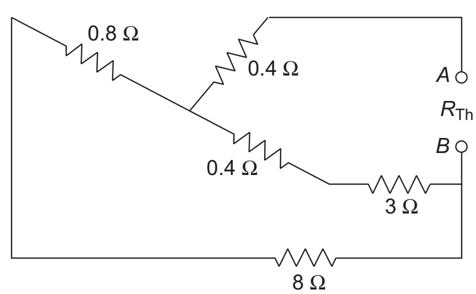


Fig. 2.598

Simplifying the network,

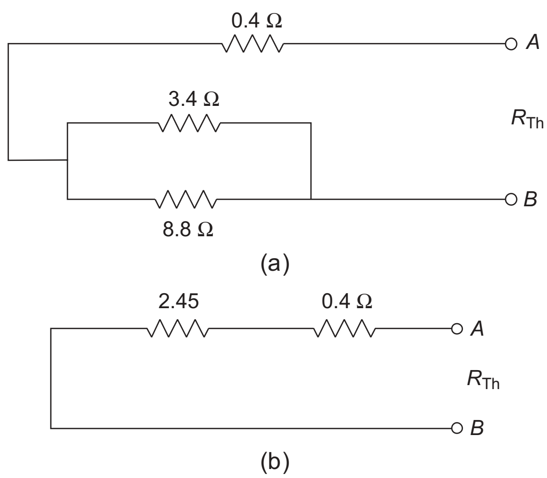


Fig. 2.599

$$R_{\text{Th}} = 2.85\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 2.85\ \Omega$$

Step IV: Calculation of P_{max}

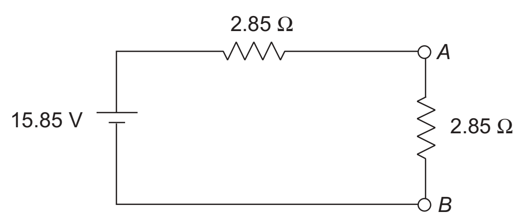


Fig. 2.600

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(15.85)^2}{4 \times 2.85} = 22.04\ \text{W}$$

Example 18

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate maximum power.

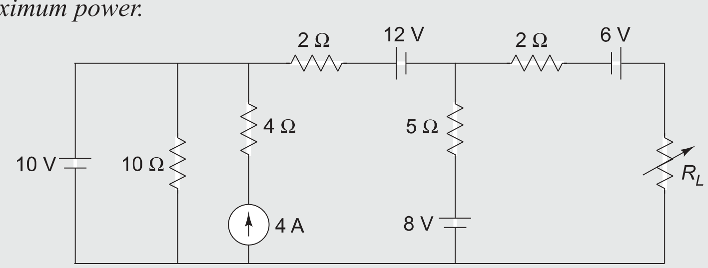


Fig. 2.601

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

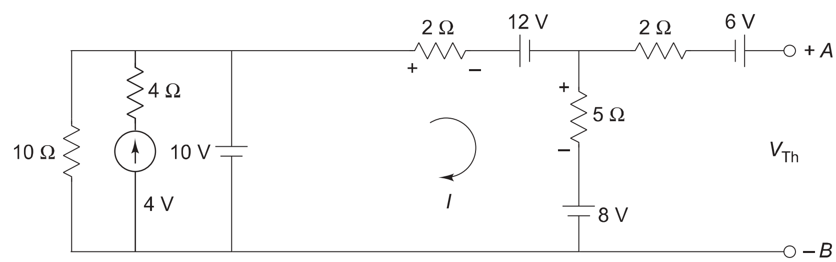


Fig. 2.602

Applying KVL to the outer path,

$$10 - 2I - 12 - 5I - 8 = 0$$

$$I = -\frac{10}{7} = -1.43 \text{ A}$$

Writing V_{Th} equation,

$$8 + 5I + 6 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 8 + 6 + 5I$$

$$= 8 + 6 + 5(-1.43)$$

$$= 6.85 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits and current source by an open circuit,

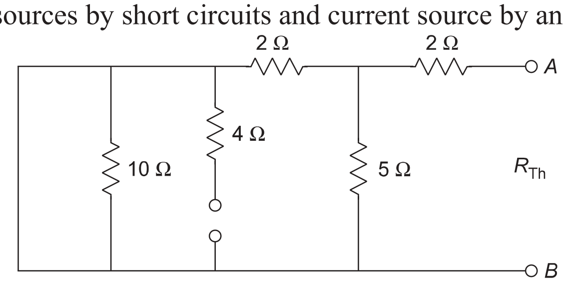


Fig. 2.603

$$R_{Th} = (2 \parallel 5) + 2$$
$$= 3.43\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.43\ \Omega$$

Step IV: Calculation of P_{max}

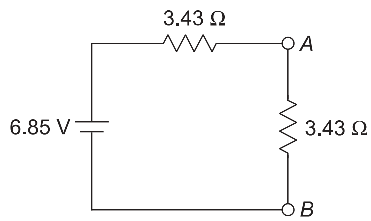


Fig. 2.604

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(6.85)^2}{4 \times 3.43} = 3.42\ \text{W}$$

Exercise 2.9

2.1 Find the value of the resistance R_L for maximum power transfer and calculate maximum power.

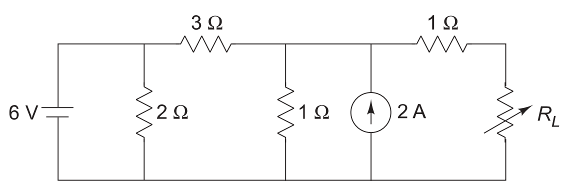


Fig. 2.605

[1.75 Ω, 1.29 W]

2.2 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

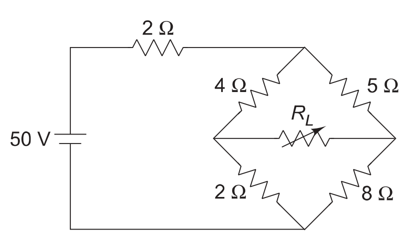


Fig. 2.606

[4.51 Ω, 4.95 W]

CHAPTER 5

Handy Circuit Analysis Techniques

KEY CONCEPTS

- Superposition: Determining the *Individual Contributions* of Different Sources to Any Current or Voltage
- Source Transformation as a Means of Simplifying Circuits
- Thévenin's Theorem
- Norton's Theorem
- Thévenin and Norton Equivalent Networks
- Maximum Power Transfer
- $\Delta \leftrightarrow Y$ Transformations for Resistive Networks
- Selecting a Particular Combination of Analysis Techniques
- Performing dc Sweep Simulations Using PSpice

INTRODUCTION

The techniques of nodal and mesh analysis described in Chap. 4 are reliable and extremely powerful methods. However, both require that we develop a complete set of equations to describe a particular circuit as a general rule, even if only one current, voltage, or power quantity is of interest. In this chapter, we investigate several different techniques for isolating specific parts of a circuit in order to simplify the analysis. After examining each of these techniques, we focus on how one might go about selecting one method over another.

5.1 LINEARITY AND SUPERPOSITION

All of the circuits which we plan to analyze can be classified as *linear circuits,* so this is a good time to be more specific in defining exactly what we mean by that. Having done this, we can then consider the most important consequence of linearity, the principle of ***superposition*****.** This principle is very basic and will appear repeatedly in our study of linear circuit analysis. As a matter of fact, the nonapplicability of superposition to nonlinear circuits is the very reason they are so difficult to analyze!

The principle of superposition states that the *response* (a desired current or voltage) in a linear circuit having more than one independent source can be obtained by adding the responses caused by the separate independent sources *acting alone*.

Linear Elements and Linear Circuits

We define a ***linear element*** as a passive element that has a linear voltage-current relationship. By a "linear voltage-current relationship"

we simply mean that multiplication of the current through the element by a constant K results in the multiplication of the voltage across the element by the same constant K. At this time, only one passive element has been defined (the resistor), and its voltage-current relationship

$$v(t) = Ri(t)$$

is clearly linear. As a matter of fact, if $v(t)$ is plotted as a function of $i(t)$, the result is a straight line.

The dependent voltage source $v_s = 0.6i_1 - 14v_2$ is linear, but $v_s = 0.6i_1^2$ and $v_s = 0.6i_1v_2$ are not.

We define a ***linear dependent source*** as a dependent current or voltage source whose output current or voltage is proportional only to the first power of a specified current *or* voltage variable in the circuit (or to the *sum* of such quantities).

We now define a ***linear circuit*** as a circuit composed entirely of independent sources, linear dependent sources, and linear elements. From this definition, it is possible to show[1] that "the response is proportional to the source," or that multiplication of all independent source voltages and currents by a constant K increases all the current and voltage responses by the same factor K (including the dependent source voltage or current outputs).

The Superposition Principle

The most important consequence of linearity is ***superposition.***

Let us explore the superposition principle by considering first the circuit of Fig. 5.1, which contains two independent sources, the current generators that force the currents i_a and i_b into the circuit. Sources are often called *forcing functions* for this reason, and the nodal voltages that they produce can be termed *response functions,* or simply *responses*. Both the forcing functions and the responses may be functions of time. The two nodal equations for this circuit are

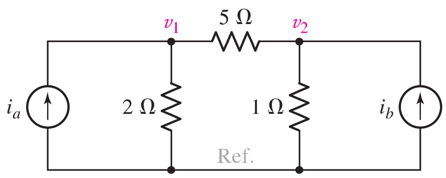


FIGURE 5.1 A circuit with two independent current sources.

$$0.7v_1 - 0.2v_2 = i_a \quad [1]$$

$$-0.2v_1 + 1.2v_2 = i_b \quad [2]$$

Now let us perform experiment x. We change the two forcing functions to i_{ax} and i_{bx}; the two unknown voltages will now be different, so we will call them v_{1x} and v_{2x}. Thus,

$$0.7v_{1x} - 0.2v_{2x} = i_{ax} \quad [3]$$

$$-0.2v_{1x} + 1.2v_{2x} = i_{bx} \quad [4]$$

We next perform experiment y by changing the source currents to i_{ay} and i_{by} and measure the responses v_{1y} and v_{2y}:

$$0.7v_{1y} - 0.2v_{2y} = i_{ay} \quad [5]$$

$$-0.2v_{1y} + 1.2v_{2y} = i_{by} \quad [6]$$

(1) The proof involves first showing that the use of nodal analysis on the linear circuit can produce only linear equations of the form

$$a_1v_1 + a_2v_2 + \cdots + a_Nv_N = b$$

where the a_i are constants (combinations of resistance or conductance values, constants appearing in dependent source expressions, 0, or ±1), the v_i are the unknown node voltages (responses), and b is an independent source value or a sum of independent source values. Given a set of such equations, if we multiply all the b's by K, then it is evident that the solution of this new set of equations will be the node voltages $Kv_1, Kv_2, \ldots, Kv_N$.

These three sets of equations describe the same circuit with three different sets of source currents. Let us *add* or "*superpose*" the last two sets of equations. Adding Eqs. [3] and [5],

$$(0.7v_{1x} + 0.7v_{1y}) - (0.2v_{2x} + 0.2v_{2y}) = i_{ax} + i_{ay} \qquad [7]$$

$$0.7v_1 \quad - \quad 0.2v_2 \quad = \quad i_a \qquad [1]$$

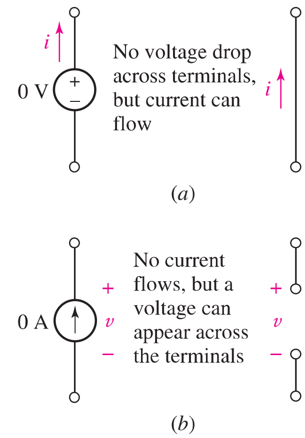
and adding Eqs. [4] and [6],

$$-(0.2v_{1x} + 0.2v_{1y}) + (1.2v_{2x} + 1.2v_{2y}) = i_{bx} + i_{by} \qquad [8]$$

$$-0.2v_1 \quad + \quad 1.2v_2 \quad = \quad i_b \qquad [2]$$

where Eq. [1] has been written immediately below Eq. [7] and Eq. [2] below Eq. [8] for easy comparison.

The linearity of all these equations allows us to compare Eq. [7] with Eq. [1] and Eq. [8] with Eq. [2] and draw an interesting conclusion. If we select i_{ax} and i_{ay} such that their sum is i_a and select i_{bx} and i_{by} such that their sum is i_b, then the desired responses v_1 and v_2 may be found by adding v_{1x} to v_{1y} and v_{2x} to v_{2y}, respectively. In other words, we can perform experiment x and note the responses, perform experiment y and note the responses, and finally add the two sets of responses. This leads to the fundamental concept involved in the superposition principle: to look at each independent source (and the response it generates) one at a time with the other independent sources "turned off" or "zeroed out."



■ **FIGURE 5.2** (*a*) A voltage source set to zero acts like a short circuit. (*b*) A current source set to zero acts like an open circuit.

If we reduce a voltage source to zero volts, we have effectively created a short circuit (Fig. 5.2*a*). If we reduce a current source to zero amps, we have effectively created an open circuit (Fig. 5.2*b*). Thus, the ***superposition theorem*** can be stated as:

> In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

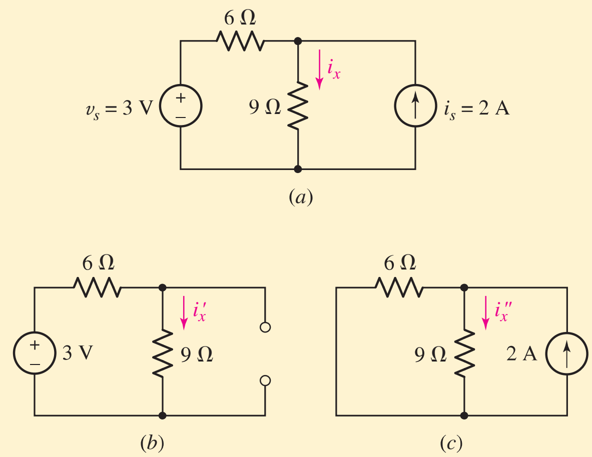
Thus, if there are N independent sources, we must perform N experiments, each having only one of the independent sources active and the others inactive/turned off/zeroed out. Note that *dependent* sources are in general active in every experiment.

There is also no reason that an independent source must assume only its given value or a zero value in the several experiments; it is necessary only for the sum of the several values to be equal to the original value. An inactive source almost always leads to the simplest circuit, however.

The circuit we have just used as an example should indicate that a much stronger theorem might be written; a *group* of independent sources may be made active and inactive collectively, if we wish. For example, suppose there are three independent sources. The theorem states that we may find a given response by considering each of the three sources acting alone and adding the three results. Alternatively, we may find the response due to the first and second sources operating with the third inactive, and then add to this the response caused by the third source acting alone. This amounts to treating several sources collectively as a sort of "supersource."

EXAMPLE 5.1

For the circuit of Fig. 5.3*a*, use superposition to determine the unknown branch current i_x.



■ **FIGURE 5.3** (*a*) An example circuit with two independent sources for which the branch current i_x is desired; (*b*) same circuit with current source open-circuited; (*c*) original circuit with voltage source short-circuited.

First set the current source equal to zero and redraw the circuit as shown in Fig. 5.3*b*. The portion of i_x due to the voltage source has been designated i_x' to avoid confusion and is easily found to be 0.2 A.

Next set the voltage source in Fig. 5.3*a* to zero and again redraw the circuit, as shown in Fig. 5.3*c*. Current division lets us determine that i_x'' (the portion of i_x due to the 2 A current source) is 0.8 A.

Now compute the total current i_x by adding the two individual components:

$$i_x = i_{x|_{3\,\text{V}}} + i_{x|_{2\,\text{A}}} = i_x' + i_x''$$

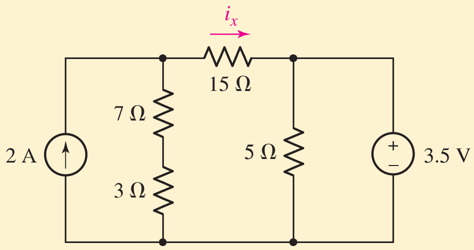
or

$$i_x = \frac{3}{6+9} + 2\left(\frac{6}{6+9}\right) = 0.2 + 0.8 = 1.0\text{ A}$$

Another way of looking at Example 5.1 is that the 3 V source and the 2 A source are each performing work on the circuit, resulting in a total current i_x flowing through the 9 Ω resistor. *However, the contribution of the 3 V source to i_x does not depend on the contribution of the 2 A source, and vice versa*. For example, if we double the output of the 2 A source to 4 A, it will now contribute 1.6 A to the total current i_x flowing through the 9 Ω resistor. However, the 3 V source will still contribute only 0.2 A to i_x, for a new total current of 0.2 + 1.6 = 1.8 A.

PRACTICE

5.1 For the circuit of Fig. 5.4, use superposition to compute the current i_x.



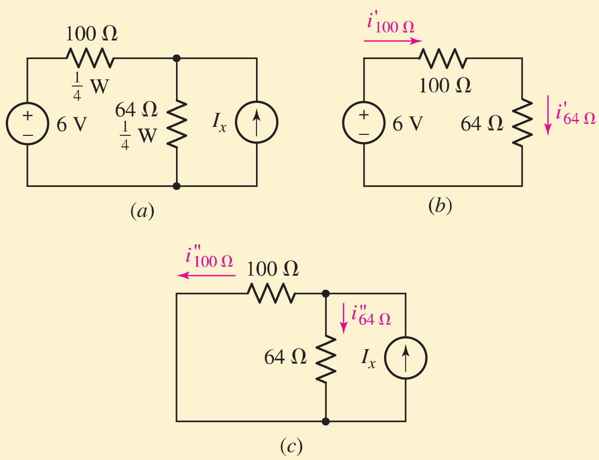
■ **FIGURE 5.4**

Ans: 660 mA.

As we will see, superposition does not generally reduce our workload when considering a particular circuit, since it leads to the analysis of several new circuits to obtain the desired response. However, it is particularly useful in identifying the significance of various parts of a more complex circuit. It also forms the basis of phasor analysis, which is introduced in Chap. 10.

EXAMPLE 5.2

Referring to the circuit of Fig. 5.5*a*, determine the maximum *positive* current to which the source I_x can be set before any resistor exceeds its power rating and overheats.



■ **FIGURE 5.5** (*a*) A circuit with two resistors each rated at $\frac{1}{4}$ W. (*b*) Circuit with only the 6 V source active. (*c*) Circuit with the source I_x active.

▶ ***Identify the goal of the problem.***

Each resistor is rated to a maximum of 250 mW. If the circuit allows this value to be exceeded (by forcing too much current through either resistor), excessive heating will occur—possibly leading to

(Continued on next page)

an accident. The 6 V source cannot be changed, so we are looking for an equation involving I_x and the maximum current through each resistor.

Collect the known information.

Based on its 250 mW power rating, the maximum current the 100 Ω resistor can tolerate is

$$\sqrt{\frac{P_{\max}}{R}} = \sqrt{\frac{0.250}{100}} = 50 \text{ mA}$$

and, similarly, the current through the 64 Ω resistor must be less than 62.5 mA.

Devise a plan.

Either nodal or mesh analysis may be applied to the solution of this problem, but superposition may give us a slight edge, since we are primarily interested in the effect of the current source.

Construct an appropriate set of equations.

Using superposition, we redraw the circuit as in Fig. 5.5*b* and find that the 6 V source contributes a current

$$i'_{100\,\Omega} = \frac{6}{100 + 64} = 36.59 \text{ mA}$$

to the 100 Ω resistor and, since the 64 Ω resistor is in series, $i'_{64\,\Omega} =$ 36.59 mA as well.

Recognizing the current divider in Fig. 5.5*c*, we note that $i''_{64\,\Omega}$ will *add* to $i'_{64\,\Omega}$, but $i''_{100\,\Omega}$ is *opposite* in direction to $i'_{100\,\Omega}$. Therefore, I_X can safely contribute $62.5 - 36.59 = 25.91$ mA to the 64 Ω resistor current, and $50 - (-36.59) = 86.59$ mA to the 100 Ω resistor current.

The 100 Ω resistor therefore places the following constraint on I_x:

$$I_x < (86.59 \times 10^{-3})\left(\frac{100 + 64}{64}\right)$$

and the 64 Ω resistor requires that

$$I_x < (25.91 \times 10^{-3})\left(\frac{100 + 64}{100}\right)$$

Attempt a solution.

Considering the 100 Ω resistor first, we see that I_x is limited to $I_x <$ 221.9 mA. The 64 Ω resistor limits I_x such that $I_x < 42.49$ mA. In order to satisfy both constraints, I_x must be less than 42.49 mA. If the value is increased, the 64 Ω resistor will overheat long before the 100 Ω resistor does.

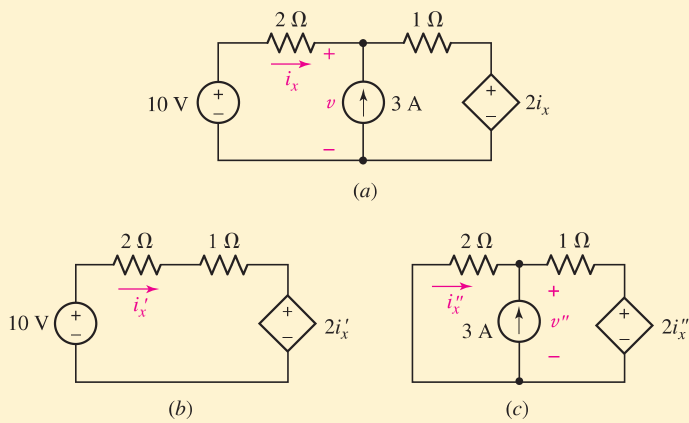
Verify the solution. Is it reasonable or expected?

One particularly useful way to evaluate our solution is to perform a dc sweep analysis in PSpice as described after the next example. An interesting question, however, is whether we would have expected the 64 Ω resistor to overheat first.

Originally we found that the 100 Ω resistor has a smaller maximum current, so it might be reasonable to expect it to limit I_x. However, because I_x *opposes* the current sent by the 6 V source through the 100 Ω resistor but *adds* to the 6 V source's contribution to the current through the 64 Ω resistor, it turns out to work the other way—it's the 64 Ω resistor that sets the limit on I_x.

EXAMPLE 5.3

In the circuit of Fig. 5.6*a*, use the superposition principle to determine the value of i_x.



■ **FIGURE 5.6** (*a*) An example circuit with two independent sources and one dependent source for which the branch current i_x is desired. (*b*) Circuit with the 3 A source open-circuited. (*c*) Original circuit with the 10 V source short-circuited.

First open-circuit the 3 A source (Fig. 5.6*b*). The single mesh equation is

$$-10 + 2i'_x + i'_x + 2i'_x = 0$$

so that

$$i'_x = 2 \text{ A}$$

Next, short-circuit the 10 V source (Fig. 5.6*c*) and write the single-node equation

$$\frac{v''}{2} + \frac{v'' - 2i''_x}{1} = 3$$

and relate the dependent-source-controlling quantity to v'':

$$v'' = 2(-i''_x)$$

Solving, we find

$$i''_x = -0.6 \text{ A}$$

and, thus,

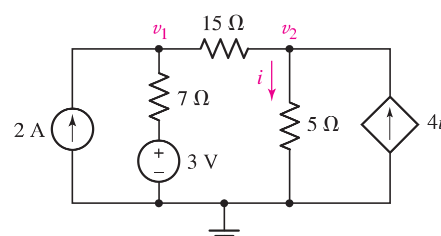
$$i_x = i'_x + i''_x = 2 + (-0.6) = 1.4 \text{ A}$$

Note that in redrawing each subcircuit, we are always careful to use some type of notation to indicate that we are not working with the original variables. This prevents the possibility of rather disastrous errors when we add the individual results.

PRACTICE

5.2 For the circuit of Fig. 5.7, use superposition to obtain the voltage across each current source.

Ans: $v_{1|2A} = 9.180$ V, $v_{2|2A} = -1.148$ V, $v_{1|3V} = 1.967$ V, $v_{2|3V} = -0.246$ V; $v_1 = 11.147$ V, $v_2 = -1.394$ V.



■ **FIGURE 5.7**

Summary of Basic Superposition Procedure

1. **Select one of the independent sources. Set all other independent sources to zero.** This means voltage sources are replaced with short circuits and current sources are replaced with open circuits. Leave dependent sources in the circuit.
2. **Relabel voltages and currents using suitable notation** (e.g., v', i_2''). Be sure to relabel controlling variables of dependent sources to avoid confusion.
3. **Analyze the simplified circuit to find the desired currents and/or voltages.**
4. **Repeat steps 1 through 3 until each independent source has been considered.**
5. **Add the partial currents and/or voltages obtained from the separate analyses.** Pay careful attention to voltage signs and current directions when summing.
6. **Do not add power quantities.** If power quantities are required, calculate only after partial voltages and/or currents have been summed.

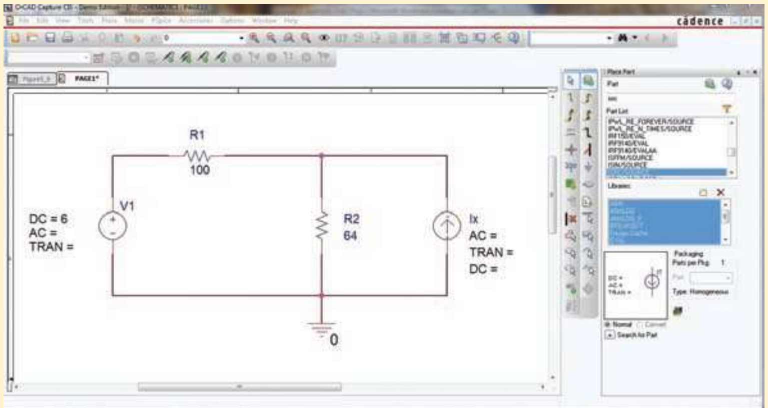
Note that step 1 may be altered in several ways. First, independent sources can be considered in groups as opposed to individually if it simplifies the analysis, as long as no independent source is included in more than one subcircuit. Second, it is technically not necessary to set sources to zero, although this is almost always the best route. For example, a 3 V source may appear in two subcircuits as a 1.5 V source, since $1.5 + 1.5 = 3$ V just as $0 + 3 = 3$ V. Because it is unlikely to simplify our analysis, however, there is little point to such an exercise.

COMPUTER-AIDED ANALYSIS

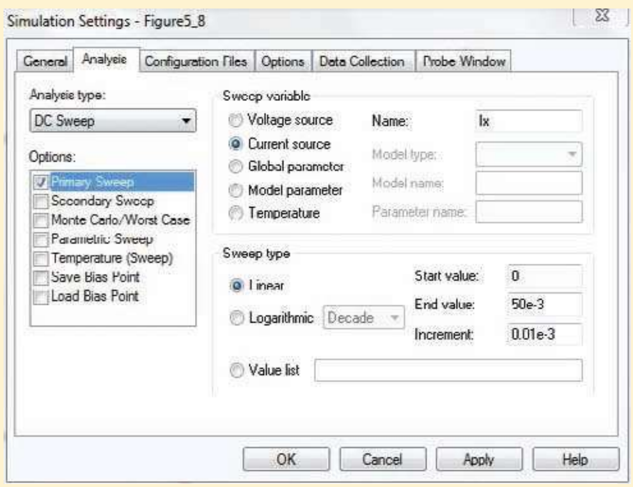
Although PSpice is extremely useful in verifying that we have analyzed a complete circuit correctly, it can also assist us in determining the contribution of each source to a particular response. To do this, we employ what is known as a *dc parameter sweep*.

Consider the circuit presented in Example 5.2, when we were asked to determine the maximum positive current that could be obtained from the current source without exceeding the power rating of either resistor in the circuit. The circuit is shown redrawn using the Orcad Capture CIS schematic tool in Fig. 5.8. Note that no value has been assigned to the current source.

After the schematic has been entered and saved, the next step is to specify the dc sweep parameters. This option allows us to specify a range of values for a voltage or current source (in the present case, the current source I_x), rather than a specific value. Selecting **New Simulation Profile** under **PSpice,** we provide a name for our profile and are then provided with the dialog box shown in Fig. 5.9.



■ **FIGURE 5.8** The circuit from Example 5.2.



■ **FIGURE 5.9** DC Sweep dialog box shown with I_x selected as the sweep variable.

Under **Analysis Type,** we pull down the **DC Sweep** option, specify the "sweep variable" as **Current Source,** and then type in I_x in the **Name** box. There are several options under Sweep Type: **Linear, Logarithmic,** and **Value List.** The last option allows us to specify each value to assign to I_x. In order to generate a smooth plot, however, we choose to perform a **Linear** sweep, with a **Start Value** of 0 mA, an **End Value** of 50 mA, and a value of 0.01 mA for the **Increment.**

After we perform the simulation, the graphical output package Probe is automatically launched. When the window appears, the horizontal axis (corresponding to our variable, I_x) is displayed, but the vertical axis variable must be chosen. Selecting **Add Trace** from the **Trace** menu, we click on **I(R1),** then type an asterisk in the **Trace Expression** box, click on **I(R1)** once again, insert yet another asterisk, and finally type in 100. This asks Probe to plot the power absorbed by the 100 Ω resistor. In a similar fashion, we repeat the process to add the power

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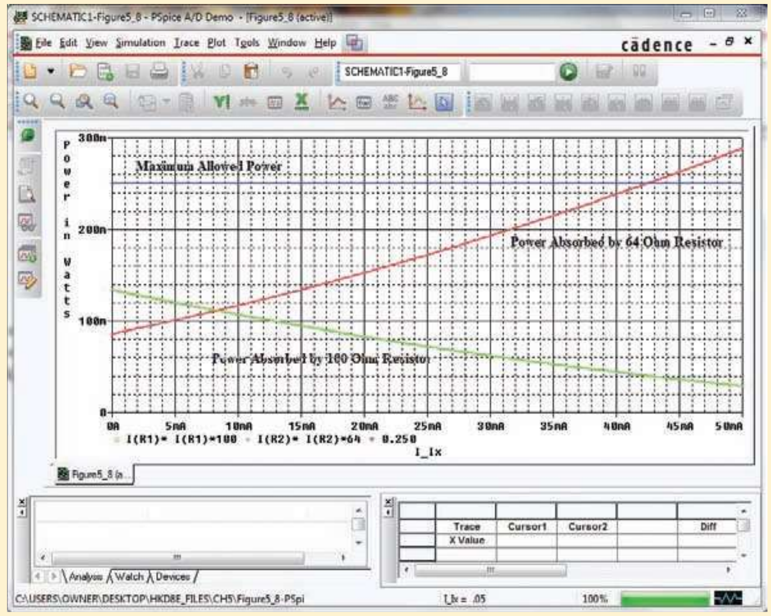
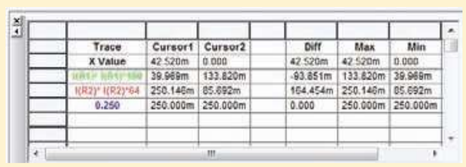


(*a*)



(*b*)

■ **FIGURE 5.10** (*a*) Probe output with text labels identifying the power absorbed by the two resistors individually. A horizontal line indicating 250 mW has also been included, as well as text labels to improve clarity. (*b*) Cursor dialog box.

absorbed by the 64 Ω resistor, resulting in a plot similar to that shown in Fig. 5.10*a*. A horizontal reference line at 250 mW was also added to the plot by typing 0.250 in the **Trace Expression** box after selecting **Add Trace** from the **Trace** menu a third time.

We see from the plot that the 64 Ω resistor *does* exceed its 250 mW power rating in the vicinity of $I_x = 43$ mA. In contrast, however, we also see that regardless of the value of the current source I_x (provided that it is between 0 and 50 mA), the 100 Ω resistor will never dissipate 250 mW; in fact, the absorbed power *decreases* with increasing current from the current source. If we desire a more precise answer, we can make use of the cursor tool, which is invoked by selecting **Trace, Cursor, Display** from the menu bar. Figure 5.10*b* shows the result of dragging cursor 1 to 42.52 A, where the 64 Ω resistor is dissipating just over its maximum rated power of 250 mW. Increased precision can be obtained by decreasing the increment value used in the dc sweep.

This technique is very useful in analyzing electronic circuits, where we might need, for example, to determine what input voltage is required to a complicated amplifier circuit in order to obtain a zero output voltage. We also notice that there are several other types of parameter sweeps that we can perform, including a dc voltage sweep. The ability to vary temperature is useful only when dealing with component models that have a temperature parameter built in, such as diodes and transistors.

Unfortunately, it usually turns out that little if any time is saved in analyzing a circuit containing one or more dependent sources by use of the superposition principle, for there must always be at least two sources in operation: one independent source and all the dependent sources.

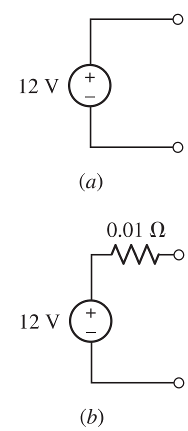
We must constantly be aware of the limitations of superposition. It is applicable only to linear responses, and thus the most common nonlinear response—power—is not subject to superposition. For example, consider two 1 V batteries in series with a 1 Ω resistor. The power delivered to the resistor is 4 W, but if we mistakenly try to apply superposition, we might say that each battery alone furnished 1 W and thus the calculated power is only 2 W. This is incorrect, but a surprisingly easy mistake to make.

5.2 SOURCE TRANSFORMATIONS

Practical Voltage Sources

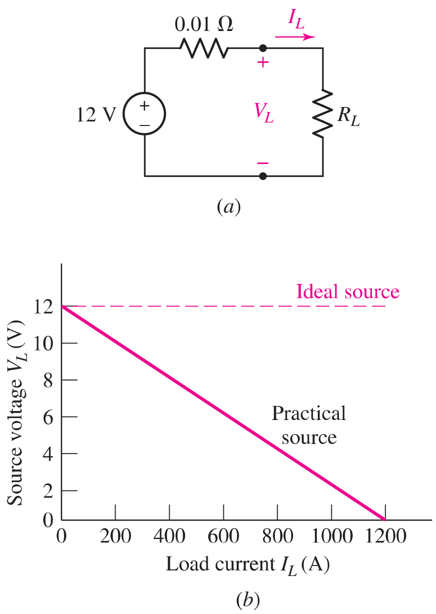
So far, we've only worked with *ideal* sources—elements whose terminal voltage is independent of the current flowing through them. To see the relevance of this fact, consider a simple independent ("ideal") 9 V source connected to a 1 Ω resistor. The 9 volt source will force a current of 9 amperes through the 1 Ω resistor (perhaps this seems reasonable enough), but the same source would apparently force 9,000,000 amperes through a 1 mΩ resistor (which hopefully does not seem reasonable). On paper, there's nothing to stop us from reducing the resistor value all the way to 0 Ω … but that would lead to a contradiction, as the source would be "trying" to maintain 9 V across a dead short, which Ohm's law tells us can't happen ($V = 9 = RI = 0$?).

What happens in the real world when we do this type of experiment? For example, if we try to start a car with the headlights already on, we most likely notice the headlights dim as the battery is asked to supply a large (~100 A or more) starter current in parallel with the current running to the headlights. If we model the 12 V battery with an ideal 12 V source as in Fig. 5.11*a*, our observation cannot be explained. Another way of saying this is that our model breaks down when the load draws a large current from the source.

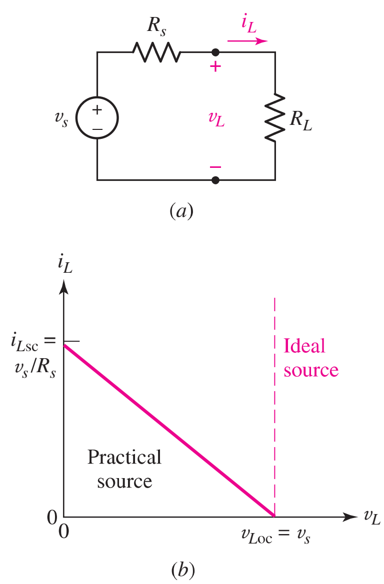


■ **FIGURE 5.11** (*a*) An ideal 12 V dc voltage source used to model a car battery. (*b*) A more accurate model that accounts for the observed reduction in terminal voltage at large currents.

To better approximate the behavior of a real device, the ideal voltage source must be modified to account for the lowering of its terminal voltage when large currents are drawn from it. Let us suppose that we observe experimentally that our car battery has a terminal voltage of 12 V when no current is flowing through it, and a reduced voltage of 11 V when 100 A is flowing. How could we model this behavior? Well, a more accurate model might be an ideal voltage source of 12 V in series with a resistor across which 1 V appears when 100 A flows through it. A quick calculation shows that the resistor must be 1 V/100 A = 0.01 Ω, and the ideal voltage source and this series resistor constitute a ***practical voltage source*** (Fig. 5.11*b*).



■ **FIGURE 5.12** (*a*) A practical source, which approximates the behavior of a certain 12 V automobile battery, is shown connected to a load resistor R_L. (*b*) The relationship between I_L and V_L is linear.



■ **FIGURE 5.13** (*a*) A general practical voltage source connected to a load resistor R_L. (*b*) The terminal voltage of a practical voltage source decreases as i_L increases and $R_L = v_L/i_L$ decreases. The terminal voltage of an ideal voltage source (also plotted) remains the same for any current delivered to a load.

Thus, we are using the series combination of two ideal circuit elements, an independent voltage source and a resistor, to model a real device.

We do not expect to find such an arrangement of ideal elements inside our car battery, of course. Any real device is characterized by a certain current-voltage relationship at its terminals, and our problem is to develop some combination of ideal elements that can furnish a similar current-voltage characteristic, at least over some useful range of current, voltage, or power.

In Fig. 5.12*a*, we show our two-piece practical model of the car battery now connected to some load resistor R_L. The terminal voltage of the practical source is the same as the voltage across R_L and is marked[2] V_L. Figure 5.12*b* shows a plot of load voltage V_L as a function of the load current I_L for this practical source. The KVL equation for the circuit of Fig. 5.12*a* may be written in terms of I_L and V_L:

$$12 = 0.01 I_L + V_L$$

and thus

$$V_L = -0.01 I_L + 12$$

This is a linear equation in I_L and V_L, and the plot in Fig. 5.12*b* is a straight line. Each point on the line corresponds to a different value of R_L. For example, the midpoint of the straight line is obtained when the load resistance is equal to the internal resistance of the practical source, or $R_L = 0.01\ \Omega$. Here, the load voltage is exactly one-half the ideal source voltage.

When $R_L = \infty$ and no current whatsoever is being drawn by the load, the practical source is open-circuited and the terminal voltage, or open-circuit voltage, is $V_{Loc} = 12$ V. If, on the other hand, $R_L = 0$, thereby short-circuiting the load terminals, then a load current or short-circuit current, $I_{Lsc} = 1200$ A, would flow. (*In practice, such an experiment would probably result in the destruction of the short circuit, the battery, and any measuring instruments incorporated in the circuit!*)

Since the plot of V_L versus I_L is a straight line for this practical voltage source, we should note that the values of V_{Loc} and I_{Lsc} uniquely determine the entire V_L–I_L curve.

The horizontal broken line of Fig. 5.12*b* represents the V_L–I_L plot for an *ideal* voltage source; the terminal voltage remains constant for any value of load current. For the practical voltage source, the terminal voltage has a value near that of the ideal source only when the load current is relatively small.

Let us now consider a *general* practical voltage source, as shown in Fig. 5.13*a*. The voltage of the ideal source is v_s, and a resistance R_s, called an *internal resistance* or *output resistance,* is placed in series with it. Again, we must note that the resistor is not really present as a separate component but merely serves to account for a terminal voltage that decreases as the load current increases. Its presence enables us to model the behavior of a physical voltage source more closely.

The linear relationship between v_L and i_L is

$$v_L = v_s - R_s i_L \qquad [9]$$

(2) From this point on we will endeavor to adhere to the standard convention of referring to strictly dc quantities using capital letters, whereas lowercase letters denote a quantity that we know to possess some time-varying component. However, in describing general theorems which apply to either dc or ac, we will continue to use lowercase to emphasize the general nature of the concept.

and this is plotted in Fig. 5.13*b*. The open-circuit voltage ($R_L = \infty$, so $i_L = 0$) is

$$v_{L\text{oc}} = v_s \qquad [10]$$

and the short-circuit current ($R_L = 0$, so $v_L = 0$) is

$$i_{L\text{sc}} = \frac{v_s}{R_s} \qquad [11]$$

Once again, these values are the intercepts for the straight line in Fig. 5.13*b*, and they serve to define it completely.

Practical Current Sources

An ideal current source is also nonexistent in the real world; there is no physical device that will deliver a constant current regardless of the load resistance to which it is connected or the voltage across its terminals. Certain transistor circuits will deliver a constant current to a wide range of load resistances, but the load resistance can always be made sufficiently large that the current through it becomes very small. Infinite power is simply never available (unfortunately).

A practical current source is defined as an ideal current source in parallel with an internal resistance R_p. Such a source is shown in Fig. 5.14*a*, and the current i_L and voltage v_L associated with a load resistance R_L are indicated. Application of KCL yields

$$i_L = i_s - \frac{v_L}{R_p} \qquad [12]$$

which is again a linear relationship. The open-circuit voltage and the short-circuit current are

$$v_{L\text{oc}} = R_p i_s \qquad [13]$$

and

$$i_{L\text{sc}} = i_s \qquad [14]$$

The variation of load current with changing load voltage may be investigated by changing the value of R_L as shown in Fig. 5.14*b*. The straight line is traversed from the short-circuit, or "northwest," end to the open-circuit termination at the "southeast" end by increasing R_L from zero to infinite ohms. The midpoint occurs for $R_L = R_p$. The load current i_L and the ideal source current are approximately equal only for small values of load voltage, which are obtained with values of R_L that are small compared to R_p.

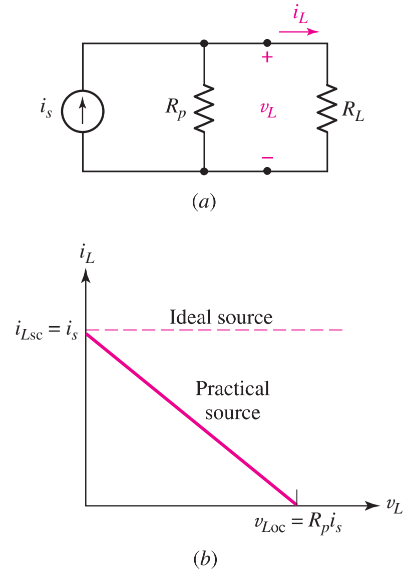
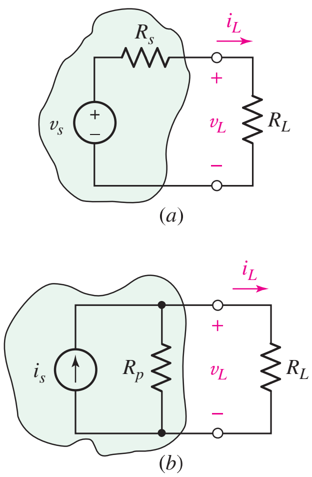


FIGURE 5.14 (*a*) A general practical current source connected to a load resistor R_L. (*b*) The load current provided by the practical current source is shown as a function of the load voltage.

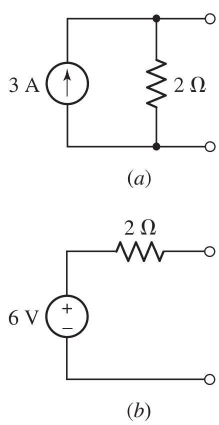
Equivalent Practical Sources

It may be no surprise that we can improve upon models to increase their accuracy; at this point we now have a practical voltage source model and also a practical current source model. Before we proceed, however, let's take a moment to compare Fig. 5.13*b* and Fig. 5.14*b*. One is for a circuit with a voltage source and the other, with a current source, *but the graphs are indistinguishable!*

It turns out that this is no coincidence. In fact, we are about to show that a practical voltage source *can be* electrically equivalent to a practical current source—meaning that a load resistor R_L connected to either will have



■ **FIGURE 5.15** (*a*) A given practical voltage source connected to a load R_L. (*b*) The equivalent practical current source connected to the same load.



■ **FIGURE 5.16** (*a*) A given practical current source. (*b*) The equivalent practical voltage source.

the same v_L and i_L. This means we can replace one practical source with the other and the rest of the circuit will not know the difference.

Consider the practical voltage source and resistor R_L shown in Fig. 5.15*a*, and the circuit composed of a practical current source and resistor R_L shown in Fig. 5.15*b*. A simple calculation shows that the voltage across the load R_L of Fig. 5.15*a* is

$$v_L = v_s \frac{R_L}{R_s + R_L} \qquad [15]$$

A similar calculation shows that the voltage across the load R_L in Fig. 5.15*b* is

$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) \cdot R_L$$

The two practical sources are electrically equivalent, then, if

$$R_s = R_p \qquad [16]$$

and

$$v_s = R_p i_s = R_s i_s \qquad [17]$$

where we now let R_s represent the internal resistance of either practical source, which is the conventional notation.

Let's try this with the practical current source shown in Fig. 5.16*a*. Since its internal resistance is 2 Ω, the internal resistance of the equivalent practical voltage source is also 2 Ω; the voltage of the ideal voltage source contained within the practical voltage source is (2)(3) = 6 V. The equivalent practical voltage source is shown in Fig. 5.16*b*.

To check the equivalence, let us visualize a 4 Ω resistor connected to each source. In both cases a current of 1 A, a voltage of 4 V, and a power of 4 W are associated with the 4 Ω load. However, we should note very carefully that the ideal current source is delivering a total power of 12 W, while the ideal voltage source is delivering only 6 W. Furthermore, the internal resistance of the practical current source is absorbing 8 W, whereas the internal resistance of the practical voltage source is absorbing only 2 W. Thus we see that the two practical sources are equivalent only with respect to what transpires at the load terminals; they are *not* equivalent internally!

EXAMPLE 5.4

Compute the current through the 4.7 kΩ resistor in Fig. 5.17*a* after transforming the 9 mA source into an equivalent voltage source.

It's not just the 9 mA source at issue, but also the resistance in parallel with it (5 kΩ). We remove these components, leaving two terminals "dangling." We then replace them with a voltage source in series with a 5 kΩ resistor. The value of the voltage source must be (0.09)(5000) = 45 V.

Redrawing the circuit as in Fig. 5.17*b*, we can write a simple KVL equation

$$-45 + 5000I + 4700I + 3000I + 3 = 0$$

which is easily solved to yield $I = 3.307$ mA.

We can check our answer of course by analyzing the circuit of Fig. 5.17*a* using either nodal or mesh techniques.

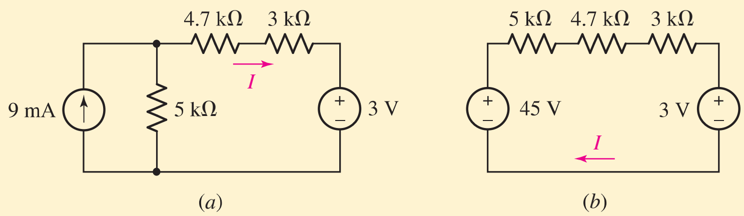


FIGURE 5.17 (*a*) A circuit with both a voltage source and a current source. (*b*) The circuit after the 9 mA source is transformed into an equivalent voltage source.

PRACTICE

5.3 For the circuit of Fig. 5.18, compute the current I_X through the 47 kΩ resistor after performing a source transformation on the voltage source.

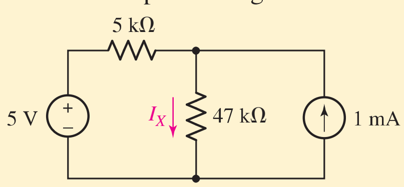


FIGURE 5.18

Ans: 192 μA.

EXAMPLE 5.5

Calculate the current through the 2 Ω resistor in Fig. 5.19*a* by making use of source transformations to first simplify the circuit.

We begin by transforming each current source into a voltage source (Fig. 5.19*b*), the strategy being to convert the circuit into a simple loop.

We must be careful to retain the 2 Ω resistor for two reasons: first, the dependent source controlling variable appears across it, and second, we desire the current flowing through it. However, we can combine the 17 Ω and 9 Ω resistors, since they appear in series. We also see that the 3 Ω and 4 Ω resistors may be combined into a single 7 Ω resistor, which can then be used to transform the 15 V source into a 15/7 A source as in Fig. 5.19*c*.

Finally, we note that the two 7 Ω resistors can be combined into a single 3.5 Ω resistor, which may be used to transform the 15/7 A current source into a 7.5 V voltage source. The result is a simple loop circuit, shown in Fig. 5.19*d*.

