Module-2

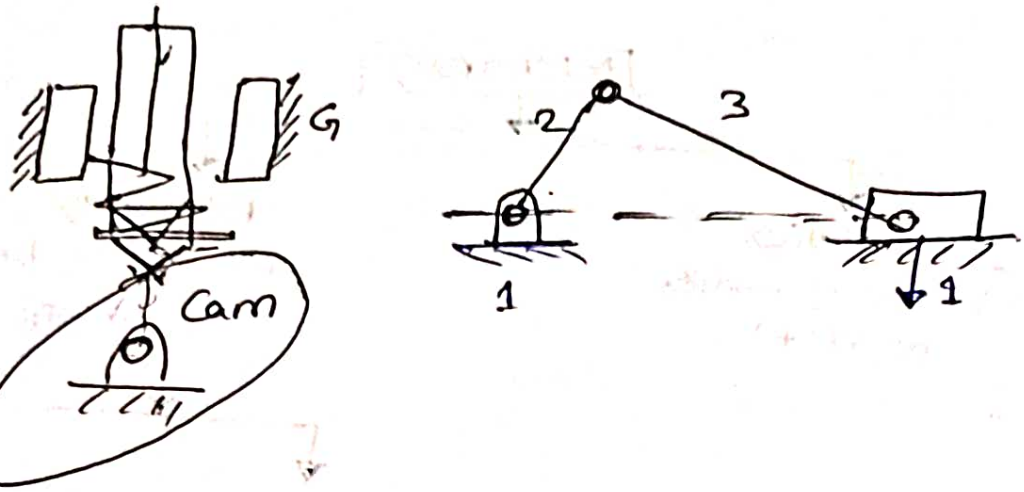
Kinematics & Kinetics of rigid bodies.

- **Mechanics**
  - Statics (study of bodies at rest)
  - Dynamics (study of bodies in motion)
    - Kinematics (without considering Forces)
    - Kinetics. (with considering forces)

- **Types of Motion in a plane**
  - Translation
    - Rectilinear (1D)
    - Curvilinear (2D)
  - Rotation
  - General motion (T+R)

- Rigid-body kinematics involves both linear & angular displacements, velocities & accelerations.
- We need to describe the motion of rigid bodies for two important reasons
  1. We frequently need to generate, transmit or control certain motions by the use of cams, gears & linkages of various types.
     - Here we must analyze the displacement, velocity & accⁿ of the motion to determine the design geometry of mechanical parts.
     - as a result of the motion generated, forces may be developed which must be accounted for in the design of the parts.

③ we must determine the motion of a rigid body caused by the forces applied to it.



Assumption of Rigid Bodies :-

- Rigid body is a system of particles for which the distances b/w the particles remains unchanged
- Thus, if each particle of such a body is located by a PV from Ref. axes attached to & rotating with the body, there will be no change in any position vector as measured from these axes
- This is, of course, an ideal case since all solid materials change shape to some extent when forces are applied to them
- If the movements associated with the changes in shape are very small compared with the movements of the body as a whole, then the