The current I can now be found using KVL:

$$-7.5 + 3.5I - 51V_x + 28I + 9 = 0$$

where

$$V_x = 2I$$

Thus,

$$I = 21.28 \text{ mA}$$

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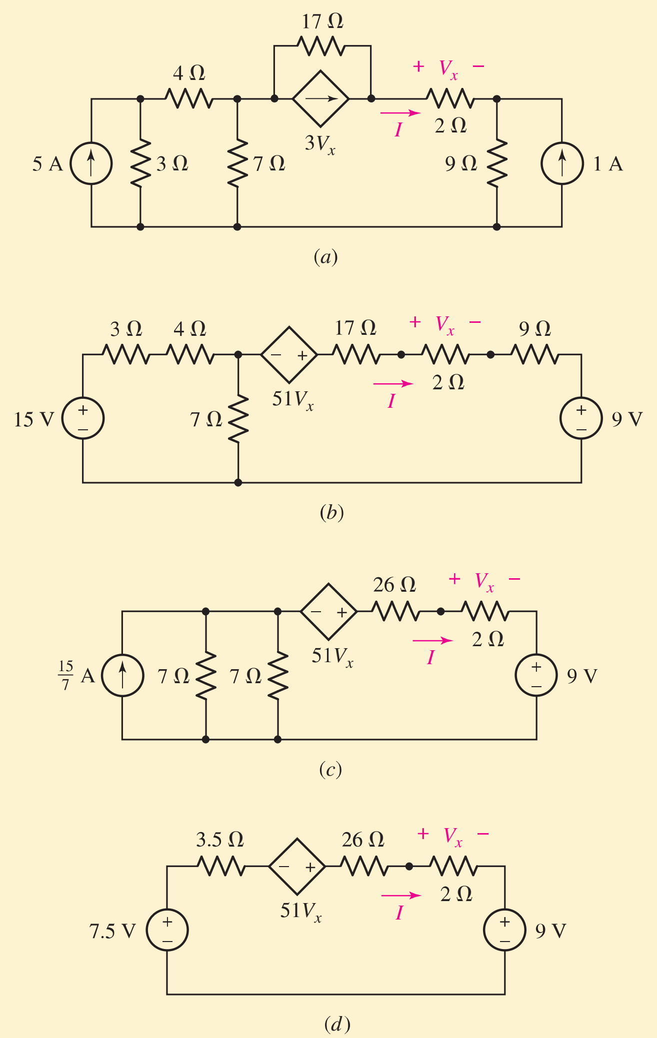
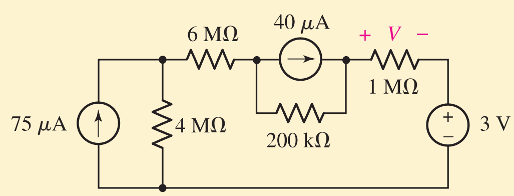


■ **FIGURE 5.19** (*a*) A circuit with two independent current sources and one dependent source. (*b*) The circuit after each source is transformed into a voltage source. (*c*) The circuit after further combinations. (*d*) The final circuit.

PRACTICE

5.4 For the circuit of Fig. 5.20, compute the voltage V across the 1 MΩ resistor using repeated source transformations.



■ **FIGURE 5.20**

Ans: 27.2 V.

Several Key Points

We conclude our discussion of practical sources and source transformations with a few observations. First, when we transform a voltage source, we must be sure that the source is in fact *in series* with the resistor under consideration. For example, in the circuit of Fig. 5.21, it is perfectly valid to perform a source transformation on the voltage source using the 10 Ω resistor, as they are in series. However, it would be incorrect to attempt a source transformation using the 60 V source and the 30 Ω resistor—a very common type of error.

In a similar fashion, when we transform a current source and resistor combination, we must be sure that they are in fact *in parallel*. Consider the current source shown in Fig. 5.22*a*. We may perform a source transformation including the 3 Ω resistor, as they are in parallel, but after the transformation there may be some ambiguity as to where to place the resistor. In such circumstances, it is helpful to first redraw the components to be transformed as in Fig. 5.22*b*. Then the transformation to a voltage source in series with a resistor may be drawn correctly as shown in Fig. 5.22*c*; the resistor may in fact be drawn above or below the voltage source.

It is also worthwhile to consider the unusual case of a current source in series with a resistor, and its dual, the case of a voltage source in parallel

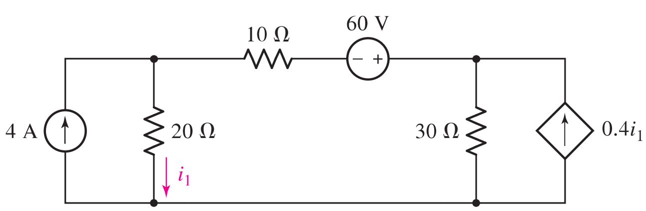


FIGURE 5.21 An example circuit to illustrate how to determine if a source transformation can be performed.

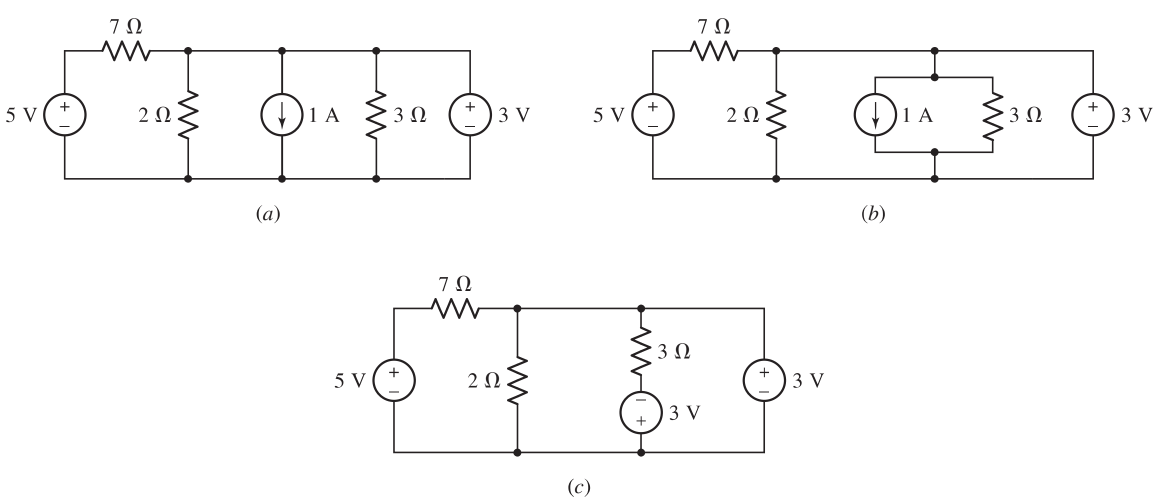


FIGURE 5.22 (*a*) A circuit with a current source to be transformed to a voltage source. (*b*) Circuit redrawn so as to avoid errors. (*c*) Transformed source/resistor combination.

with a resistor. Let's start with the simple circuit of Fig. 5.23*a*, where we are interested only in the voltage across the resistor marked R_2. We note that regardless of the value of resistor R_1, $V_{R_2} = I_x R_2$. Although we might be tempted to perform an inappropriate source transformation on such a circuit, in fact *we may simply omit resistor* R_1 (provided that it is of no interest to us itself). A similar situation arises with a voltage source in parallel with a resistor, as depicted in Fig. 5.23*b*. Again, if we are only interested in some quantity regarding resistor R_2, we may find ourselves tempted to perform some strange (and incorrect) source transformation on the voltage source and resistor R_1. In reality, we may omit resistor R_1 from our circuit as far as resistor R_2 is concerned—its presence does not alter the voltage across, the current through, or the power dissipated by resistor R_2.

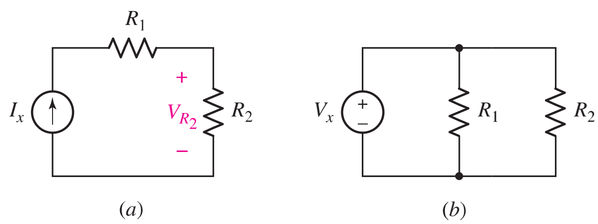


FIGURE 5.23 (*a*) Circuit with a resistor R_1 in series with a current source. (*b*) A voltage source in parallel with two resistors.

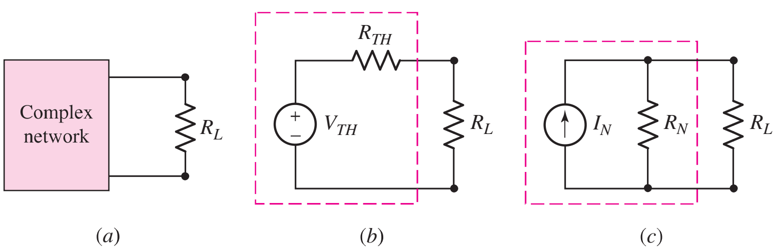
Summary of Source Transformation

1. **A common goal in source transformation is to end up with either all current sources or all voltage sources in the circuit.** This is especially true if it makes nodal or mesh analysis easier.
2. **Repeated source transformations can be used to simplify a circuit by allowing resistors and sources to eventually be combined.**
3. **The resistor value does not change during a source transformation, but it is not the same resistor.** This means that currents or voltages associated with the original resistor are irretrievably lost when we perform a source transformation.
4. **If the voltage or current associated with a particular resistor is used as a controlling variable for a dependent source, it should not be included in any source transformation.** The original resistor must be retained in the final circuit, untouched.
5. **If the voltage or current associated with a particular element is of interest, that element should not be included in any source transformation.** The original element must be retained in the final circuit, untouched.
6. **In a source transformation, the head of the current source arrow corresponds to the "+" terminal of the voltage source.**
7. **A source transformation on a current source and resistor requires that the two elements be in parallel.**
8. **A source transformation on a voltage source and resistor requires that the two elements be in series.**

5.3 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Now that we have been introduced to source transformations and the superposition principle, it is possible to develop two more techniques that will greatly simplify the analysis of many linear circuits. The first of these theorems is named after L. C. Thévenin, a French engineer working in telegraphy who published the theorem in 1883; the second may be considered a corollary of the first and is credited to E. L. Norton, a scientist with the Bell Telephone Laboratories.

Let us suppose that we need to make only a partial analysis of a circuit. For example, perhaps we need to determine the current, voltage, and power delivered to a single "load" resistor by the remainder of the circuit, which may consist of a sizable number of sources and resistors (Fig. 5.24*a*). Or, perhaps we wish to find the response for different values of the load resistance. Thévenin's theorem tells us that it is possible to replace everything except the load resistor with an independent voltage source in series with a resistor (Fig. 5.24*b*); the response measured *at the load resistor* will be unchanged. Using Norton's theorem, we obtain an equivalent composed of an independent current source in parallel with a resistor (Fig. 5.24*c*).



■ **FIGURE 5.24** (*a*) A complex network including a load resistor R_L. (*b*) A Thévenin equivalent network connected to the load resistor R_L. (*c*) A Norton equivalent network connected to the load resistor R_L.

Thus, one of the main uses of Thévenin's and Norton's theorems is the replacement of a large part of a circuit, often a complicated and uninteresting part, with a very simple equivalent. The new, simpler circuit enables us to make rapid calculations of the voltage, current, and power which the original circuit is able to deliver to a load. It also helps us to choose the best value of this load resistance. In a transistor power amplifier, for example, the Thévenin or Norton equivalent enables us to determine the maximum power that can be taken from the amplifier and delivered to the speakers.

EXAMPLE 5.6

Consider the circuit shown in Fig. 5.25*a*. Determine the Thévenin equivalent of network *A*, and compute the power delivered to the load resistor R_L.

The dashed regions separate the circuit into networks *A* and *B*; our main interest is in network *B*, which consists only of the load resistor R_L. Network *A* may be simplified by making repeated source transformations.

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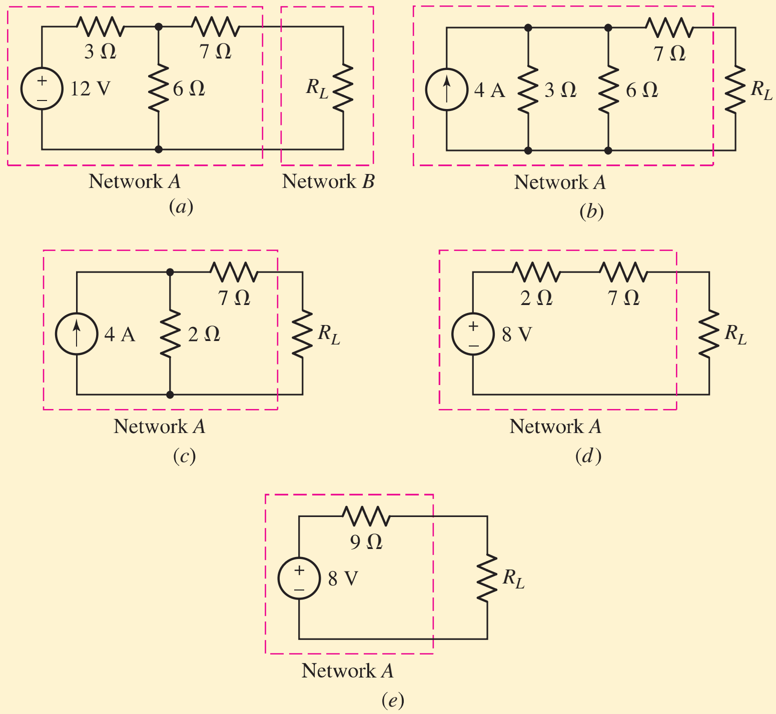


FIGURE 5.25 (*a*) A circuit separated into two networks. (*b*)–(*d*) Intermediate steps to simplifying network *A*. (*e*) The Thévenin equivalent circuit.

We first treat the 12 V source and the 3 Ω resistor as a practical voltage source and replace it with a practical current source consisting of a 4 A source in parallel with 3 Ω (Fig. 5.25*b*). The parallel resistances are then combined into 2 Ω (Fig. 5.25*c*), and the practical current source that results is transformed back into a practical voltage source (Fig. 5.25*d*). The final result is shown in Fig. 5.25*e*.

From the viewpoint of the load resistor R_L, this network *A* (the Thévenin equivalent) is equivalent to the original network *A*; from our viewpoint, the circuit is much simpler, and we can now easily compute the power delivered to the load:

$$P_L = \left(\frac{8}{9 + R_L}\right)^2 R_L$$

Furthermore, we can see from the equivalent circuit that the maximum voltage that can be obtained across R_L is 8 V and corresponds to $R_L = \infty$. A quick transformation of network *A* to a practical current source (the Norton equivalent) indicates that the maximum current that may be delivered to the load is 8/9 A, which occurs when $R_L = 0$. Neither of these facts is readily apparent from the original circuit.

PRACTICE

5.5 Using repeated source transformations, determine the Norton equivalent of the highlighted network in the circuit of Fig. 5.26.

Ans: 1 A, 5 Ω.

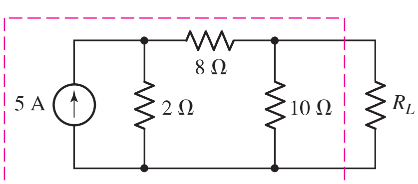


FIGURE 5.26

Thévenin's Theorem

Using the technique of source transformation to find a Thévenin or Norton equivalent network worked well enough in Example 5.6, but it can rapidly become impractical in situations where dependent sources are present or the circuit is composed of a large number of elements. An alternative is to employ Thévenin's theorem (or Norton's theorem) instead. We will state the theorem[3] as a somewhat formal procedure and then consider various ways to make the approach more practical depending on the situation we face.

A Statement of Thévenin's Theorem

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** Network *A* is the network to be simplified; *B* will be left untouched.
2. **Disconnect network *B*.** Define a voltage v_{oc} as the voltage now appearing across the terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged.
4. **Connect an independent voltage source with value v_{oc} in series with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

Note that if either network contains a dependent source, *its control variable must be in the same network*.

Let us see if we can apply Thévenin's theorem successfully to the circuit we considered in Fig. 5.25. We have already found the Thévenin equivalent of the circuit to the left of R_L in Example 5.6, but we want to see if there is an easier way to obtain the same result.

EXAMPLE 5.7

Use Thévenin's theorem to determine the Thévenin equivalent for that part of the circuit in Fig. 5.25*a* to the left of R_L.

We begin by disconnecting R_L, and note that no current flows through the 7 Ω resistor in the resulting partial circuit shown in Fig. 5.27*a*. Thus, V_{oc} appears across the 6 Ω resistor (with no current through the 7 Ω resistor there is no voltage drop across it), and voltage division enables us to determine that

$$V_{oc} = 12\left(\frac{6}{3+6}\right) = 8 \text{ V}$$

(3) A proof of Thévenin's theorem in the form in which we have stated it is rather lengthy, and therefore it has been placed in Appendix 3, where the curious may peruse it.

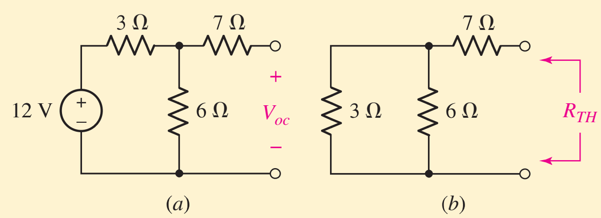


FIGURE 5.27 (*a*) The circuit of Fig. 5.25*a* with network *B* (the resistor R_L) disconnected and the voltage across the connecting terminals labeled as V_{oc}. (*b*) The independent source in Fig. 5.25*a* has been killed, and we look into the terminals where network *B* was connected to determine the effective resistance of network *A*.

Turning off network *A* (i.e., replacing the 12 V source with a short circuit) and looking back into the dead network, we see a 7 Ω resistor connected in series with the parallel combination of 6 Ω and 3 Ω (Fig. 5.27*b*).

Thus, the inactive network can be represented here by a 9 Ω resistor, referred to as the ***Thévenin equivalent resistance*** of network *A*. The Thévenin equivalent then is V_{oc} in series with a 9 Ω resistor, which agrees with our previous result.

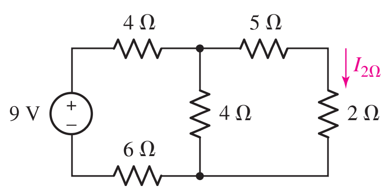


FIGURE 5.28

PRACTICE

5.6 Use Thévenin's theorem to find the current through the 2 Ω resistor in the circuit of Fig. 5.28. (*Hint:* Designate the 2 Ω resistor as network *B*.)

Ans: $V_{TH} = 2.571$ V, $R_{TH} = 7.857$ Ω, $I_{2\,\Omega} = 260.8$ mA.

A Few Key Points

The equivalent circuit we have learned how to obtain is completely independent of network *B*: we have been instructed to first remove network *B* and then measure the open-circuit voltage produced by network *A*, an operation that certainly does not depend on network *B* in any way. The *B* network is mentioned only to indicate that an equivalent for *A* may be obtained no matter what arrangement of elements is connected to the *A* network; the *B* network represents this general network.

There are several points about the theorem which deserve emphasis.

- The only restriction that we must impose on *A* or *B* is that all *dependent* sources in *A* have their control variables in *A*, and similarly for *B*.
- No restrictions are imposed on the complexity of *A* or *B*; either one may contain any combination of independent voltage or current sources, linear dependent voltage or current sources, resistors, or any other circuit elements which are linear.
- The dead network *A* can be represented by a single equivalent resistance R_{TH}, which we will call the Thévenin equivalent resistance.

This holds true whether or not dependent sources exist in the inactive *A* network, an idea we will explore shortly.

- A Thévenin equivalent consists of two components: a voltage source in series with a resistance. Either may be zero, although this is not usually the case.

Norton's Theorem

Norton's theorem bears a close resemblance to Thévenin's theorem and may be stated as follows:

A Statement of Norton's Theorem

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** Network *A* is the network to be simplified; *B* will be left untouched. As before, if either network contains a dependent source, *its controlling variable must be in the same network.*
2. **Disconnect network *B*, and short the terminals of *A*.** Define a current i_{sc} as the current now flowing through the shorted terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged.
4. **Connect an independent current source with value i_{sc} in parallel with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

The Norton equivalent of a linear network is the Norton current source i_{sc} in parallel with the Thévenin resistance R_{TH}. Thus, we see that in fact it is possible to obtain the Norton equivalent of a network by performing a source transformation on the Thévenin equivalent. This results in a direct relationship between v_{oc}, i_{sc}, and R_{TH}:

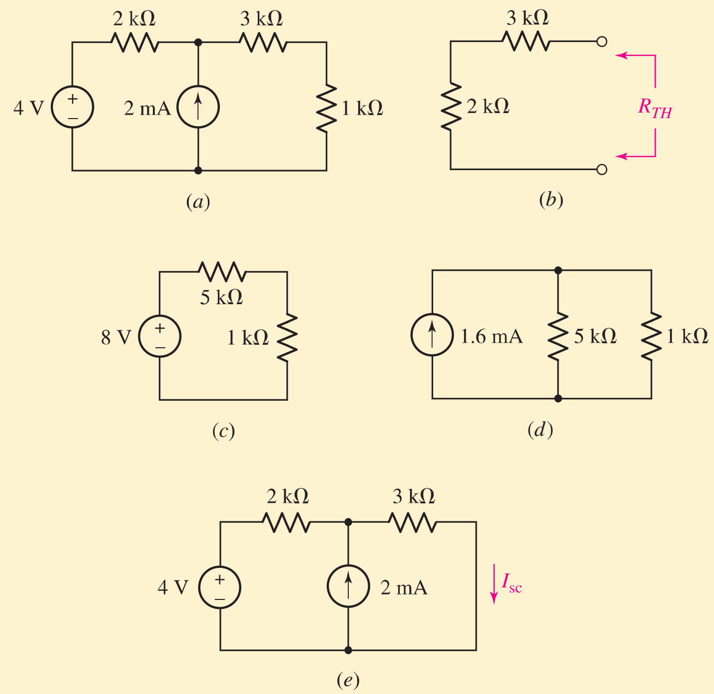
$$v_{oc} = R_{TH} i_{sc} \qquad [18]$$

In circuits containing dependent sources, we will often find it more convenient to determine either the Thévenin or Norton equivalent by finding both the open-circuit voltage and the short-circuit current and then determining the value of R_{TH} as their quotient. It is therefore advisable to become adept at finding both open-circuit voltages and short-circuit currents, even in the simple problems that follow. If the Thévenin and Norton equivalents are determined independently, Eq. [18] can serve as a useful check.

Let's consider three different examples of the determination of a Thévenin or Norton equivalent circuit.

EXAMPLE 5.8

Find the Thévenin and Norton equivalent circuits for the network faced by the 1 kΩ resistor in Fig. 5.29*a*.



■ **FIGURE 5.29** (*a*) A given circuit in which the 1 kΩ resistor is identified as network *B*. (*b*) Network *A* with all independent sources killed. (*c*) The Thévenin equivalent is shown for network *A*. (*d*) The Norton equivalent is shown for network *A*. (*e*) Circuit for determining I_{sc}.

From the wording of the problem statement, network *B* is the 1 kΩ resistor, so network *A* is everything else.

Choosing to find the Thévenin equivalent of network *A* first, we apply superposition, noting that no current flows through the 3 kΩ resistor once network *B* is disconnected. With the current source set to zero, $V_{oc|_{4\,V}} = 4$ V. With the voltage source set to zero,

$$V_{oc|_{2\,mA}} = (0.002)(2000) = 4 \text{ V. Thus, } V_{oc} = 4 + 4 = 8 \text{ V.}$$

To find R_{TH}, set both sources to zero as in Fig. 5.29*b*. By inspection, $R_{TH} = 2\text{ k}\Omega + 3\text{ k}\Omega = 5\text{ k}\Omega$. The complete Thévenin equivalent, with network *B* reconnected, is shown in Fig. 5.29*c*.

The Norton equivalent is found by a simple source transformation of the Thévenin equivalent, resulting in a current source of 8/5000 = 1.6 mA in parallel with a 5 kΩ resistor (Fig. 5.29*d*).

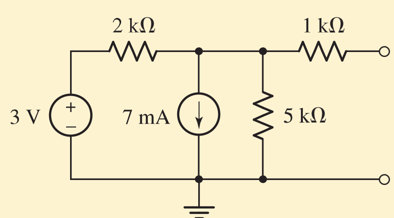
Check: Find the Norton equivalent directly from Fig. 5.29*a*. Removing the 1 kΩ resistor and shorting the terminals of network *A*, we find I_{sc} as shown in Fig. 5.29*e* by superposition and current division:

$$I_{sc} = I_{sc}|_{4\,V} + I_{sc}|_{2\,mA} = \frac{4}{2+3} + (2)\frac{2}{2+3}$$
$$= 0.8 + 0.8 = 1.6 \text{ mA}$$

which completes the check.

PRACTICE

5.7 Determine the Thévenin and Norton equivalents of the circuit of Fig. 5.30.



■ **FIGURE 5.30**

Ans: −7.857 V, −3.235 mA, 2.429 kΩ.

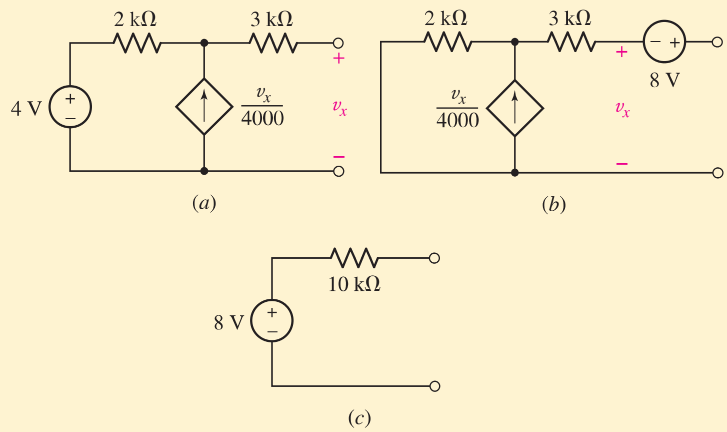
When Dependent Sources Are Present

Technically speaking, there does not always have to be a "network *B*" for us to invoke either Thévenin's theorem or Norton's theorem; we could instead be asked to find the equivalent of a network with two terminals not yet connected to another network. If there *is* a network *B* that we do not want to involve in the simplification procedure, however, we must use a little caution if it contains dependent sources. In such situations, the controlling variable and the associated element(s) must be included in network *B* and excluded from network *A*. Otherwise, there will be no way to analyze the final circuit because the controlling quantity will be lost.

If network *A* contains a dependent source, then again we must ensure that the controlling variable and its associated element(s) cannot be in network *B*. Up to now, we have only considered circuits with resistors and independent sources. Although technically speaking it is correct to leave a dependent source in the "inactive" network when creating a Thévenin or Norton equivalent, in practice this does not result in any kind of simplification. What we really want is an independent voltage source in series with a single resistor, or an independent current source in parallel with a single resistor—in other words, a two-component equivalent. In the following examples, we consider various means of reducing networks with dependent sources and resistors into a single resistance.

EXAMPLE 5.9

Determine the Thévenin equivalent of the circuit in Fig. 5.31*a*.



■ **FIGURE 5.31** (*a*) A given network whose Thévenin equivalent is desired. (*b*) A possible, but rather useless, form of the Thévenin equivalent. (*c*) The best form of the Thévenin equivalent for this linear resistive network.

To find V_{oc} we note that $v_x = V_{oc}$ and that the dependent source current must pass through the 2 kΩ resistor, since no current can flow through the 3 kΩ resistor. Using KVL around the outer loop:

$$-4 + 2 \times 10^3\left(-\frac{v_x}{4000}\right) + 3 \times 10^3(0) + v_x = 0$$

and

$$v_x = 8\text{ V} = V_{oc}$$

By Thévenin's theorem, then, the equivalent circuit could be formed with the inactive *A* network in series with an 8 V source, as shown in Fig. 5.31*b*. This is correct, but not very simple and not very helpful; in the case of linear resistive networks, we really want a simpler equivalent for the inactive *A* network, namely, R_{TH}.

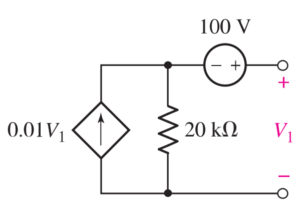
The dependent source prevents us from determining R_{TH} directly for the inactive network through resistance combination; we therefore seek I_{sc}. Upon short-circuiting the output terminals in Fig. 5.31*a*, it is apparent that $V_x = 0$ and the dependent current source is not active. Hence, $I_{sc} = 4/(5 \times 10^3) = 0.8$ mA. Thus,

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{8}{0.8 \times 10^{-3}} = 10\text{ k}\Omega$$

and the acceptable Thévenin equivalent of Fig. 5.31*c* is obtained.

PRACTICE

5.8 Find the Thévenin equivalent for the network of Fig. 5.32. (*Hint:* a quick source transformation on the dependent source might help.)



■ **FIGURE 5.32**

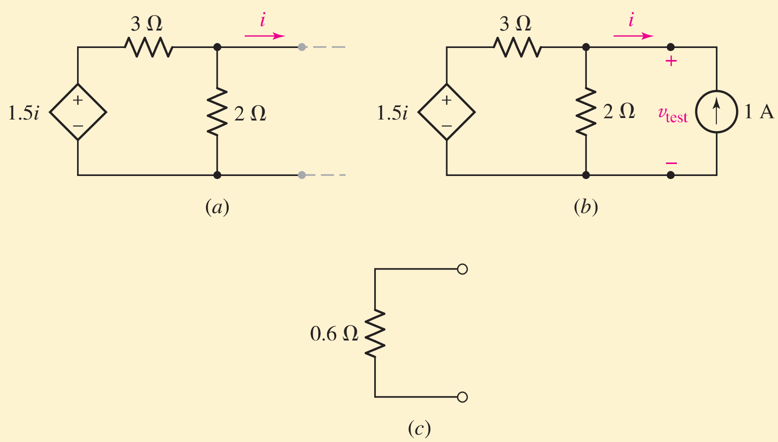
Ans: −502.5 mV, −100.5 Ω.

Note: a negative resistance might seem strange—and it is! Such a thing is physically possible only if, for example, we do a bit of clever electronic circuit design to create something that behaves like the dependent current source we represented in Fig. 5.32.

As another example, let us consider a network having a dependent source but no independent source.

EXAMPLE 5.10

Find the Thévenin equivalent of the circuit shown in Fig. 5.33*a*.



■ **FIGURE 5.33** (*a*) A network with no independent sources. (*b*) A hypothetical measurement to obtain R_{TH}. (*c*) The Thévenin equivalent to the original circuit.

The rightmost terminals are already open-circuited, hence $i = 0$. Consequently, the dependent source is inactive, so $v_{oc} = 0$.

We next seek the value of R_{TH} represented by this two-terminal network. However, we cannot find v_{oc} and i_{sc} and take their quotient, for there is no independent source in the network and both v_{oc} and i_{sc} are zero. Let us, therefore, be a little tricky.

We apply a 1 A source externally, measure the voltage v_{test} that results, and then set $R_{TH} = v_{test}/1$. Referring to Fig. 5.33*b*, we see that $i = -1$ A. Applying nodal analysis,

$$\frac{v_{test} - 1.5(-1)}{3} + \frac{v_{test}}{2} = 1$$

so that

$$v_{test} = 0.6 \text{ V}$$

and thus

$$R_{TH} = 0.6\ \Omega$$

The Thévenin equivalent is shown in Fig. 5.33*c*.

A Quick Recap of Procedures

We have now looked at three examples in which we determined a Thévenin or Norton equivalent circuit. The first example (Fig. 5.29) contained only independent sources and resistors, and several different methods could have been applied to it. One would involve calculating R_{TH} for the inactive network and then V_{oc} for the live network. We could also have found R_{TH} and I_{sc}, or V_{oc} and I_{sc}.

PRACTICAL APPLICATION

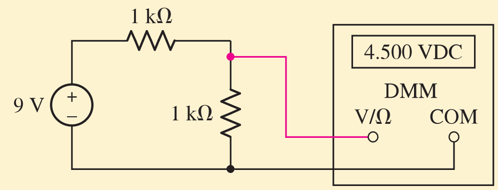
The Digital Multimeter

One of the most common pieces of electrical test equipment is the DMM, or digital multimeter (Fig. 5.34), which is designed to measure voltage, current, and resistance values.

■ **FIGURE 5.34** A handheld digital multimeter.

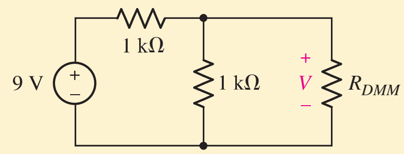
In a voltage measurement, two leads from the DMM are connected across the appropriate circuit element, as depicted in Fig. 5.35. The positive reference terminal of the meter is typically marked "V/Ω," and the negative reference terminal—often referred to as the *common terminal*—is typically designated by "COM." The convention is to use a red lead for the positive reference terminal and a black lead for the common terminal.

From our discussion of Thévenin and Norton equivalents, it may now be apparent that the DMM has its own



■ **FIGURE 5.35** A DMM connected to measure voltage.

Thévenin equivalent resistance. This Thévenin equivalent resistance will appear in parallel with our circuit, and its value can affect the measurement (Fig. 5.36). The DMM does not supply power to the circuit to measure voltage, so its Thévenin equivalent consists of only a resistance, which we will name R_{DMM}.



■ **FIGURE 5.36** DMM in Fig. 5.35 shown as its Thévenin equivalent resistance, R_{DMM}.

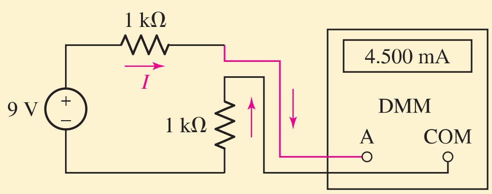
The input resistance of a good DMM is typically 10 MΩ or more. The measured voltage V thus appears across $1\ \text{k}\Omega \| 10\ \text{M}\Omega = 999.9\ \Omega$. Using voltage division, we find that $V = 4.4998$ volts, slightly less than the expected value of 4.5 volts. Thus, the finite input resistance of the voltmeter introduces a small error in the measured value.

In the second example (Fig. 5.31), both independent and dependent sources were present, and the method we used required us to find V_{oc} and I_{sc}. We could not easily find R_{TH} for the inactive network because the dependent source could not be made inactive.

The last example did not contain any independent sources, and therefore the Thévenin and Norton equivalents do not contain an independent source. We found R_{TH} by applying 1 A and finding $v_{\text{test}} = 1 \times R_{TH}$. We could also apply 1 V and determine $i = 1/R_{TH}$. These two related techniques can be applied to any circuit with dependent sources, *as long as all independent sources are set to zero first*.

Two other methods have a certain appeal because they can be used for any of the three types of networks considered. In the first, simply replace network B with a voltage source v_s, define the current leaving its positive terminal as i, analyze network A to obtain i, and put the equation in the form $v_s = ai + b$. Then, $a = R_{TH}$ and $b = v_{\text{oc}}$.

To measure current, the DMM must be placed in series with a circuit element, generally requiring that we cut a wire (Fig. 5.37). One DMM lead is connected to the common terminal of the meter, and the other lead is placed in a connector usually marked "A" to signify current measurement. Again, the DMM does not supply power to the circuit in this type of measurement.



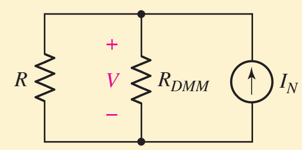
■ **FIGURE 5.37** A DMM connected to measure current.

We see that the Thévenin equivalent resistance (R_{DMM}) of the DMM is in series with our circuit, so its value can affect the measurement. Writing a simple KVL equation around the loop,

$$-9 + 1000I + R_{\text{DMM}}I + 1000I = 0$$

Note that since we have reconfigured the meter to perform a current measurement, the Thévenin equivalent resistance is not the same as when the meter is configured to measure voltages. In fact, we would ideally like R_{DMM} to be 0 Ω for current measurements, and ∞ for voltage measurements. If R_{DMM} is now 0.1 Ω, we see that the measured current I is 4.4998 mA, which is only slightly different from the expected value of 4.5 mA. Depending on the number of digits that can be displayed by the meter, we may not even notice the effect of nonzero DMM resistance on our measurement.

The same meter can be used to determine resistance, provided no independent sources are active during the measurement. Internally, a known current is passed through the resistor being measured, and the voltmeter circuitry is used to measure the resulting voltage. Replacing the DMM with its Norton equivalent (which now includes an active independent current source to generate the predetermined current), we see that R_{DMM} appears in parallel with our unknown resistor R (Fig. 5.38).



■ **FIGURE 5.38** DMM in resistance measurement configuration replaced by its Norton equivalent, showing R_{DMM} in parallel with the unknown resistor R to be measured.

As a result, the DMM actually measures $R \| R_{\text{DMM}}$. If $R_{\text{DMM}} = 10$ MΩ and $R = 10$ Ω, $R_{\text{measured}} = 9.99999$ Ω, which is more than accurate enough for most purposes. However, if $R = 10$ MΩ, $R_{\text{measured}} = 5$ MΩ. The input resistance of a DMM therefore places a practical upper limit on the values of resistance that can be measured, and special techniques must be used to measure larger resistances. We should note that if a digital multimeter is *programmed* with knowledge of R_{DMM}, it is possible to compensate and allow measurement of larger resistances.

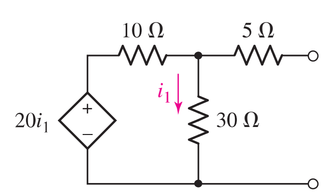
We could also apply a current source i_s, let its voltage be v, and then determine $i_s = cv - d$, where $c = 1/R_{TH}$ and $d = i_{\text{sc}}$ (the minus sign arises from assuming both current source arrows are directed into the same node). Both of these last two procedures are universally applicable, but some other method can usually be found that is easier and more rapid.

Although we are devoting our attention almost entirely to the analysis of linear circuits, it is good to know that Thévenin's and Norton's theorems are both valid if network B is nonlinear; only network A must be linear.

PRACTICE

5.9 Find the Thévenin equivalent for the network of Fig. 5.39. (*Hint:* Try a 1 V test source.)

Ans: $I_{\text{test}} = 50$ mA so $R_{TH} = 20$ Ω.



■ **FIGURE 5.39** See Practice Problem 5.9.

5.4 MAXIMUM POWER TRANSFER

A very useful power theorem may be developed with reference to a practical voltage or current source. For the practical voltage source (Fig. 5.40), the power delivered to the load R_L is

$$p_L = i_L^2 R_L = \frac{v_s^2 R_L}{(R_s + R_L)^2} \qquad [19]$$

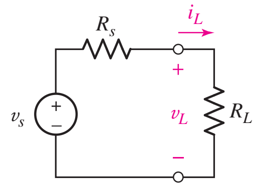


FIGURE 5.40 A practical voltage source connected to a load resistor R_L.

To find the value of R_L that absorbs maximum power from the given practical source, we differentiate with respect to R_L:

$$\frac{dp_L}{dR_L} = \frac{(R_s + R_L)^2 v_s^2 - v_s^2 R_L (2)(R_s + R_L)}{(R_s + R_L)^4}$$

and equate the derivative to zero, obtaining

$$2R_L(R_s + R_L) = (R_s + R_L)^2$$

or

$$R_s = R_L$$

Since the values $R_L = 0$ and $R_L = \infty$ both give a minimum ($p_L = 0$), and since we have already developed the equivalence between practical voltage and current sources, we have therefore proved the following ***maximum power transfer theorem:***

> An independent voltage source in series with a resistance R_s, or an independent current source in parallel with a resistance R_s, delivers maximum power to a load resistance R_L such that $R_L = R_s$.

An alternative way to view the maximum power theorem is possible in terms of the Thévenin equivalent resistance of a network:

> A network delivers maximum power to a load resistance R_L when R_L is equal to a the Thévenin equivalent resistance of the network.

Thus, the maximum power transfer theorem tells us that a 2 Ω resistor draws the greatest power (4.5 W) from either practical source of Fig. 5.16, whereas a resistance of 0.01 Ω receives the maximum power (3.6 kW) in Fig. 5.11.

There is a distinct difference between *drawing* maximum power from a *source* and *delivering* maximum power to a *load*. If the load is sized such that its Thévenin resistance is equal to the Thévenin resistance of the network to which it is connected, it will receive maximum power from that network. *Any change to the load resistance will reduce the power delivered to the load*. However, consider just the Thévenin equivalent of the network itself. We draw the maximum possible power from the voltage source by drawing the maximum possible current—which is achieved by shorting the network terminals! However, in this extreme example *we deliver zero power* to the "load"—a short circuit in this case—as $p = i^2 R$, and we just set $R = 0$ by shorting the network terminals.

A minor amount of algebra applied to Eq. [19] coupled with the maximum power transfer requirement that $R_L = R_s = R_{TH}$ will provide

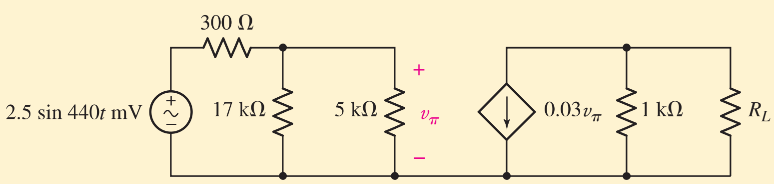
$$p_{\max}\big|_{\text{delivered to load}} = \frac{v_s^2}{4R_s} = \frac{v_{TH}^2}{4R_{TH}}$$

where v_{TH} and R_{TH} recognize that the practical voltage source of Fig. 5.40 can also be viewed as a Thévenin equivalent of some specific source.

It is also not uncommon for the maximum power theorem to be misinterpreted. It is designed to help us select an optimum load in order to maximize power absorption. If the load resistance is already specified, however, the maximum power theorem is of no assistance. If for some reason we can affect the size of the Thévenin equivalent resistance of the network connected to our load, setting it equal to the load does not guarantee maximum power transfer to our predetermined load. A quick consideration of the power lost in the Thévenin resistance will clarify this point.

EXAMPLE 5.11

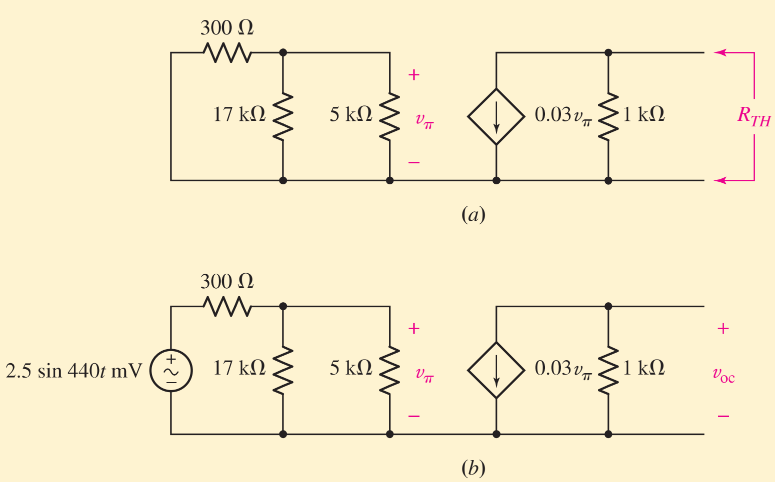
The circuit shown in Fig. 5.41 is a model for the common-emitter bipolar junction transistor amplifier. Choose a load resistance so that maximum power is transferred to it from the amplifier, and calculate the actual power absorbed.



■ **FIGURE 5.41** A small-signal model of the common-emitter amplifier, with the load resistance unspecified.

Since it is the load resistance we are asked to determine, the maximum power theorem applies. The first step is to find the Thévenin equivalent of the rest of the circuit.

We first determine the Thévenin equivalent resistance, which requires that we remove R_L and short-circuit the independent source as in Fig. 5.42*a*.



■ **FIGURE 5.42** (*a*) Circuit with R_L removed and independent source short-circuited. (*b*) Circuit for determining v_{TH}.

(Continued on next page)

Since $v_\pi = 0$, the dependent current source is an open circuit, and $R_{TH} = 1$ kΩ. This can be verified by connecting an independent 1 A current source across the 1 kΩ resistor; v_π will still be zero, so the dependent source remains inactive and hence contributes nothing to R_{TH}.

In order to obtain maximum power delivered into the load, R_L should be set to $R_{TH} = 1$ kΩ.

To find v_{TH} we consider the circuit shown in Fig. 5.42*b*, which is Fig. 5.41 with R_L removed. We may write

$$v_{oc} = -0.03v_\pi(1000) = -30v_\pi$$

where the voltage v_π may be found from simple voltage division:

$$v_\pi = (2.5 \times 10^{-3} \sin 440t)\left(\frac{3864}{300 + 3864}\right)$$

so that our Thévenin equivalent is a voltage −69.6 sin 440*t* mV in series with 1 kΩ.

The maximum power is given by

$$p_{max} = \frac{v_{TH}^2}{4R_{TH}} = \boxed{1.211 \sin^2 440t\ \mu\text{W}}$$

PRACTICE

5.10 Consider the circuit of Fig. 5.43.

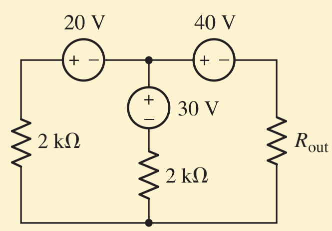


FIGURE 5.43

(*a*) If $R_{out} = 3$ kΩ, find the power delivered to it.

(*b*) What is the maximum power that can be delivered to any R_{out}?

(*c*) What two different values of R_{out} will have exactly 20 mW delivered to them?

Ans: 230 mW; 306 mW; 59.2 kΩ and 16.88 Ω.

5.5 DELTA-WYE CONVERSION

We saw previously that identifying parallel and series combinations of resistors can often lead to a significant reduction in the complexity of a circuit. In situations where such combinations do not exist, we can often make use of source transformations to enable such simplifications. There is another useful technique, called **Δ-*Y* (*delta-wye*)** conversion, that arises out of network theory.

Consider the circuits in Fig. 5.44. There are no series or parallel combinations that can be made to further simplify any of the circuits (note that 5.44*a* and 5.44*b* are identical, as are 5.44*c* and 5.44*d*), and without any

Example 4.2

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

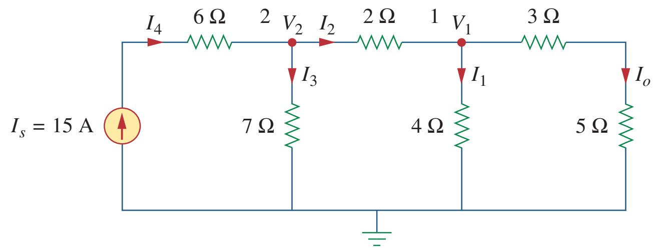


Figure 4.4
For Example 4.2.

Solution:
If $I_o = 1$ A, then $V_1 = (3 + 5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

Practice Problem 4.2

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

Answer: 16 V.

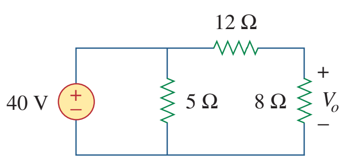


Figure 4.5
For Practice Prob. 4.2.

4.3 Superposition

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis as in Chapter 3. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition*.

The idea of superposition rests on the linearity property.

Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.

The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.

Other terms such as *killed*, *made inactive*, *deadened*, or *set equal to zero* are often used to convey the same idea.

With these in mind, we apply the superposition principle in three steps:

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: It may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Example 4.3

Use the superposition theorem to find v in the circuit of Fig. 4.6.

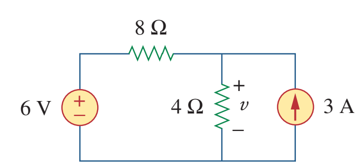


Figure 4.6 For Example 4.3.

Solution:
Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

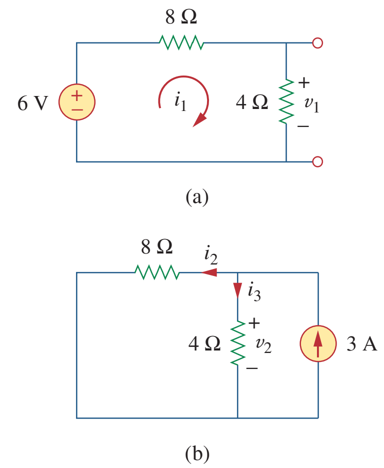


Figure 4.7
For Example 4.3: (a) calculating v_1, (b) calculating v_2.

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Practice Problem 4.3

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.

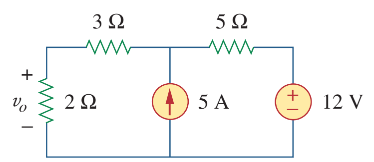


Figure 4.8
For Practice Prob. 4.3.

Answer: 7.4 V.

Example 4.4

Find i_o in the circuit of Fig. 4.9 using superposition.

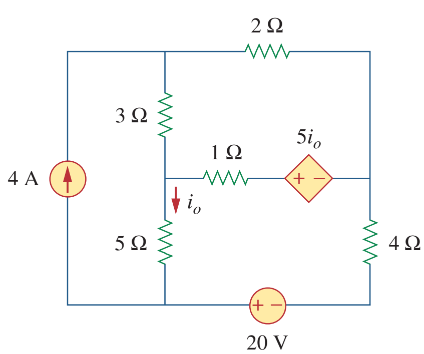


Figure 4.9
For Example 4.4.

Solution:
The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i'_o + i''_o \tag{4.4.1}$$

where i'_o and i''_o are due to the 4-A current source and 20-V voltage source respectively. To obtain i'_o, we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i'_o. For loop 1,

$$i_1 = 4 \text{ A} \tag{4.4.2}$$

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$

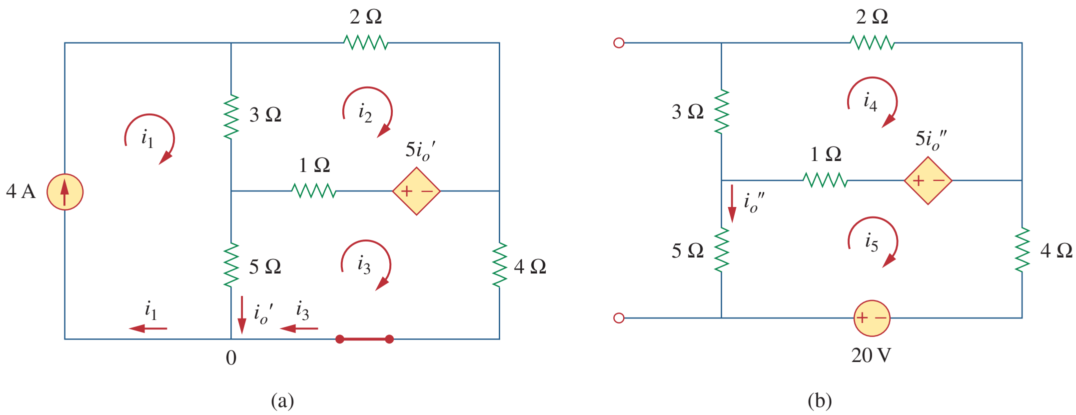


Figure 4.10
For Example 4.4: Applying superposition to (a) obtain i'_o, (b) obtain i''_o.

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \tag{4.4.4}$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \tag{4.4.5}$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \tag{4.4.6}$$

$$i_2 + 5i'_o = 20 \tag{4.4.7}$$

which can be solved to get

$$i'_o = \frac{52}{17} \text{ A} \tag{4.4.8}$$

To obtain i''_o, we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_o = 0 \tag{4.4.9}$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0 \tag{4.4.10}$$

But $i_5 = -i''_o$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i''_o = 0 \tag{4.4.11}$$

$$i_4 + 5i''_o = -20 \tag{4.4.12}$$

which we solve to get

$$i''_o = -\frac{60}{17} \text{ A} \tag{4.4.13}$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706 \text{ A}$$

Practice Problem 4.4

Use superposition to find v_x in the circuit of Fig. 4.11.

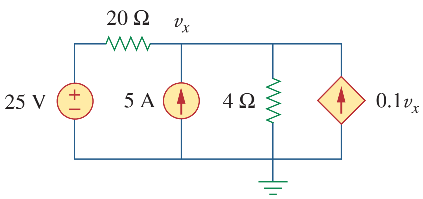


Figure 4.11
For Practice Prob. 4.4.

Answer: $v_x = 31.25$ V.

Example 4.5

For the circuit in Fig. 4.12, use the superposition theorem to find i.

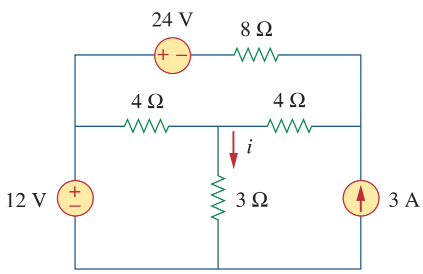


Figure 4.12
For Example 4.5.

Solution:
In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3$ Ω. Thus,

$$i_1 = \frac{12}{6} = 2 \text{ A}$$

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \tag{4.5.1}$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \tag{4.5.2}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \tag{4.5.3}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \tag{4.5.4}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

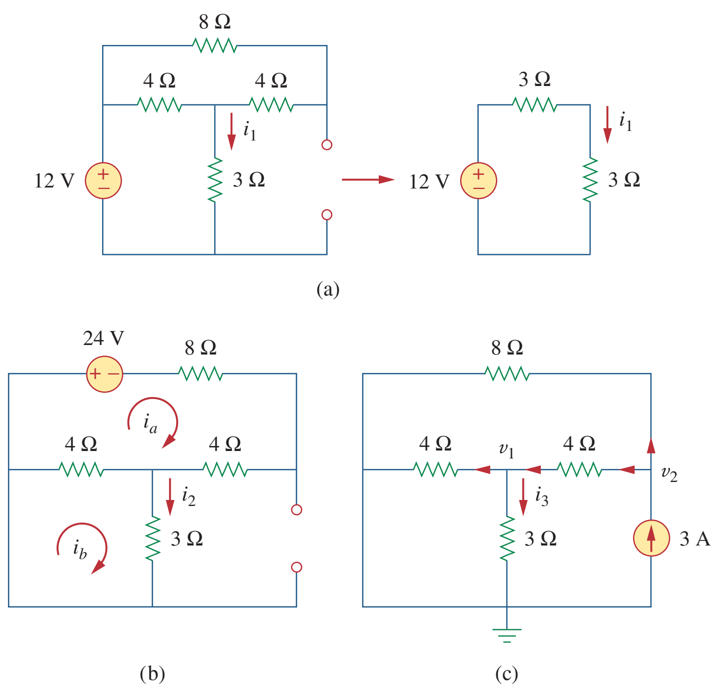


Figure 4.13
For Example 4.5.

Practice Problem 4.5

Find I in the circuit of Fig. 4.14 using the superposition principle.

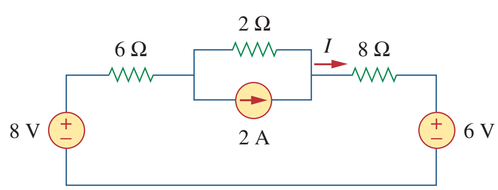


Figure 4.14
For Practice Prob. 4.5.

Answer: 375 mA.

4.4 Source Transformation

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. *Source transformation* is another tool for simplifying circuits. Basic to these tools is the concept of *equivalence*. We recall that an equivalent circuit is one whose v-i characteristics are identical with the original circuit.

In Section 3.6, we saw that node-voltage (or mesh-current) equations can be obtained by mere inspection of a circuit when the sources are all independent current (or all independent voltage) sources. It is therefore expedient in circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in Fig. 4.15. Either substitution is known as a *source transformation*.

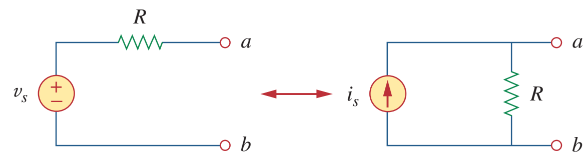


Figure 4.15
Transformation of independent sources.

> A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

The two circuits in Fig. 4.15 are equivalent—provided they have the same voltage-current relation at terminals *a-b*. It is easy to show that they are indeed equivalent. If the sources are turned off, the equivalent resistance at terminals *a-b* in both circuits is R. Also, when terminals *a-b* are short-circuited, the short-circuit current flowing from *a* to *b* is $i_{sc} = v_s/R$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the right-hand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R} \tag{4.5}$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. 4.16, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa where we make sure that Eq. (4.5) is satisfied.

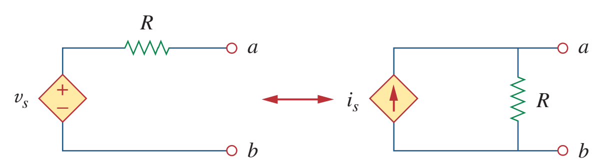


Figure 4.16
Transformation of dependent sources.

Like the wye-delta transformation we studied in Chapter 2, a source transformation does not affect the remaining part of the circuit. When applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis. However, we should keep the following points in mind when dealing with source transformation.

1. Note from Fig. 4.15 (or Fig. 4.16) that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from Eq. (4.5) that source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source. More will be said on ideal and nonideal sources in Section 4.10.1.

Example 4.6

Use source transformation to find v_o in the circuit of Fig. 4.17.

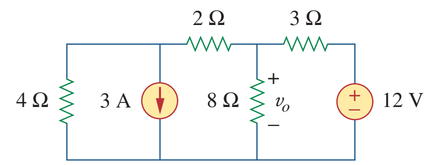


Figure 4.17
For Example 4.6.

Solution:
We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

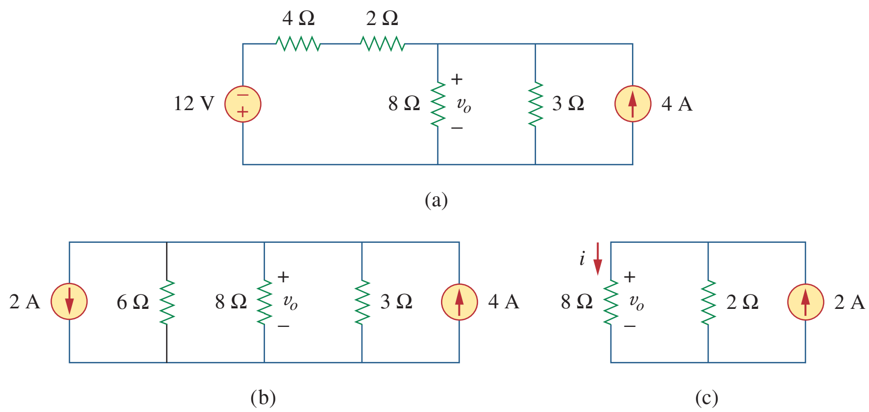


Figure 4.18
For Example 4.6.

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2 + 8}(2) = 0.4 \text{ A}$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

Practice Problem 4.6

Find i_o in the circuit of Fig. 4.19 using source transformation.

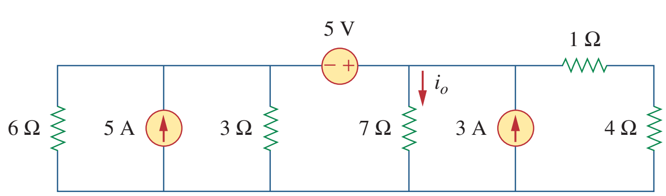


Figure 4.19
For Practice Prob. 4.6.

Answer: 1.78 A.

Example 4.7

Find v_x in Fig. 4.20 using source transformation.

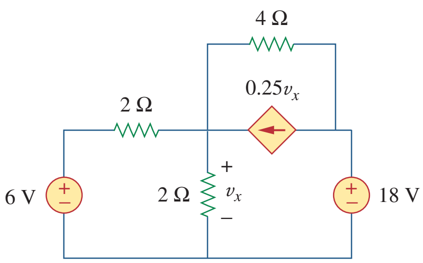


Figure 4.20
For Example 4.7.

Solution:
The circuit in Fig. 4.20 involves a voltage-controlled dependent current source. We transform this dependent current source as well as the 6-V independent voltage source as shown in Fig. 4.21(a). The 18-V voltage source is not transformed because it is not connected in series with any resistor. The two 2-Ω resistors in parallel combine to give a 1-Ω resistor, which is in parallel with the 3-A current source. The current source is transformed to a voltage source as shown in Fig. 4.21(b). Notice that the terminals for v_x are intact. Applying KVL around the loop in Fig. 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0 \tag{4.7.1}$$

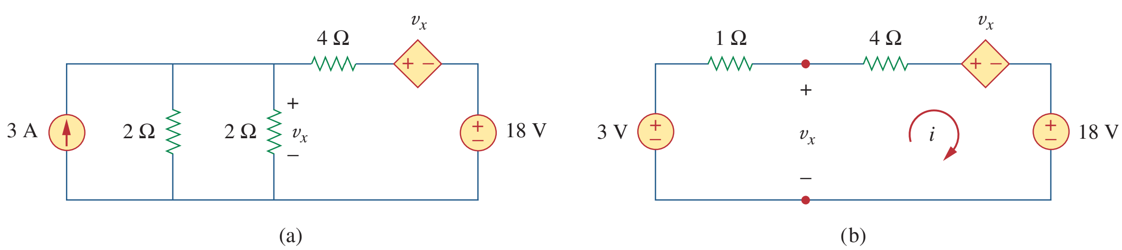


Figure 4.21
For Example 4.7: Applying source transformation to the circuit in Fig. 4.20.

Applying KVL to the loop containing only the 3-V voltage source, the 1-Ω resistor, and v_x yields

$$-3 + 1i + v_x = 0 \quad \Rightarrow \quad v_x = 3 - i \tag{4.7.2}$$

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 - i = 0 \quad \Rightarrow \quad i = -4.5 \text{ A}$$

Alternatively, we may apply KVL to the loop containing v_x, the 4-Ω resistor, the voltage-controlled dependent voltage source, and the 18-V voltage source in Fig. 4.21(b). We obtain

$$-v_x + 4i + v_x + 18 = 0 \quad \Rightarrow \quad i = -4.5 \text{ A}$$

Thus, $v_x = 3 - i = 7.5$ V.

Practice Problem 4.7

Use source transformation to find i_x in the circuit shown in Fig. 4.22.

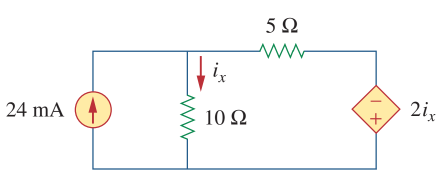


Figure 4.22
For Practice Prob. 4.7.

Answer: 7.059 mA.

4.5 Thevenin's Theorem

It often occurs in practice that a particular element in a circuit is variable (usually called the *load*) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in Fig. 4.23(a) can be replaced by that in Fig. 4.23(b). (The load in Fig. 4.23 may be a single resistor or another circuit.) The circuit to the left of the terminals *a-b* in Fig. 4.23(b) is known as the *Thevenin equivalent circuit*; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

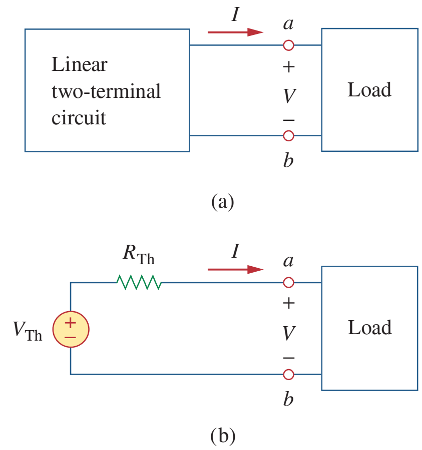


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

> **Thevenin's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

The proof of the theorem will be given later, in Section 4.7. Our major concern right now is how to find the Thevenin equivalent voltage V_{Th} and resistance R_{Th}. To do so, suppose the two circuits in Fig. 4.23 are equivalent. Two circuits are said to be *equivalent* if they have the same voltage-current relation at their terminals. Let us find out what will make the two circuits in Fig. 4.23 equivalent. If the terminals *a-b* are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals *a-b* in Fig. 4.23(a) must be equal to the voltage source V_{Th} in Fig. 4.23(b), since the two circuits are equivalent. Thus V_{Th} is the open-circuit voltage across the terminals as shown in Fig. 4.24(a); that is,

$$V_{Th} = v_{oc} \tag{4.6}$$

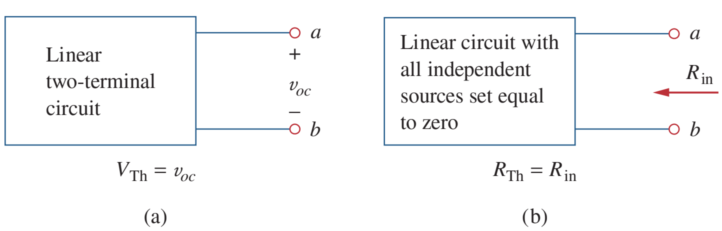


Figure 4.24
Finding V_{Th} and R_{Th}.

Again, with the load disconnected and terminals *a-b* open-circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals *a-b* in Fig. 4.23(a) must be equal to R_{Th} in Fig. 4.23(b) because the two circuits are equivalent. Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off, as shown in Fig. 4.24(b); that is,

$$R_{Th} = R_{in} \tag{4.7}$$

To apply this idea in finding the Thevenin resistance R_{Th}, we need to consider two cases.

■ **CASE 1** If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b, as shown in Fig. 4.24(b).

■ **CASE 2** If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals a and b and determine the resulting current i_o. Then $R_{Th} = v_o/i_o$, as shown in Fig. 4.25(a). Alternatively, we may insert a current source i_o at terminals a-b as shown in Fig. 4.25(b) and find the terminal voltage v_o. Again $R_{Th} = v_o/i_o$. Either of the two approaches will give the same result. In either approach we may assume any value of v_o and i_o. For example, we may use $v_o = 1$ V or $i_o = 1$ A, or even use unspecified values of v_o or i_o.

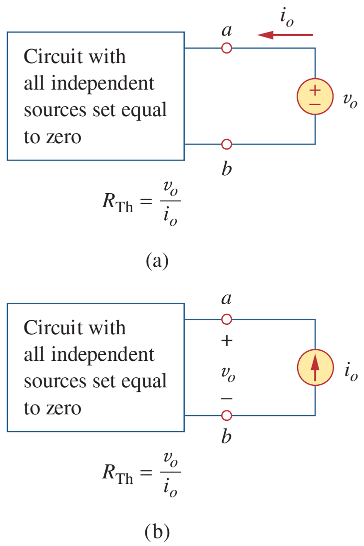


Figure 4.25
Finding R_{Th} when circuit has dependent sources.

Later we will see that an alternative way of finding R_{Th} is $R_{Th} = v_{oc}/i_{sc}$.

It often occurs that R_{Th} takes a negative value. In this case, the negative resistance ($v = -iR$) implies that the circuit is supplying power. This is possible in a circuit with dependent sources; Example 4.10 will illustrate this.

Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load R_L, as shown in Fig. 4.26(a). The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig. 4.26(b). From Fig. 4.26(b), we obtain

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \tag{4.8a}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th} \tag{4.8b}$$

Note from Fig. 4.26(b) that the Thevenin equivalent is a simple voltage divider, yielding V_L by mere inspection.

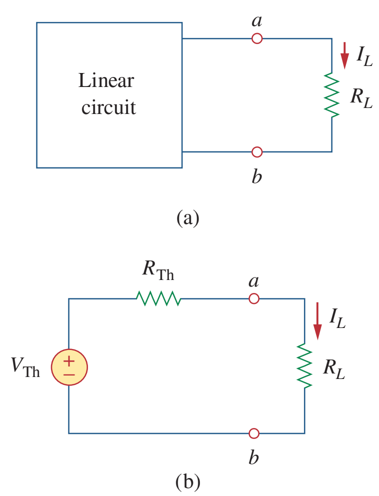


Figure 4.26
A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6$, 16, and 36 Ω.

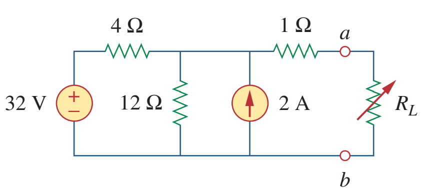


Figure 4.27
For Example 4.8.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

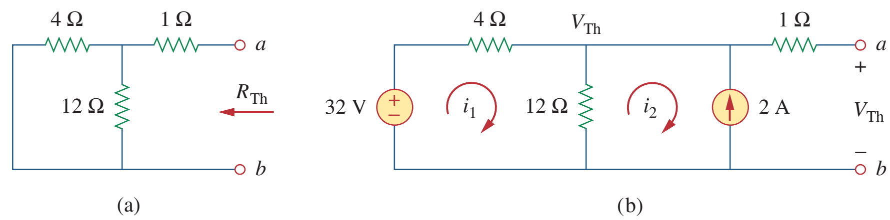


Figure 4.28
For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \quad \Rightarrow \quad V_{Th} = 30\text{ V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

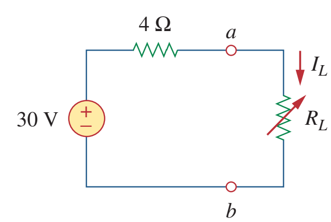


Figure 4.29
The Thevenin equivalent circuit for Example 4.8.

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3\text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5\text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75\text{ A}$$

Practice Problem 4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit of Fig. 4.30. Then find I.

Answer: $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, $I = 1.5$ A.

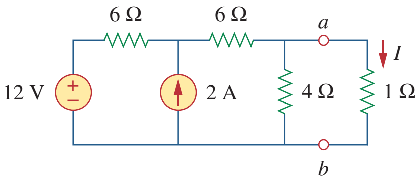


Figure 4.30
For Practice Prob. 4.8.

Example 4.9

Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals a-b.

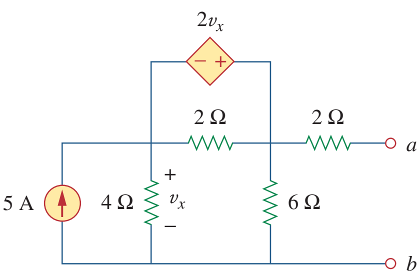


Figure 4.31
For Example 4.9.

Solution:
This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as indicated in Fig. 4.32(a). We may set $v_o = 1$ V to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{Th} = 1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_o, and obtain $R_{Th} = v_o/1$.)

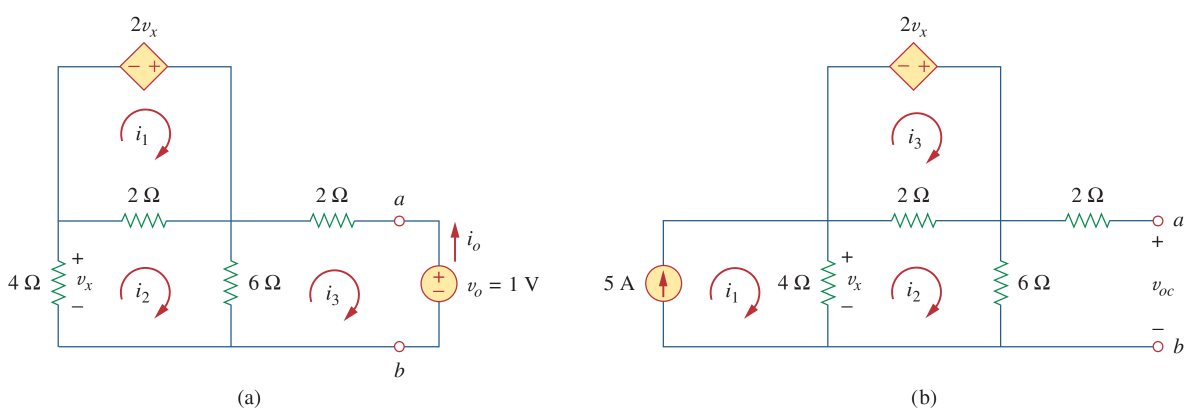


Figure 4.32
Finding R_{Th} and V_{Th} for Example 4.9.

Applying mesh analysis to loop 1 in the circuit of Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \tag{4.9.1}$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \tag{4.9.2}$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \tag{4.9.3}$$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{\text{Th}} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

To get V_{Th}, we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get

$$i_1 = 5 \quad \textbf{(4.9.4)}$$

$$-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2 \quad \textbf{(4.9.5)}$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

or

$$12i_2 - 4i_1 - 2i_3 = 0 \quad \textbf{(4.9.6)}$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20\text{ V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

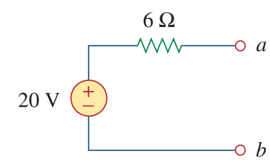


Figure 4.33
The Thevenin equivalent of the circuit in Fig. 4.31.

Practice Problem 4.9

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

Answer: $V_{\text{Th}} = 5.333$ V, $R_{\text{Th}} = 444.4$ mΩ.

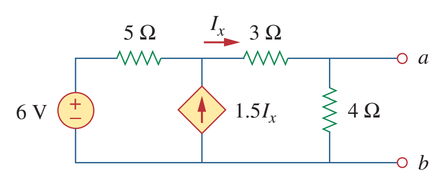


Figure 4.34
For Practice Prob. 4.9.

Example 4.10

Determine the Thevenin equivalent of the circuit in Fig. 4.35(a) at terminals *a-b*.

Solution:

1. **Define.** The problem is clearly defined; we are to determine the Thevenin equivalent of the circuit shown in Fig. 4.35(a).
2. **Present.** The circuit contains a 2-Ω resistor in parallel with a 4-Ω resistor. These are, in turn, in parallel with a dependent current source. It is important to note that there are no independent sources.
3. **Alternative.** The first thing to consider is that, since we have no independent sources in this circuit, we must excite the circuit externally. In addition, when you have no independent sources you will not have a value for V_{Th}; you will only have to find R_{Th}.

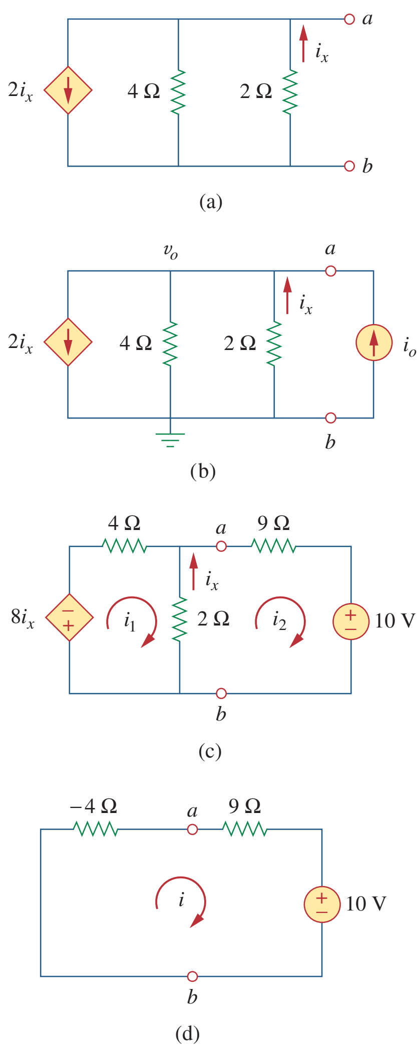


Figure 4.35
For Example 4.10.

The simplest approach is to excite the circuit with either a 1-V voltage source or a 1-A current source. Since we will end up with an equivalent resistance (either positive or negative), I prefer to use the current source and nodal analysis which will yield a voltage at the output terminals equal to the resistance (with 1 A flowing in, v_o is equal to 1 times the equivalent resistance).

As an alternative, the circuit could also be excited by a 1-V voltage source and mesh analysis could be used to find the equivalent resistance.

4. **Attempt.** We start by writing the nodal equation at a in Fig. 4.35(b) assuming $i_o = 1$ A.

$$2i_x + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0 \qquad \textbf{(4.10.1)}$$

Since we have two unknowns and only one equation, we will need a constraint equation.

$$i_x = (0 - v_o)/2 = -v_o/2 \qquad \textbf{(4.10.2)}$$

Substituting Eq. (4.10.2) into Eq. (4.10.1) yields

$$2(-v_o/2) + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0$$
$$= (-1 + \tfrac{1}{4} + \tfrac{1}{2})v_o - 1 \qquad \text{or} \qquad v_o = -4 \text{ V}$$

Since $v_o = 1 \times R_{\text{Th}}$, then $R_{\text{Th}} = v_o/1 = \mathbf{-4\ \Omega}$.

The negative value of the resistance tells us that, according to the passive sign convention, the circuit in Fig. 4.35(a) is supplying power. Of course, the resistors in Fig. 4.35(a) cannot supply power (they absorb power); it is the dependent source that supplies the power. This is an example of how a dependent source and resistors could be used to simulate negative resistance.

5. **Evaluate.** First of all, we note that the answer has a negative value. We know this is not possible in a passive circuit, but in this circuit we do have an active device (the dependent current source). Thus, the equivalent circuit is essentially an active circuit that can supply power.

Now we must evaluate the solution. The best way to do this is to perform a check, using a different approach, and see if we obtain the same solution. Let us try connecting a 9-Ω resistor in series with a 10-V voltage source across the output terminals of the original circuit and then the Thevenin equivalent. To make the circuit easier to solve, we can take and change the parallel current source and 4-Ω resistor to a series voltage source and 4-Ω resistor by using source transformation. This, with the new load, gives us the circuit shown in Fig. 4.35(c).

We can now write two mesh equations.

$$8i_x + 4i_1 + 2(i_1 - i_2) = 0$$
$$2(i_2 - i_1) + 9i_2 + 10 = 0$$

Note, we only have two equations but have 3 unknowns, so we need a constraint equation. We can use

$$i_x = i_2 - i_1$$

This leads to a new equation for loop 1. Simplifying leads to

$$(4 + 2 - 8)i_1 + (-2 + 8)i_2 = 0$$

or

$$-2i_1 + 6i_2 = 0 \quad \text{or} \quad i_1 = 3i_2$$
$$-2i_1 + 11i_2 = -10$$

Substituting the first equation into the second gives

$$-6i_2 + 11i_2 = -10 \quad \text{or} \quad i_2 = -10/5 = \mathbf{-2\ A}$$

Using the Thevenin equivalent is quite easy since we have only one loop, as shown in Fig. 4.35(d).

$$-4i + 9i + 10 = 0 \quad \text{or} \quad i = -10/5 = \mathbf{-2\ A}$$

6. **Satisfactory?** Clearly we have found the value of the equivalent circuit as required by the problem statement. Checking does validate that solution (we compared the answer we obtained by using the equivalent circuit with one obtained by using the load with the original circuit). We can present all this as a solution to the problem.

Practice Problem 4.10

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Answer: $V_{Th} = 0$ V, $R_{Th} = -7.5\ \Omega$.

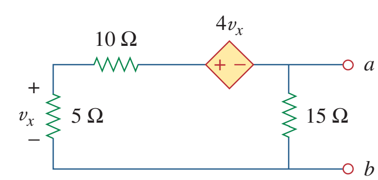


Figure 4.36
For Practice Prob. 4.10.

4.6 Norton's Theorem

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

> **Norton's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. 4.37(a) can be replaced by the one in Fig. 4.37(b).

The proof of Norton's theorem will be given in the next section. For now, we are mainly concerned with how to get R_N and I_N. We find R_N in the same way we find R_{Th}. In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{Th} \tag{4.9}$$

To find the Norton current I_N, we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 4.37. It is evident

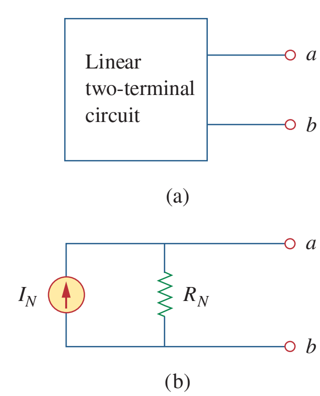


Figure 4.37
(a) Original circuit, (b) Norton equivalent circuit.

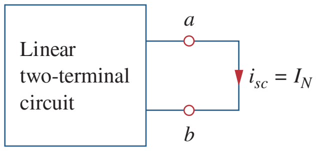


Figure 4.38
Finding Norton current I_N.

that the short-circuit current in Fig. 4.37(b) is I_N. This must be the same short-circuit current from terminal a to b in Fig. 4.37(a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \tag{4.10}$$

shown in Fig. 4.38. Dependent and independent sources are treated the same way as in Thevenin's theorem.

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$ as in Eq. (4.9), and

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}} \tag{4.11}$$

The Thevenin and Norton equivalent circuits are related by a source transformation.

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since V_{Th}, I_N, and R_{Th} are related according to Eq. (4.11), to determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage v_{oc} across terminals a and b.
- The short-circuit current i_{sc} at terminals a and b.
- The equivalent or input resistance R_{in} at terminals a and b when all independent sources are turned off.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. Example 4.11 will illustrate this. Also, since

$$V_{Th} = v_{oc} \tag{4.12a}$$

$$I_N = i_{sc} \tag{4.12b}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N \tag{4.12c}$$

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent, of a circuit which contains at least one independent source.

Example 4.11

Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a*-*b*.

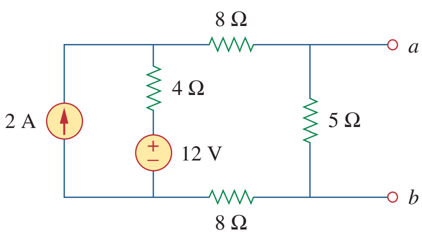


Figure 4.39
For Example 4.11.

Solution:
We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

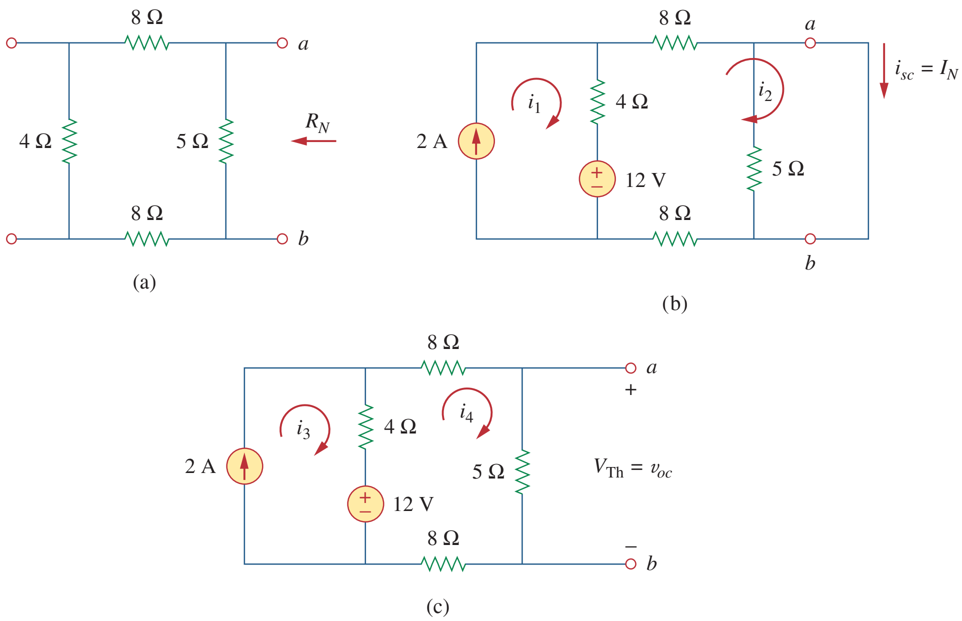


Figure 4.40
For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2\text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8\text{ A}$$

and

$$v_{oc} = V_{Th} = 5i_4 = 4\text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.12c) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

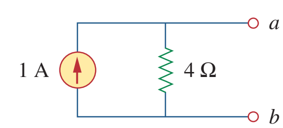


Figure 4.41
Norton equivalent of the circuit in Fig. 4.39.

Practice Problem 4.11

Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals *a*-*b*.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.

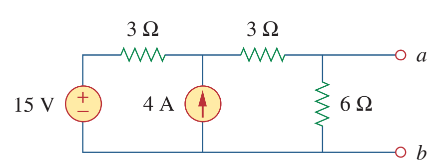


Figure 4.42
For Practice Prob. 4.11.

Example 4.12

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals *a-b*.

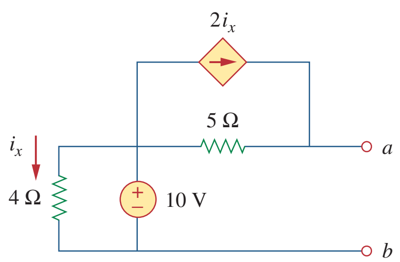


Figure 4.43
For Example 4.12.

Solution:

To find R_N, we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the terminals. We obtain the circuit in Fig. 4.44(a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = 0$. At node *a*, $i_o = \frac{1v}{5\Omega} = 0.2$ A, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

To find I_N, we short-circuit terminals *a* and *b* and find the current i_{sc}, as indicated in Fig. 4.44(b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10}{4} = 2.5\ \text{A}$$

At node *a*, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7\ \text{A}$$

Thus,

$$I_N = 7\ \text{A}$$

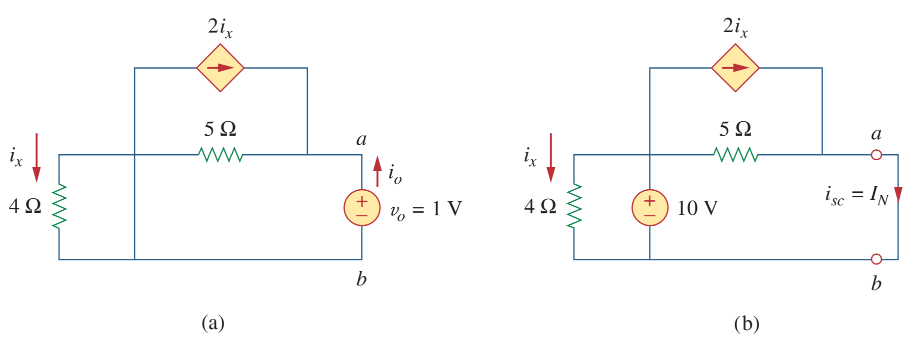


Figure 4.44
For Example 4.12: (a) finding R_N, (b) finding I_N.

Practice Problem 4.12

Find the Norton equivalent circuit of the circuit in Fig. 4.45 at terminals *a-b*.

Answer: $R_N = 1\ \Omega$, $I_N = 10$ A.

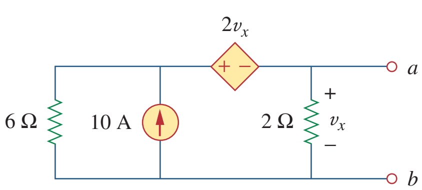


Figure 4.45
For Practice Prob. 4.12.

4.7 †Derivations of Thevenin's and Norton's Theorems

In this section, we will prove Thevenin's and Norton's theorems using the superposition principle.

Consider the linear circuit in Fig. 4.46(a). It is assumed that the circuit contains resistors and dependent and independent sources. We have access to the circuit via terminals a and b, through which current from an external source is applied. Our objective is to ensure that the voltage-current relation at terminals a and b is identical to that of the Thevenin equivalent in Fig. 4.46(b). For the sake of simplicity, suppose the linear circuit in Fig. 4.46(a) contains two independent voltage sources v_{s1} and v_{s2} and two independent current sources i_{s1} and i_{s2}. We may obtain any circuit variable, such as the terminal voltage v, by applying superposition. That is, we consider the contribution due to each independent source including the external source i. By superposition, the terminal voltage v is

$$v = A_0 i + A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2} \tag{4.13}$$

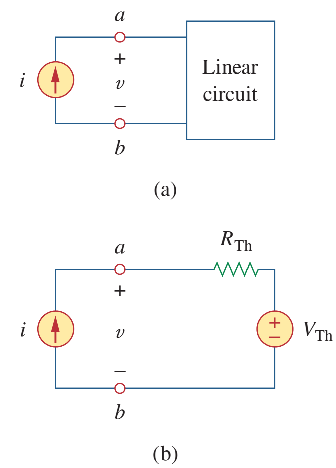


Figure 4.46
Derivation of Thevenin equivalent: (a) a current-driven circuit, (b) its Thevenin equivalent.

where A_0, A_1, A_2, A_3, and A_4 are constants. Each term on the right-hand side of Eq. (4.13) is the contribution of the related independent source; that is, $A_0 i$ is the contribution to v due to the external current source i, $A_1 v_{s1}$ is the contribution due to the voltage source v_{s1}, and so on. We may collect terms for the internal independent sources together as B_0, so that Eq. (4.13) becomes

$$v = A_0 i + B_0 \tag{4.14}$$

where $B_0 = A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2}$. We now want to evaluate the values of constants A_0 and B_0. When the terminals a and b are open-circuited, $i = 0$ and $v = B_0$. Thus, B_0 is the open-circuit voltage v_{oc}, which is the same as V_{Th}, so

$$B_0 = V_{\text{Th}} \tag{4.15}$$

When all the internal sources are turned off, $B_0 = 0$. The circuit can then be replaced by an equivalent resistance R_{eq}, which is the same as R_{Th}, and Eq. (4.14) becomes

$$v = A_0 i = R_{\text{Th}} i \quad \Rightarrow \quad A_0 = R_{\text{Th}} \tag{4.16}$$

Substituting the values of A_0 and B_0 in Eq. (4.14) gives

$$v = R_{\text{Th}} i + V_{\text{Th}} \tag{4.17}$$

which expresses the voltage-current relation at terminals a and b of the circuit in Fig. 4.46(b). Thus, the two circuits in Fig. 4.46(a) and 4.46(b) are equivalent.

When the same linear circuit is driven by a voltage source v as shown in Fig. 4.47(a), the current flowing into the circuit can be obtained by superposition as

$$i = C_0 v + D_0 \tag{4.18}$$

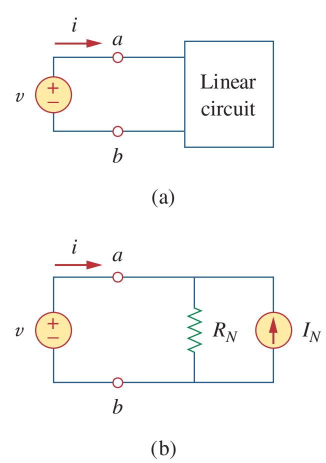


Figure 4.47
Derivation of Norton equivalent: (a) a voltage-driven circuit, (b) its Norton equivalent.

where $C_0 v$ is the contribution to i due to the external voltage source v and D_0 contains the contributions to i due to all internal independent sources. When the terminals a-b are short-circuited, $v = 0$ so that

$i = D_0 = -i_{sc}$, where i_{sc} is the short-circuit current flowing out of terminal a, which is the same as the Norton current I_N, i.e.,

$$D_0 = -I_N \tag{4.19}$$

When all the internal independent sources are turned off, $D_0 = 0$ and the circuit can be replaced by an equivalent resistance R_{eq} (or an equivalent conductance $G_{eq} = 1/R_{eq}$), which is the same as R_{Th} or R_N. Thus Eq. (4.19) becomes

$$i = \frac{v}{R_{Th}} - I_N \tag{4.20}$$

This expresses the voltage-current relation at terminals a-b of the circuit in Fig. 4.47(b), confirming that the two circuits in Fig. 4.47(a) and 4.47(b) are equivalent.

4.8 Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 4.48, the power delivered to the load is

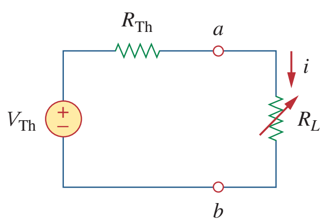


Figure 4.48
The circuit used for maximum power transfer.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \tag{4.21}$$

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in Fig. 4.49. We notice from Fig. 4.49 that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and ∞. We now want to show that this maximum power occurs when R_L is equal to R_{Th}. This is known as the *maximum power theorem.*

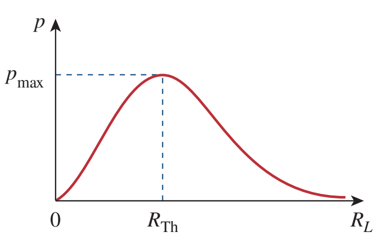


Figure 4.49
Power delivered to the load as a function of R_L.

> **Maximum power** is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in Eq. (4.21) with respect to R_L and set the result equal to zero. We obtain

$$\begin{aligned}\frac{dp}{dR_L} &= V_{Th}^2\left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] \\ &= V_{Th}^2\left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0\end{aligned}$$

This implies that

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L) \qquad \textbf{(4.22)}$$

which yields

$$\boxed{R_L = R_{\text{Th}}} \qquad \textbf{(4.23)}$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that Eq. (4.23) gives the maximum power by showing that $d^2p/dR_L^2 < 0$.

The source and load are said to be *matched* when $R_L = R_{\text{Th}}$.

The maximum power transferred is obtained by substituting Eq. (4.23) into Eq. (4.21), for

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}} \qquad \textbf{(4.24)}$$

Equation (4.24) applies only when $R_L = R_{\text{Th}}$. When $R_L \neq R_{\text{Th}}$, we compute the power delivered to the load using Eq. (4.21).

Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

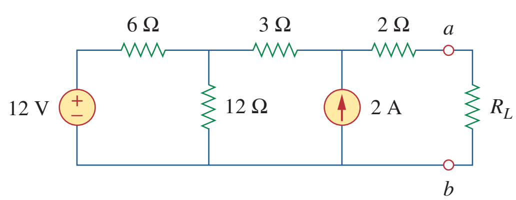


Figure 4.50
For Example 4.13.

Solution:
We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{\text{Th}} = 2 + 3 + 6 \,\|\, 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

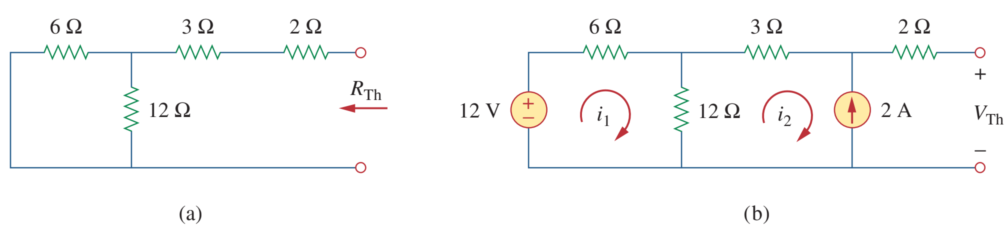


Figure 4.51
For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

Practice Problem 4.13

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

Answer: 4.222 Ω, 2.901 W.

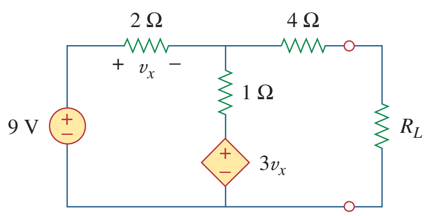


Figure 4.52
For Practice Prob. 4.13.

4.9 Verifying Circuit Theorems with *PSpice*

In this section, we learn how to use *PSpice* to verify the theorems covered in this chapter. Specifically, we will consider using DC Sweep analysis to find the Thevenin or Norton equivalent at any pair of nodes in a circuit and the maximum power transfer to a load. The reader is advised to read Section D.3 of Appendix D in preparation for this section.

To find the Thevenin equivalent of a circuit at a pair of open terminals using *PSpice*, we use the schematic editor to draw the circuit and insert an independent probing current source, say, Ip, at the terminals. The probing current source must have a part name ISRC. We then perform a DC Sweep on Ip, as discussed in Section D.3. Typically, we may let the current through Ip vary from 0 to 1 A in 0.1-A increments. After saving and simulating the circuit, we use Probe to display a plot of the voltage across Ip versus the current through Ip. The zero intercept of the plot gives us the Thevenin equivalent voltage, while the slope of the plot is equal to the Thevenin resistance.

To find the Norton equivalent involves similar steps except that we insert a probing independent voltage source (with a part name VSRC), say, Vp, at the terminals. We perform a DC Sweep on Vp and let Vp vary from 0 to 1 V in 0.1-V increments. A plot of the current through Vp versus the voltage across Vp is obtained using the Probe menu after simulation. The zero intercept is equal to the Norton current, while the slope of the plot is equal to the Norton conductance.

To find the maximum power transfer to a load using *PSpice* involves performing a DC parametric Sweep on the component value of R_L in Fig. 4.48 and plotting the power delivered to the load as a function of R_L. According to Fig. 4.49, the maximum power occurs

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 14$$

$$-\frac{1}{0.5}V_1 + \left(\frac{1}{0.5} + \frac{1}{2}\right)V_2 - \frac{1}{2}V_3 = 14$$

$$-2\,V_1 + 2.5\,V_2 - 0.5\,V_3 = 14 \qquad \ldots\text{(ii)}$$

Nodes 3 and 4 will form a supernode,

Writing voltage equation for the supernode,

$$V_3 - V_4 = 0.2V_y = 0.2(V_4 - V_1)$$

$$0.2V_1 + V_3 - 1.2V_4 = 0 \qquad \ldots\text{(iii)}$$

Applying KCL to the supernode,

$$\frac{V_3 - V_2}{2} - 0.5V_x + \frac{V_4}{1} + \frac{V_4 - V_1}{2.5} = 0$$

$$\frac{V_3 - V_2}{2} - 0.5(V_2 - V_1) + V_4 + \frac{V_4 - V_1}{2.5} = 0$$

$$\left(0.5 - \frac{1}{2.5}\right)V_1 - \left(\frac{1}{2} + 0.5\right)V_2 + \frac{1}{2}V_3 + \left(1 + \frac{1}{2.5}\right)V_4 = 0$$

$$0.1\,V_1 - V_2 + 0.5\,V_3 + 1.4\,V_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (i), (ii), (iii) and (iv),

$$V_1 = -12 \text{ V}$$

$$V_2 = -4 \text{ V}$$

$$V_3 = 0$$

$$V_4 = -2 \text{ V}$$

2.7 SUPERPOSITION THEOREM

It states that '*in a linear network containing more than one independent source and dependent source, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned. The independent current sources are represented by infinite resistances, i.e., open circuits.

The dependent sources are not sources but dissipative components—hence they are active at all times. A dependent source has zero value only when its control voltage or current is zero.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the network shown in Fig. 2.61. Suppose we have to find current I_4 through resistor R_4.

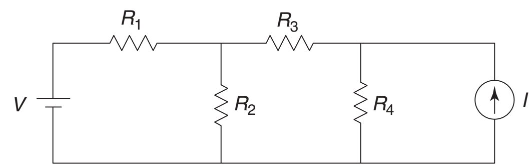


Fig. 2.61 *Network to illustrate superposition theorem*

The current flowing through resistor R_4 due to constant voltage source V is found to be say I_4' (with proper direction), representing constant current source with infinite resistance, i.e., open circuit.

The current flowing through resistor R_4 due to constant current source I is found to be say I_4'' (with proper direction), representing the constant voltage source with zero resistance or short circuit.

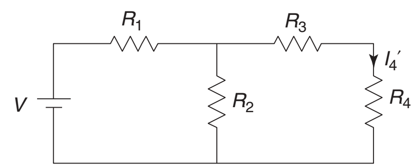


Fig. 2.62 *When voltage source V is acting alone*

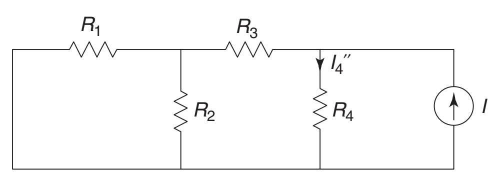


Fig. 2.63 *When current source I is acting alone*

The resultant current I_4 through resistor R_4 is found by superposition theorem.

$$I_4 = I_4' + I_4''$$

Steps to be followed in Superposition Theorem

1. Find the current through the resistance when only one independent source is acting, replacing all other independent sources by respective internal resistances.
2. Find the current through the resistance for each of the independent sources.
3. Find the resultant current through the resistance by the superposition theorem considering magnitude and direction of each current.

Example 2.45 *Find the current through the 4 Ω resistor in Fig. 2.64.*

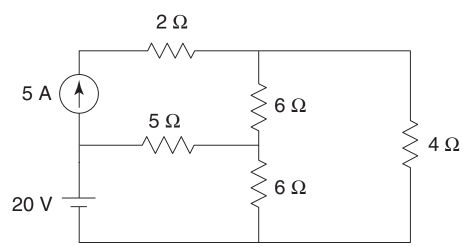


Fig. 2.64

Solution

Step I When the 5 A source is acting alone (Fig. 2.65)

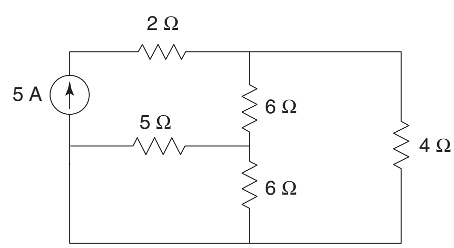


Fig. 2.65

By series-parallel reduction technique (Fig. 2.66),

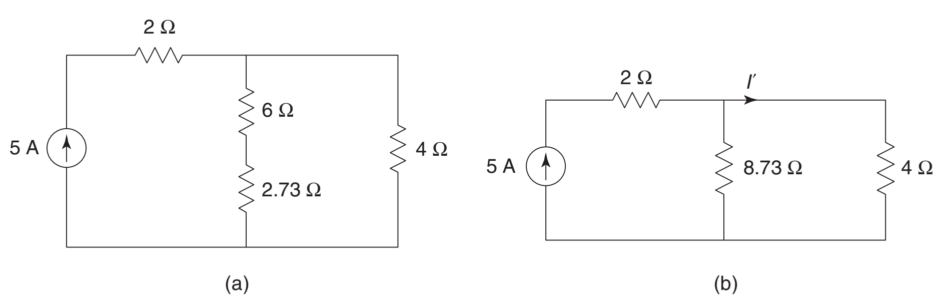


Fig. 2.66

$$I' = 5 \times \frac{8.73}{8.73 + 4} = 3.43\ \text{A}(\downarrow)$$

Step II When the 20 V source is acting alone (Fig. 2.67)

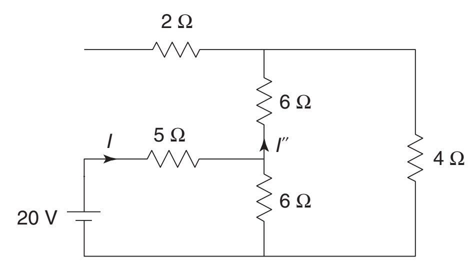


Fig. 2.67

By series-parallel reduction technique (Fig. 2.68),

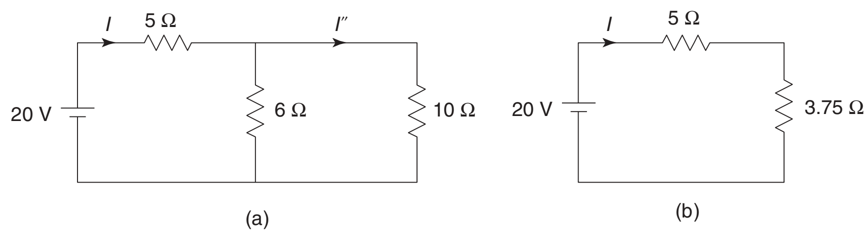


Fig. 2.68

$$I = \frac{20}{5+3.75} = 2.29 \text{ A}$$

From Fig. 2.68(a), by current-division rule,

$$I'' = 2.29 \times \frac{6}{6+10} = 0.86 \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.43 + 0.86 = 4.29 \text{ A } (\downarrow)$$

Example 2.46 *Find the current through the 3 Ω resistor in Fig. 2.69.*

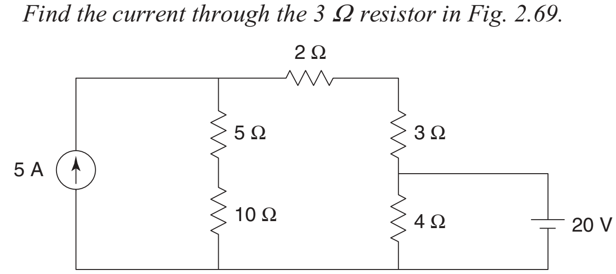


Fig. 2.69

Solution

Step I When the 5 A source is acting alone (Fig. 2.70)

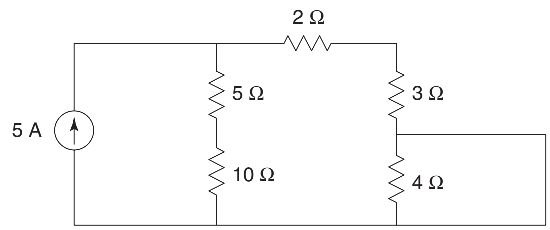


Fig. 2.70

By series-parallel reduction technique (Fig. 2.71),

$$I' = 5 \times \frac{15}{15+2+3} = 3.75\ \text{A}(\downarrow)$$

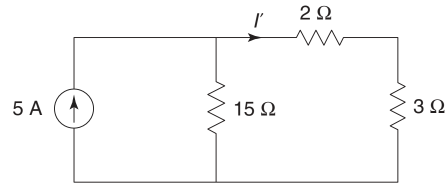


Fig. 2.71

Step II When the 20 V source is acting alone (Fig. 2.72)

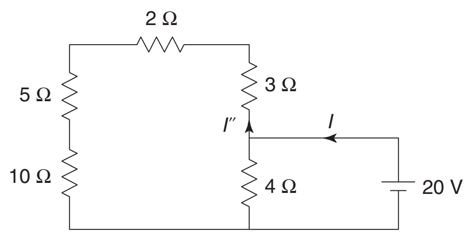


Fig. 2.72

By series-parallel reduction technique (Fig. 2.73),

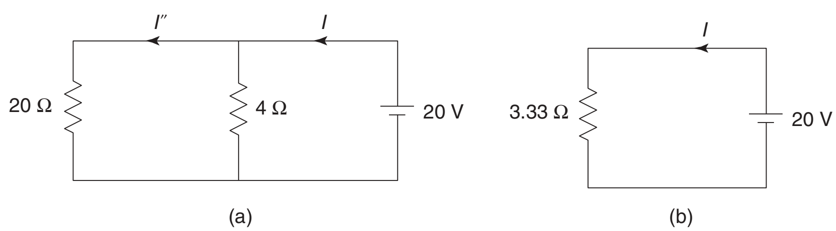


Fig. 2.73

$$I = \frac{20}{3.33} = 6\ \text{A}$$

From Fig. 2.73(a), by current-division rule,

$$I'' = 6 \times \frac{4}{20+4} = 1\ \text{A}(\uparrow) = -1\ \text{A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.75 - 1 = 2.75\ \text{A}\ (\downarrow)$$

Example 2.47 *Find the current in the 1 Ω resistors in Fig. 2.74.*

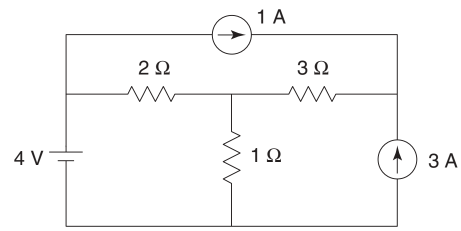


Fig. 2.74

Solution

Step I When the 4 V source is acting alone (Fig. 2.75)

$$I' = \frac{4}{2+1} = 1.33 \text{ A}(\downarrow)$$

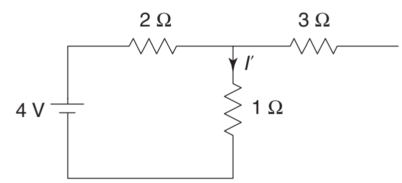


Fig. 2.75

Step II When the 3 A source is acting alone (Fig. 2.76)
By current-division rule,

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A} (\downarrow)$$

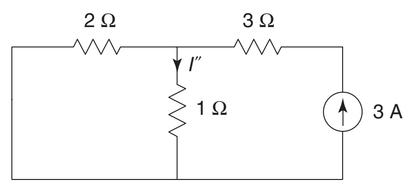


Fig. 2.76

Step III When the 1 A source is acting alone (Fig. 2.77)

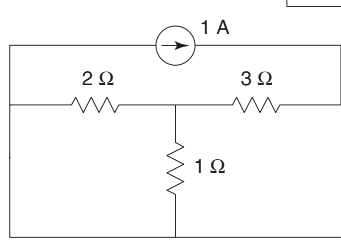


Fig. 2.77

Redrawing the network (Fig. 2.78),
By current-division rule,

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A}(\downarrow)$$

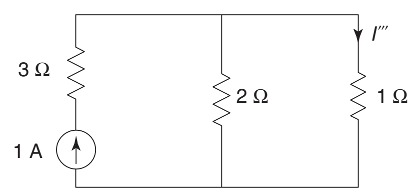


Fig. 2.78

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 1.33 + 2 + 0.66 = 4 \text{ A} (\downarrow)$$

Example 2.48 *Find the voltage V_{AB} in Fig. 2.79.*

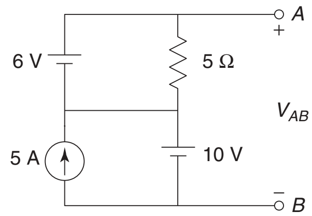


Fig. 2.79

Solution

Step I When the 6 V source is acting alone (Fig. 2.80)

$V'_{AB} = 6$ V

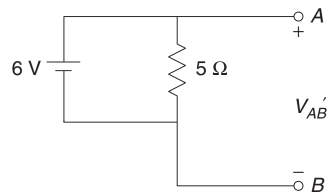


Fig. 2.80

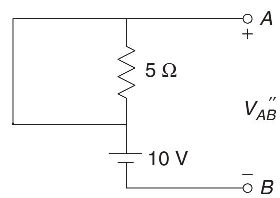


Fig. 2.81

Step II When the 10 V source is acting alone (Fig. 2.81)
Since the resistor of 5 Ω is shorted, the voltage across it is zero.

$$V''_{AB} = 10 \text{ V}$$

Step III When the 5 A source is acting alone (Fig. 2.82)
Due to short circuit in both the parts,

$$V'''_{AB} = 0$$

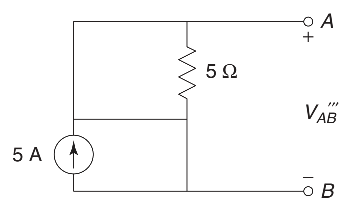


Fig. 2.82

Step IV By superposition theorem,

$$V_{AB} = V'_{AB} + V''_{AB} + V'''_{AB} = 6 + 10 + 0 = 16 \text{ V}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.49 *Find the current through the 6 Ω resistor in Fig. 2.83.*

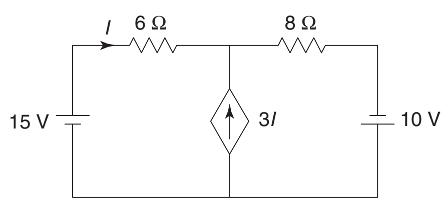


Fig. 2.83

Solution

Step I When the 15 V source is acting alone (Fig. 2.84)
From Fig. 2.84,

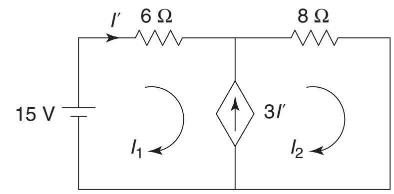


Fig. 2.84

$$I' = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3I' = 3I_1$$

$$4I_1 - I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$15 - 6I_1 - 8I_2 = 0$$

$$6I_1 + 8I_2 = 15 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.39\text{A}$$
$$I_2 = 1.57\text{A}$$
$$I' = I_1 = 0.39 \text{ A } (\rightarrow)$$

Step II When the 10 V source is acting alone (Fig. 2.85)
From Fig. 2.85,

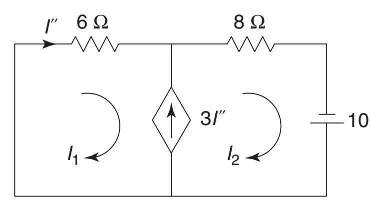


Fig. 2.85

$$I'' = I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3I'' = 3I_1$$
$$4I_1 - I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-6I_1 - 8I_2 + 10 = 0$$
$$6I_1 + 8I_2 = 10 \qquad \text{.....(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.26 \text{ A}$$
$$I_2 = 1.05 \text{ A}$$
$$I'' = I_1 = 0.26 \text{ A} \quad (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 0.39 + 0.26 = 0.65 \text{ A } (\rightarrow)$$

Example 2.50 *Find the current I_x in Fig. 2.86.*

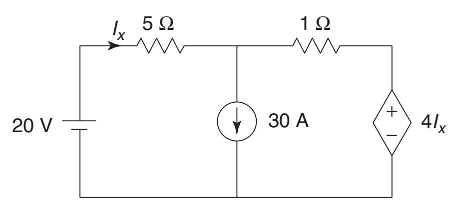


Fig. 2.86

Solution

Step I When the 30 A source is acting alone (Fig. 2.87)
From Fig. 2.87,

$$I_x' = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_1 - I_2 = 30 \qquad \ldots\text{(ii)}$$

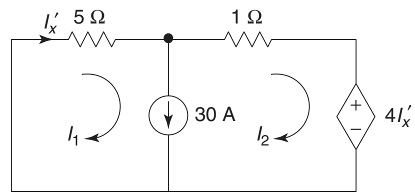


Fig. 2.87

Applying KVL to the outer path of the supermesh,

$$-5I_1 - 1I_2 - 4I_x' = 0$$
$$-5I_1 - I_2 - 4I_1 = 0$$
$$9I_1 + I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 3\text{ A}$$
$$I_2 = -27\text{ A}$$
$$I_x' = I_1 = 3\text{ A }(\rightarrow)$$

Step II When the 20 V source is acting alone (Fig. 2.88)
Applying KVL to the mesh,

$$20 - 5I_x'' - 1I_x'' - 4I_x'' = 0$$
$$I_x'' = 2\text{ A}(\rightarrow)$$

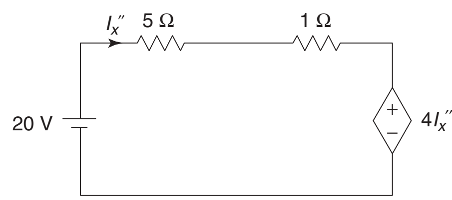


Fig. 2.88

Step III By superposition theorem,

$$I_x = I_x' + I_x'' = 3 + 2 = 5\text{ A}(\rightarrow)$$

Example 2.51 *Find the current I_1 in Fig. 2.89.*

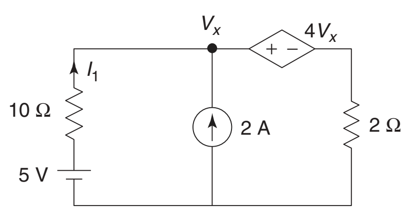


Fig. 2.89

Solution

Step I When the 5 V source is acting alone (Fig. 2.90)
From Fig. 2.90,

$$V_x = 5 - 10I_1'$$

Applying KVL to the mesh,

$$5 - 10I_1' - 4V_x - 2I_1' = 0$$

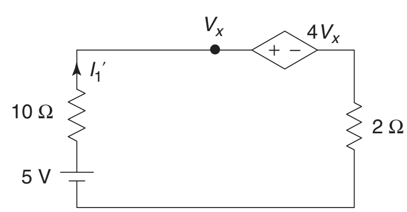


Fig. 2.90

$$5 - 10I_1' - 4(5 - 10I_1') - 2I_1' = 0$$
$$5 - 10I_1' - 20 + 40I_1' - 2I_1' = 0$$
$$I_1' = \frac{15}{28} = 0.54 \text{ A}(\uparrow)$$

Step II When the 2 A source is acting alone (Fig. 2.91)

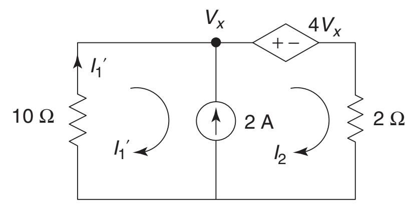


Fig. 2.91

From Fig. 2.91,

$$V_x = -10I_1' \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1' = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-10I_1' - 4V_x - 2I_2 = 0$$
$$-10I_1' - 4(-10I_1') - 2I_2 = 0$$
$$30I_1' - 2I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.14 \text{ A } (\uparrow)$$
$$I_2 = 2.14 \text{ A}$$

Step III By superposition theorem,

$$I_1 = I_1' + I_1'' = 0.54 + 0.14 = 0.68 \text{ A } (\uparrow)$$

Example 2.52 *Determine the current through the 10 Ω resistor in Fig. 2.92.*

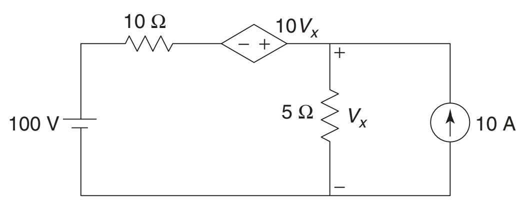


Fig. 2.92

Solution

Step I When the 100 V source is acting alone (Fig. 2.93)

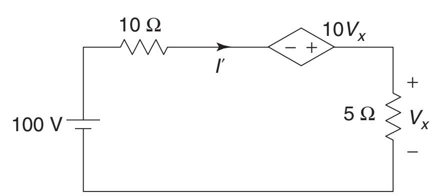


Fig. 2.93

From Fig. 2.93,

$$V_x = 5I'$$

Applying KVL to the mesh,

$$100 - 10I' + 10V_x - 5I' = 0$$
$$100 - 10I' + 10(5I') - 5I' = 0$$

$$I' = -2.86 \text{ A } (\rightarrow)$$

Step II When the 10 A source is acting alone (Fig. 2.94)
From Fig. 2.94,

$$V_x = 5(I_1 - I_2) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$-10I_1 + 10V_x - 5(I_1 - I_2) = 0$$
$$-10I_1 + 10\{5(I_1 - I_2)\} - 5(I_1 - I_2) = 0$$
$$35I_1 - 45I_2 = 0 \qquad \ldots\text{(ii)}$$

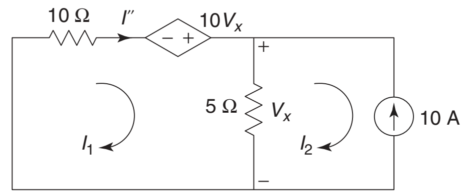


Fig. 2.94

For Mesh 2,

$$I_2 = -10 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -12.86 \text{ A}$$
$$I_2 = -10 \text{ A}$$
$$I'' = I_1 = -12.86 \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = -2.86 - 12.86 = -15.72 \text{ A } (\rightarrow)$$

Example 2.53 *Find the current I in the network of Fig. 2.95.*

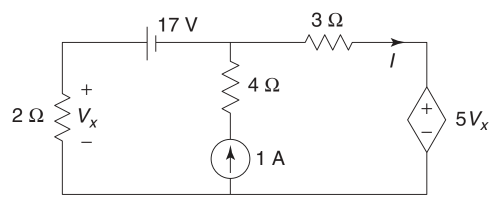


Fig. 2.95

Solution

Step I When the 17 V source is acting alone (Fig. 2.96)
From Fig. 2.96,

$$V_x = -2I'$$

Applying KVL to the mesh,

$$-2I' - 17 - 3I' - 5V_x = 0$$
$$-2I' - 17 - 3I' - 5(-2I') = 0$$
$$I' = 3.4 \text{ A } (\rightarrow)$$

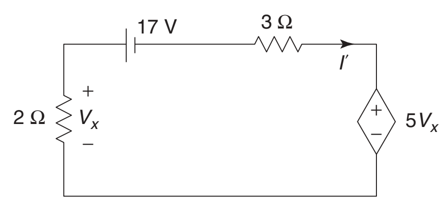


Fig. 2.96

Step II When the 1 A source is acting alone (Fig. 2.97)
From Fig. 2.97,

$$V_x = -2I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \ldots\text{(ii)}$$

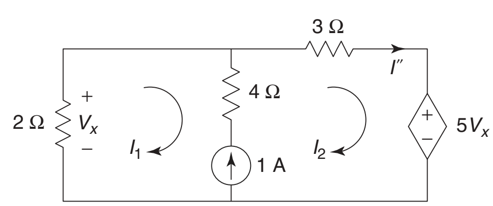


Fig. 2.97

Applying KVL to the outer path of the supermesh,

$$-2I_1 - 3I_2 - 5V_x = 0$$

$$-2I_1 - 3I_2 - 5(-2I_1) = 0$$

$$8I_1 - 3I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.6 \text{ A}$$

$$I_2 = 1.6 \text{ A}$$

$$I'' = I_2 = 1.6 \text{ A} (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.4 + 1.6 = 5 \text{ A} (\rightarrow)$$

Example 2.54 *Find the voltage V_1 in Fig. 2.98.*

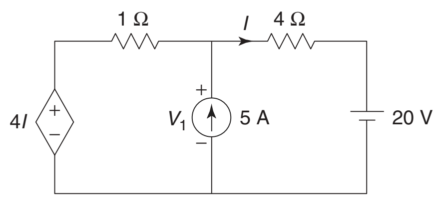


Fig. 2.98

Solution

Step I When the 5 A source is acting alone (Fig. 2.99)

From Fig. 2.99,

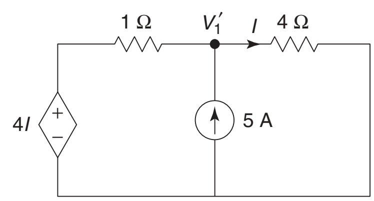


Fig. 2.99

$$I = \frac{V_1'}{4}$$

Applying KCL at Node 1,

$$\frac{V_1' - 4I}{1} + \frac{V_1'}{4} = 5$$

$$V_1' - 4\left(\frac{V_1'}{4}\right) + \frac{V_1'}{4} = 5$$

$$V_1' = 20 \text{ V}$$

Step II When the 20 V source is acting alone (Fig. 2.100)

Applying KVL to the mesh,

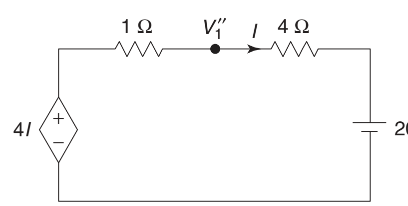


Fig. 2.100

$$4I - I - 4I - 20 = 0$$

$$I = -20 \text{ A}$$

$$V_1'' = 4I - 1(I) = 3I = 3(-20) = -60 \text{ V}$$

Step III By superposition theorem,

$$V_1 = V_1' + V_1'' = 20 - 60 = -40 \text{ V}$$

Example 2.55 *Find the current in the 6 Ω resistor in Fig. 2.101.*

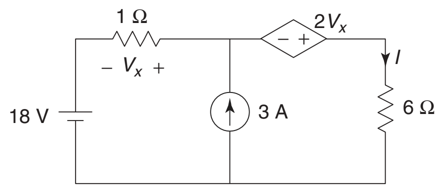


Fig. 2.101

Solution

Step I When the 18 V source is acting alone (Fig. 2.102)

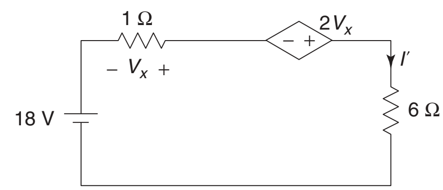


Fig. 2.102

From Fig. 2.102,

$$V_x = -I'$$

Applying KVL to the mesh,

$$18 - I' + 2V_x - 6I' = 0$$
$$18 - I' - 2I' - 6I' = 0$$
$$I' = 2 \text{ A } (\downarrow)$$

Step II When the 3 A source is acting alone (Fig. 2.103)

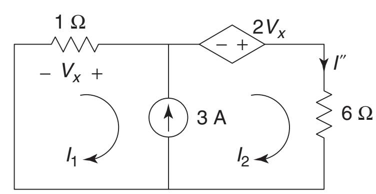


Fig. 2.103

From Fig. 2.103,

$$V_x = -1\, I_1 = -I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3 \qquad \text{...(ii)}$$

Applying KVL to the outerpath of the supermesh,

$$-1I_1 + 2V_x - 6I_2 = 0$$
$$-I_1 + 2(-I_1) - 6I_2 = 0$$
$$3I_1 + 6I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -2 \text{ A}$$
$$I_2 = 1 \text{ A}$$
$$I'' = I_2 = 1 \text{ A } (\downarrow)$$

Step III By superposition theorem,

$$I_{6\,\Omega} = I' + I'' = 2 + 1 = 3 \text{ A } (\downarrow)$$

Example 2.56 *Find the current I_y in Fig. 2.104.*

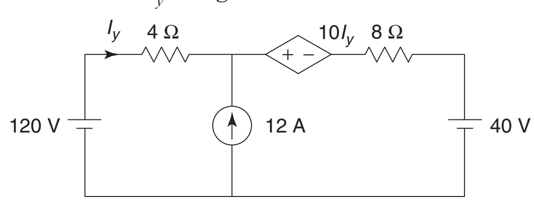


Fig. 2.104

Solution

Step I When the 120 V source is acting alone (Fig. 2.105)

Applying KVL to the mesh,

$$120 - 4I_y' - 10I_y' - 8I_y' = 0$$

$$I_y' = 5.45 \text{ A } (\rightarrow)$$

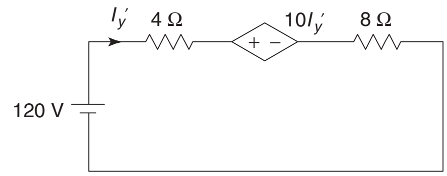


Fig. 2.105

Step II When the 12 A source is acting alone (Fig. 2.106)

From Fig. 2.106,

$$I_y'' = I_1 \quad \text{...(i)}$$

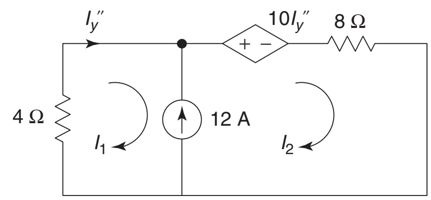


Fig. 2.106

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 12 \quad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-4I_1 - 10I_y'' - 8I_2 = 0$$

$$-4I_1 - 10I_1 - 8I_2 = 0$$

$$14I_1 + 8I_2 = 0 \quad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -4.36 \text{ A}$$

$$I_2 = 7.64 \text{ A}$$

$$Iy'' = I_1 = -4.36 \text{ A } (\rightarrow)$$

Step III When the 40 V source is acting alone (Fig. 2.107)

Applying KVL to the mesh,

$$-4\,I_y''' - 10I_y''' - 8I_y''' - 40 = 0$$

$$I_y''' = -\frac{40}{22} = -1.82 \text{ A } (\rightarrow)$$

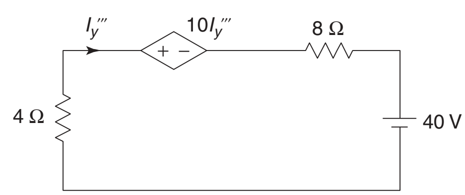


Fig. 2.107

Step IV By superposition theorem,

$$I_y = I_y' + I_y'' + I_y''' = 5.45 - 4.36 - 1.82 = -0.73 \text{ A } (\rightarrow)$$

Example 2.57 *Find the voltage V_x in Fig. 2.108.*

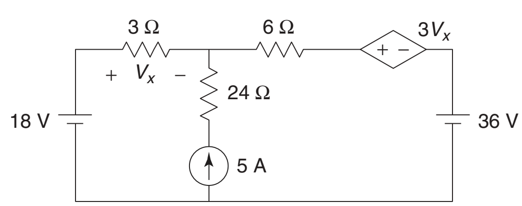


Fig. 2.108

Solution

Step I When the 18 V source is acting alone (Fig. 2.109)

From Fig. 2.109,

$$V_x' = 3I$$

Applying KVL to the mesh,

$$18 - 3I - 6I - 3V_x' = 0$$
$$18 - 3I - 6I - 3\,(3I) = 0$$
$$I = 1\text{ A}$$
$$V_x' = 3\text{ V}$$

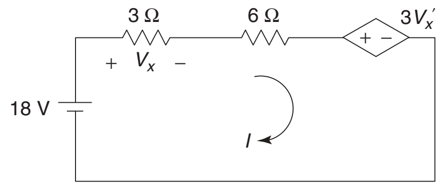


Fig. 2.109

Step II When the 5 A source is acting alone (Fig. 2.110)

From Fig. 2.110,

$$V_x'' = -3I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 5 \qquad \text{...(ii)}$$

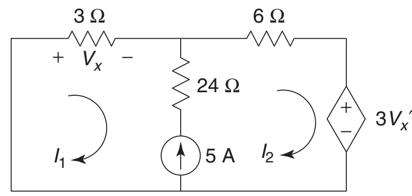


Fig. 2.110

Applying KVL to the outer path of the supermesh,

$$-3I_1 - 6I_2 - 3V_x'' = 0$$
$$-3I_1 - 6I_2 - 3(3I_1) = 0$$
$$12I_1 + 6I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -1.67\text{A}$$
$$I_2 = 3.33\text{ A}$$
$$V_x'' = 3I_1 = 3(-1.67) = -5\text{ V}$$

Step III When the 36 V source is acting alone (Fig. 2.111)

From Fig. 2.111,

$$V_x''' = -3I$$

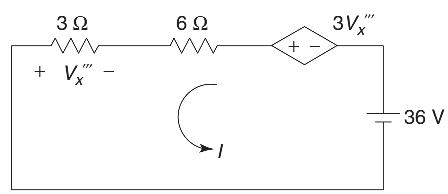


Fig. 2.111

Applying KVL to the mesh,

$$36 + 3V_x''' - 6I - 3I = 0$$

$$36 + 3V_x'' \quad 6\left(\frac{-V_x'''}{3}\right) - 3\left(\frac{-V_x'''}{3}\right) = 0$$
$$36 + 3V_x''' + 2V_x''' + V_x''' = 0$$
$$V_x''' = -6\text{ V}$$

Step IV By superposition theorem,

$$V_x = V_x' + V_x'' + V_x''' = 3 - 5 - 6 = -8\text{ V}$$

Example 2.58 *Find the voltage V in the network of Fig. 2.112.*

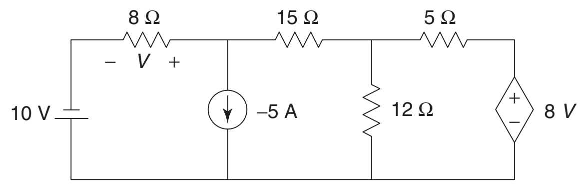


Fig. 2.112

Solution

Step I When the 10 V source is acting alone (Fig. 2.113)

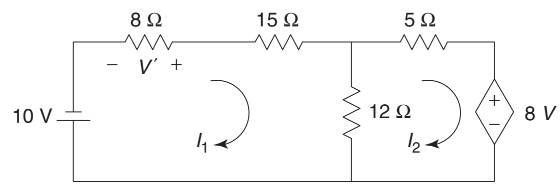


Fig. 2.113

From Fig. 2.113,

$$V' = -8I_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-10 - 8I_1 - 15\,I_1 - 12\,(I_1 - I_2) = 0$$

$$35I_1 - 12I_2 = -10 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 5I_2 - 8V' = 0$$

$$-12I_2 + 12I_1 - 5I_2 - 8(-8I_1) = 0$$

$$76I_1 - 17I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.54 \text{ A}$$

$$I_2 = 2.4 \text{ A}$$

$$V' = -8I_1 = -8(0.54) = -4.32 \text{ V}$$

Step II When the –5 A source is acting alone (Fig. 2.114)

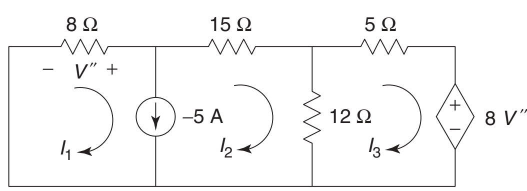


Fig. 2.114

From Fig. 2.114,

$$V'' = -8I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_1 - I_2 = -5 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-8I_1 - 15I_2 - 12(I_2 - I_3) = 0$$

$$-8I_1 - 27I_2 + 12I_3 = 0 \quad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-12(I_3 - I_2) - 5I_3 - 8V'' = 0$$

$$-12I_3 + 12I_2 - 5I_3 - 8(-8I_1) = 0$$

$$64I_1 + 12I_2 - 17I_3 = 0 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 4.97 \text{ A}$$

$$I_2 = 9.97 \text{ A}$$

$$I_3 = 25.74 \text{ A}$$

$$V'' = -8I_1 = -8(-4.97) = -39.76 \text{ V}$$

Step III By superposition theorem,

$$V = V' + V'' = -4.32 - 39.76 = -44.08 \text{ V}$$

Example 2.59 *For the network shown in Fig. 2.115, find the voltage V_0.*

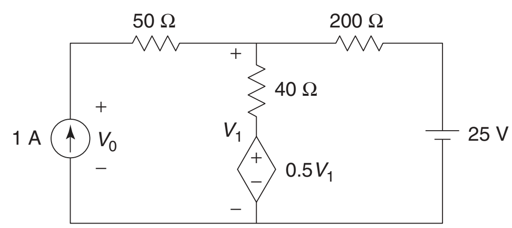


Fig. 2.115

Solution

Step I When the 1 A source is acting alone (Fig. 2.116)

From Fig. 2.116,

$$V_1 = 200\,I_2 \quad \text{...(i)}$$

For Mesh 1,

$$I_1 = 1 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$0.5V_1 - 40(I_2 - I_1) - 200\,I_2 = 0$$

$$0.5(200I_2) - 40I_2 + 40I_1 - 200I_2 = 0$$

$$40I_1 - 140\,I_2 = 0 \quad \text{...(iii)}$$

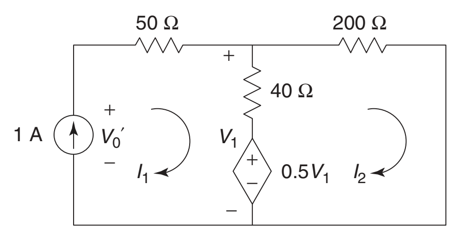


Fig. 2.116

Solving Eqs (ii) and (iii),

$$I_1 = 1 \text{ A}$$

$$I_2 = 0.29 \text{ A}$$

$$V_0' - 50\;I_1 - 200I_2 = 0$$

$$V_0' - 50(1) - 200(0.29) = 0$$

$$V_0' = 108 \text{ V}$$

Step II When the 25 V source is acting alone (Fig. 2.117)
From Fig. 2.117,

$$V_1 - 200I - 25 = 0$$

$$V_1 = 200I + 25 \qquad \text{...(i)}$$

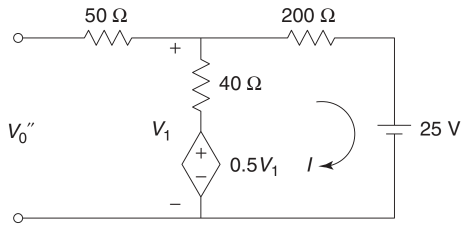


Fig. 2.117

Applying KVL to Mesh 1,

$$0.5V_1 - 40I - 200I - 25 = 0$$

$$0.5(200I + 25) - 40I - 200I - 25 = 0$$

$$I = -0.09 \text{ A}$$

$$V_0'' = V_1 = 200I + 25 = 200(-0.09) + 25 = 7 \text{ V}$$

Step III By superposition theorem,

$$V_0 = V_0' + V_0'' = 108 + 7 = 115 \text{ V}$$

Example 2.60 *For the network shown in Fig. 2.118, find the voltage V_x.*

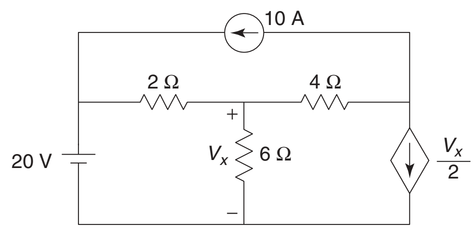


Fig. 2.118

Solution

Step I When the 20 V source is acting alone (Fig. 2.119)
From Fig. 2.119,

$$V_x' = 6(I_1 - I_2) \qquad \text{...(i)}$$

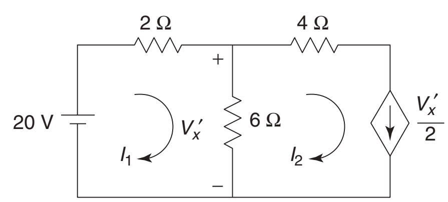


Fig. 2.119

Applying KVL to Mesh 1,

$$20 - 2I_1 - 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 = 20 \qquad \text{...(ii)}$$

For Mesh 2,

$$I_2 = \frac{V_x'}{2} = \frac{6(I_1 - I_2)}{2} = 3I_1 - 3I_2$$

$$3I_1 - 4I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 5.71 \text{ A}$$

$$I_2 = 4.29 \text{ A}$$

$$V_x' = 6(I_1 - I_2) = 6(5.71 - 4.29) = 8.52 \text{ V}$$

Step II When the 10 A source is acting alone (Fig. 2.120)

From Fig. 2.120,

$$V_x'' = 6(I_1 - I_2) \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-2(I_1 - I_3) - 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 - 2I_3 = 0 \quad \text{...(ii)}$$

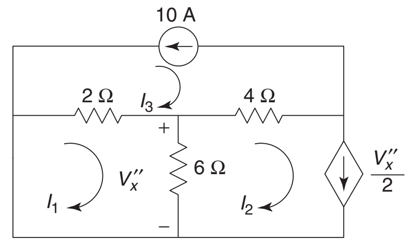


Fig. 2.120

For Mesh 2,

$$I_2 = \frac{V_x''}{2} = \frac{6(I_1 - I_2)}{2} = 3I_1 - 3I_2$$

$$3I_1 - 4I_2 = 0 \quad \text{...(iii)}$$

For Mesh 3,

$$I_3 = -10 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -5.71 \text{ A}$$

$$I_2 = -4.29 \text{ A}$$

$$I_3 = -10 \text{ A}$$

$$V_x'' = 6(I_1 - I_2) = 6(-5.71 + 4.29) = -8.52 \text{ V}$$

Step III By superposition theorem,

$$V_x = V_x' + V_x'' = 8.52 - 8.52 = 0$$

Example 2.61 *Calculate the current I in the network shown in Fig. 2.121.*

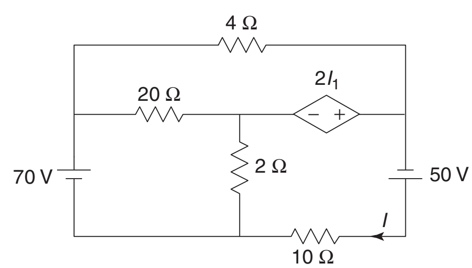


Fig. 2.121

Solution

Step I When the 70 V source is acting alone (Fig. 2.122)

From Fig. 2.122,

$$I' = I_3 \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$70 - 20(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$-20I_1 + 22I_2 - 2I_3 = 70 \quad \text{...(iii)}$$

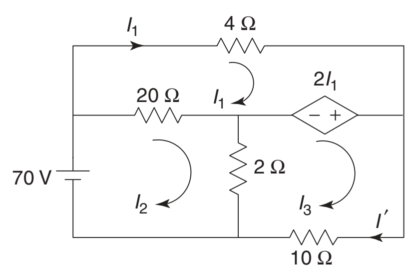


Fig. 2.122

Applying KVL to Mesh 3,

$$-2(I_3 - I_2) + 2I_1 - 10I_3 = 0$$

$$2I_1 + 2I_2 - 12I_3 = 0 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 8.94 \text{ A}$$
$$I_2 = 11.62 \text{ A}$$
$$I_3 = 3.43 \text{ A}$$
$$I' = I_3 = 3.43 \text{ A} (\leftarrow)$$

Step II When the 50 V source is acting alone (Fig. 2.123)
From Fig. 2.123,

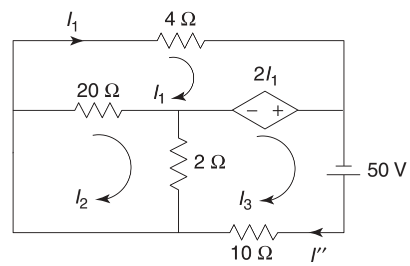


Fig. 2.123

$$I'' = I_3 \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-20(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$-20I_1 + 22I_2 - 2I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_2) + 2I_1 + 50 - 10I_3 = 0$$

$$2I_1 + 2I_2 - 12I_3 = -50 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii), and (iv),

$$I_1 = 1.06 \text{ A}$$
$$I_2 = 1.38 \text{ A}$$
$$I_3 = 4.57 \text{ A}$$
$$I'' = I_3 = 4.57 \text{ A} (\leftarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.43 + 4.57 = 8 \text{ A} (\leftarrow)$$

Example 2.62 *Find the voltage V_0 in the network of Fig. 2.124.*

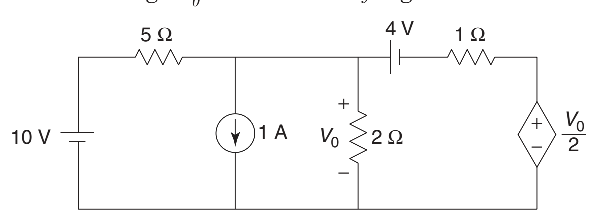


Fig. 2.124

Solution

Step I When the 10 V source is acting alone (Fig. 2.125)

Applying KCL at the node,

$$\frac{V_0'-10}{5}+\frac{V_0'}{2}+\frac{V_0'-\frac{V_0'}{2}}{1}=0$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0'=2$$

$$V_0'=1.67\text{ V}$$

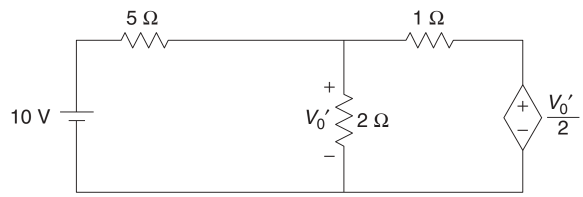


Fig. 2.125

Step II When the 1A current source is acting alone (Fig. 2.126)

Applying KCL at the node,

$$\frac{V_0''}{5}+1+\frac{V_0''}{2}+\frac{V_0''-\frac{V_0''}{2}}{1}$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0''=-1$$

$$V_0''=-0.83\text{ V}$$

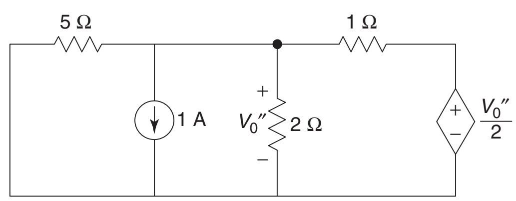


Fig. 2.126

Step III When the 4 V source is acting alone (Fig. 2.127)

Applying KCL at the node,

$$\frac{V_0'''}{5}+\frac{V_0'''}{2}+\frac{V_0'''-4-\frac{V_0'''}{2}}{1}=0$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0'''=4$$

$$V_0'''=3.33\text{ V}$$

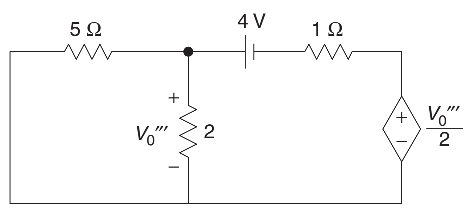


Fig. 2.127

Step IV By superposition theorem,

$$V_0 = V_0' + V_0'' + V_0''' \quad 1.67-0.83+3.33=4.17\text{ V}$$

2.8 THEVENIN'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit*.'

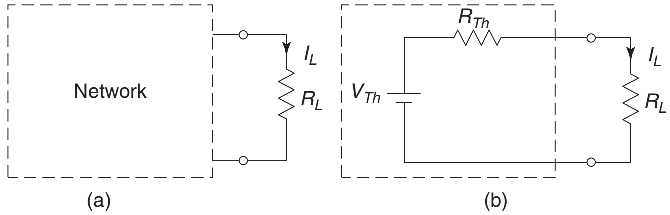


Fig. 2.128 *Network illustrating Thevenin's theorem*

Explanation Consider a simple network as shown in Fig. 2.129.

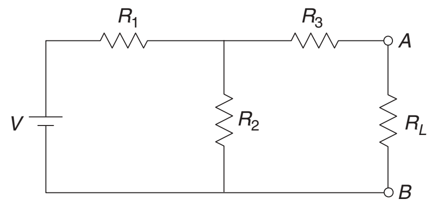


Fig. 2.129 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate open circuit voltage V_{Th} across points A and B as shown in Fig. 2.130.

$$V_{Th} = \frac{R_2}{R_1 + R_2} V$$

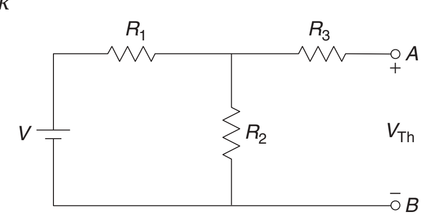


Fig. 2.130 *Calculation of V_{Th}*

For finding series resistance R_{Th}, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 2.131.

$$R_{Th} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

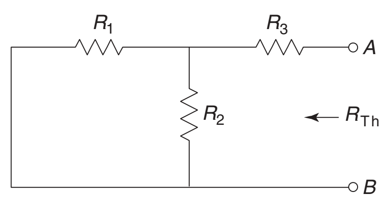


Fig. 2.131 *Calculation of R_{Th}*

Thevenin's equivalent network is shown in Fig. 2.132.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

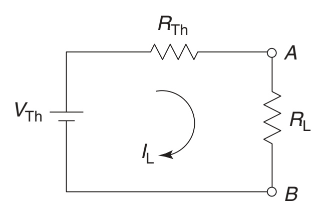


Fig. 2.132 *Thevenin's equivalent network*

If the network contains both independent and dependent sources, Thevenin's resistance R_{Th} is calculated as,

$$R_{Th} = \frac{V_{Th}}{I_N}$$

where I_N is the short-circuit current which would flow in a short circuit placed across the terminals A and B. Dependent sources are active at all times. They have zero values only when the control voltage or current is zero. R_{Th} may be negative in some cases which indicates negative resistance region of the device, i.e., as voltage increases, current decreases in the region and vice-versa.

If the network contains only dependent sources then

$$V_{Th} = 0$$
$$I_N = 0$$

For finding R_{Th} in such a network, a known voltage V is applied across the terminals A and B and current is calculated through the path AB.

$$R_{Th} = \frac{V}{I}$$

or a known current source I is connected across the terminals A and B and voltage is calculated across the terminals A and B.

$$R_{Th} = \frac{V}{I}$$

Thevenin's equivalent network for such a network is shown in Fig. 2.133.

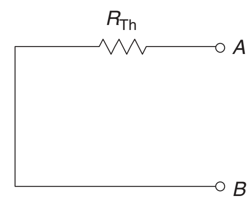


Fig. 2.133 *Thevenin's equivalent network*

Steps to be Followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Example 2.63 *Determine the current through the 24 Ω resistor in Fig. 2.134.*

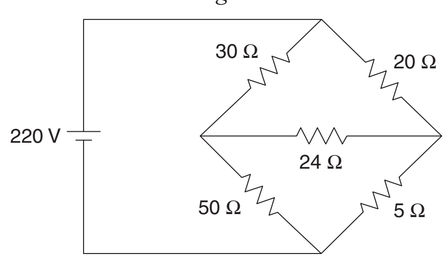


Fig. 2.134

Solution

Step I Calculation of V_{Th} (Fig. 2.135)

$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

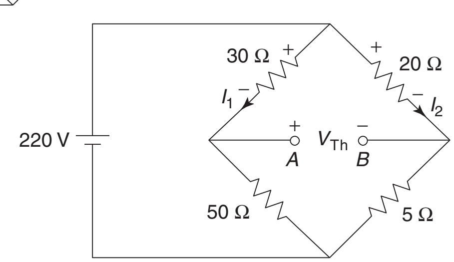


Fig. 2.135

Writing the V_{Th} equation,

$$V_{Th} + 30I_1 - 20I_2 = 0$$

$$V_{Th} = 20I_2 - 30I_1 = 20(8.8) - 30(2.75) = 93.5 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.136)

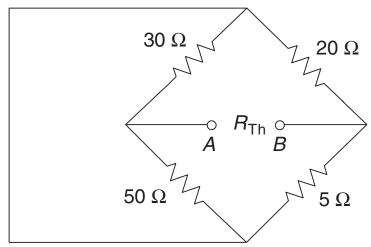


Fig. 2.136

Redrawing the circuit (Fig. 2.137),

$$R_{Th} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75 \ \Omega$$

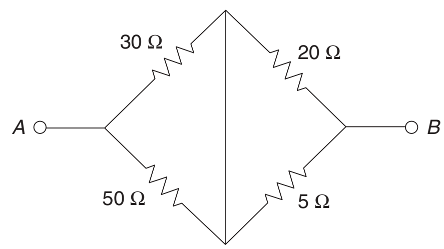


Fig. 2.137

Step III Calculation of I_L (Fig. 2.138)

$$I_L = \frac{93.5}{22.75 + 24} = 2 \text{ A}$$

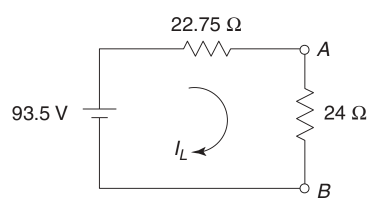


Fig. 2.138

Example 2.64 *Find the current through the 20 Ω resistor in Fig. 2.139.*

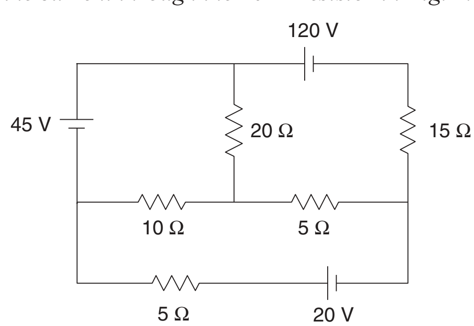


Fig. 2.139

Solution

Step I Calculation of V_{Th} (Fig. 2.140)

Applying KVL to Mesh 1,

$$45-120-15I_1-5(I_1-I_2)-10(I_1-I_2)=0 \quad \text{...(i)}$$
$$30I_1-15I_2=-75$$

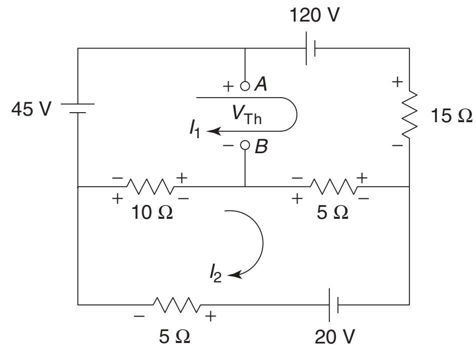


Fig. 2.140

Applying KVL to Mesh 2,

$$20-5I_2-10(I_2-I_1)-5(I_2-I_1)=0 \quad \text{...(ii)}$$
$$-15I_1+20I_2=20$$

Solving Eqs (i) and (ii),

$$I_1=-3.2 \text{ A}$$
$$I_2=-1.4 \text{ A}$$

Writing the V_{Th} equation,

$$45-V_{Th}-10(I_1-I_2)=0$$
$$V_{Th}=45-10(I_1-I_2)=45-10[-3.2-(-1.4)]=63 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.141)

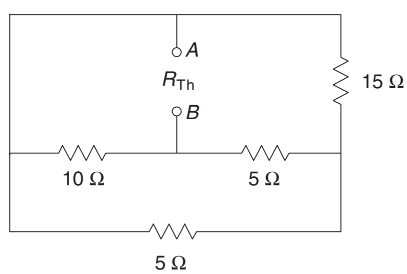


Fig. 2.141

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network (Fig. 2.142),

$$R_1=\frac{10\times5}{20}=2.5\ \Omega$$
$$R_2=\frac{10\times5}{20}=2.5\ \Omega$$
$$R_3=\frac{5\times5}{20}=1.25\ \Omega$$

Simplifying the network (Fig. 2.143 and Fig. 2.144),

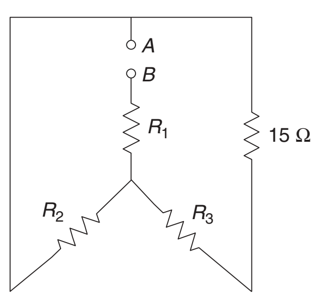


Fig. 2.142

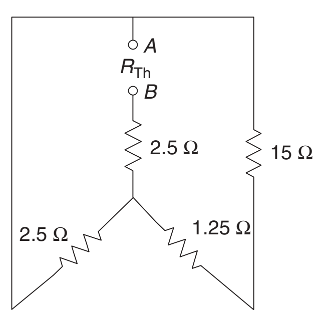


Fig. 2.143

$$R_{Th} = (16.25 \,||\, 2.5) + 2.5 = 4.67\ \Omega$$

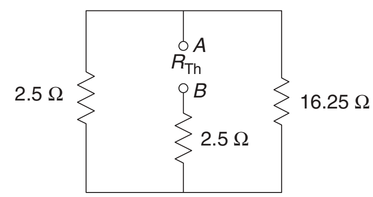


Fig. 2.144

Step III Calculation of I_L (Fig. 2.145)

$$I_L = \frac{63}{4.67+20} = 2.55\ \text{A}$$

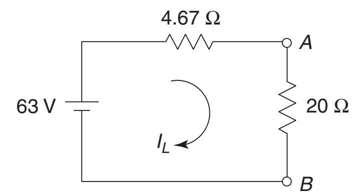


Fig. 2.145

Example 2.65 *Find the current through the 10 Ω resister in Fig. 2.146.*

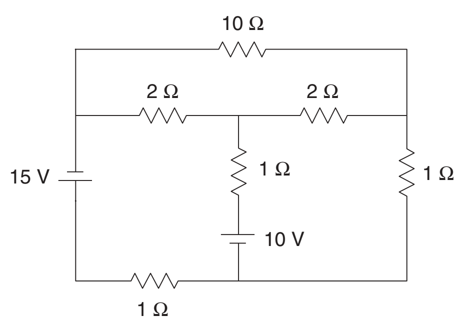


Fig. 2.146

Solution

Step I Calculation of V_{Th} (Fig. 2.147)

Applying KVL to Mesh 1,

$$-15 - 2I_1 - 1(I_1 - I_2) - 10 - 1I_1 = 0 \qquad \text{...(i)}$$
$$4I_1 - I_2 = -25$$

Applying KVL to Mesh 2,

$$10 - 1(I_2 - I_1) - 2I_2 - 1I_2 = 0 \qquad \text{...(ii)}$$
$$-I_1 + 4I_2 = 10$$

Solving Eqs (i) and (ii),

$$I_1 = -6\ \text{A}$$
$$I_2 = 1\ \text{A}$$

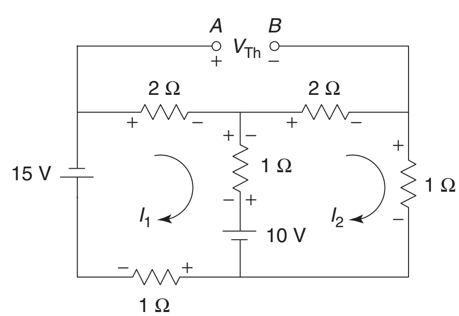


Fig. 2.147

Writing the V_{Th} equation,

$$-V_{\text{Th}} + 2I_2 + 2I_1 = 0$$

$$V_{\text{Th}} = 2I_1 + 2I_2 = 2(-6) + 2(1) = -10\text{ V}$$

$$= 10\text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II Calculation of R_{Th} (Fig. 2.148)

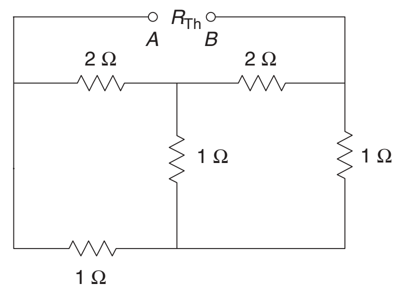


Fig. 2.148

Converting the star network formed by resistors of $2\,\Omega, 2\,\Omega$ and $1\,\Omega$ into an equivalent delta network (Fig. 2.149),

$$R_1 = 2 + 2 + \frac{2 \times 2}{1} = 8\,\Omega$$

$$R_2 = 2 + 1 + \frac{2 \times 1}{2} = 4\,\Omega$$

$$R_3 = 2 + 1 + \frac{2 \times 1}{2} = 4\,\Omega$$

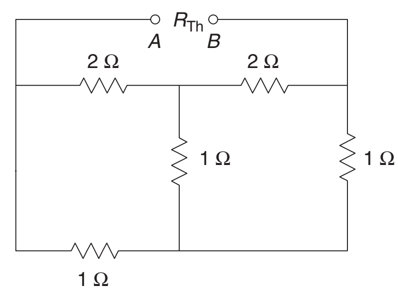


Fig. 2.149

Simplifying the network (Fig. 2.150),

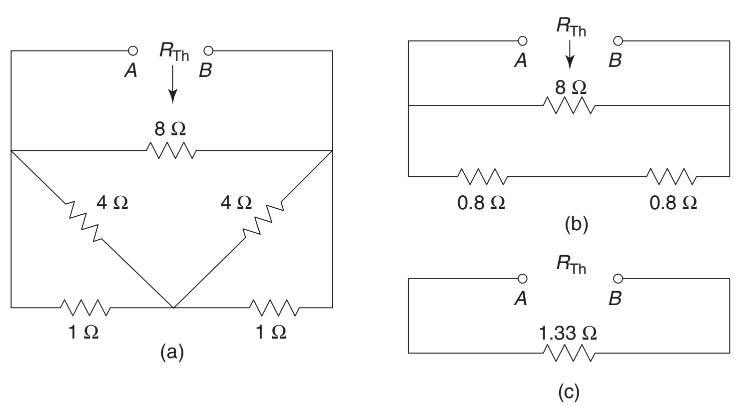


Fig. 2.150

$$R_{Th} = 1.33\ \Omega$$

Step III Calculation of I_L (Fig. 2.151)

$$I_L = \frac{10}{1.33+10} = 0.88\ \text{A}\ (\uparrow)$$

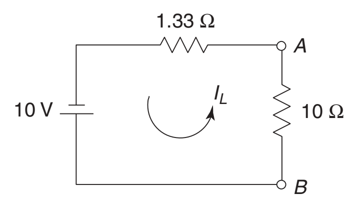


Fig. 2.151

Example 2.66 *Find the current through the 1 Ω resistor in Fig. 2.152.*

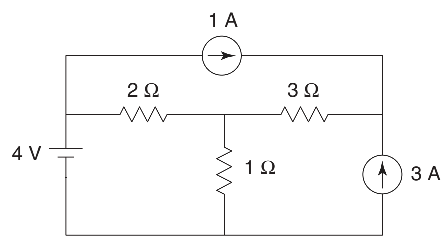


Fig. 2.152

Solution

Step I Calculation of V_{Th} (Fig. 2.153)

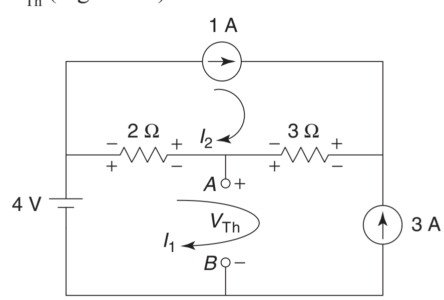


Fig. 2.153

Writing the current equations for Meshes 1 and 2,

$$I_1 = -3$$
$$I_2 = 1$$

Writing the V_{Th} equation,

$$4 - 2(I_1 - I_2) - V_{Th} = 0$$
$$V_{Th} = 4 - 2(I_1 - I_2) = 4 - 2(-4) = 12\ \text{V}$$

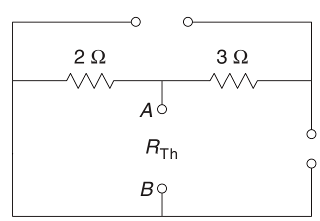


Fig. 2.154

Step II Calculation of R_{Th} (Fig. 2.154)

$$R_{Th} = 2\ \Omega$$

Step III Calculation of I_L (Fig. 2.155)

$$I_L = \frac{12}{2+1} = 4\ \text{A}$$

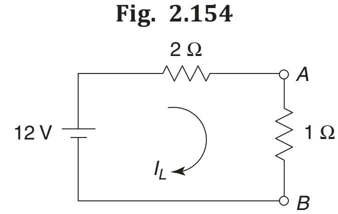


Fig. 2.155

EXAMPLES WITH DEPENDENT SOURCES

Example 2.67 *Obtain the Thevenin equivalent network for the given network of Fig. 2.156 at terminals A and B.*

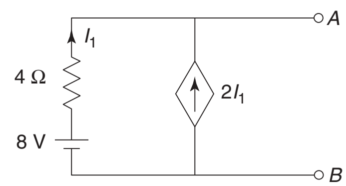


Fig. 2.156

Solution

Step I Calculation of V_{Th} (Fig. 2.157)

From Fig. 2.157,

$$I_1 = -2I_1$$
$$3I_1 = 0$$
$$I_1 = 0$$

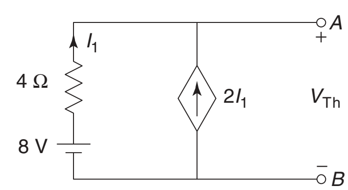


Fig. 2.157

Writing the V_{Th} equation,

$$8 - 0 - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 8 \text{ V}$$

Step II Calculation of I_N (Fig. 2.158),

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 2I_1$$
$$3I_1 - I_2 = 0 \qquad \text{...(i)}$$

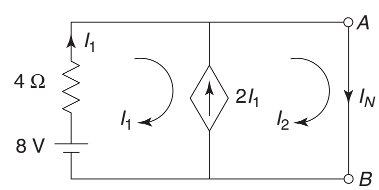


Fig. 2.158

Applying KVL to the outer path of the supermesh,

$$8 - 4I_1 = 0$$
$$I_1 = 2 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 6 \text{ A}$$
$$I_N = I_2 = 6 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{8}{6} = 1.33\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.159)

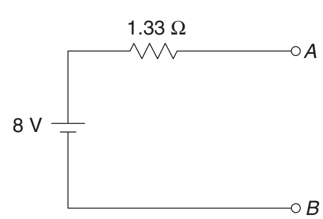


Fig. 2.159

Example 2.68 *Find Thevenin's equivalent network of Fig. 2.160.*

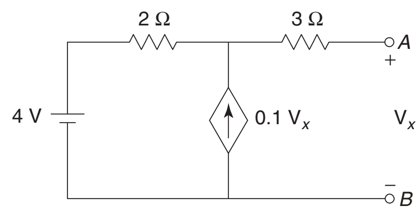


Fig. 2.160

Solution

Step I Calculation of V_{Th} (Fig. 2.161)

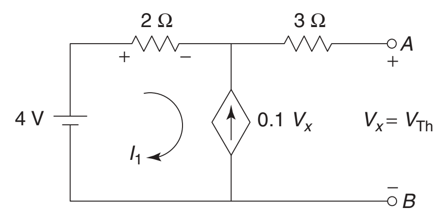


Fig. 2.161

$$V_x = V_{Th}$$

$$I_1 = -0.1V_x$$

Writing the V_{Th} equation,

$$4 - 2I_1 - V_x = 0$$

$$4 - 2(-0.1V_x) - V_x = 0$$

$$4 - 0.8V_x = 0$$

$$V_x = 5 \text{ V}$$

$$V_x = V_{Th} = 5 \text{ V}$$

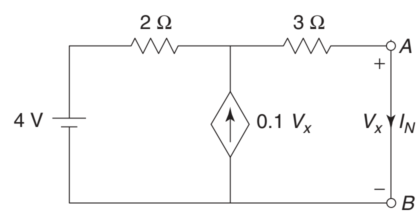


Fig. 2.162

Step II Calculation of I_N (Fig. 2.162)
From Fig. 2.162,

$$V_x = 0$$

The dependent source 0.1 V_x depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e., 0.1 $V_x = 0$ as shown in Fig. 2.163.

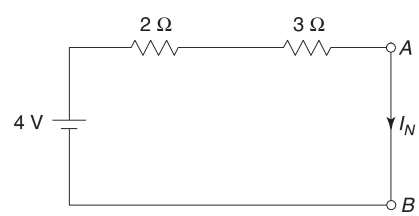


Fig. 2.163

$$I_N = \frac{4}{2+3} = 0.8 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{5}{0.8} = 6.25\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.164)

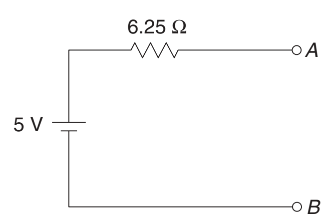


Fig. 2.164

Example 2.69 *Obtain the Thevenin equivalent network of Fig. 2.165 for the terminals A and B.*

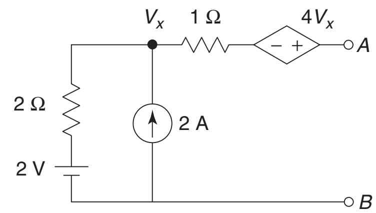


Fig. 2.165

Solution

Step I Calculation of V_{Th} (Fig. 2.166)

From Fig. 2.166,

$$2 - 2I_1 - V_x = 0$$
$$V_x = 2 - 2I_1$$

For Mesh 1,

$$I_1 = -2 \text{ A}$$
$$V_x = 2 - 2(-2) = 6 \text{ V}$$

Writing the V_{Th} equation,

$$2 - 2I_1 - 0 + 4V_x - V_{\text{Th}} = 0$$
$$2 - 2(-2) - 0 + 4(6) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 30 \text{ V}$$

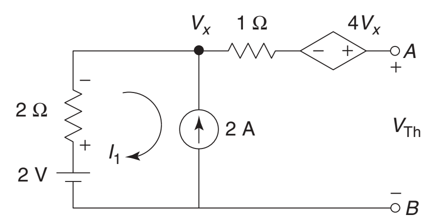


Fig. 2.166

Step II Calculation of I_N (Fig. 2.167)

From Fig. 2.167,

$$V_x = 2 - 2I_1 \quad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh,

Writing current equation for the supermesh

$$I_2 - I_1 = 2 \quad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$2 - 2I_1 - 1I_2 + 4V_x = 0$$
$$2 - 2I_1 - I_2 + 4(2 - 2I_1) = 0$$
$$10I_1 + I_2 = 10 \quad \text{...(iii)}$$

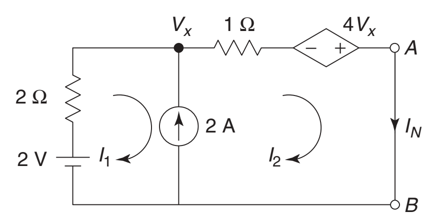


Fig. 2.167

Solving Eqs (ii) and (iii),

$$I_1 = 0.73 \text{ A}$$
$$I_2 = 2.73 \text{ A}$$
$$I_N = I_2 = 2.73 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{30}{2.73} = 10.98\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.168)

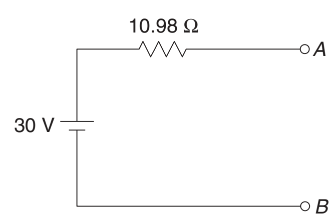


Fig. 2.168

Example 2.70 *Find the Thevenin equivalent network of Fig. 2.169 for the terminals A and B.*

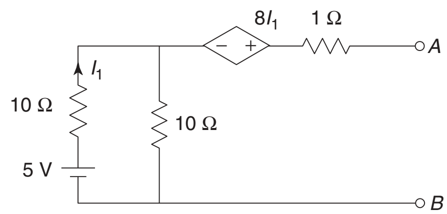


Fig. 2.169

Solution

Step I Calculation of V_{Th} (Fig. 2.170)
Applying KVL to the mesh,

$$5 - 10I_1 - 10I_1 = 0$$
$$I_1 = \frac{5}{20} = 0.25 \text{ A}$$

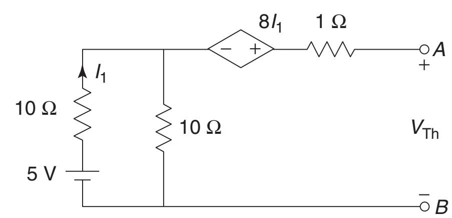


Fig. 2.170

Writing the V_{Th} equation,

$$5 - 10I_1 + 8I_1 - 0 - V_{Th} = 0$$
$$V_{Th} = 5 - 2I_1 = 5 - 2(0.25) = 4.5 \text{ V}$$

Step II Calculation of I_N (Fig. 2.171)
Applying KVL to Mesh 1,

$$5 - 10I_1 - 10(I_1 - I_2) = 0$$
$$20I_1 - 10I_2 = 5 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-10(I_2 - I_1) + 8I_1 - 1I_2 = 0$$
$$18I_1 - 11I_2 = 0 \qquad \text{...(ii)}$$

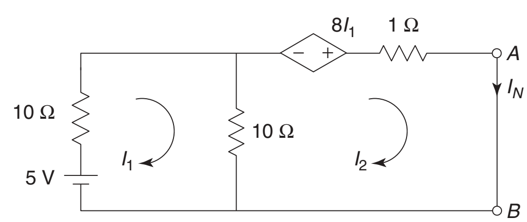


Fig. 2.171

Solving Eqs (i) and (ii),

$$I_1 = 1.375 \text{ A}$$
$$I_2 = 2.25 \text{ A}$$
$$I_N = I_2 = 2.25 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{4.5}{2.25} = 2\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.172)

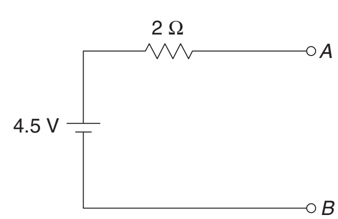


Fig. 2.172

Example 2.71 *Find V_{Th} and R_{Th} between terminals A and B of the network shown in Fig. 2.173.*

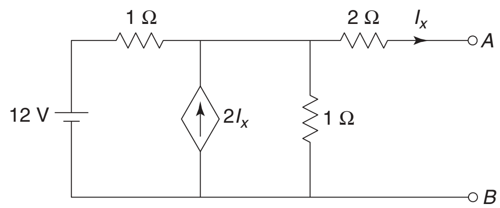


Fig. 2.173

Solution

Step I Calculation of V_{Th} (Fig. 2.174)

$$I_x = 0$$

The dependent source $2I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e., $2I_x = 0$ as shown in Fig. 2.174.

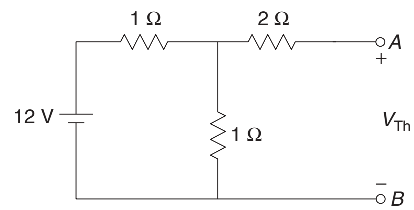


Fig. 2.174

Writing the V_{Th} equation,

$$V_{Th} = 12 \times \frac{1}{1+1} = 6 \text{ V}$$

Step II Calculation of I_N (Fig. 2.175)
From Fig. 2.175,

$$I_x = \frac{V_1}{2}$$

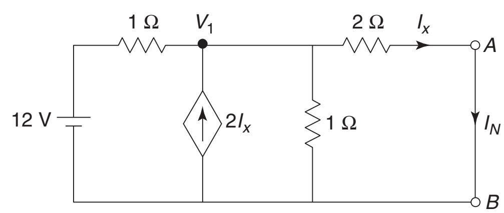


Fig. 2.175

Applying KCL at Node 1,

$$\frac{V_1 - 12}{1} + \frac{V_1}{1} + \frac{V_1}{2} = 2I_x$$

$$V_1 + V_1 + \frac{V_1}{2} - 12 = 2\left(\frac{V_1}{2}\right)$$

$$V_1 = 8 \text{ V}$$

$$I_N = \frac{V_1}{2} = \frac{8}{2} = 4 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{6}{4} = 1.5\ \Omega$$

Example 2.72 *Obtain the Thevenin equivalent network of Fig. 2.176 for the given network at terminals a and b .*

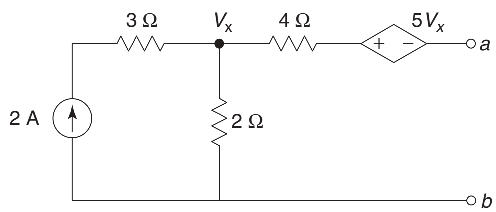


Fig. 2.176

Solution

Step I Calculation of V_{Th} (Fig. 2.177)

Applying KCL at Node x,

$$2 = \frac{V_x}{2}$$

$$V_x = 4 \text{ V}$$

Writing the V_{Th} equation,

$$V_{Th} = V_x - 5V_x = -4V_x$$

$$= -16 \text{ V} \quad \text{(the terminal } a \text{ is negative w.r.t. } b)$$

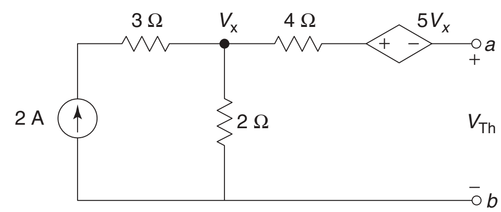


Fig. 2.177

Step II Calculation of I_N (Fig. 2.178)

Applying KCL at Node x,

$$2 = \frac{V_x}{2} + \frac{V_x - 5V_x}{4}$$

$$2 = \frac{V_x}{2} - V_x = -\frac{V_x}{2}$$

$$V_x = -4 \text{ V}$$

$$I_N = \frac{V_x - 5V_x}{4} = -V_x = 4 \text{ A}$$

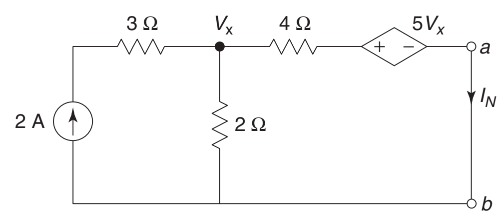


Fig. 2.178

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{-16}{4} = -4\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.179)

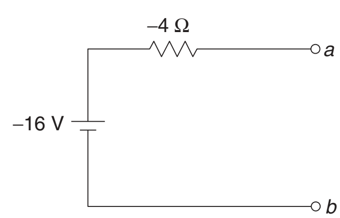


Fig. 2.179

Example 2.73 *Obtain the Thevenin equivalent network of Fig. 2.180 for the given network.*

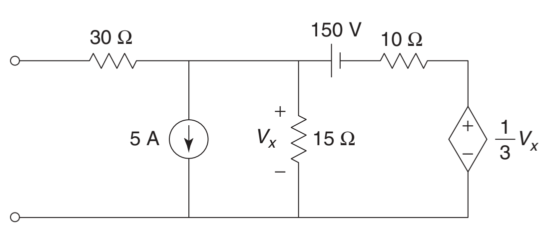


Fig. 2.180

Solution

Step I Calculation of V_{Th} (Fig. 2.181)

From Fig. 2.181,

$$V_x = V_{Th}$$

Applying KCL at the node,

$$\frac{V_x - 150 - \frac{1}{3}V_x}{10} + \frac{V_x}{15} + 5 = 0$$

$$V_x = 75 \text{ V}$$

$$V_{Th} = 75 \text{ V}$$

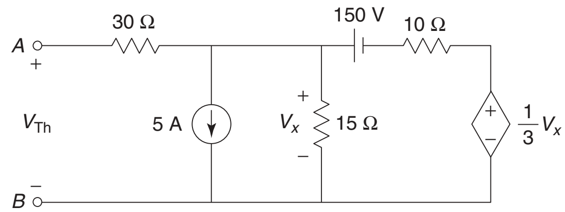


Fig. 2.181

Step II Calculation of I_N (Fig. 2.182)

Applying KCL at Node *x*,

$$\frac{V_x}{30} + 5 + \frac{V_x}{15} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = 0$$

$$\frac{V_x}{30} + \frac{V_x}{15} + \frac{V_x}{10} - \frac{V_x}{30} = 15 - 5$$

$$V_x = 60 \text{ V}$$

$$I_N = \frac{V_x}{30} = \frac{60}{30} = 2 \text{ A}$$

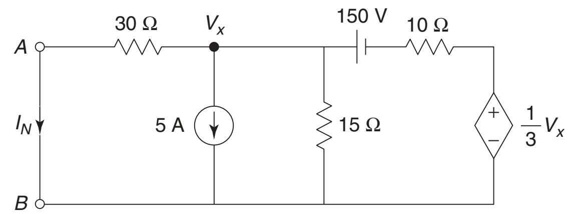


Fig. 2.182

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{75}{2} = 37.5 \ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.183)

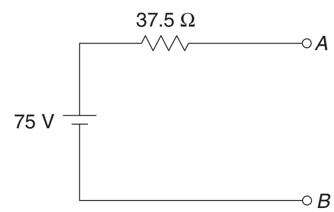


Fig. 2.183

Example 2.74 *Find the Thevenin's equivalent network of the network to the left of A-B in the Fig. 2.184.*

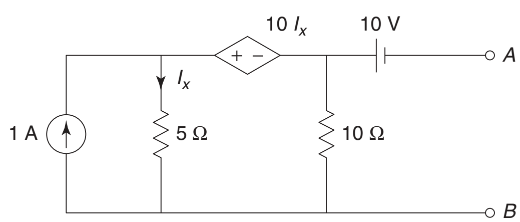


Fig. 2.184

Solution

Step I Calculation of V_{Th} (Fig. 2.185)

From Fig. 2.185,

$$I_x = I_1 - I_2 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 1 \qquad \ldots\text{(ii)}$$

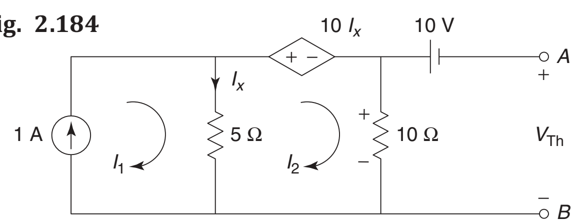


Fig. 2.185

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 10I_x - 10I_2 = 0$$

$$-5(I_2 - I_1) - 10(I_1 - I_2) - 10I_2 = 0$$

$$5I_1 + 5I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 1 \text{ A}$$

$$I_2 = -1 \text{ A}$$

$$I_x = I_1 - I_2 = 1 - (-1) = 2 \text{ A}$$

Writing the V_{Th} equation,

$$10I_2 - 10 - V_{\text{Th}} = 0$$

$$10(-1) - 10 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -20 \text{ V}$$

Step II Calculation of I_N (Fig. 2.186)

From Fig. 2.186,

$$I_x = I_1 - I_2 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 1 \qquad \ldots\text{(ii)}$$

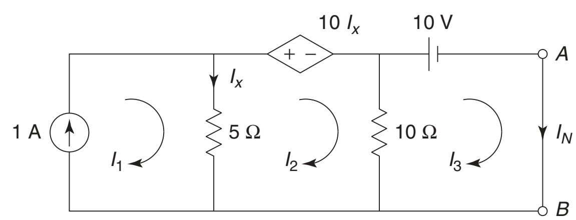


Fig. 2.186

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 10I_x - 10(I_2 - I_3) = 0$$

$$-5(I_2 - I_1) - 10(I_1 - I_2) - 10(I_2 - I_3) = 0$$

$$-5I_1 - 5I_2 + 10I_3 = 0 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$-10(I_3 - I_2) - 10 = 0$$

$$10I_2 - 10I_3 = 10 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 1 \text{ A}$$

$$I_2 = 3 \text{ A}$$

$$I_3 = 2 \text{ A}$$

$$I_N = I_3 = 2 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{-20}{2} = -10\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.187)

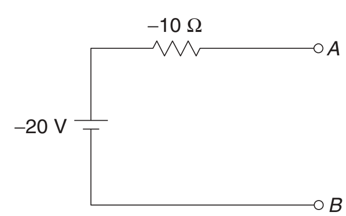


Fig. 2.187

Example 2.75 *Find Thevenin's equivalent network at terminals A and B in the network of Fig. 2.188.*

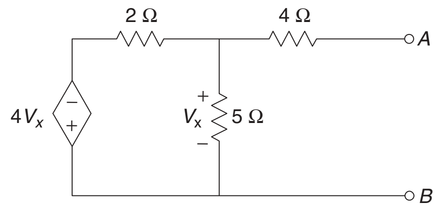


Fig. 2.188

Solution

Since the network does not contain any independent source,

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

But the R_{Th} can be calculated by applying a known voltage source of 1 V at the terminals A and B as shown in Fig. 2.189.

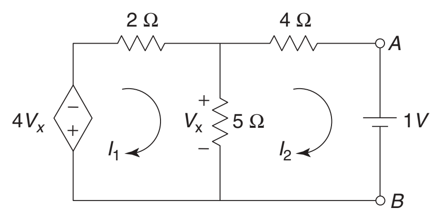


Fig. 2.189

$$R_{\text{Th}} = \frac{V}{I} = \frac{1}{I}$$

From Fig. 2.189,

$$V_x = 5(I_1 - I_2) \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4V_x - 2I_1 - 5(I_1 - I_2) = 0$$

$$-4\left[5(I_1 - I_2)\right] - 2I_1 - 5I_1 + 5I_2 = 0$$

$$-27I_1 + 25I_2 = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 4I_2 - 1 = 0$$

$$5I_1 - 9I_2 = 1 \quad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -0.21 \text{ A}$$

$$I_2 = -0.23 \text{ A}$$

Hence, current supplied by voltage source of 1 V is 0.23 A.

$$R_{\text{Th}} = \frac{1}{0.23} = 4.35\ \Omega$$

Hence, Thevenin's equivalent network is shown in Fig. 2.190.

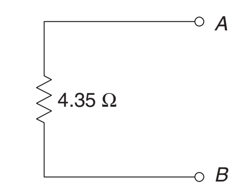


Fig. 2.190

Example 2.76 *Find the current in the 9 Ω resistor in Fig. 2.191.*

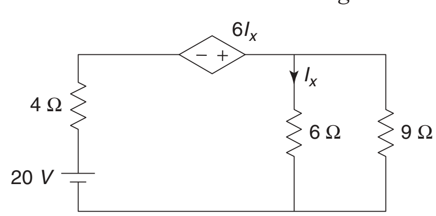


Fig. 2.191

Solution

Step I Calculation of V_{Th} (Fig. 2.192)
Applying KVL to the mesh,

$$20 - 4I_x + 6I_x - 6I_x = 0$$
$$I_x = 5 \text{ A}$$

Writing the V_{Th} equation,

$$6I_x - V_{Th} = 0$$
$$6(5) - V_{Th} = 0$$
$$V_{Th} = 30 \text{ V}$$

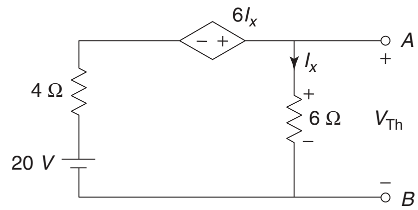


Fig. 2.192

Step II Calculation of I_N (Fig. 2.193).
From Fig. 2.193,

$$I_x = 0$$

The dependent source $6I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e., $6I_x = 0$ as shown in Fig. 2.194.

$$I_N = \frac{20}{4} = 5 \text{ A}$$

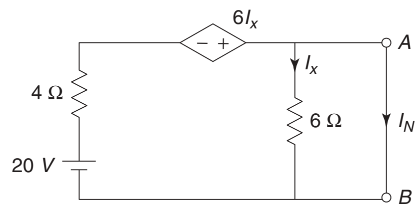


Fig. 2.193

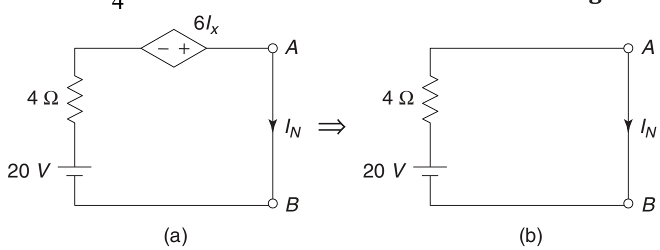


Fig. 2.194

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{30}{5} = 6\ \Omega$$

Step IV Calculation of I_L (Fig. 2.195)

$$I_L = \frac{30}{6+9} = 2 \text{ A}$$

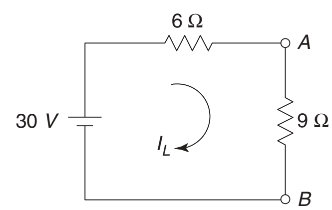


Fig. 2.195

Example 2.77 *Determine the current in the 16 Ω resistor in Fig. 2.196.*

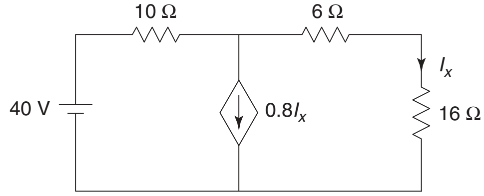


Fig. 2.196

Solution

Step I Calculation of V_{Th} (Fig. 2.197)
From Fig. 2.197,

$$I_x = 0$$

The dependent source $0.8I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, as shown in Fig. 2.198.

i.e.,
$$0.8I_x = 0$$
$$V_{Th} = 40\text{ V}$$

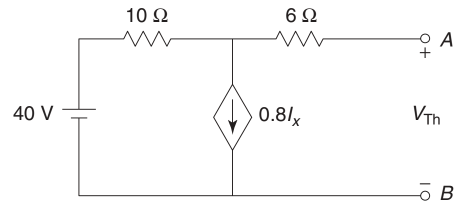


Fig. 2.197

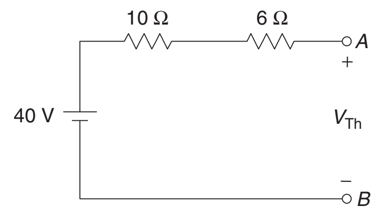


Fig. 2.198

Step II Calculation of I_N (Fig. 2.199)
From Fig. 2.199,

$$I_x = I_2 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh,
Writing current equation for the supermesh,

$$I_1 - I_2 = 0.8\, I_x = 0.8\, I_2$$
$$I_1 - 1.8\, I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$40 - 10\, I_1 - 6\, I_2 = 0$$
$$10\, I_1 + 6\, I_2 = 40 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 3\text{ A}$$
$$I_2 = \frac{5}{3}\text{ A}$$
$$I_N = I_2 = \frac{5}{3}\text{ A}$$

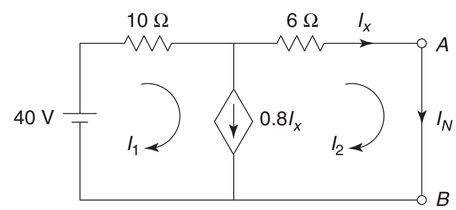


Fig. 2.199

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{40}{\frac{5}{3}} = 24\ \Omega$$

Step IV Calculation of I_L (Fig. 2.200)

$$I_L = \frac{40}{24 + 16} = 1\text{ A}$$

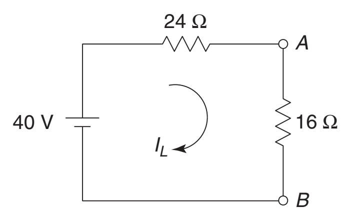


Fig. 2.200

Example 2.78 *Find the current in the 6 Ω resistor in Fig. 2.201.*

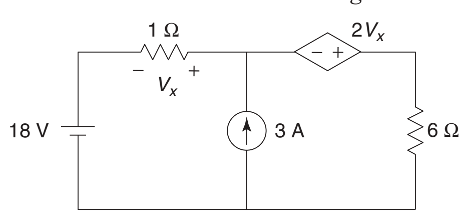


Fig. 2.201

Solution

Step I Calculation of V_{Th} (Fig. 2.202)

From Fig. 2.202,

$$V_x = -1I_1 = -I_1 \qquad \ldots(i)$$

For Mesh 1,

$$I_1 = -3 \text{ A} \qquad \ldots(ii)$$

$$V_x = 3 \text{ V}$$

Writing the V_{Th} equation,

$$18 - 1\,I_1 + 2\,V_x - V_{Th} = 0$$

$$18 + 3 + 2(3) - V_{Th} = 0$$

$$V_{Th} = 27 \text{ V}$$

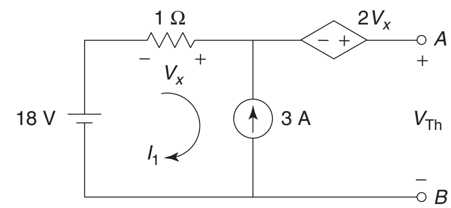


Fig. 2.202

Step II Calculation of I_N (Fig. 2.203)

From Fig. 2.203,

$$V_x = -I_1 \qquad \ldots(i)$$

Meshes 1 and 2 will form a supermesh,

Writing current equation for supermesh,

$$I_2 - I_1 = 3 \qquad \ldots(ii)$$

Applying KVL to the outer path of the supermesh,

$$18 - 1I_1 + 2\,V_x = 0$$

$$18 - I_1 + 2(-I_1) = 0 \qquad \ldots(iii)$$

$$I_1 = 6 \text{ A}$$

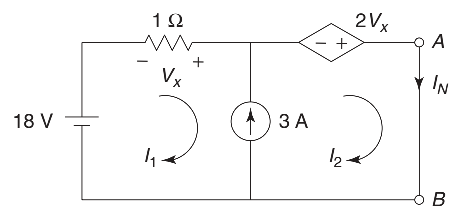


Fig. 2.203

Solving Eqs (ii) and (iii),

$$I_2 = 9 \text{ A}$$

$$I_N = I_2 = 9 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{27}{9} = 3\ \Omega$$

Step IV Calculation of I_L (Fig. 2.204)

$$I_L = \frac{27}{3+6} = 3 \text{ A}$$

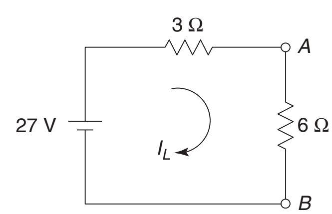


Fig. 2.204

Example 2.79 *Find the current in the 10 Ω resistor.*

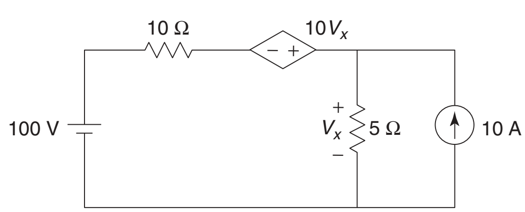


Fig. 2.205

Solution

Step I Calculation of V_{Th} (Fig. 2.206)

From Fig. 2.206,

$$V_x = 10 \times 5 = 50\text{ V}$$

Writing the V_{Th} equation,

$$100 - V_{Th} + 10V_x - V_x = 0$$
$$100 - V_{Th} + 9V_x = 0$$
$$100 - V_{Th} + 9(50) = 0$$
$$V_{Th} = 550\text{ V}$$

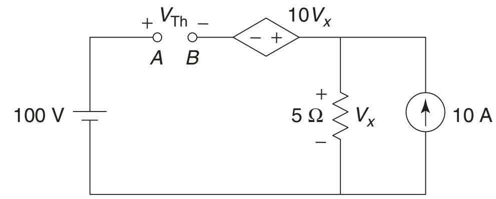


Fig. 2.206

Step II Calculation of I_N (Fig. 2.207)

From Fig. 2.207,

$$V_x = 5(I_N + 10)$$

Applying KVL to Mesh 1,

$$100 + 10V_x - V_x = 0$$
$$V_x = -\frac{100}{9}$$
$$-\frac{100}{9} = 5I_N + 50$$
$$I_N = -\frac{550}{45}\text{ A}$$

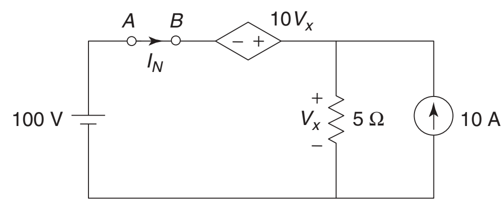


Fig. 2.207

Step III Calculation of R_{Th}

$$R_{Th} = \frac{550}{-\frac{550}{45}} = -45\ \Omega$$

Step IV Calculation of I_L (Fig. 2.208)

$$I_L = \frac{550}{-45 + 10} = -\frac{110}{7}\text{ A}$$

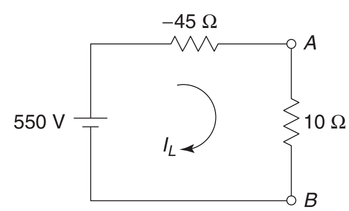


Fig. 2.208

2.9 NORTON'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.*' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.

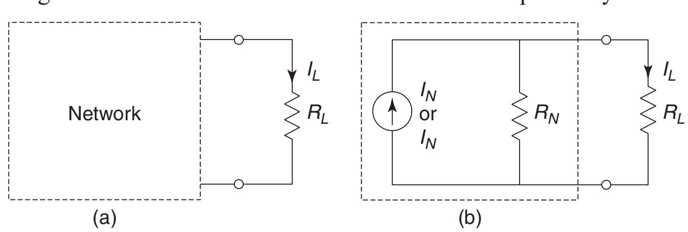


Fig. 2.209 *Network illustrating Norton's theorem*

Explanation Consider a simple network as shown in Fig. 2.210.

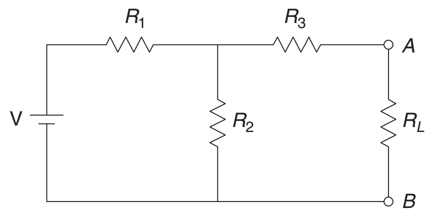


Fig. 2.210 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate short circuit current I_{SC} or I_N which would flow in a short circuit placed across terminals A and B as shown in Fig. 2.211.

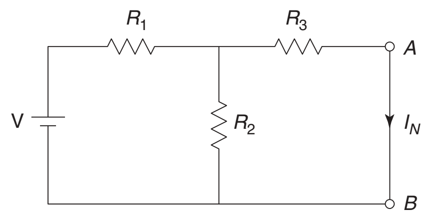


Fig. 2.211 *Calculation of I_N*

For finding parallel resistance R_N, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 2.212.

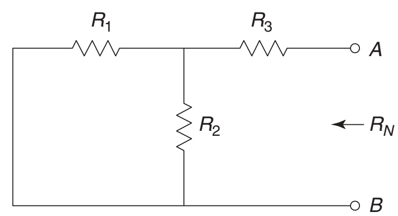


Fig. 2.212 *Calculation of R_N*

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Norton's equivalent network is shown in Fig. 2.213.

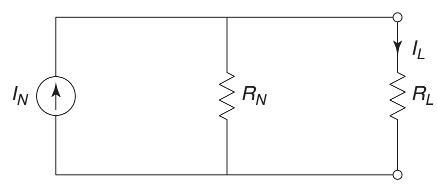


Fig. 2.213 *Norton's equivalent network*

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

If the network contains both independent and dependent sources, Norton's resistances R_N is calculated as

$$R_N = \frac{V_{\text{Th}}}{I_N}$$

where V_{Th} is the open-circuit voltage across terminals A and B. If the network contains only dependent sources, then

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

To find R_{Th} in such network, a known voltage V or current I is applied across the terminals A and B, and the current I or the voltage V is calculated respectively.

$$R_N = \frac{V}{I}$$

Norton's equivalent network for such a network is shown in Fig. 2.214.

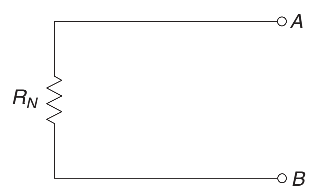


Fig. 2.214 *Norton's equivalent network*

Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{SC} or I_N.
3. Find the resistance R_N as seen from points *A* and *B*.
4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_L by current–division rule.

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 2.80 *Find the current through the 10 Ω resistor in Fig. 2.215.*

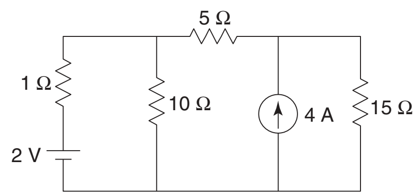


Fig. 2.215

Solution

Step I Calculation of I_N (Fig. 2.216)

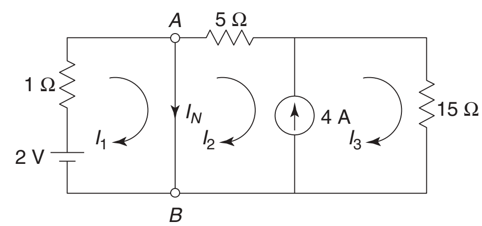


Fig. 2.216

Applying KVL to Mesh 1,

$$2 - 1I_1 = 0$$
$$I_1 = 2 \qquad \text{...(i)}$$

Meshes 2 and 3 will form a supermesh.
Writing the current equation for the supermesh,

$$I_3 - I_2 = 4 \qquad \text{...(ii)}$$

Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 2\text{ A}$$
$$I_2 = -3\text{ A}$$
$$I_3 = 1\text{A}$$
$$I_N = I_1 - I_2 = 2 - (-3) = 5\text{A}$$

Step II Calculation of R_N (Fig. 2.217)

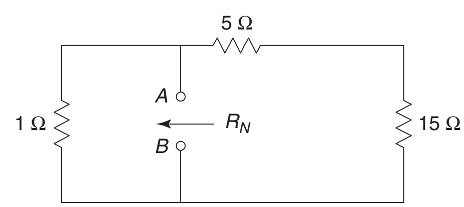


Fig. 2.217

$$R_N = 1\,||\,(5+15) = 0.95\ \Omega$$

Step III Calculation of I_L (Fig. 2.218)

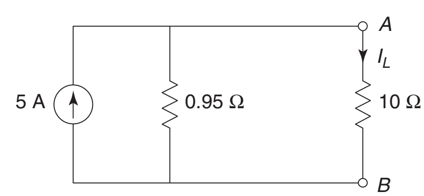


Fig. 2.218

$$I_L = 5 \times \frac{0.95}{0.95+10} = 0.43\text{A}$$

Example 2.81 *Find the current through the 10 Ω resistor in Fig. 2.219.*

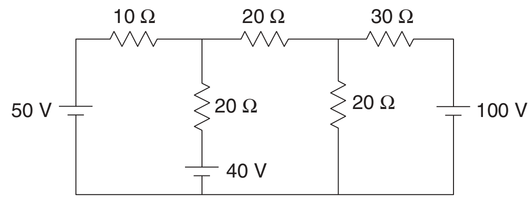


Fig. 2.219

Solution

Step I Calculation of I_N (Fig. 2.220)

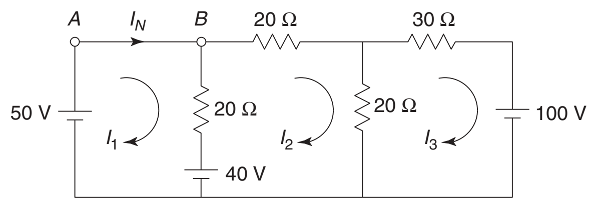


Fig. 2.220

Applying KVL to Mesh 1,

$$50-20(I_1-I_2)-40=0$$
$$20I_1-20I_2=10 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$40-20(I_2-I_1)-20I_2-20(I_2-I_3)=0$$
$$-20I_1+60I_2-20I_3=40 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-20(I_3-I_2)-30I_3-100=0$$
$$-20I_2+50I_3=-100 \quad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=0.81\text{A}$$
$$I_N=I_1=0.81\text{A}$$

Step II Calculation of R_N (Fig. 2.221)

$$R_N=[(20\,||\,30)+20]\,||\,20=12.3\,\Omega$$

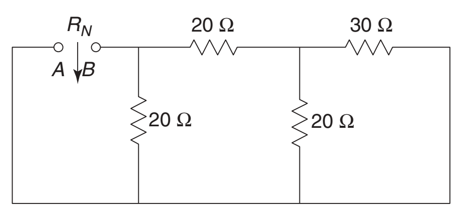


Fig. 2.221

Step III Calculation of I_L (Fig. 2.222)

$$I_L=0.81\times\frac{12.3}{12.3+10}=0.45\,\text{A}$$

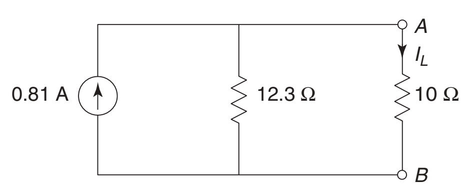


Fig. 2.222

Example 2.82 *Find the current through the 8 Ω resistor in Fig. 2.223.*

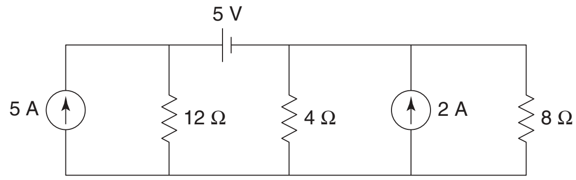


Fig. 2.223

Solution

Step I Calculation of I_N (Fig. 2.224)

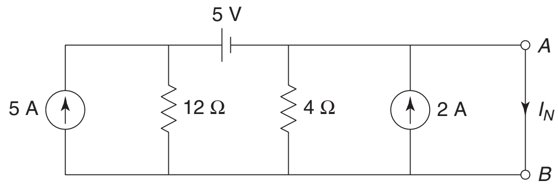


Fig. 2.224

The resistor of the 4 Ω gets shorted as it is in parallel with the short circuit. Simplifying the network by source transformation (Fig. 2.225),

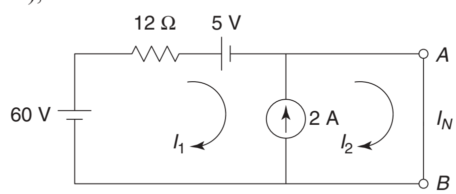


Fig. 2.225

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 2 \qquad \text{...(i)}$$

Applying KVL to the supermesh,

$$60 - 12I_1 - 5 = 0$$

$$12I_1 = 55 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 4.58\,\text{A}$$

$$I_2 = 6.58\,\text{A}$$

$$I_N = I_2 = 6.58\,\text{A}$$

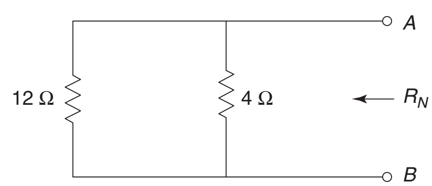


Fig. 2.226

Step II Calculation of R_N (Fig. 2.226)

$$R_N = 12 \,||\, 4 = 3\,\Omega$$

Step III Calculation of I_L (Fig. 2.227)

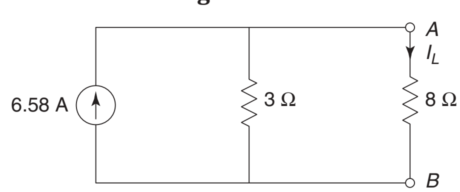


Fig. 2.227

$$I_L = 6.58 \times \frac{3}{3+8} = 1.79\,\text{A}$$

Example 2.83 *Find the current through the 1 Ω resistor in Fig. 2.228.*

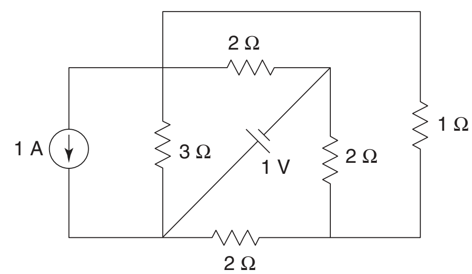


Fig. 2.228

Solution

Step I Calculation of I_N (Fig. 2.229)

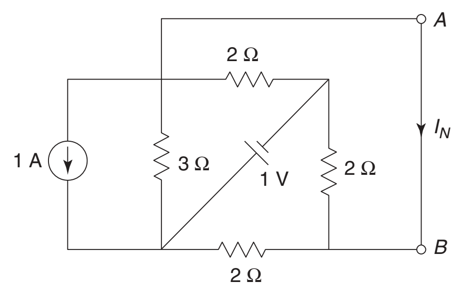


Fig. 2.229

By source transformation (Fig. 2.230),

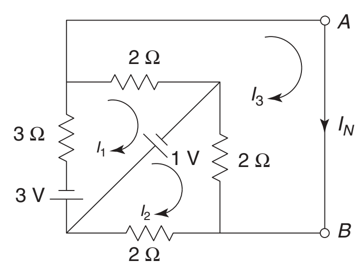


Fig. 2.230

Applying KVL to Mesh 1,

$$-3-3I_1-2(I_1-I_3)+1=0$$

$$5I_1-2I_3=-2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-1-2(I_2-I_3)-2I_2=0$$
$$4I_2-2I_3=-1 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3-I_1)-2(I_3-I_2)=0$$
$$-2I_1-2I_2+4I_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=-0.64\,\text{A}$$
$$I_2=-0.55\,\text{A}$$
$$I_3=-0.59\,\text{A}$$
$$I_N=I_3=-0.59\,\text{A}$$

Step II Calculation of R_N (Fig. 2.231)

$$R_N=2.2\,\Omega$$

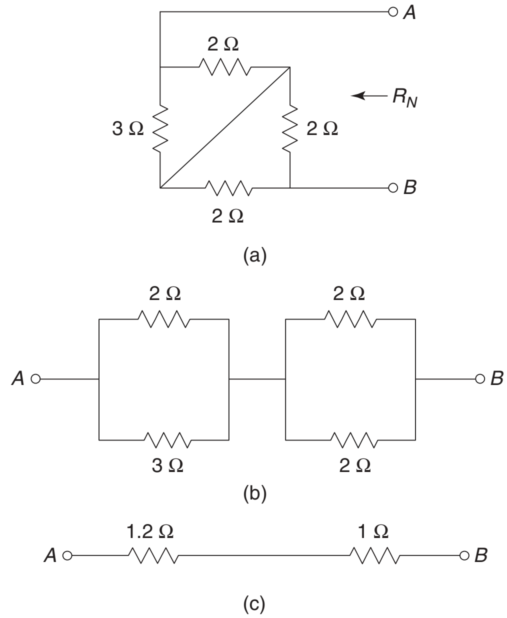


Fig. 2.231

Step III Calculation of I_L (Fig. 2.232)

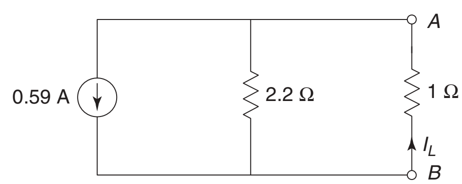


Fig. 2.232

$$I_L=0.59\times\frac{2.2}{2.2+1}=0.41\,\text{A}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.84 *Find Norton's equivalent network across terminals A and B of Fig. 2.233.*

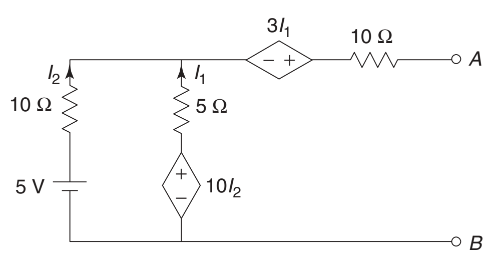


Fig. 2.233

Solution

Step I Calculation of V_{Th} (Fig. 2.234)

From Fig. 2.234,

$$I_2 = I_x$$
$$I_1 = -I_x$$

Applying KVL to the mesh,

$$5 - 10I_x - 5I_x - 10I_2 = 0$$
$$5 - 10I_x - 5I_x - 10I_x = 0$$
$$I_x = 0.2 \text{ A}$$
$$I_1 = -0.2 \text{ A}$$

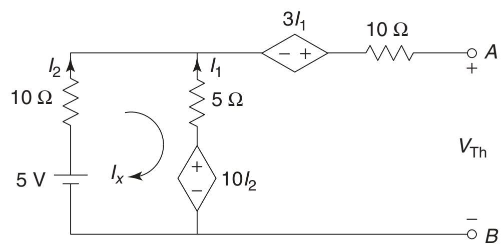


Fig. 2.234

Writing the V_{Th} equation,

$$5 - 10I_x + 3I_1 - V_{Th} = 0$$
$$5 - 10(0.2) + 3(-0.2) - V_{Th} = 0$$
$$V_{Th} = 2.4 \text{ V}$$

Step II Calculation of I_N (Fig. 2.235)

From Fig. 2.235,

$$I_2 = I_x \qquad \ldots\text{(i)}$$
$$I_1 = I_y - I_x \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$5 - 10I_x - 5(I_x - I_y) - 10I_2 = 0$$
$$5 - 10I_x - 5I_x + 5I_y - 10I_x = 0$$
$$25I_x - 5I_y = 5 \qquad \ldots\text{(iii)}$$

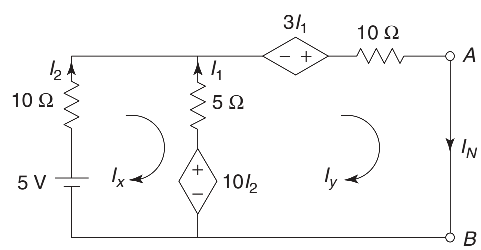


Fig. 2.235

Applying KVL to Mesh 2,

$$10I_2 - 5(I_y - I_x) + 3I_1 - 10I_y = 0$$
$$10I_x - 5I_y + 5I_x + 3(I_y - I_x) - 10I_y = 0$$
$$12I_x - 12I_y = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (iii) and (iv),

$$I_x = 0.25 \text{ A}$$
$$I_y = 0.25 \text{ A}$$
$$I_N = I_y = 0.25 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{2.4}{0.25} = 9.6\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.236)

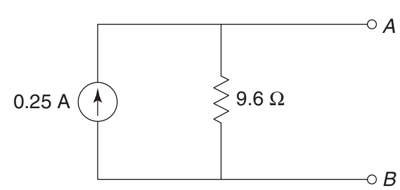


Fig. 2.236

Example 2.85 *For the network shown in Fig. 2.237, find Norton's equivalent network.*

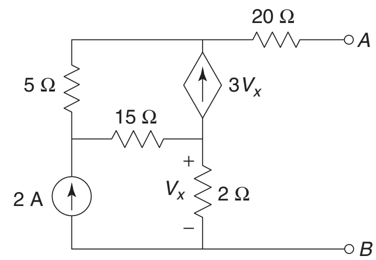


Fig. 2.237

Solution

Step I Calculation of V_{Th} (Fig. 2.238)

From Fig. 2.238,

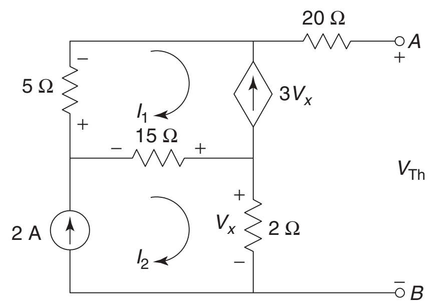


Fig. 2.238

$$V_x = 2I_2 \quad \text{...(i)}$$

For Mesh 1,

$$I_1 = -3V_x = -3(2I_2) = -6I_2 \quad \text{...(ii)}$$

For Mesh 2,

$$I_2 = 2 \quad \text{...(iii)}$$

$$I_1 = -6I_2 = -6(2) = -12 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 0 + 5I_1 + 15(I_1 - I_2) - 2I_2 = 0$$
$$V_{Th} + 5(-12) + 15(-12 - 2) - 2(2) = 0$$
$$V_{Th} = 274 \text{ V}$$

Step II Calculation of I_N (Fig. 2.239)

From Fig. 2.239,

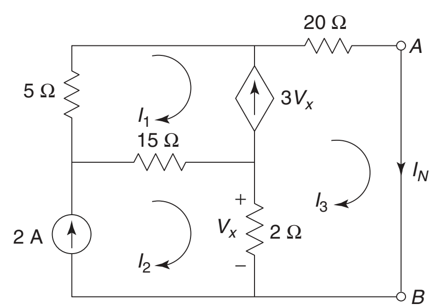


Fig. 2.239

$$V_x = 2(I_2 - I_3) \quad \text{...(i)}$$

For Mesh 2,

$$I_2 = 2 \quad \text{...(ii)}$$

Meshes 1 and 3 will form a supermesh.

Writing the current equation for the supermesh,

$$I_3 - I_1 = 3V_x = 3[2(I_2 - I_3)] = 6I_2 - 6I_3$$

$$I_1 + 6I_2 - 7I_3 = 0 \quad \text{...(iii)}$$

Applying KVL to the outer path of the supermesh,

$$-5I_1 - 20I_3 - 2(I_3 - I_2) - 15(I_1 - I_2) = 0$$
$$-20I_1 + 17I_2 - 22I_3 = 0 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -0.16 \text{ A}$$
$$I_2 = 2 \text{ A}$$
$$I_3 = 1.69 \text{ A}$$
$$I_N = I_3 = 1.69 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{274}{1.69} = 162.13\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.240)

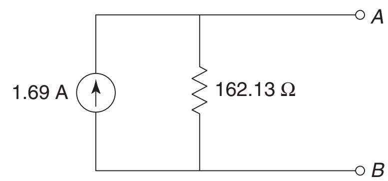


Fig. 2.240

Example 2.86 *Obtain Norton's equivalent network across A-B in the network of Fig. 2.241.*

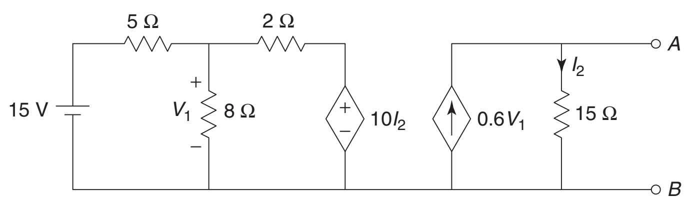


Fig. 2.241

Solution

Step I Calculation of V_{Th} (Fig. 2.242)

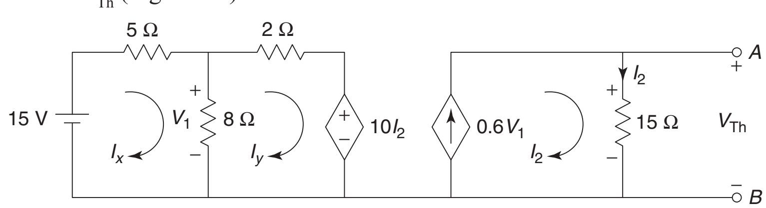


Fig. 2.242

From Fig. 2.242,

$$V_1 = 8(I_x - I_y) \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$15 - 5I_x - 8(I_x - I_y) = 0$$
$$13I_x - 8I_y = 15 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-8(I_y - I_x) - 2I_y - 10I_2 = 0$$
$$8I_x - 10I_y - 10I_2 = 0 \quad \text{...(iii)}$$

For Mesh 3,

$$I_2 = 0.6V_1 = 0.6\left[8(I_x - I_y)\right]$$

$$4.8I_x - 4.8I_y - I_2 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_x = 3.28 \text{ A}$$
$$I_y = 3.45 \text{ A}$$
$$I_2 = -0.83 \text{ A}$$

Writing the V_{Th} equation,

$$15I_2 - V_{\text{Th}} = 0$$
$$15(-0.83) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = -12.45 \text{ V}$$

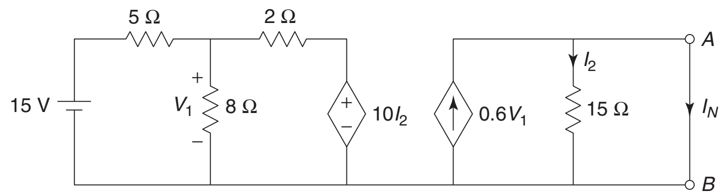


Fig. 2.243

Step II Calculation of I_N (Fig. 2.243)

From Fig. 2.243,

$$I_2 = 0$$

The dependent source of $10\,I_2$ depends on the controlling variable I_2. When $I_2 = 0$, the dependent source vanishes, i.e. $10I_2 = 0$ as shown in Fig. 2.244.

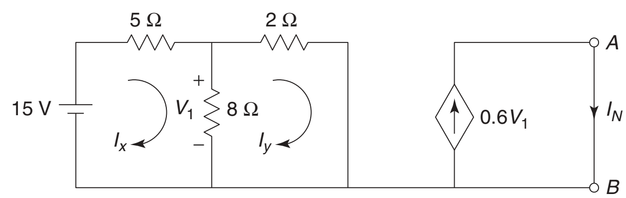


Fig. 2.244

From Fig. 2.244,

$$V_1 = 8(I_x - I_y) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$15 - 5I_x - 8(I_x - I_y) = 0$$
$$13I_x - 8I_y = 15 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-8(I_y - I_x) - 2I_y = 0$$
$$-8I_x + 10I_y = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_x = 2.27 \text{ A}$$
$$I_y = 1.82 \text{ A}$$
$$V_1 = 8(I_x - I_y) = 8(2.27 - 1.82) = 3.6 \text{ V}$$

For Mesh 3,

$$I_N = 0.6V_1 = 0.6(3.6) = 2.16 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-12.45}{2.16} = -5.76\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.245)

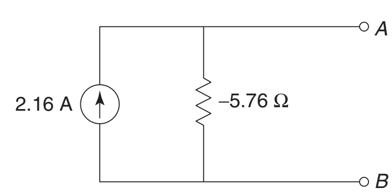


Fig. 2.245

Example 2.87 *Find Norton's equivalent network of Fig. 2.246.*

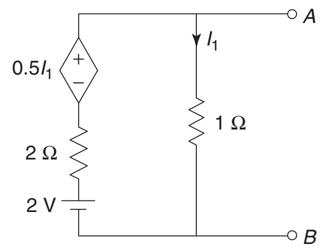


Fig. 2.246

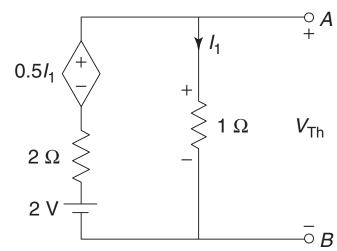


Fig. 2.247

Solution

Step I Calculation of V_{Th} (Fig. 2.247)

Applying KVL to the mesh,

$$2 - 2I_1 + 0.5I_1 - 1I_1 = 0$$

$$2 - 2.5I_1 = 0$$

$$I_1 = 0.8 \text{ A}$$

Writing the V_{Th} equation,

$$1I_1 - V_{Th} = 0$$

$$1(0.8) - V_{Th} = 0$$

$$V_{Th} = 0.8 \text{ V}$$

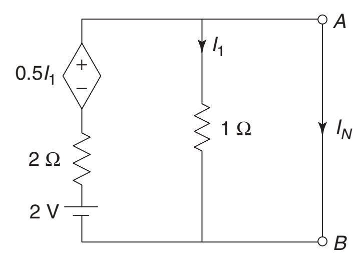


Fig. 2.248

Step II Calculation of I_N (Fig. 2.248)

When a short circuit is placed across the 1 Ω resistor, it gets shorted.

$$I_1 = 0$$

The dependent source of $0.5I_1$ depends on the controlling variable I_1. When $I_1 = 0$, the dependent source vanishes, i.e. $0.5\, I_1 = 0$ as shown in Fig. 2.249.

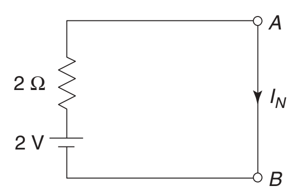


Fig. 2.249

$$I_N = \frac{2}{2} = 1 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{0.8}{1} = 0.8\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.250)

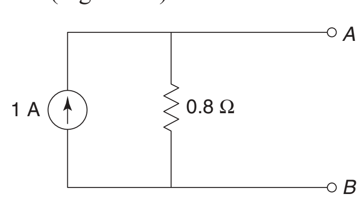


Fig. 2.250

Example 2.88 *Find Norton's equivalent network at the terminals A and B of Fig. 2.251.*

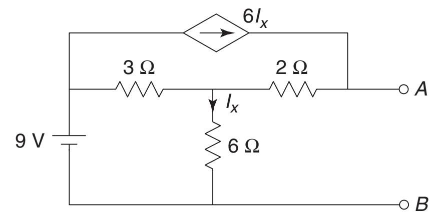


Fig. 2.251

Solution

Step I Calculation of V_{Th} (Fig. 2.252)

From Fig. 2.252,

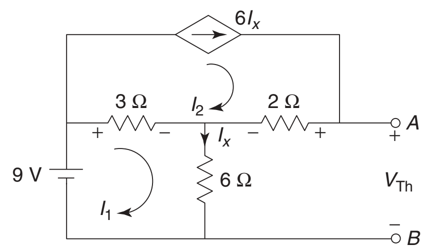


Fig. 2.252

$$I_x = I_1 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$9 - 3(I_1 - I_2) - 6I_1 = 0$$
$$9I_1 - 3I_2 = 9 \quad \ldots\text{(ii)}$$

For Mesh 2,

$$I_2 = 6I_x = 6I_1$$
$$6I_1 - I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -1 \text{ A}$$
$$I_2 = -6 \text{ A}$$

Writing the V_{Th} equation,

$$9 - 3(I_1 - I_2) + 2I_2 - V_{Th} = 0$$
$$9 - 3(-1 + 6) + 2(-6) - V_{Th} = 0$$
$$V_{Th} = -18 \text{ V}$$

Step II Calculation of I_N (Fig. 2.253)

From Fig. 2.253,

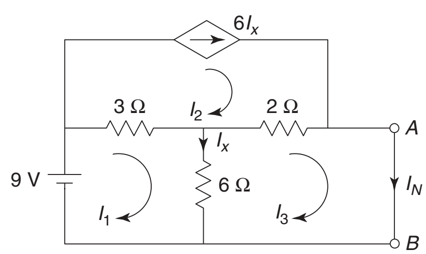


Fig. 2.253

$$I_x = I_1 - I_3 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$9 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$
$$9I_1 - 3I_2 - 6I_3 = 9 \quad \ldots\text{(ii)}$$

For Mesh 2,

$$I_2 = 6I_x = 6(I_1 - I_3)$$
$$6I_1 - I_2 - 6I_3 = 0 \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) - 2(I_3 - I_2) = 0$$
$$-6I_1 - 2I_2 + 8I_3 = 0 \quad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 5\text{ A}$$
$$I_2 = 3\text{ A}$$
$$I_3 = 4.5\text{ A}$$
$$I_N = I_3 = 4.5\text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-18}{4.5} = -4\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.254)

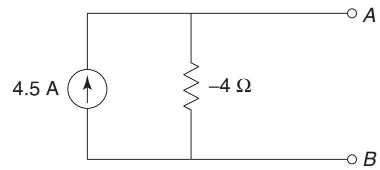


Fig. 2.254

Example 2.89 *Find Norton's equivalent network to the left of terminal A-B in Fig. 2.255.*

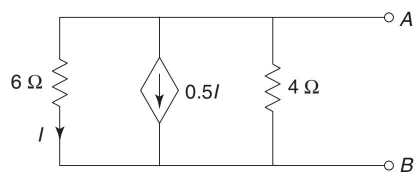


Fig. 2.255

Solution Since the network does not contain any independent source,

$$V_{\text{Th}} = 0$$
$$I_N = 0$$

But R_N can be calculated by applying a known current source of 1 A at the terminals A and B as shown in Fig. 2.256.

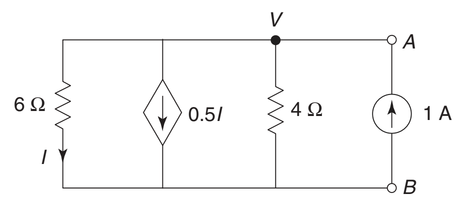


Fig. 2.256

From Fig. 2.256,

$$I = \frac{V}{6}$$

Applying KCL at the node,

$$\frac{V}{6} + 0.5I + \frac{V}{4} = 1$$

$$\frac{V}{6} + 0.5\left(\frac{V}{6}\right) + \frac{V}{4} = 1$$

$$\left(\frac{1}{6} + \frac{0.5}{6} + \frac{1}{4}\right)V = 1$$

$$V = 2$$

$$R_N = \frac{V}{1} = \frac{2}{1} = 2\ \Omega$$

Hence, Norton's equivalent network is shown in Fig. 2.257.

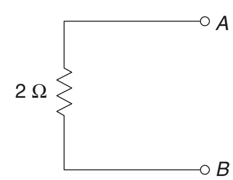


Fig. 2.257

Example 2.90 *Find the current through the 2 Ω resistor in the network shown in Fig. 2.258.*

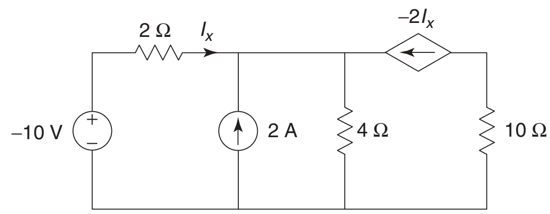


Fig. 2.258

Solution

Step I Calculation of V_{Th} (Fig. 2.259)

From Fig. 2.259,

$$I_x = 0$$

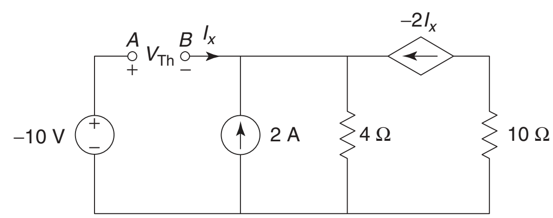


Fig. 2.259

The dependent source of –2 I_x depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e. $-2I_x = 0$ as shown in Fig. 2.260.

$$I_1 = 2$$

Writing the V_{Th} equation,

$$-10 - V_{Th} - 4I_1 = 0$$

$$-10 - V_{Th} - 4(2) = 0$$

$$V_{Th} = -18 \text{ V}$$

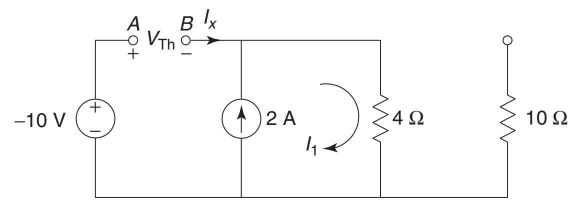


Fig. 2.260

Step II Calculation of I_N (Fig. 2.261)

From Fig. 2.261,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Mesh 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

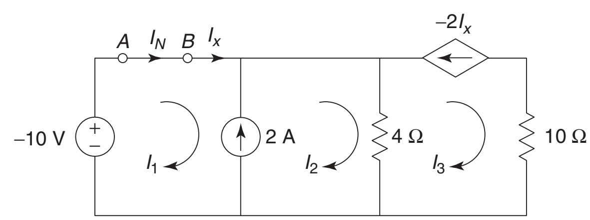


Fig. 2.261

Applying KVL to the outer path of the supermesh,

$$-10 - 4(I_2 - I_3) = 0$$

$$-4I_2 + 4I_3 = 10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = -(-2I_x) = 2I_x = 2I_1$$

$$2I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 4.5 \text{ A}$$
$$I_2 = 6.5 \text{ A}$$
$$I_3 = 9 \text{ A}$$
$$I_N = I_1 = 4.5 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-18}{4.5} = -4\ \Omega$$

Step IV Calculation of I_L (Fig. 2.262)

$$I_L = 4.5 \times \frac{-4}{-4+2} = 9 \text{ A}$$

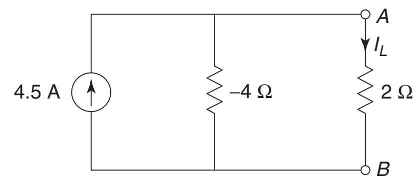


Fig. 2.262

Example 2.91 *Find the current through the 2Ω resistor in the network of Fig. 2.263.*

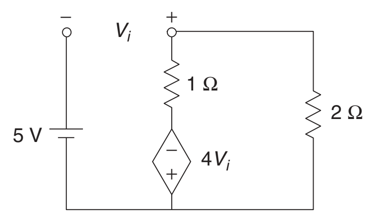


Fig. 2.263

Solution

Step I Calculation of V_{Th} (Fig. 2.264)

From Fig. 2.264,

$$5 + V_i + 4V_i = 0$$
$$V_i = -1 \text{ V}$$

Writing the V_{Th} equation,

$$-4V_i - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = -4V_i = -4(-1) = 4 \text{ V}$$

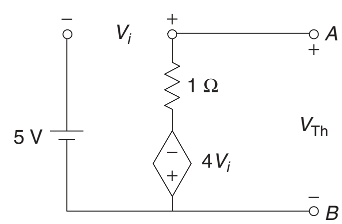


Fig. 2.264

Step II Calculation of I_N (Fig. 2.265)

From Fig. 2.265,

$$5 + V_i = 0$$
$$V_i = -5 \text{ V}$$

Applying KVL to the mesh,

$$-4V_i - 1I_N = 0$$
$$I_N = -4V_i = -4(-5) = 20 \text{ A}$$

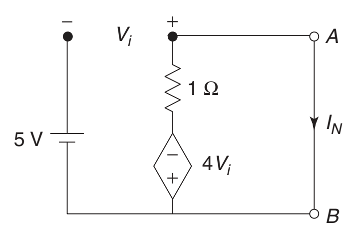


Fig. 2.265

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{4}{20} = 0.2\ \Omega$$

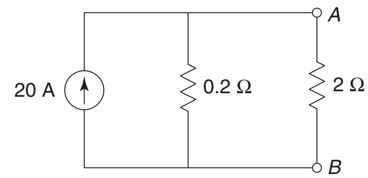


Fig. 2.266

Step IV Calculation of I_L (Fig. 2.266)

$$I_L = 20 \times \frac{0.2}{0.2+2} = 1.82\ \text{A}$$

Example 2.92 *Find the current in the 2 Ω resistor in the network of Fig. 2.267.*

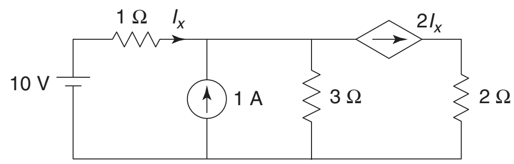


Fig. 2.267

Solution

Step I Calculation of V_{Th} (Fig. 2.268)

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \ldots\text{(i)}$$

Applying KVL to the outer path of the supermesh,

$$10 - 1I_1 - 3I_2 = 0$$

$$I_1 + 3I_2 = 10 \qquad \ldots\text{(ii)}$$

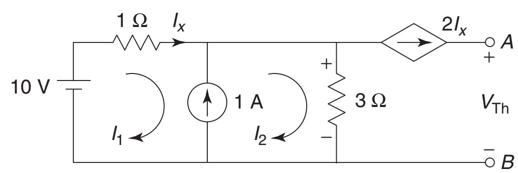


Fig. 2.268

Solving Eqs (i) and (ii),

$$I_1 = 1.75\ \text{A}$$

$$I_2 = 2.75\ \text{A}$$

Writing the V_{Th} equation,

$$3I_2 - V_{Th} = 0$$

$$3(2.75) - V_{Th} = 0$$

$$V_{Th} = 8.25\ \text{V}$$

Step II Calculation of I_N (Fig. 2.269)

From Fig. 2.269,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \ldots\text{(ii)}$$

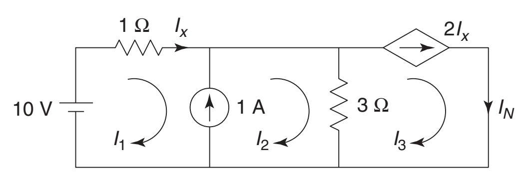


Fig. 2.269

Applying KVL to the outer path of the supermesh,

$$10 - 1I_1 - 3(I_2 - I_3) = 0$$

$$I_1 + 3I_2 - 3I_3 = 10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = 2I_x = 2I_1$$

$$2I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -3.5 \text{ A}$$

$$I_2 = -2.5 \text{ A}$$

$$I_3 = -7 \text{ A}$$

$$I_N = I_3 = -7 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{8.25}{-7} = -1.18\ \Omega$$

Step IV Calculation of I_L (Fig. 2.270)

$$I_L = -7 \times \frac{-1.18}{-1.18 + 2} = 10.07 \text{ A}$$

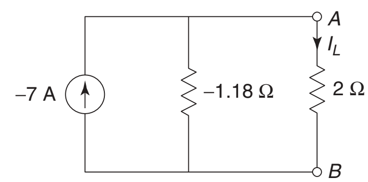


Fig. 2.270

Example 2.93 *Find the current through the 10 Ω resistor for the network of Fig. 2.271.*

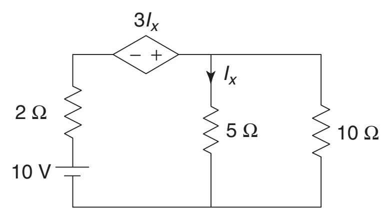


Fig. 2.271

Solution

Step I Calculation of V_{Th} (Fig. 2.272)
Applying KVL to the mesh,

$$10 - 2I_x + 3I_x - 5I_x = 0$$

$$I_x = 2.5 \text{ A}$$

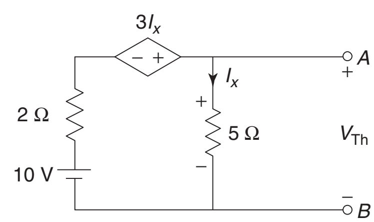


Fig. 2.272

Writing the V_{Th} equation,

$$5I_x - V_{Th} = 0$$

$$5(2.5) - V_{Th} = 0$$

$$V_{Th} = 12.5 \text{ V}$$

Step II Calculation of I_N (Fig. 2.273)
From Fig. 2.273,

$$I_x = 0$$

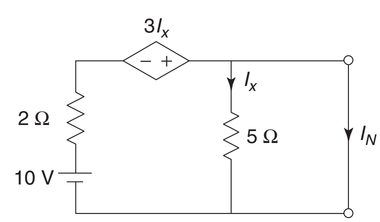


Fig. 2.273

The dependent source of 3 I_x depends on the controlling variable I_x. When $I_x = 0$, the dependent source 3 I_x vanishes, i.e. 3 $I_x = 0$ as shown in Fig. 2.274.

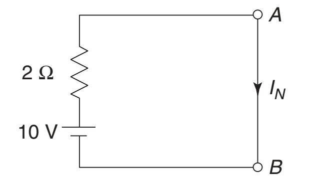


Fig. 2.274

$$I_N = \frac{10}{2} = 5 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{12.5}{5} = 2.5 \ \Omega$$

Step IV Calculation of I_L (Fig. 2.275)

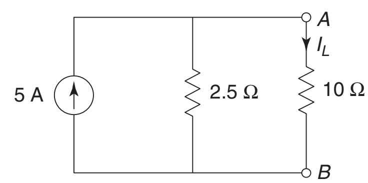


Fig. 2.275

$$I_L = 5 \times \frac{2.5}{2.5+10} = 1 \text{ A}$$

Example 2.94 *Find the current through the 5 Ω resistor in the network of Fig. 2.276.*

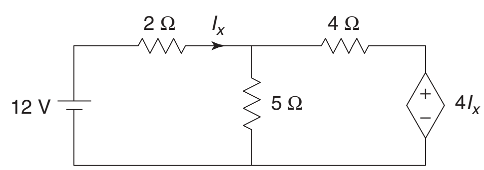


Fig. 2.276

Solution

Step I Calculation of V_{Th} (Fig. 2.277)

Applying KVL to the mesh,

$$12 - 2I_x - 4I_x - 4I_x = 0$$
$$12 - 10I_x = 0$$
$$I_x = 1.2 \text{ A}$$

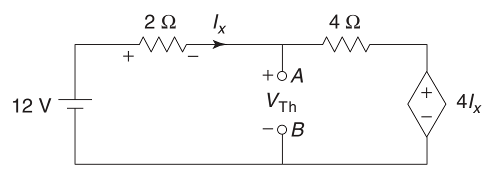


Fig. 2.277

Writing the V_{Th} equation,

$$12 - 2I_x - V_{Th} = 0$$
$$12 - 2(1.2) - V_{Th} = 0$$
$$V_{Th} = 9.6 \text{ V}$$

Step II Calculation of I_N (Fig. 2.278)

From Fig. 2.278,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$12 - 2I_1 = 0$$
$$I_1 = 6 \text{ A} \qquad \ldots\text{(ii)}$$

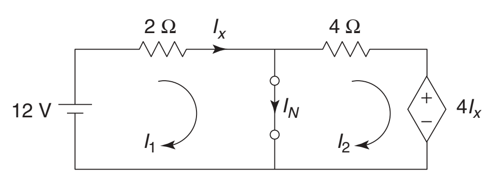


Fig. 2.278

Applying KVL to Mesh 2,

$$-4I_2 - 4I_x = 0$$
$$-4I_2 - 4I_1 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_2 = -6 \text{ A}$$
$$I_N = I_1 - I_2 = 6 - (-6) = 12 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{9.6}{12} = 0.8\ \Omega$$

Step IV Calculation of I_L (Fig. 2.279)

$$I_L = 12 \times \frac{0.8}{0.8+5} = 1.66\ \text{A}$$

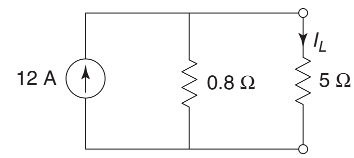


Fig. 2.279

Example 2.95 *Find the current through the 10 Ω resistor for the network of Fig. 2.280.*

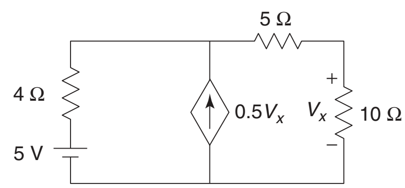


Fig. 2.280

Solution

Step I Calculation of V_{Th} (Fig. 2.281)

For the mesh,

$$I = -0.5V_x = -0.5V_{\text{Th}}$$

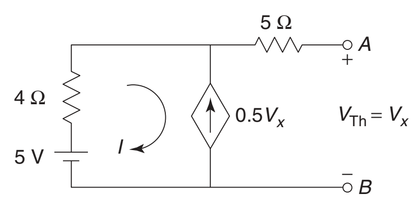


Fig. 2.281

Writing the V_{Th} equation,

$$5 - 4I - 0 - V_{\text{Th}} = 0$$

$$5 - 4(-0.5V_{\text{Th}}) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -5\ \text{V}$$

Step II Calculation of I_N (Fig. 2.282)

From Fig. 2.282,

$$V_x = 0$$

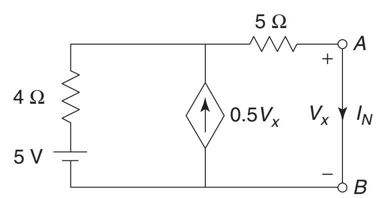


Fig. 2.282

The dependent source of 0.5 V_x depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e. 0.5 $V_x = 0$ as shown in Fig. 2.283.

$$I_N = \frac{5}{4+5} = \frac{5}{9}\ \text{A}$$

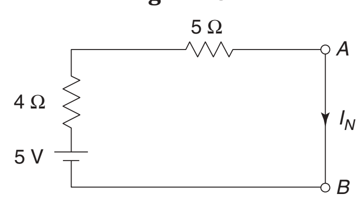


Fig. 2.283

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-5}{\frac{5}{9}} = -9\ \Omega$$

Step IV Calculation of I_L (Fig. 2.284)

$$I_L = \frac{5}{9} \times \frac{-9}{-9+10} = -5\ \text{A}$$

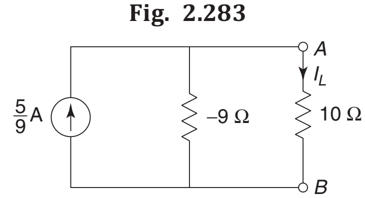


Fig. 2.284

Example 2.96 *Find the current through the 10 Ω resistor in the network shown in Fig. 2.285.*

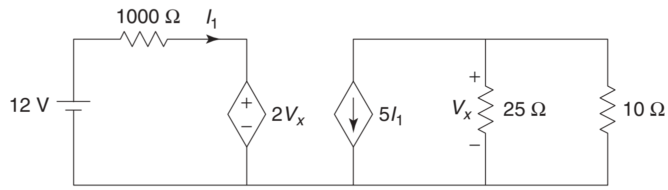


Fig. 2.285

Solution

Step I Calculation of V_{Th} (Fig. 2.286)

From Fig. 2.286,

$$V_x = -25(5I_1) = -125\,I_1 \quad \ldots\text{(i)}$$

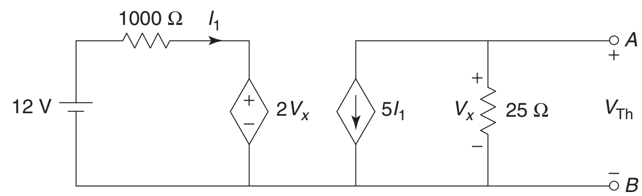


Fig. 2.286

Applying KVL to Mesh 1,

$$12 - 1000I_1 - 2V_x = 0$$

$$12 - 1000I_1 - 2(-125I_1) = 0 \quad \ldots\text{(ii)}$$

$$I_1 = 0.016 \text{ A}$$

$$V_x = -125I_1 = -125(0.016) = -2 \text{ V}$$

Writing the V_{Th} equation,

$$V_{Th} = V_x = -2 \text{ V}$$

Step II Calculation of I_N (Fig. 2.287)

From Fig. 2.287,

$$V_x = 0$$

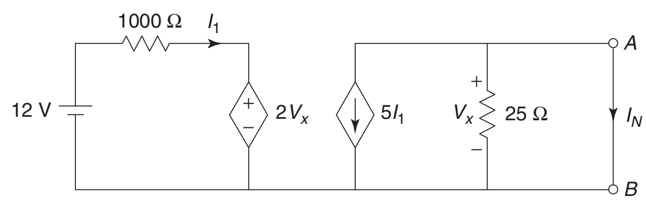


Fig. 2.287

The dependent source of $2V_x$ depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e. $2\,V_x = 0$, as shown in Fig. 2.288.

$$I_1 = \frac{12}{1000} = 0.012 \text{ A}$$

$$I_N = -5I_1 = -5(0.012) = -0.06 \text{ A}$$

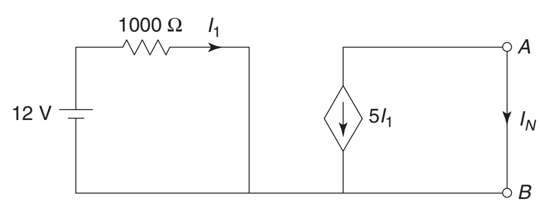


Fig. 2.288

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{-2}{-0.06} = 33.33\ \Omega$$

Step IV Calculation of I_L (Fig. 2.289)

$$I_L = -0.06 \times \frac{33.33}{33.33 + 10} = -0.046 \text{ A}$$

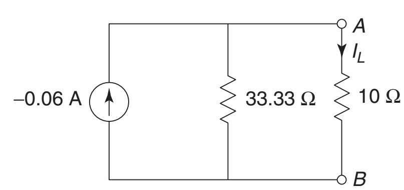


Fig. 2.289

Example 2.97 *Find the current through the 5 Ω resistor for the network of Fig. 2.290.*

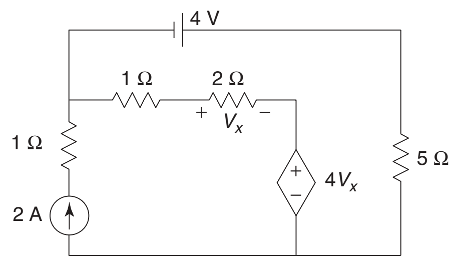


Fig. 2.290

Solution

Step I Calculation of V_{Th} (Fig. 2.291)

From Fig. 2.291,

$$V_x = 2I \qquad \ldots\text{(i)}$$

For the mesh,

$$I = 2 \qquad \ldots\text{(ii)}$$

$$V_x = 2(2) = 4 \text{ V}$$

Writing the V_{Th} equation,

$$4V_x + 2I + 1I + 4 - V_{Th} = 0$$

$$4(4) + 2(2) + 2 + 4 - V_{Th} = 0$$

$$V_{Th} = 26 \text{ V}$$

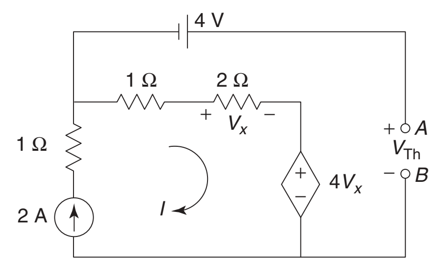


Fig. 2.291

Step II Calculation of I_N (Fig. 2.292)

From Fig. 2.292,

$$V_x = 2(I_1 - I_2) \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$4V_x - 2(I_2 - I_1) - 1(I_2 - I_1) + 4 = 0$$

$$4[2(I_1 - I_2)] - 2I_2 + 2I_1 - I_2 + I_1 + 4 = 0$$

$$11I_1 - 11I_2 = -4 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$

$$I_2 = 2.36 \text{ A}$$

$$I_N = I_2 = 2.36 \text{ A}$$

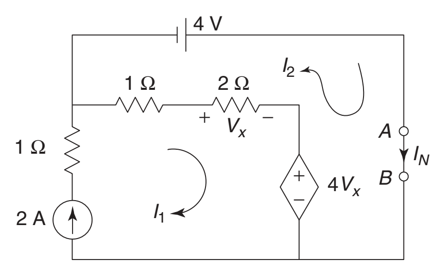


Fig. 2.292

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{26}{2.36} = 11.02 \ \Omega$$

Step IV Calculation of I_L (Fig. 2.293)

$$I_L = 2.36 \times \frac{11.02}{11.02 + 5} = 1.62 \text{ A}$$

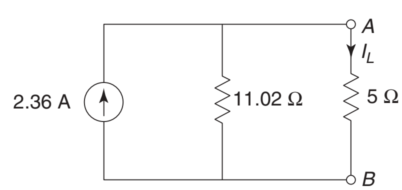


Fig. 2.293

Example 2.98 *Find the current through the 1 Ω resistor in the network of Fig. 2.294.*

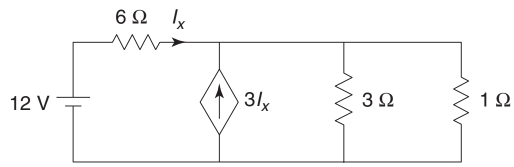


Fig. 2.294

Solution

Step I Calculation of V_{Th} (Fig. 2.295)

From Fig. 2.295,

$$I_x = I_1 \quad \ldots\text{(i)}$$

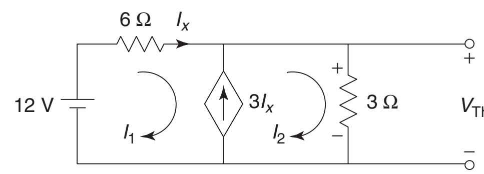


Fig. 2.295

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 3I_x = 3I_1$$

$$4I_1 - I_2 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$12 - 6I_1 - 3I_2 = 0$$

$$6I_1 + 3I_2 = 12 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.67 \text{ A}$$

$$I_2 = 2.67 \text{ A}$$

Writing the V_{Th} equation,

$$3I_2 - V_{Th} = 0$$

$$3(2.67) - V_{Th} = 0$$

$$V_{Th} = 8 \text{ V}$$

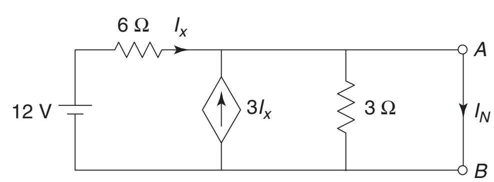


Fig. 2.296

Step II Calculation of I_N (Fig. 2.296)

When a short circuit is placed across a 3 Ω resistor, it gets shorted as shown in Fig. 2.297.

From Fig. 2.297,

$$I_x = I_1 \quad \ldots\text{(i)}$$

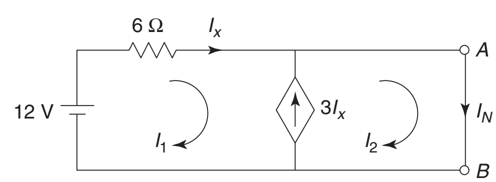


Fig. 2.297

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 3I_x = 3I_1$$

$$4I_1 - I_2 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$12 - 6I_1 = 0$$

$$I_1 = 2 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$
$$I_2 = 8 \text{ A}$$
$$I_N = I_2 = 8 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{8}{8} = 1\ \Omega$$

Step IV Calculation of I_L (Fig. 2.298)

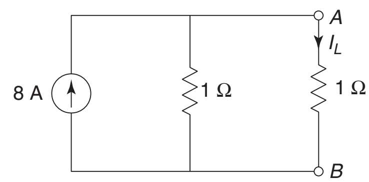


Fig. 2.298

$$I_L = 8 \times \frac{1}{1+1} = 4 \text{ A}$$

Example 2.99 *Find the current through the 1.6 Ω resistor in the network of Fig. 2.299.*

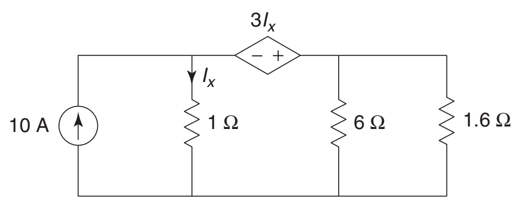


Fig. 2.299

Solution

Step I Calculation of V_{Th} (Fig. 2.300)

From Fig. 2.300,

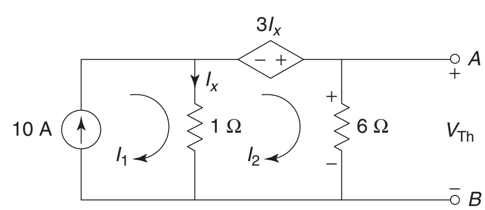


Fig. 2.300

$$I_x = I_1 - I_2 \quad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 10 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + 3I_x - 6I_2 = 0$$
$$-I_2 + I_1 + 3(I_1 - I_2) - 6I_2 = 0$$
$$4I_1 - 10I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 4 \text{ A}$$

Writing the V_{Th} equation,

$$6I_2 - V_{Th} = 0$$
$$6(4) - V_{Th} = 0$$
$$V_{Th} = 24 \text{ V}$$

Step II Calculation of I_N (Fig. 2.301)

When a short circuit is placed across the 3 Ω resistor, it gets shorted as shown in Fig. 2.302.

From Fig. 2.302,

$$I_x = I_1 - I_2 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = 10 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + 3I_x = 0$$
$$-I_2 + I_1 + 3(I_1 - I_2) = 0$$
$$4I_1 - 4I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 10 \text{ A}$$
$$I_N = I_2 = 10 \text{ A}$$

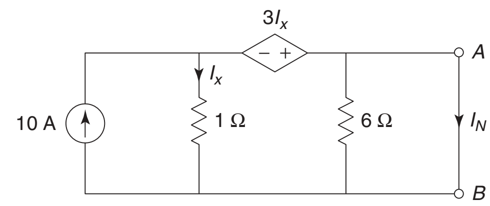


Fig. 2.301

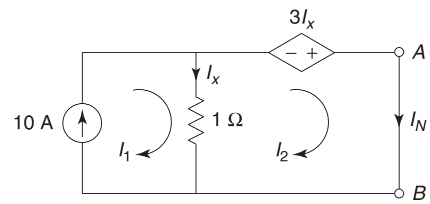


Fig. 2.302

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{24}{10} = 2.4\ \Omega$$

Step IV Calculation of I_L (Fig. 2.303)

$$I_L = 10 \times \frac{2.4}{2.4 + 1.6} = 6 \text{ A}$$

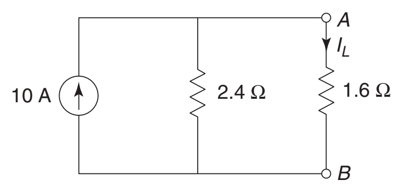


Fig. 2.303

2.10 MAXIMUM POWER TRANSFER THEOREM

It states that '*the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.*'

Proof From Fig. 2.304,

$$I = \frac{V}{R_s + R_L}$$

Power delivered to the load $R_L = P = I^2\ R_L = \dfrac{V^2 R_L}{(R_s + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_s + R_L)^2} R_L$$

$$= \frac{V^2[(R_s + R_L)^2 - (2R_L)(R_s + R_L)]}{(R_s + R_L)^4}$$

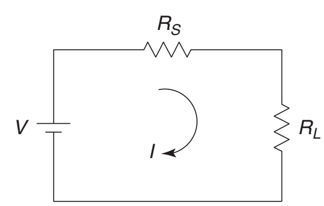


Fig. 2.304 *Network illustrating maximum power transfer theorem*

$$(R_s + R_L)^2 - 2\,R_L(R_s + R_L) = 0$$

$$R_s^{\,2} + R_L^{\,2} + 2R_sR_L - 2R_LR_s - 2R_L^2 = 0$$

$$R_s = R_L$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{\text{Th}}$$

5. Find the maximum power (Fig. 2.305).

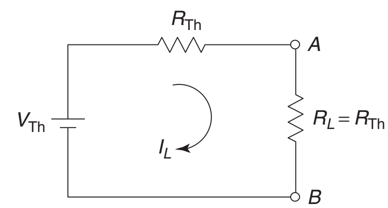


Fig. 2.305 *Thevenin's equivalent network*

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{V_{\text{Th}}}{2R_{\text{Th}}}$$

$$P_{\max} = I_L^2\,R_L = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}^2} \times R_{\text{Th}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

Example 2.100 *For the value of resistance R_L in Fig. 2.306 for maximum power transfer and calculate the maximum power.*

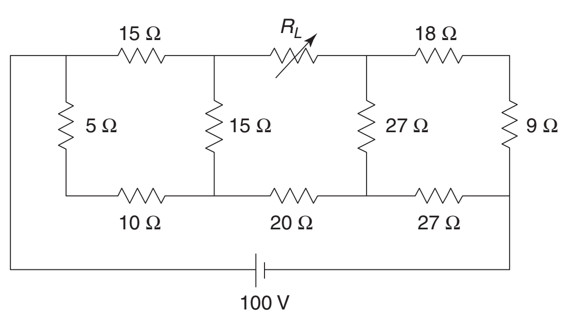


Fig. 2.306

Solution

Step I Calculation of V_{Th} (Fig. 2.307)

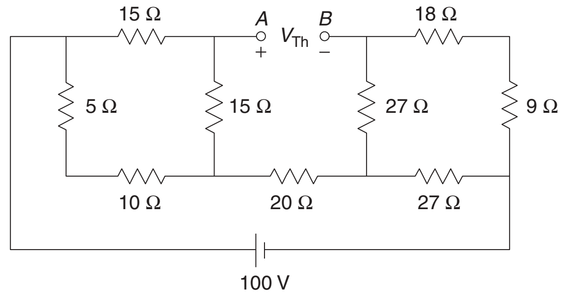


Fig. 2.307

By star-delta transformation (Fig. 2.308),

$$I = \frac{100}{5+5+20+9+9} = 2.08 \text{ A}$$

Writing the V_{Th} equation,

$$100 - 5I - V_{Th} - 9I = 0$$

$$V_{Th} = 100 - 14I$$

$$= 100 - 14(2.08)$$

$$= 70.88 \text{ V}$$

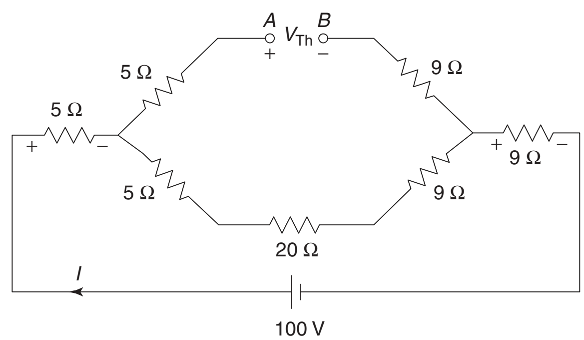


Fig. 2.308

Step II Calculation of R_{Th} (Fig. 2.309)

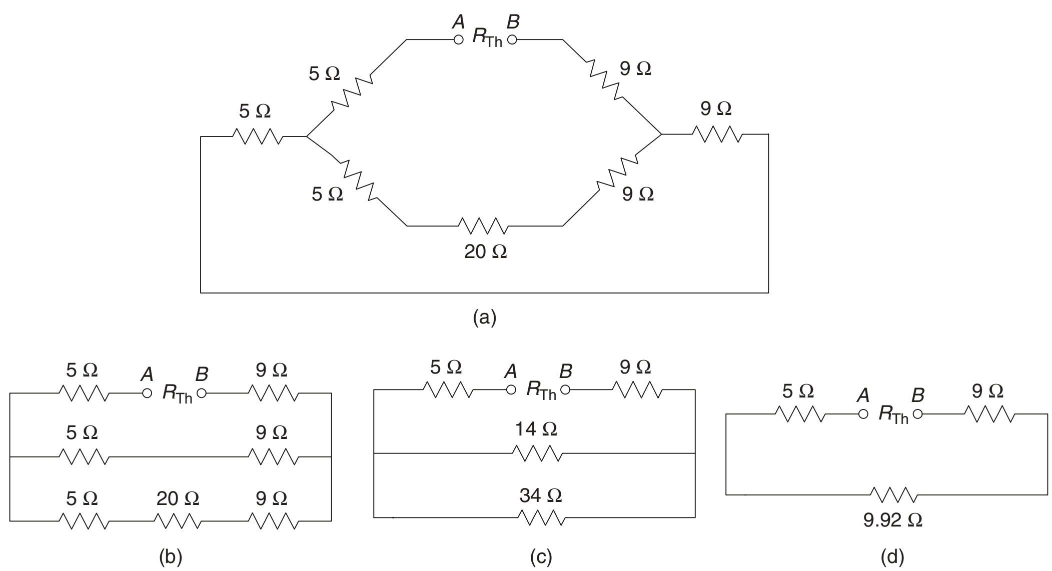


Fig. 2.309

$$R_{Th} = 23.92\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 23.92\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.310)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(70.88)^2}{4 \times 23.92} = 52.51\ \text{W}$$

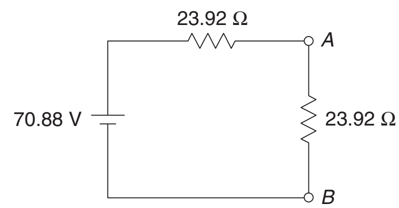


Fig. 2.310

Example 2.101 *For the value of resistance R_L in Fig. 2.311 for maximum power transfer and calculate the maximum power.*

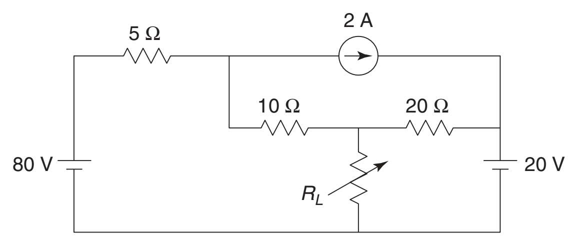


Fig. 2.311

Solution

Step I Calculation of V_{Th} (Fig. 2.312)

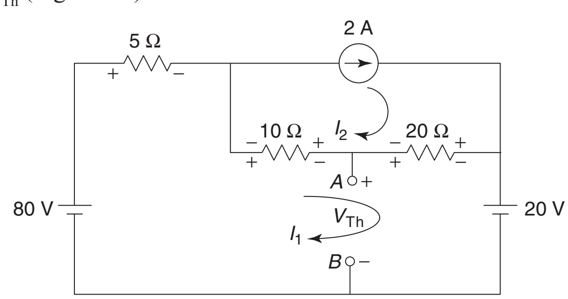


Fig. 2.312

Applying KVL to Mesh 1,

$$80 - 5I_1 - 10(I_1 - I_2) - 20(I_1 - I_2) - 20 = 0$$

$$35I_1 - 30I_2 = 60 \qquad \text{...(i)}$$

Writing the current equation for Mesh 2,

$$I_2 = 2 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 3.43 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 20\,(I_1 - I_2) - 20 = 0$$

$$V_{Th} = 20(3.43 - 2) + 20 = 48.6 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.313)

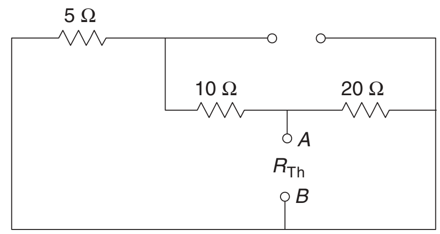


Fig. 2.313

$$R_{Th} = 15 \,\|\, 20 = 8.57\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 8.57\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.314)

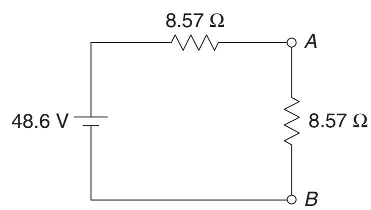


Fig. 2.314

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(48.6)^2}{4 \times 8.57} = 68.9 \text{ W}$$

Example 2.102 *For the value of resistance R_L in Fig. 2.315 for maximum power transfer and calculate the maximum power.*

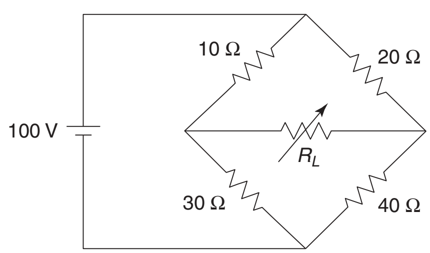


Fig. 2.315

Solution

Step I Calculation of V_{Th} (Fig. 2.316)

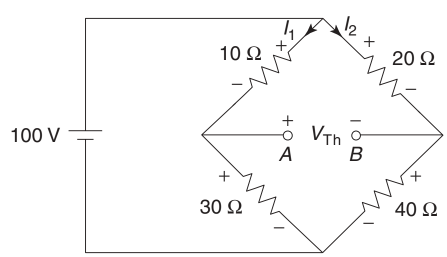


Fig. 2.316

$$I_1 = \frac{100}{10 + 30} = 2.5 \text{ A}$$

$$I_2 = \frac{100}{20 + 40} = 1.66 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} + 10\,I_1 - 20 I_2 = 0$$

$$V_{Th} = 20 I_2 - 10 I_1 = 20(1.66) - 10(2.5) = 8.2 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.317)

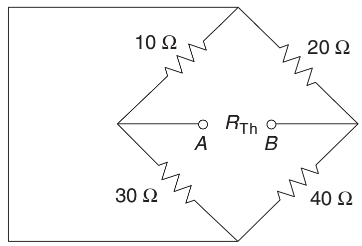


Fig. 2.317

Redrawing the network (Fig. 2.318),

$$R_{Th} = (10 \parallel 30) + (20 \parallel 40) = 20.83\ \Omega$$

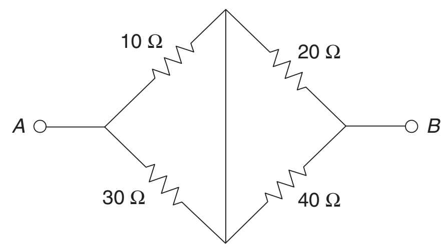


Fig. 2.318

Step III Value of R_L

For maximum power transfer,

$$R_L = R_{Th} = 20.83\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.319)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8.2)^2}{4 \times 20.83} = 0.81\ \text{W}$$

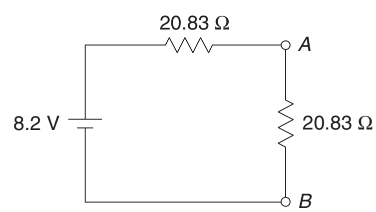


Fig. 2.319

Example 2.103 *For the value of resistance R_L in Fig. 2.320 for maximum power transfer and calculate the maximum power.*

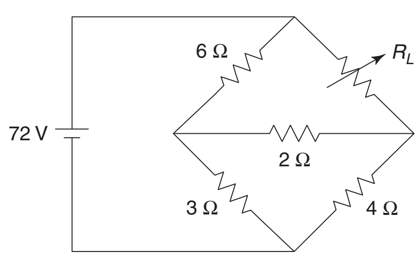


Fig. 2.320

Solution

Step I Calculation of V_{Th} (Fig. 2.321)

Applying KVL to Mesh 1,

$$72 - 6I_1 - 3(I_1 - I_2) = 0$$
$$9I_1 - 3I_2 = 72 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 4I_2 = 0$$
$$-3I_1 + 9I_2 = 0 \quad \ldots\text{(ii)}$$

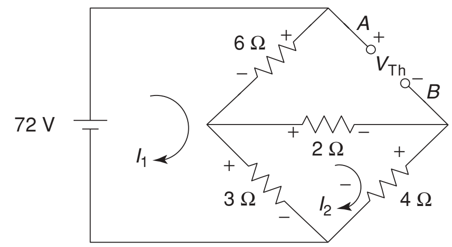


Fig. 2.321

Solving Eqs (i) and (ii),

$$I_1 = 9 \text{ A}$$
$$I_2 = 3 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 6I_1 - 2I_2 = 0$$
$$V_{Th} = 6I_1 + 2I_2 = 6(9) + 2(3) = 60 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.322)

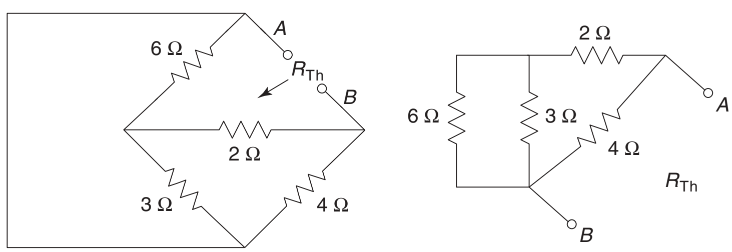


Fig. 2.322

$$R_{Th} = [(6 \parallel 3) + 2] \parallel 4 = 2\ \Omega$$

Step III Calculation of R_L
For maximum power transfer,

$$R_L = R_{Th} = 2\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.323)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 2} = 450 \text{ W}$$

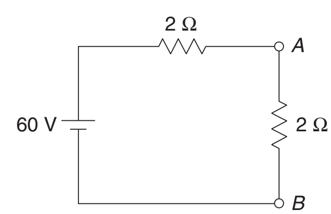


Fig. 2.323

EXAMPLES WITH DEPENDENT SOURCES

Example 2.104 *For the network shown in Fig. 2.324, find the value of R_L for maximum power transfer. Also, calculate maximum power.*

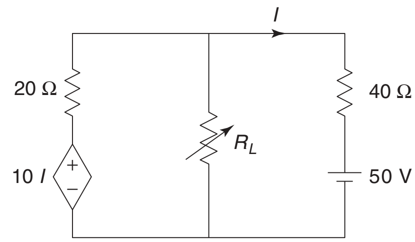


Fig. 2.324

Solution

Step I Calculation of V_{Th} (Fig. 2.325)

Applying KVL to the mesh,

$$10I - 20I - 40I - 50 = 0$$
$$I = -1 \text{ A}$$

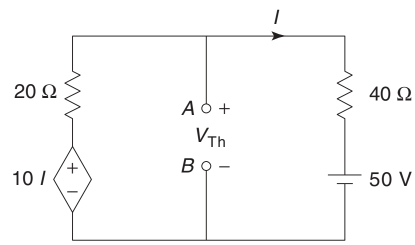


Fig. 2.325

Writing the V_{Th} equation,

$$V_{Th} - 40I - 50 = 0$$
$$V_{Th} - 40(-1) - 50 = 0$$
$$V_{Th} = 10 \text{ V}$$

Step II Calculation of I_N (Fig. 2.326)

From Fig. 2.326,

$$I = I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$10I - 20I_1 = 0$$
$$10I_2 - 20I_1 = 0 \qquad \ldots\text{(ii)}$$

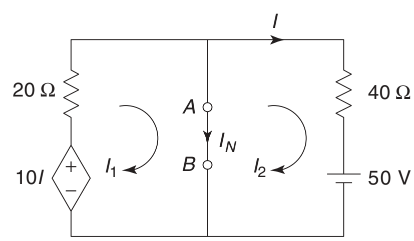


Fig. 2.326

Applying KVL to Mesh 2,

$$-40I_2 - 50 = 0$$
$$I_2 = -1.25 \text{ A} \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = -0.625 \text{ A}$$
$$I_N = I_1 - I_2 = -0.625 + 1.25 = 0.625 \text{ A}$$

Step III Calculation of R_N

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{10}{0.625} = 16\ \Omega$$

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 16\ \Omega$$

Step V Calculation of P_{max} (Fig. 2.327)

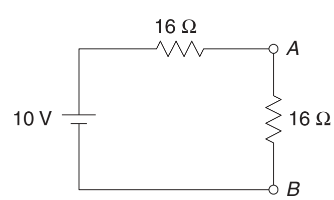


Fig. 2.327

$$P_{max} = \frac{V_{Th}}{4R_{Th}} = \frac{(10)^2}{4 \times 16} = 1.56 \text{ W}$$

Example 2.105 *For the network shown in Fig. 2.328, calculate the maximum power that may be dissipated in the load resistor R_L.*

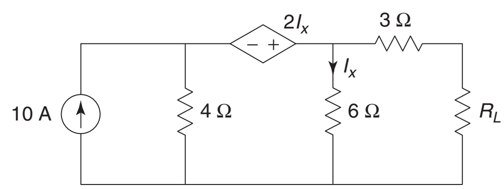


Fig. 2.328

Solution

Step I Calculation of V_{Th} (Fig. 2.329)

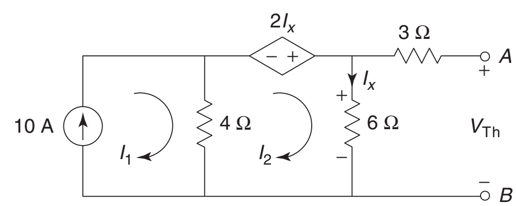


Fig. 2.329

From Fig. 2.329,

$$I_x = I_2 \quad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 10 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) + 2I_x - 6I_2 = 0$$
$$-4I_2 + 4I_1 + 2I_2 - 6I_2 = 0$$
$$4I_1 - 8I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 5 \text{ A}$$

Writing the V_{Th} equation,

$$6I_2 - 0 - V_{Th} = 0$$
$$V_{Th} = 6I_2 = 6(5) = 30 \text{ V}$$

Step II Calculation of I_N (Fig. 2.330)

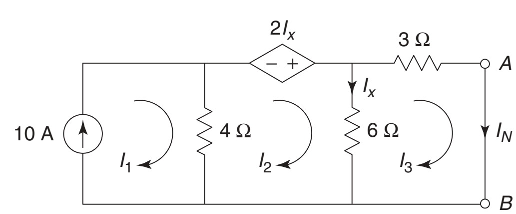


Fig. 2.330

From Fig. 2.330,

$$I_x = I_2 - I_3 \quad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 10 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) + 2I_x - 6(I_2 - I_3) = 0$$
$$-4I_2 + 4I_1 + 2(I_2 - I_3) - 6I_2 + 6I_3 = 0$$
$$4I_1 - 8I_2 + 4I_3 = 0 \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_2) - 3I_3 = 0$$

$$6I_2 - 9I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 10 \text{ A}$$

$$I_2 = 7.5 \text{ A}$$

$$I_3 = 5 \text{ A}$$

$$I_N = I_3 = 5 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{30}{5} = 6\ \Omega$$

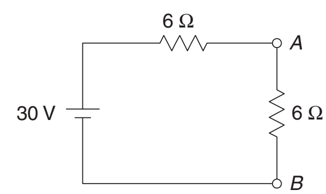


Fig. 2.331

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 6\ \Omega$$

Step V Calculation of P_{max} (Fig. 2.331)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(30)^2}{4 \times 16} = 37.5 \text{ W}$$

Example 2.106 *For the network shown in Fig. 2.332, find the value of R_L for maximum power transfer. Also, find maximum power.*

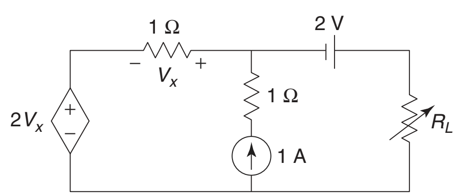


Fig. 2.332

Solution

Step I Calculation of V_{Th} (Fig. 2.333)

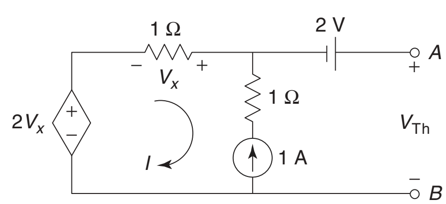


Fig. 2.333

From Fig. 2.333,

$$V_x = -1I = -I \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I = -1 \qquad \ldots\text{(ii)}$$

$$V_x = 1 \text{ V}$$

Writing the V_{Th} equation,

$$2V_x - 1I + 2 - V_{Th} = 0$$
$$2(1) - (-1) + 2 - V_{Th} = 0$$
$$V_{Th} = 5 \text{ V}$$

Step II Calculation of I_N (Fig. 2.334)

From Fig. 2.334,

$$V_x = -1I_1 = -I_1 \quad \ldots\text{(i)}$$

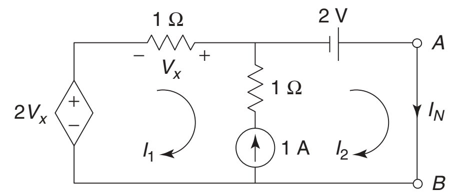


Fig. 2.334

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$2V_x - 1I_1 + 2 = 0$$
$$2(-I_1) - I_1 + 2 = 0$$
$$3I_1 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.67 \text{ A}$$
$$I_2 = 1.67 \text{ A}$$
$$I_N = I_2 = 1.67 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{5}{1.67} = 3\ \Omega$$

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3\ \Omega$$

Step V Calculation of P_{max} (Fig. 2.335)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5)^2}{4 \times 3} = 2.08 \text{ W}$$

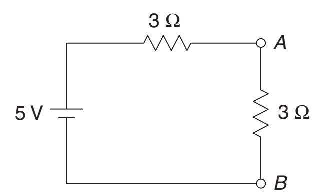


Fig. 2.335

Example 2.107 *What will be the value of R_L in Fig. 2.336 to get maximum power delivered to it? What is the value of this power?*

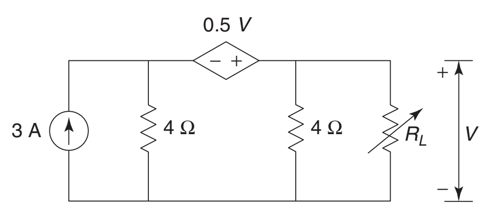


Fig. 2.336

Solution

Step I Calculation of V_{Th} (Fig. 2.337)
By source transformation,
From Fig. 2.337,

$$V_{Th} = 4I$$

Applying KVL to the mesh,

$$12 - 4I + 0.5V_{Th} - 4I = 0$$

$$12 - V_{Th} + 0.5V_{Th} - V_{Th} = 0$$

$$V_{Th} = 8 \text{ V}$$

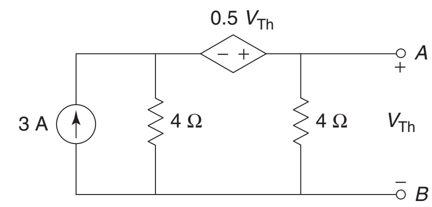


Fig. 2.337

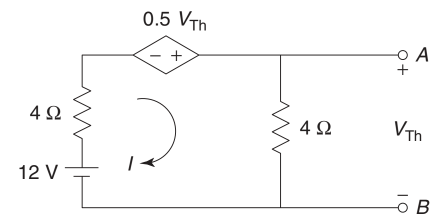


Fig. 2.338

Step II Calculation of I_N (Fig. 2.338)

If two terminals *A* and *B* are shorted, the 4 Ω resistor gets shorted.

$$V = 0$$

Dependent source 0.5 *V* depends on the controlling variable *V*. When *V* = 0, the dependent source vanishes, i.e. 0.5 *V* = 0 as shown in Fig. 2.339 and Fig. 2.340.

$$I_N = \frac{12}{4} = 3 \text{ A}$$

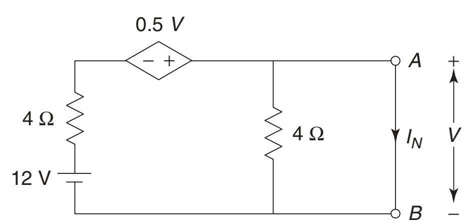


Fig. 2.339

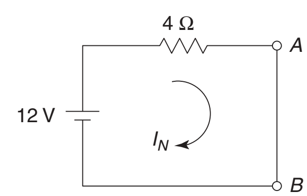


Fig. 2.340

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{8}{3} = 2.67\ \Omega$$

Step IV *Calculation of* R_L
For maximum power transfer,

$$R_L = R_{Th} = 2.67\ \Omega$$

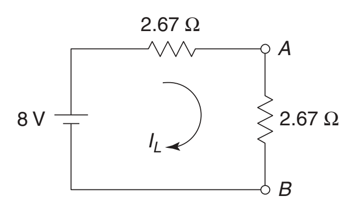


Fig. 2.341

Step V Calculation of P_{max} (Fig. 2.341)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8)^2}{4 \times 2.67} = 6 \text{ W}$$

Solved Exercises

Superposition Theorem

Problem 4.1 ***Calculate the voltage* V *across the resistor* R *by using the superposition theorem.***

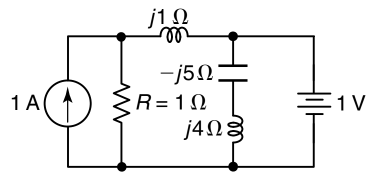


Fig. 4.30

Solution We consider two cases:

Case (1) When the 1-A current source is acting alone

For Fig. 4.31(a), the voltage across the resistor $R = 1\ \Omega$ is, $V' = \dfrac{j}{1+j}$.

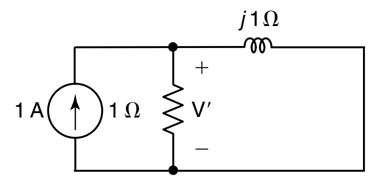


Fig. 4.31 (a) *Circuit with current source acting alone*

Case (2) When the 1-V voltage source is acting alone

For Fig. 4.31(b), the current through the resistor $I'' = \dfrac{1}{1+j}$

and hence, the voltage across the resistor $R = 1\ \Omega$ is $V'' = I'' \times 1 = \dfrac{1}{1+j}$.

So, by the superposition theorem, total voltage across the resistor when both the sources are acting simultaneously is,

$$V = (V' + V'') = \frac{j}{1+j} + \frac{1}{1+j} = 1\,\text{V}$$

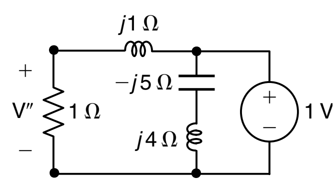


Fig. 4.31 (b) *Circuit with voltage source acting alone*

Problem 4.2 ***Use the superposition theorem on the circuit shown in Fig. 4.32 to find 'I'.***

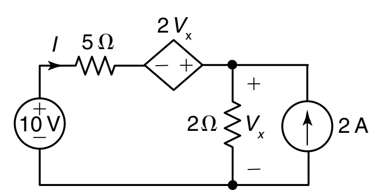


Fig. 4.32

Solution We consider two cases:

Case(1) When the 10-V voltage source is acting alone

For Fig. 4.33(a), by KVL, $5i' - 2v_x' + 2i' = 10$ with $v_x' = -2i'$

$$\Rightarrow 7i' + 4i' = 10 \;\Rightarrow\; i' = {}^{10}\!/_{11}\,\text{A}$$

Case (2) When 1-V voltage source is acting alone

For Fig 4.33(b), by KCL at the node (x)

$$2 = i_x + i'' = -\frac{v_x'}{2} + i''$$

But loop analysis in the left loop gives

$$5i'' + 3v_x'' = 0 \quad \text{or,} \quad i'' = -\frac{3}{5}v_x''$$

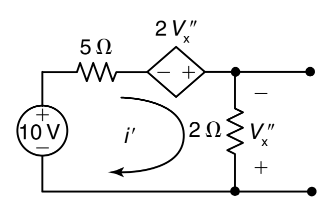
(i)


Fig. 4.33 (a) *Voltage source acting alone*

From (i), $2 = -\frac{v''_x}{2} - \frac{3}{5}v''_x \Rightarrow v''_x = -\frac{20}{11}$

$$\therefore i'' = -\frac{3}{5} \times \left(-\frac{20}{11}\right) = \frac{12}{11}\text{A}$$

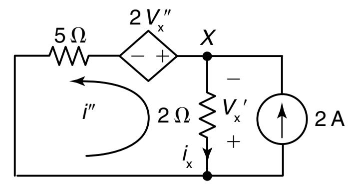


Fig. 4.33 (b) *Current source acting alone*

So, by the superposition theorem total current, when both the sources are acting simultaneously, is,

$$I = (i' - i'') = \left(\frac{10}{11} - \frac{12}{11}\right) = -\frac{2}{11}\text{A}$$

Problem 4.3 ***Determine the current in the capacitor branch by the superposition theorem.***

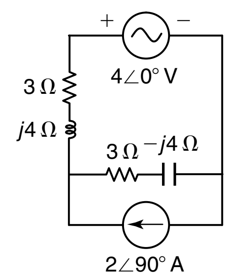


Fig. 4.34

Solution **When the voltage source is acting alone**

Here, the current in the capacitor branch is

$$I' = \frac{4\angle 0°}{(3+j4)+(3-j4)} = \frac{2}{3}\angle 0°\ \text{A}$$

When the current source is acting alone

Here, the current in the capacitor branch is

$$I'' = 2\angle 90° \times \frac{(3+j4)}{(3+j4)+(3-j4)} = \left(-\frac{4}{3} + j1\right)\text{A}$$

∴ total current when both the sources are acting simultaneously is

$$I = (I' + I'') = \left(\frac{2}{3} - \frac{4}{3} + j1\right) = \left(-\frac{2}{3} + j1\right)$$

$$= 1.2\angle 123.7°\ \text{A}$$

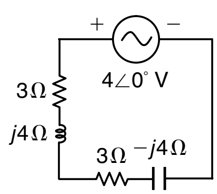


Fig. 4.35 (a) *When voltage source acting alone*

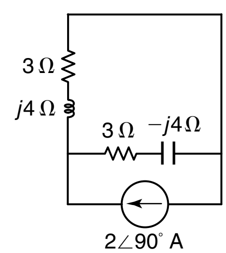


Fig. 4.35 (b) *When current source acting alone*

Problem 4.4 ***Find the current*** i_0 ***using superposition theorem.***

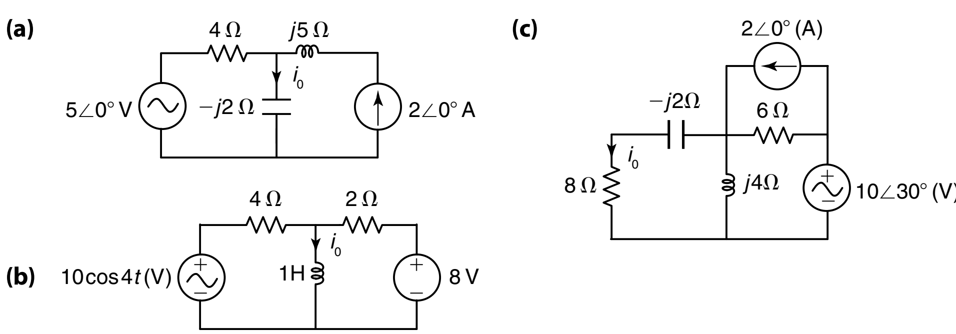


Fig. 4.36

Solution **(a) When the voltage source is acting alone**

The current in this case is

$$i_0' = \frac{5}{4-j2} = \left(1 + j\frac{1}{2}\right)\text{A}$$

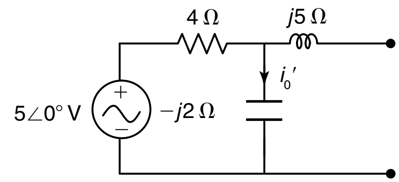


Fig. 4.37 (a) *Voltage source acting alone*

When the current source is acting alone

In this case, the current is,

$$i_0'' = 2\angle 0^\circ \times \frac{4}{4-j2} = \left(\frac{8}{5} + j\frac{14}{5}\right)\text{A}$$

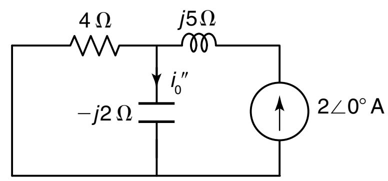


Fig. 4.37 (b) *Current source acting alone*

∴ by the superposition theorem, total current is

$$i_0 = (i_0' + i_0'') = \left(1 + \frac{8}{5}\right) + j\left(\frac{1}{2} + \frac{4}{5}\right) = 2.9\angle 26.56^\circ \text{ A}$$

(b) When the dc source is acting alone

Equivalent impedance, $Z = \left(\frac{j4 \times 4}{4+j4} + 2\right) = \frac{2+j6}{1+j}$

∴ main current, $I = \frac{8}{Z} = \frac{8(1+j)}{2+j6} = \frac{4(1+j)}{1+j3}$

∴ the current, $i_0' = I \times \frac{4}{4+j4} = \frac{4(1+j)}{1+j3} \times \frac{4}{4+j4} = \left(\frac{2}{5} - j\frac{6}{5}\right)\text{A}$

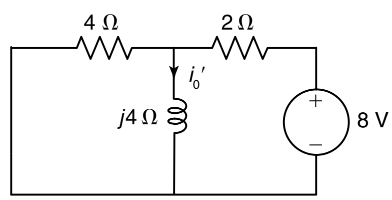


Fig. 4.38 (a) *dc source acting alone*

When the ac source is acting alone

Equivalent impedance, $Z = 4 + \left(\frac{j4 \times 2}{2+j4}\right) = \frac{4+j6}{1+j2}$

∴ main current,

$$I = \frac{10\angle 0^\circ}{Z} = 10\angle 0^\circ \frac{(1+j2)}{4+j6} = \frac{10+j20}{4+j6}$$

∴ the current,

$$i_0'' = I \times \frac{2}{2+j4} = \frac{10(1+j2)}{4+j6} \times \frac{1}{1+j2} = \left(\frac{10}{13} - j\frac{15}{13}\right)\text{A}$$

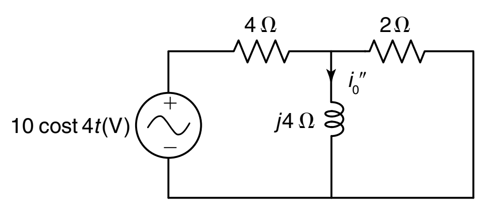


Fig. 4.38 (b) *ac source acting alone*

∴ by the superposition theorem, total current is, $i_0 = (i_0' + i_0'') = \left(\frac{2}{5} + \frac{10}{13}\right) - j\left(\frac{6}{5} + \frac{15}{13}\right) = 2.63\angle -63.58^\circ$ A

(c) When the voltage source is acting alone

Equivalent impedance, $Z = \frac{j4(8-j2)}{8+j2} + 6 = \frac{28+j22}{4+j}$

$\therefore$ main current, $I = \dfrac{10\angle 30°\ (4+j)}{28+j22} = \dfrac{(8.66+j5)(4+j)}{28+j22}$

$\therefore$ the current,

$$i_0' = I \times \frac{8-j2}{8+j2} = \frac{8.66+j5}{56+j44} = 0.14\angle -8.16°\ \text{A}$$

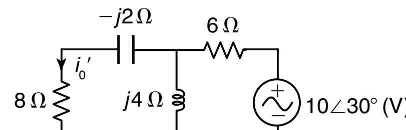


Fig. 4.39 *Voltage source acting alone*

When the current source is acting alone

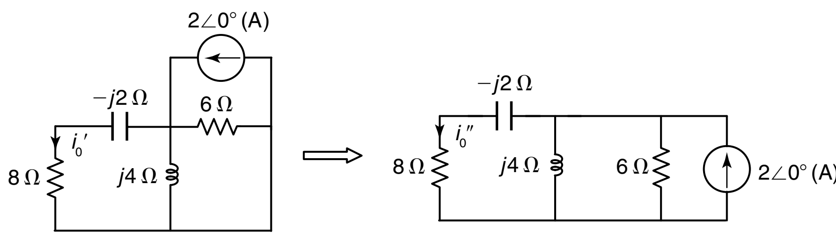


Fig. 4.40

where, $Z = \dfrac{j4\times 6}{6+j4} = \dfrac{j12}{3+j2}$

$\therefore$ the current,

$$i_0'' = 2\angle 0° \times \frac{Z}{8-j2+Z} = \frac{j12}{12+j11} = 0.73\angle 47.49°\text{A}$$

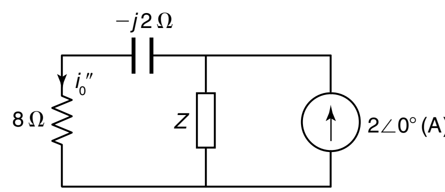


Fig. 4.41

$\therefore$ by the superposition theorem, total current is

$$i_0 = (i_0' + i_0'') = (0.14\angle -8.16° + 0.73\angle 47.49°) = (0.631 + j0.518) = 0.81\angle 39.38°\ \text{A}$$

Problem 4.5 ***Find v_0 using the superposition theorem.***

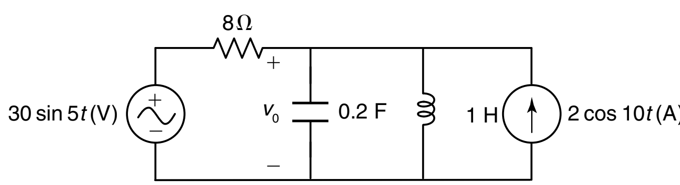


Fig. 4.42

Solution **(a) When the voltage source is acting alone**

Here, $X_C = \dfrac{-j}{5\times 0.2} = -j1\,\Omega$ and $X_L = j\times 5\times 1 = j5\,\Omega$

By KCL,

$$-\frac{30-v_0'}{8} + \frac{v_0'}{-j1} + \frac{v_0'}{j5} = 0 \Rightarrow v_0' = \frac{30}{8(0.125+j0.8)} = 4.631\angle -81.12°\ \text{(V)}$$

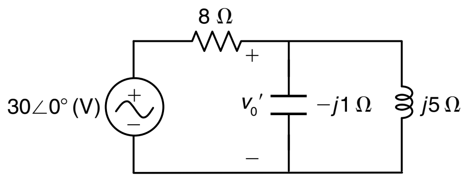


Fig. 4.43 (a) *Voltage source acting alone*

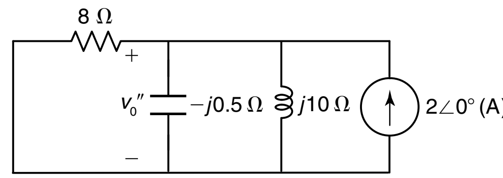


Fig. 4.43 (b) *Current source acting alone*

When the current source is acting alone

Here, $X_C = \dfrac{-j}{10 \times 0.2} = -j0.5\,\Omega$ and $X_L = j \times 10 \times 1 = j10\,\Omega$

By KCL, $2 = v_0''\left(\dfrac{1}{8} + \dfrac{1}{j10} + \dfrac{1}{-j0.5}\right) \Rightarrow v_0'' = \dfrac{2}{0.125 + j1.9} = 1.051\angle -86.24°\ (\text{V})$

By the superposition theorem, when both the sources are acting simultaneously, the voltage is

$v_0 = (v_0' + v_0'') = 4.631\sin(5t - 81.12°) + 1.051\cos(10t - 86.24°)\ (\text{V})$

Problem 4.6 ***Find i_0 and i from the circuit of Fig. 4.44 using superposition theorem.***

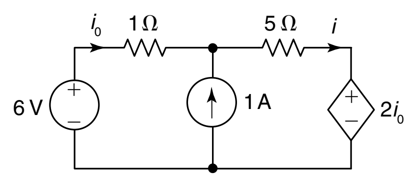


Fig. 4.44

Solution **When the 6-V source is acting alone**

The circuit is shown.

Here, $i_0' = i'$

By KVL, $6i' + 2i' = 6 \Rightarrow i' = i_0' = \dfrac{6}{8} = \dfrac{3}{4}\,\text{A} = 0.75\,\text{A}$

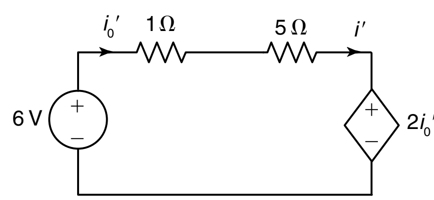


Fig. 4.45 (a) *6-V Source acting alone*

When the 1-A source is acting alone

By KCL, we get, $1 = i'' - i_0'' \Rightarrow i'' = 1 + i_0''$

By KVL for the supermesh,

$$1 \times i_0'' + 5i'' + 2i_0'' = 0 \quad \text{or,} \quad 3i_0'' + 5i'' = 0$$

$$\text{or,} \quad 3i_0'' + 5(1 + i_0'') = 0 \quad \text{or,} \quad i_0'' = -\frac{5}{4} = -1.25\,\text{A}$$

$\therefore i'' = 1 - 1.25 = -0.25\,\text{A}$

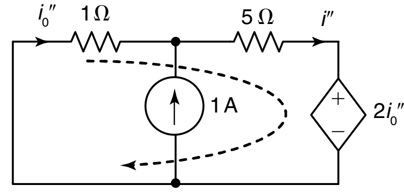


Fig. 4.45 (b) *1-A source acting alone*

By the superposition theorem, the total currents when both the sources are acting simultaneously is given as

$$\left.\begin{aligned} i &= (i' + i'') = (0.75 - 0.25) = 0.5\,\text{A} \\ i_0 &= (i_0' + i_0'') = (0.75 - 1.25) = -0.5\,\text{A} \end{aligned}\right\}$$

Problem 4.7 ***Using the superposition theorem, calculate the current through the (2 + j3) Ω impedance branch of the circuit shown in Fig. 4.46.***

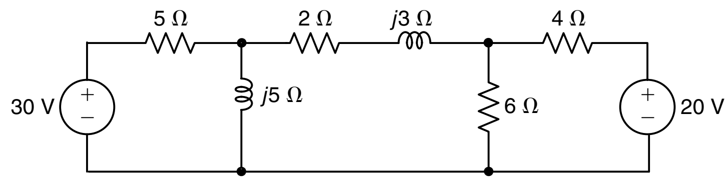


Fig. 4.46

Solution **Case (I) When the 30-V source is acting alone**

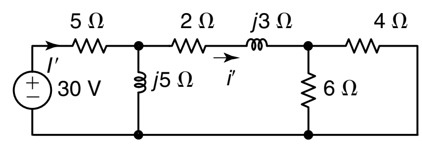


Fig. 4.47

$$\text{Impedance, } Z = 5 + \frac{(4.4 + j3) \times j5}{4.4 + j3 + j5} = (6.32 + j2.6)\ \Omega$$

$$\therefore I' = \frac{30}{Z} = \frac{30}{6.32 + j2.6} = (4.06 - j1.67)\ \text{A}$$

$$i' = I' \times \frac{j5}{4.4 + j3 + j5} = (2.39 + j0.27)\ \text{A}$$

Case (II) when the 20-V source is acting alone

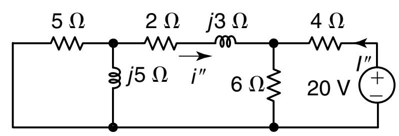


Fig. 4.48

$$\text{Impedance, } Z = 4 + \frac{(4.5 + j5.5) \times 6}{4.5 + j5.5 + 6} = (7.31 + j1.41)\ \Omega$$

$$\therefore I'' = \frac{20}{Z} = \frac{20}{7.31 + j1.41} = (2.64 - 0.509)\ \text{A}$$

$$i'' = -I'' \times \frac{6}{4.5 + j5.5 + 6} = -(1.064 - j0.848)\ \text{A}$$

By the superposition theorem, total current flowing through the (2 + j3) impedance is

$$i = (i' + i'') = (2.39 + j0.27) - (1.064 - j0.848) = (1.325 + j1.117)\ \text{A} = 1.733\angle 40.14^\circ\ \text{A}$$

Problem 4.8 ***Using the superposition theorem, find*** $\mathbf{V}_{AB}$.

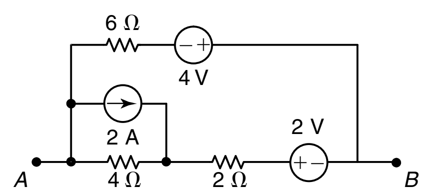


Fig. 4.49

Solution We consider three cases:

Case (I) When the 2-V source is acting alone

The circuit is shown Fig. 4.50.

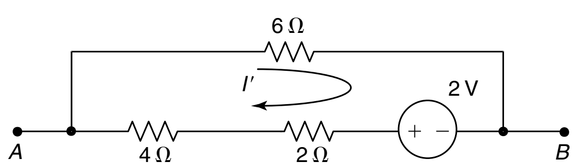


Fig. 4.50

For this circuit, the current in the loop is obtained as $I' = \frac{2}{12} = \frac{1}{6}$ A

$\therefore$ the voltage between A and B is $V'_{AB} = I' \times 6 = \frac{1}{6} \times 6 = 1$ V

Case (II) When the 4-V source is acting alone

The circuit is shown in Fig. 4.51.

In this circuit, the loop current is obtained as

$$I'' = \frac{4}{12} = \frac{1}{3} \text{ A}$$

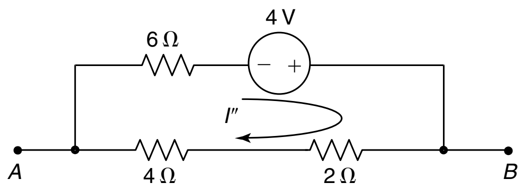


Fig. 4.51

$\therefore$ voltage between A and B is,

$$V''_{AB} = -I'' \times 6 = -\frac{1}{3} \times 6 = -2 \text{ V}$$

Case (III) When the 2-A source is acting alone

The circuit is shown in Fig. 4.52.

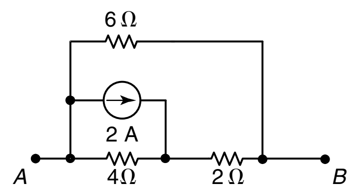


Fig. 4.52

We convert the current source into its equivalent voltage source as shown in Fig. 4.53.

The loop current is $I''' = \frac{8}{12} = \frac{2}{3}$ A

$\therefore$ voltage between A and B is

$$V'''_{AB} = -I''' \times 6 = -\frac{2}{3} \times 6 = -4 \text{ V}$$

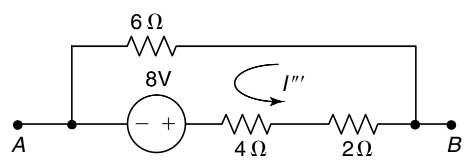


Fig. 4.53

$\therefore$ voltage between A and B when all the sources are acting simultaneously is given by superposition theorem as

$$V_{AB} = V'_{AB} + V''_{AB} + V'''_{AB} = (1 - 2 - 4) = -5 \text{ V}$$

Problem 4.9 ***Find the current* i *in the circuit shown in the Fig. 4.54 using the superposition theorem.***

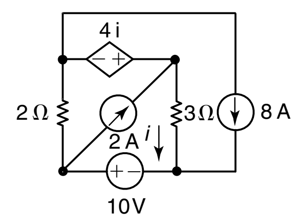


Fig. 4.54

Solution We consider the three cases:

Case (I) When the 10-V source is acting alone

The circuit is shown in Fig. 4.55.

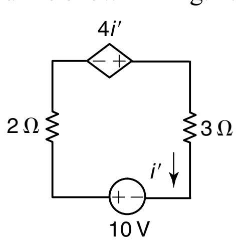


Fig. 4.55

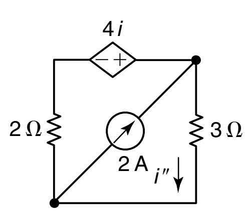


Fig. 4.56

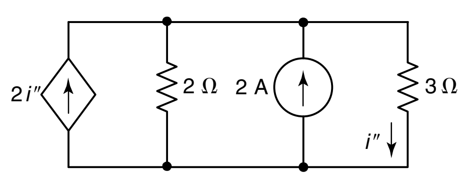


Fig. 4.57

By KVL for the loop, we get, $-4i'+3i'-10+2i'=0 \;\Rightarrow\; i'=10\text{ A}$

Case (II) When the 2-A source is acting alone

The circuit is shown in Fig. 4.56.

We convert the dependent voltage source into its equivalent dependent current source as shown in Fig. 4.57.

The total current $(2 + 2i'')$ is divided into two paths, resistors 2 Ω and 3 Ω.

∴ by current divider rule, current through the 3-Ω resistor is

$$i''=\left(\frac{2}{2+3}\right)\times(2+2i'') \;\Rightarrow\; i''=4\text{ A}$$

Case (III) When the 8-A source is acting alone

The circuit is shown in Fig. 4.58.

By KVL for the loop, we get,

$$-4i'''+3(I-8)+2I=0$$

where, $i'''=(I-8)$ or, $I=(i'''+8)$

$$\Rightarrow\; -4i'''+3i''+2(i'''+8)=0 \;\Rightarrow\; i=-16\text{ A}$$

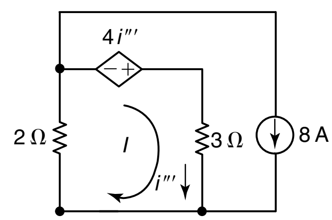


Fig. 4.58

∴ current when all the sources are acting simultaneously is given by the superposition theorem as

$$i=(i'+i''+i''')=(10+4-16)=-2\text{ A}$$

Problem 4.10 ***Using the superposition theorem determine V₁, the voltage across the 3-ohm resistor in Fig. 4.59.***

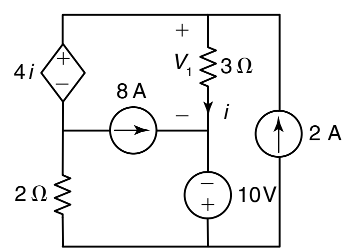


Fig. 4.59

Solution **Case (I) When the 8-A current source is acting alone**

By KVL for the supermesh, $3i'+2i_1-4i'=0 \;\Rightarrow\; i_1=\frac{1}{2}i'$

By KCL at the node x,

$$i_1=(8+i') \;\Rightarrow\; \frac{1}{2}i'=8+i' \;\Rightarrow\; i'=-16\text{ A}$$

$$\therefore V_1'=3i'=3\times(-16)=-48\text{ V}$$

Case (II) When the 2-A current source is acting alone

By KVL,

$$3(i_2+2)+2i_2-4i''=0 \;\Rightarrow\; 5i_2+6-4i''=0$$

Now, $i''=(i_2+2)$

$$\therefore 5i_2+6-4(i_2+2)=0 \;\Rightarrow\; i_2=2\text{ A}$$

$$\therefore i''=(i_2+2)=(2+2)=4\text{ A}$$

$$\therefore V_1''=3i''=3\times4=12\text{ V}$$

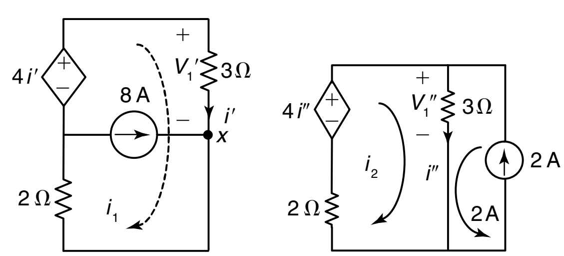


Fig. 4.60 (a) **Fig. 4.60 (b)**

Case (III) When the 10-V voltage source is acting alone

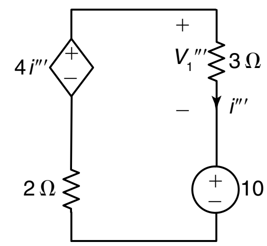


Fig. 4.60 (c)

By KVL, $3i''' - 10 + 2i''' - 4i''' = 0 \;\Rightarrow\; i''' = 10\text{ A}$

$$\therefore V_1''' = 10 \times 3 = 30\text{ V}$$

When all the sources are acting simultaneously, by the superposition theorem the voltage is given as

$$V_1 = (V_1' + V_1'' + V_1''') = (-48 + 12 + 30) = -6\text{ V}$$

Problem 4.11 ***For the network shown in Fig. 4.61 calculate the current throughout the impedance (3 + j4)Ω using superposition theorem.***

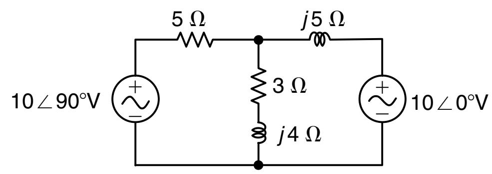


Fig. 4.61

Solution **When the 10∠90° V is acting alone**

$$\text{Main current, } I = \frac{10\angle 90^\circ}{5 + \dfrac{(3+j4)j5}{3+j4+j5}} = \frac{j10(3+j9)}{-5+j60}$$

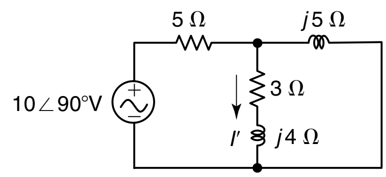


Fig. 4.62

$$\therefore I' = I \times \frac{j5}{3+j9} = \frac{j10 \times j5}{-5+j60} = \frac{-10}{-1+j12}$$

When the 10∠0° V is acting alone

$$\text{Main current, } I = \frac{10\angle 0^\circ}{j5 + \dfrac{(3+j4)5}{3+j4+5}} = \frac{10(8+j4)}{-5+j60}$$

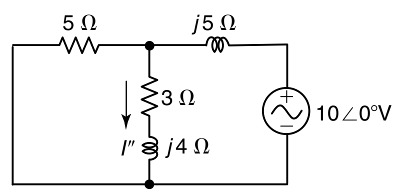


Fig. 4.63

$$\therefore I'' = I \times \frac{5}{8+j4} = \frac{10 \times 5}{-5+j60} = \frac{10}{-1+j12}$$

When both the sources are acting simultaneously, by the superposition theorem, the total current flowing through the impedance (3 + j4) is

$$I = (I' + I'') = \frac{-10}{-1+j12} + \frac{10}{-1+j12} = 0\text{ A}$$

Problem 4.12 ***Using the superposition theorem, determine the current in the 4-Ω resistor in the network shown in Fig. 4.64.***

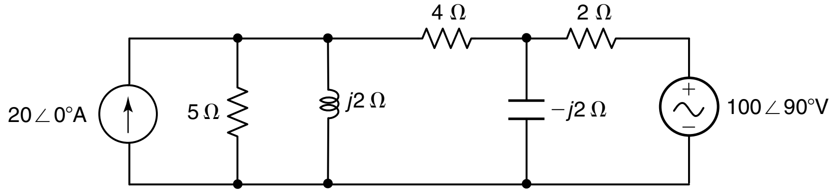


Fig. 4.64

Solution **Case (I) When the $20\angle 0°$A source is acting alone**

The circuit is shown in Fig. 4.65.

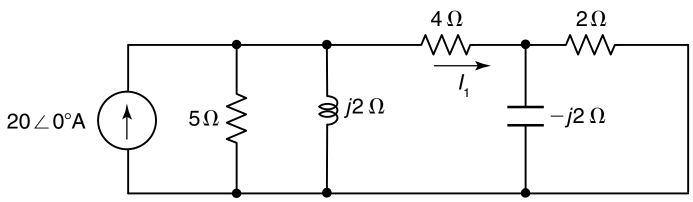


Fig. 4.65

Reducing the parallel combination, the simplified circuit is shown in Fig. 4.66.

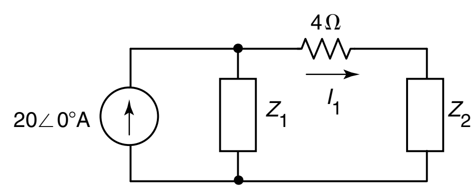


Fig. 4.66

$$Z_1 = \frac{5 \times j2}{5 + j2} = 1.857\angle 68.2° = (0.69 + j1.72)\,\Omega$$

$$Z_2 = \frac{2 \times (-j2)}{2 - j2} = (1 - j1)\Omega = 1.414\angle -45°\ \Omega$$

By current division rule, the current through the 4-Ω resistor is

$$I_1 = 20\angle 0° \times \frac{Z_1}{Z_1 + 4 + Z_2} = 20\angle 0° \times \frac{1.857\angle 68.2°}{0.69 + j1.72 + 4 + 1 - j1} = 6.48\angle 61° = (3.14 + j5.66)\,\text{A}$$

Case (II) When the $100\angle 90°$ V source is acting alone

Here, the current source is open-circuited. Combining the parallel connection of 5 Ω and $j2$ Ω the simplified circuit is shown in Fig. 4.67.

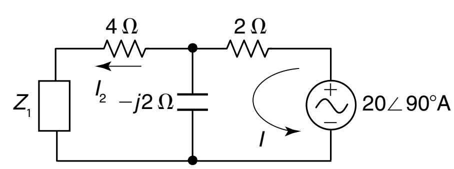


Fig. 4.67

By KVL for the two loops, we get,

$$(4 + 0.69 + j1.72 - j2)I_2 + j2I = 0$$

$$\Rightarrow \quad (4.69 - j0.28)I_2 + j2I = 0 \qquad \text{(i)}$$

and,
$$j2I_2 + (2 - j2)I = 100\angle 90° = j100 \qquad \text{(ii)}$$

Solving (i) and (ii), we get

$$I_2 = \frac{\begin{vmatrix} 0 & j2 \\ j100 & (2 - j2) \end{vmatrix}}{\begin{vmatrix} (4.69 - 0.28) & j2 \\ j2 & (2 - j2) \end{vmatrix}} = \frac{200}{-12.83 + j9.93} = 12.33\angle 37.75°\ (\text{A}) = (9.75 + j7.55)\,\text{A}$$

By superposition theorem, when both the sources are acting simultaneously, the current through the 4-Ω resistor is

$$I = I_1 - I_2 = (3.14 + j5.66) - (9.75 + j7.55) = (-6.61 - j1.9) = 6.89\angle -163.67°\ \text{A}$$

The direction of the current is from right to left.

Problem 4.13 ***Find I in the Fig. 4.68 using the superposition theorem.***

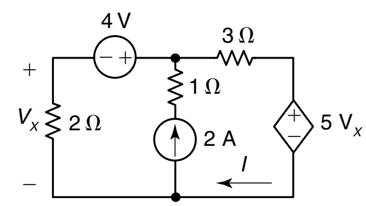


Fig. 4.68

Solution **When the 4-V voltage source is acting alone**

The circuit is shown in Fig. 4.69.

Here, by KVL,

$$-4+3I'+5V_x'-V_x'=0$$

or, $$3I'+4\times(-2I')=4 \quad [\because V_x'=-2I']$$

or, $$I'=-\frac{4}{5}\text{ A}=-0.8\text{ A}$$

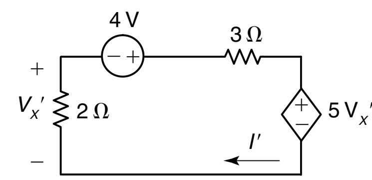


Fig. 4.69

When the 2-A current source is acting alone

The circuit is shown in Fig. 4.70.

By KCL, $2=\frac{V_x''}{2}+\frac{V_x''-5V_x''}{3} \Rightarrow V_x''=-\frac{12}{5}=-2.4\text{ V}$

$$\therefore I''=\frac{V_x''-5V_x''}{3}=\frac{-\frac{12}{5}-5\times\left(-\frac{12}{5}\right)}{3}=\frac{16}{5}=3.2\text{ A}$$

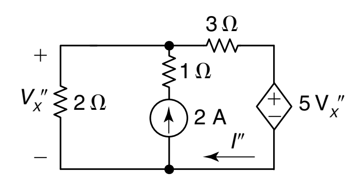


Fig. 4.70

When both the sources are acting simultaneously, the current by superposition theorem is given as $I=(I'+I'')=(-0.8+3.2)=2.4$ A

Thevenin's and Norton's Theorem

Problem 4.14 ***Draw the Thevenin's equivalent of the circuit in Fig. 4.71 and find the load current,* i.**

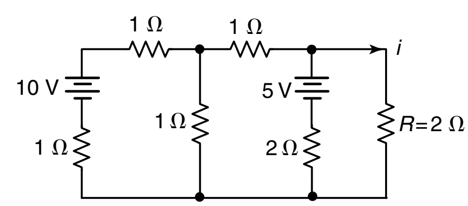


Fig. 4.71

Solution Open-circuiting the terminals, by KVL for two meshes,

$$3i_1-i_2=10 \quad\text{and}\quad -i_1+4i_2=-5$$

Solving, $i_1=\frac{5}{11}$, and $i_2=-\frac{5}{11}$

$$\therefore V_{oc}=(5+2i_2)=\left(5-\frac{10}{11}\right)=\frac{45}{11}\text{ V}$$

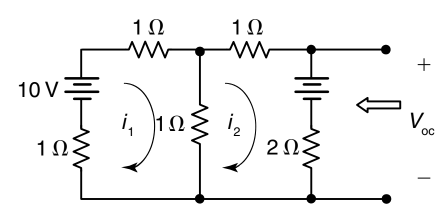


Fig. 4.72 (a)

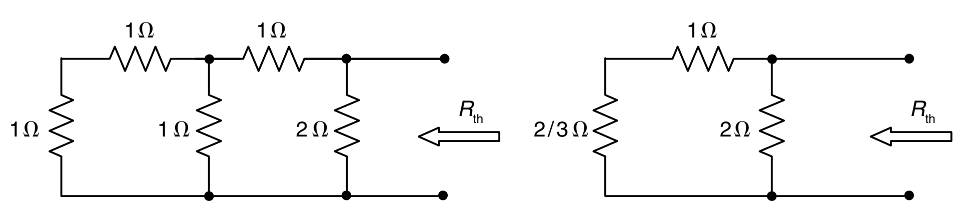


Fig. 4.72 (b)

Equivalent resistance, $R_{th} = \dfrac{\frac{5}{3}\times 2}{5/3 + 2} = \dfrac{10}{11}\,\Omega$

So, the load current is, $i = \dfrac{V_{oc}}{R_{th}+2} = \dfrac{45/11}{10/11 + 2} = \dfrac{45}{32} = 1.40625\text{ A}$

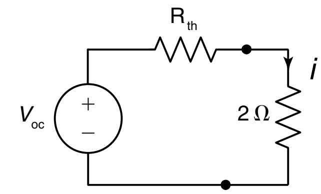


Fig. 4.73

Problem 4.15 ***Find I, in the given figure, using Thevenin's theorem.***

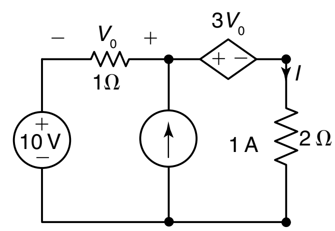


Fig. 4.74

Solution Removing the 2-Ω resistor,

By KVL for the supermesh, $-10 - v_0 + 3v_0 + v_{0c} = 0 \;\Rightarrow\; v_{0c} = 10 - 2v_0$

But, due to open-circuit, the 1-A source will circulate through the1-Ω resistor.

$$\therefore\; v_0 = 1\times 1 = 1\text{ V}$$

$$\therefore V_{0c} = (10-2) = 8\text{ V}$$

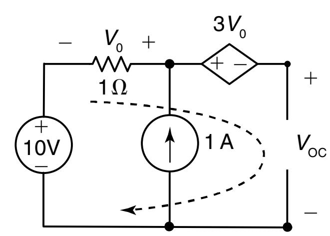


Fig. 4.75 (a)

Let's short circuit the terminals *x–y*,

By KVL,

$$-10 - v_0 + 3v_0 = 0 \quad\text{or,}\quad v_0 = 5$$

But, by KCL at the node (a),

$$\frac{v_0}{1} = 1 - I_{sc}$$

$$\Rightarrow\; I_{sc} = (1 - v_0) = -4\text{ A} \quad (\text{e.g., current is flowing from } y \text{ to } x)$$

$$\therefore\quad R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8}{4} = 2\,\Omega$$

So, the current through the 2-Ω resistor, $I = \dfrac{8}{2+2} = 2\text{ A}$

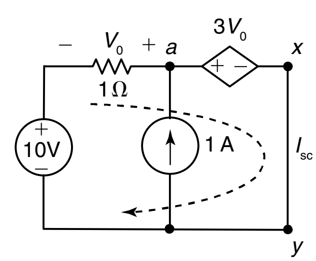


Fig. 4.75 (b)

Problem 4.16 ***By the iterative use of Thevenin's theorem, reduce the circuit shown in Fig. 4.76 to a single emf acting in series with a single resistor. Hence, calculate the current in the 10-Ω resistor connected across* XY.**

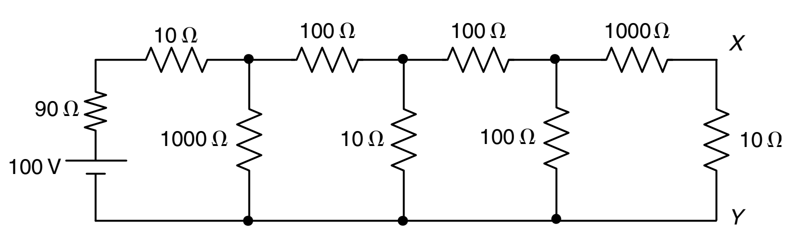


Fig. 4.76

Solution Consider the section of the network to the left of A–B: By use of Theremin's theorem, this portion is reduced to the form of Fig. 4.77 (b).

$$\therefore R_{th} = \frac{1000 \times 100}{1000 + 100} = \frac{1000}{11}\,\Omega$$

$$\therefore V_{th} = \frac{100 \times 1000}{1100} = \frac{1000}{11}\,\text{V}$$

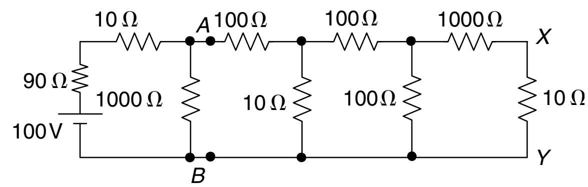


Fig. 4.77 (a)

Applying Thevenin's theorem to the section left of CD of Fig. 4.77 (b),

$$\therefore R_{th} = \frac{\left(1000/11\right) \times 10}{\left(2100/11\right) + 10} = \frac{2100}{221}\,\Omega$$

$$\therefore V_{th} = \frac{\left(1000/11\right) \times 10}{\left(2100/11\right) + 10} = \frac{1000}{221}\,\text{V}$$

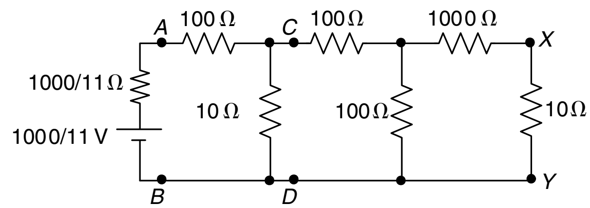


Fig. 4.77 (b)

Applying Theremin's theorem to the section left of EF of Fig. 4.77 (c),

$$\therefore R_{th} = \frac{\left(24200/221\right) \times 100}{\left(24200/221\right) + 100} = \frac{24200}{463}\,\Omega$$

$$\therefore V_{th} = \frac{\left(1000/221\right) \times 100}{\left(24200/221\right) + 100} = \frac{1000}{463}\,\text{V}$$

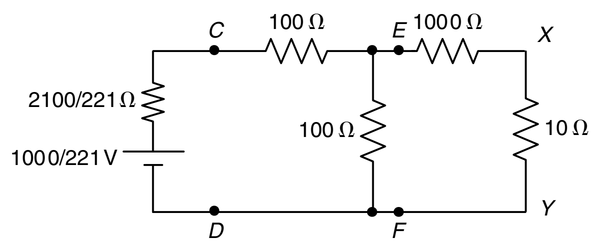


Fig. 4.77 (c)

Section left to XY is put as in Fig. 4.77 (d).

$$\therefore R_{th} = \frac{24200}{463} + 1000 = \frac{487200}{463}\,\Omega$$

$$V_{th} = \frac{\left(1000/463\right) \times 1000}{\left(24200/463\right) + 1000} = \frac{1000}{4872}\,\text{V}$$

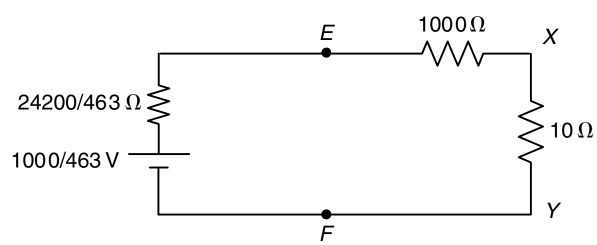


Fig. 4.77 (d)

Hence, the current in the 10-Ω resistor is

$$I = \frac{\left(1000/487.2\right)}{\left(487200/436\right) + 10} = 0.0193\,\text{A}$$

Problem 4.17 ***In the operational-amplifier circuit shown in Fig. 4.78 find I, in the R = 4-kΩ resistor, using Thevenin's theorem.***

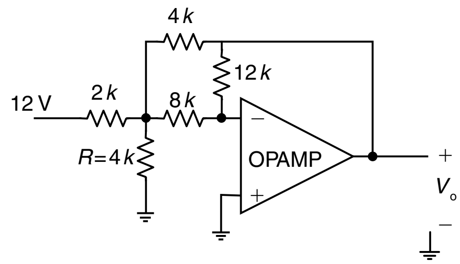


Fig. 4.78

Solution Open- circuiting the 4-kΩ resistor,

Here, $e_2 = 0, \quad e_3 = V_0$

$$\frac{e_1 - 12}{2\times10^3} + \frac{e_1 - V_0}{4\times10^3} + \frac{e_1}{8\times10^3} = 0 \Rightarrow 7e_1 = (48 + 2V_0) \qquad \text{(i)}$$

$$\frac{0 - e_1}{8\times10^3} + \frac{0 - V_0}{12\times10^3} = 0 \Rightarrow V_0 = -\frac{3}{2}e_1 \qquad \text{(ii)}$$

From (i) and (ii), ⇒ $e_1 = 4{\cdot}8 \text{ V} = e_{oc}$

Now, we connect a 1-A current source at the place of the 4-kΩ resistor.

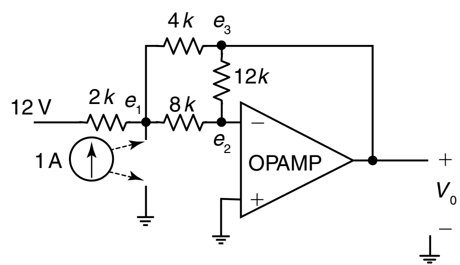


Fig. 4.79

By KCL at the node (1),

$$\frac{e_1}{2\times10^3} + \frac{e_1 - V_0}{4\times10^3} + \frac{e_1}{8\times10^3} = 1 \Rightarrow 7e_1 = 8000 + 2V_0$$

By KCL at the node (2),

$$V_0 = -\frac{3}{2}e_1 \Rightarrow 7e_1 = 8000 + 2\left(-\frac{3}{2}e_1\right) \Rightarrow e_1 = 800 \text{ V}$$

$$\therefore R_{th} = \frac{e_1}{1} = 800\,\Omega$$

$$\therefore i = \frac{4{\cdot}8}{4000 + 800} = \frac{4{\cdot}8}{4{\cdot}8\times10^3} = 1 \text{ mA}$$

Problem 4.18 ***Find Thevenin's equivalent about AB for the circuit shown in Fig. 4.80.***

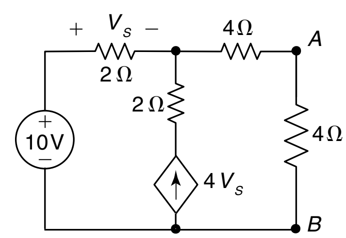


Fig. 4.80

Solution Open-circuiting the 4-Ω resistor by KCL,

$$\frac{V_{oc} - 10}{2} = 4v_s = 4(10 - V_{oc}) \Rightarrow V_{oc} = 10 \text{ V}$$

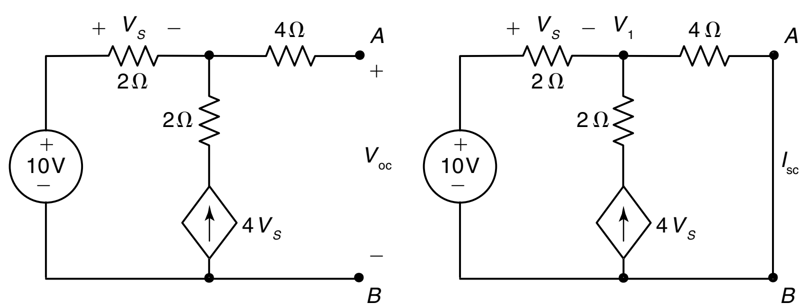


Fig. 4.81

Short-circuiting the terminals AB, by KCL

$$\frac{V_1-10}{2}+\frac{V_1}{4}=4v_s=4(10-V_1)$$

$$V_1=\frac{180}{19}=9\cdot 47\text{ V}$$

$$\therefore\quad I_{sc}=\frac{9\cdot 47}{4}=2\cdot 368\text{A}$$

$$\therefore\quad R_{th}=\frac{V_{th}}{I_{sc}}=4\cdot 22\,\Omega$$

Problem 4.19 ***In the network, determine the steady current in the 8-Ω inductor using Thevenin's theorem.***

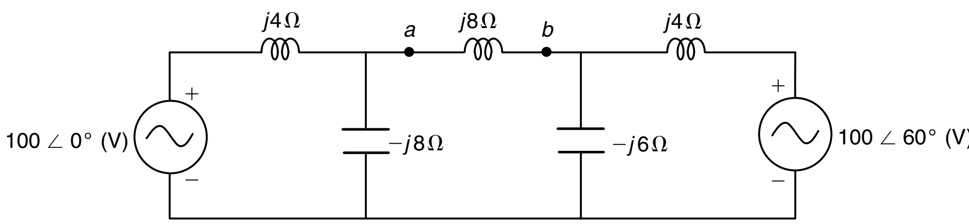


Fig. 4.82

Solution With a-b open-circuited,

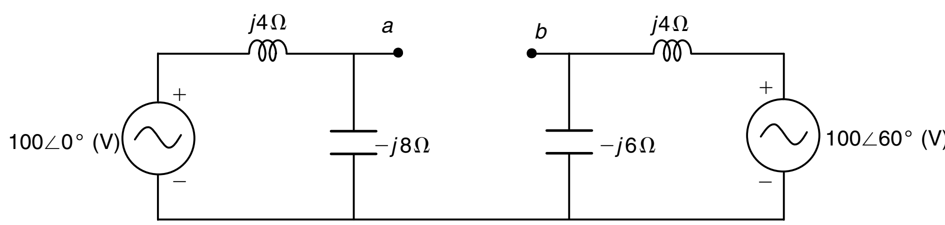


Fig. 4.83

$$V_a=\frac{100\angle 0^\circ}{j4-j8}(-j8)=200\angle 0^\circ\text{ V}$$

$$V_b=\frac{100\angle 60^\circ}{j4-j6}(-j6)=300\angle 60^\circ\text{ V}$$

$$\therefore\ V_{th}=(V_a-V_b)=200\angle 0^\circ-300\angle 60^\circ=(50-j259.81)\text{ V}$$

$$\therefore\ Z_{th}=\frac{(j4)(-j8)}{j4-j8}+\frac{(j4)(-j6)}{j4-j6}=j20\,\Omega$$

$\therefore$ current in the 8-Ω inductor, $i=\dfrac{V_{th}}{Z_{th}+Z_L}=\dfrac{(50-j259.81)}{j20+j8}=9.45\angle -169.1^\circ\text{ A}$

Problem 4.20 ***Obtain Thevenin's equivalent circuit with respect to terminals A–B in the networks shown below.***

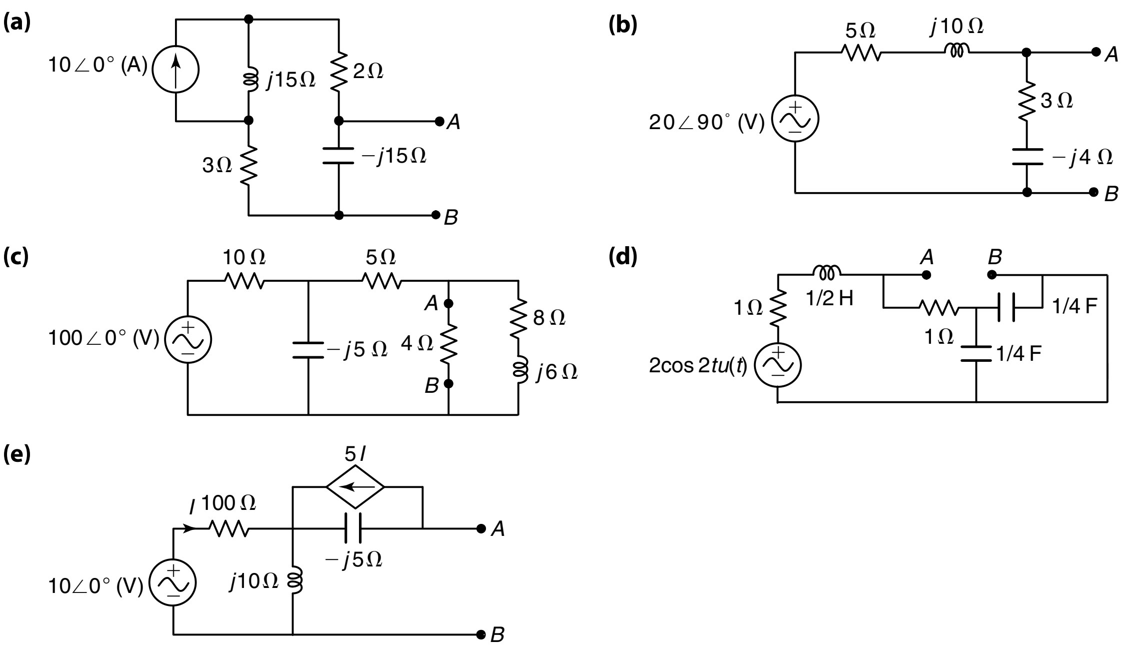


Fig. 4.84

Solution

(a) With A–B open, the current is

$$I = \frac{10\angle 0^\circ}{5 - j5 + j15} \times j15 = \frac{150\angle 90^\circ}{5 + j10}$$

Thevenin voltage

$$V_{th} = V_{AB} = I \times (-j5) = \frac{150\angle 90^\circ}{5 + j10} \times (5\angle -90^\circ) = 67.08\angle -63.4^\circ \text{ V}$$

Thevenin impedance,

$$Z_{th} = Z_{AB} = \frac{-j5 \times (5 + j15)}{-j5 + 5 + j15} = 7.07\angle -81.86^\circ \ \Omega$$

Thus, the Thevenin's equivalent circuit is shown in Fig. 4.85 (b).

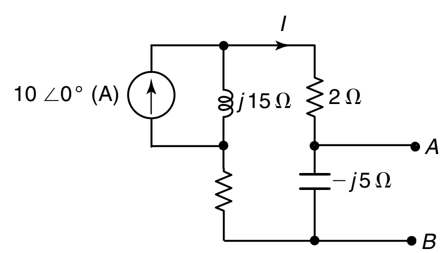


Fig. 4.85 (a)

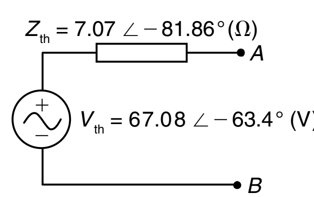


Fig. 4.85 (b)

(b) Here, Thevenin voltage,

$$V_{th} = \frac{20\angle 90^\circ}{5 + j10 + 3 - j4} \times (3 - j4) = \frac{j120(3 - j4)}{8 + j6}$$

$$V_{th} = \frac{50\angle 36.87^\circ}{5\angle 36.87^\circ} = 10\angle 0^\circ \ \text{(V)}$$

Thevenin impedance,

$$Z_{\text{th}} = \frac{(5+j10)\times(3-j4)}{(5+j10)+(3-j4)} = \frac{11.8\angle 63.43^\circ \times 5\angle -53.13^\circ}{10\angle 36.87^\circ} = 5.59\angle -26.56^\circ\,(\Omega)$$

Thus, the Thevenin's equivalent circuit is shown in Fig. 4.86 (b).

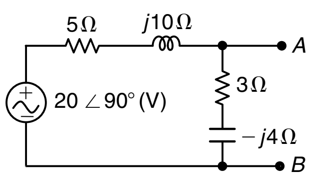


Fig. 4.86 (a)

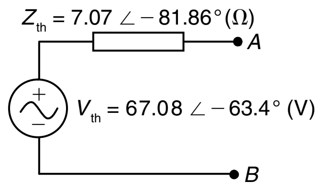


Fig. 4.86 (b)

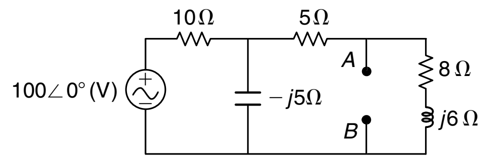


Fig. 4.87

(c) Here, with A–B open, the equivalent impedance,

$$Z = 10 + \frac{-j5\times(13+j6)}{-j5+(13+j6)} = \frac{160-j55}{13+j1}\,\Omega = 12.98\angle -23.37^\circ\ (\Omega)$$

$\therefore$ main current, $\quad I = \dfrac{100\angle 0^\circ}{Z} = \dfrac{100\angle 0^\circ}{12.98\angle -23.37^\circ} = 7.7\angle 23.37^\circ\ (\text{A})$

$\therefore$ Thevenin voltage,

$$V_{\text{th}} = I\times\left(\frac{-j5}{-j5+5+8+j6}\right)\times(8+j6) = 7.7\angle 23.37^\circ \times\left(\frac{-j5}{13+j1}\right)\times(8+j6) = 29.553\angle -34.16^\circ\ (\text{V})$$

$\therefore$ Thevenin impedance, $\quad Z_{\text{th}} = \left[\dfrac{10\times(-j5)}{10-j5}+5\right] \| (8+j6) = 5.33\angle -0.5^\circ\ (\Omega)$

(d) The circuit is redrawn as shown in Fig. 4.88, considering two capacitors in parallel.

$$\therefore\ C_{eq} = (C_1 + C_2) = \left(\frac{1}{4}+\frac{1}{4}\right) = \frac{1}{2}F$$

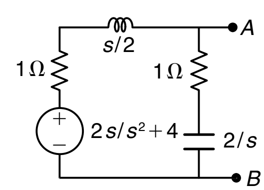


Fig. 4.88

Thevenin voltage is given as

$$V_{\text{th}}(s) = \frac{2s}{s^2+4}\times\frac{\left(1+2/s\right)}{\left(1+2/s+1+s/2\right)} = \frac{4s}{(s^2+4)(s+2)}\ (\text{V})$$

$\therefore$ Thevenin impedance, $Z_{\text{th}}(s) = \left(1+2/s\right) \| \left(1+s/2\right) = 1\,\Omega$

(e) **To find V_{th}**

With A–B open, the current of the dependent source can flow through the capacitor only.

$$\therefore I = \frac{10\angle 0^\circ}{100+j10} = 0.09995\angle -5.7^\circ\ (\text{A})$$

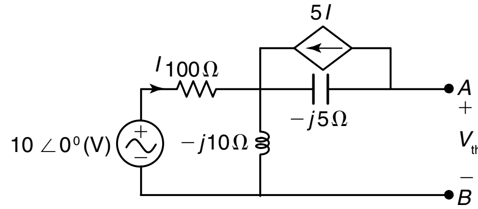


Fig. 4.89

∴ Thevenin voltage,

$$V_{th} = V_{AB} = (I \times j10) - \{5I \times (-j5)\} = j35I$$
$$= j35 \times 0.09995\angle -5.7^\circ = 3.48\angle 84.3^\circ \text{ (V)}$$

To find I_N

Converting the dependent current source into the voltage source, by KVL,

$$10\angle 0^0 = (100 + j10)I - j10I_N$$

and $-(-j25I) = -j10I + I_N(j10 - j5)$

Solving for I_N, $I_N = 0.6\angle 31^\circ$ (A)

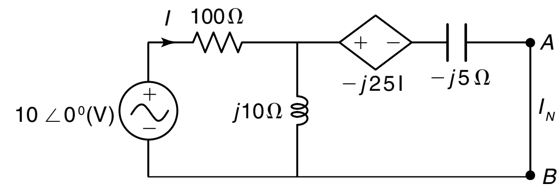


Fig. 4.90

∴ Thevenin impedance,

$$Z_{th} = \frac{V_{th}}{I_N} = \frac{3.48\angle 84.3^\circ}{0.6\angle 31^\circ} = 5.8\angle 53.3^\circ \,(\Omega)$$

Problem 4.21 ***Find V_0 using Thevenin's theorem***

Solution **To find V_{th}**

Removing the 2-Ω resistor and open circuiting the terminals and then converting the dependent current source into dependent voltage source, we redraw the circuit as follows.

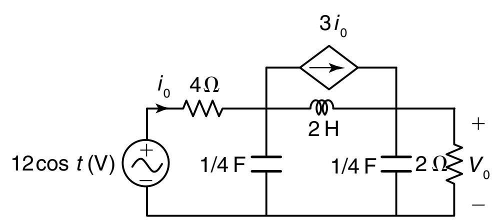


Fig. 4.91

By KVL for the two loops, (here, $i_0 = I_1$)

$$(4 - j4)I_1 + j4I_2 = -12 \quad \text{and} \quad -j2I_1 + (-j6)I_2 = 0$$

Solving for I_2,

$$I_2 = \frac{\begin{vmatrix} (4-j4) & -12 \\ -j2 & 0 \end{vmatrix}}{\begin{vmatrix} (4-j4) & j4 \\ -j2 & -j6 \end{vmatrix}} = \frac{-j24}{-j24 - 24 - 8}$$
$$= \frac{j3}{4 + j3} = 0.6\angle 53.13^\circ \text{ (A)}$$

Therefore, Thevenin voltage is

$$V_{th} = I_2 \times (-j8) = \frac{24}{4 + j3} = 4.8\angle -36.87^\circ \text{ (V)}$$

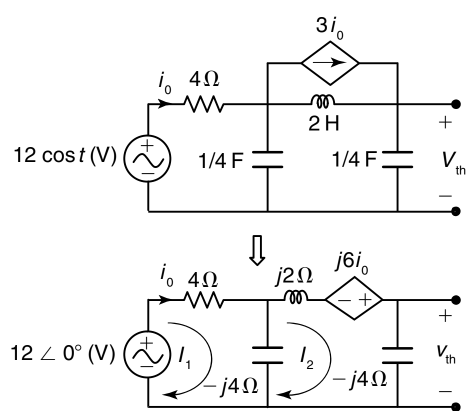


Fig. 4.92

To find I_N

Removing the 2-Ω resistor and short-circuiting the terminals and then converting the dependent current source into dependent voltage source, we redraw the circuit as shown in Fig. 4.92 (b)

By KVL for the two loops,

$$(4-j4)I_1 + j4I_2 = -12$$
$$-j2I_1 + (-j2)I_2 = 0$$

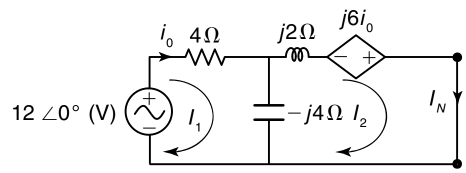


Fig. 4.92 (b)

Solving for I_2,

$$I_2 = I_N = \frac{\begin{vmatrix}(4-j4) & -12\\ -j2 & 0\end{vmatrix}}{\begin{vmatrix}(4-j4) & j4\\ -j2 & -j2\end{vmatrix}} = \frac{-j24}{-8-j8-8} = \frac{j3}{2+j} = 1.341\angle 63.435° \text{ (A)}$$

Therefore, Thevenin impedance is,

$$Z_{th} = \frac{V_{th}}{I_N} = \frac{4.8\angle -36.87°}{1.341\angle 63.435°} 3.58\angle -100.3° \text{ (Ω)}$$

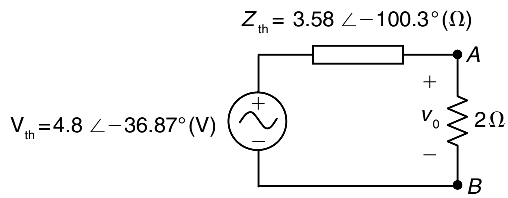


Fig. 4.93

Thus, Thevenin's equivalent circuit becomes as shown in Fig. 4.93.

Thus, the required voltage,

$$v_0 = \left(\frac{V_{th}}{Z_{th}+2}\right)\times 2 = \left(\frac{4.8\angle -36.87°}{3.58\angle -100.3° + 2}\right)\times 2 = 1.27\angle 32° \text{ (V)}$$

Problem 4.22 ***Obtain the Norton's equivalent circuit with respect to the terminals* AB *for the network shown in Fig. 4.94.***

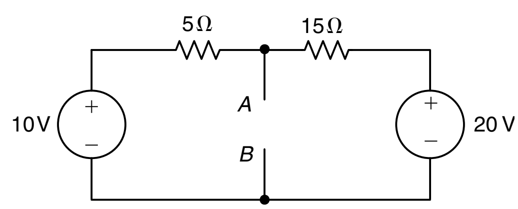


Fig. 4.94

Solution Removing the source,

$$\therefore Z_{eq} = \frac{5\times 15}{5+15} = \frac{75}{20} = 3\cdot 75\,\Omega$$

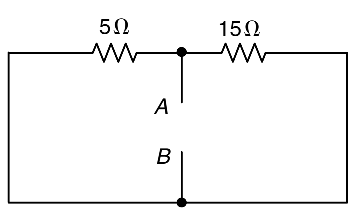


Fig. 4.94 (b)

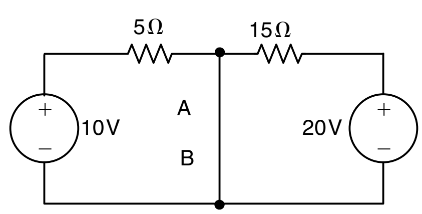


Fig. 4.95

Short-circuiting *AB*,

$$I_{sc} = \frac{10}{5} + \frac{20}{15} = 3.33\text{ A}$$

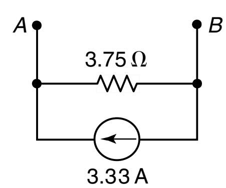


Fig. 4.96

So, Norton's equivalent circuit is shown in Fig.

Problem 4.23 ***Replace the circuit in Fig. 4.97 with the Thevenin's equivalent circuit across* A *and* B.**

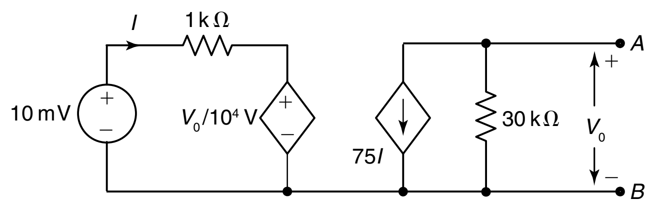


Fig. 4.97

Solution By KVL for the left-hand side loop,

$$1\times10^3\times I+\frac{V_0}{10^4}=10\times10^{-3} \tag{i}$$

In the right-hand side loop, the dependent current source current will circulate in the resistor. By KVL,

$$V_0=30\times10^3\times(-75I)=-225\times10^4\,I \tag{ii}$$

Substituting the value of I from (ii) in (i), we get,

$$\Rightarrow\ 1\times10^3\times\left(-\frac{V_0}{225\times10^4}\right)+\frac{V_0}{10^4}=10\times10^{-3}$$

$$\Rightarrow\ -4.44\times10^{-4}V_0+1\times10^{-4}V_0=10\times10^{-3}$$

$$\Rightarrow\ V_0=-\frac{10\times10^{-3}}{3.44\times10^{-4}}=-29\text{ V}$$

Now, short circuiting the terminals A and B, we get by KVL to left-hand-side loop,

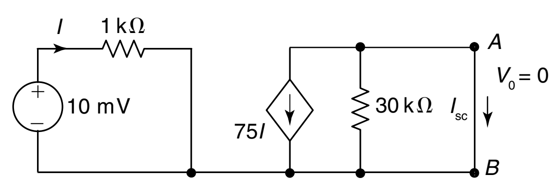


Fig. 4.98

$$1\times10^3\times I+0=10\times10^{-3}\ \Rightarrow\ I=1\times10^{-5}\text{ A}$$

Also, from right-hand side loop on the short circuit,

$$I_{sc}=-75I=-75\times1\times10^{-5}=-75\times10^{-5}\text{ A}$$

Thus, the Thevenin equivalent impedance is given as

$$Z_{th}=\frac{V_{oc}}{I_{sc}}=\frac{-29}{-75\times10^{-5}}=38.67\text{ k}\Omega$$

Thevenin's equivalent circuit is shown in the Fig. 4.99.

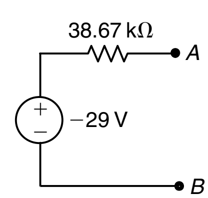


Fig. 4.99

Problem 24 ***Find the Thevenin's equivalent between terminals* a *and* b *of the circuit shown in Fig. 4.100.***

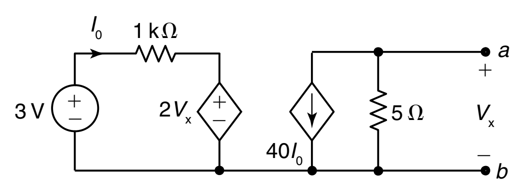


Fig. 4.100

Solution By KVL for the right-hand side mesh,

$$V_{oc}=V_x=(-40I_0)\times50=-2000I_0 \tag{i}$$

From the left-hand side loop,

$$I_0 = \frac{3-2V_x}{1000} = \frac{3-2V_{oc}}{1000} \qquad \text{(ii)}$$

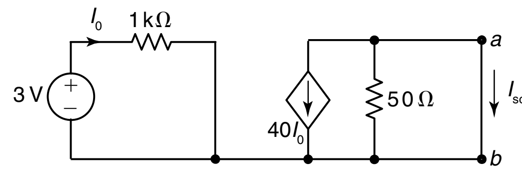


Fig. 4.101

From (*i*) and (*ii*), we get,

$$V_{oc} = -2000\left(\frac{3-2V_{oc}}{1000}\right) \Rightarrow V_{oc} = 2\text{ V}$$

To determine the Thevenin's impedance, we short circuit the terminals *a* and *b*.

Here,

$$I_{sc} = -40I_0 = -40 \times \left(\frac{3}{1000}\right) = -0.12$$

$$\therefore R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{2}{0.12} = 16.67\ \Omega$$

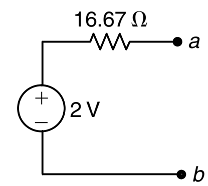


Fig. 4.102

Thevenin's equivalent circuit is shown in Fig. 4.102.

Problem 25 ***In the network shown in Fig. 4.103 the switch is closed at time t = 0. Assuming all the initial currents and voltages as zero, find the current through the inductor L_2 by the use of Norton's theorem.***

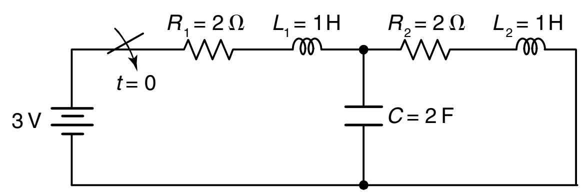


Fig. 4.103

Solution The network for $t > 0$ in Laplace domain is shown in Fig. 4.104.

The equivalent network reduces to one as shown in Fig. 4.105.

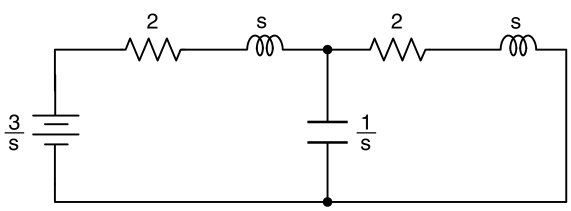


Fig. 4.104

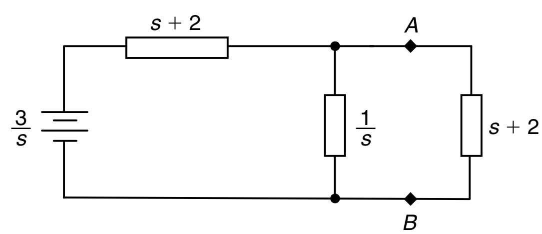


Fig. 4.105

To find the current in L_2, we have to find Thevenin's equivalent circuit across the terminals *A* and *B*. The impedance between terminals *A* and *B* is given as

$$Z_{th} = Z_{AB} = \frac{(s+2)\times\frac{1}{s}}{s+2+\frac{1}{s}} = \frac{(s+2)}{s^2+2s+1} = \frac{(s+2)}{(s+1)^2}$$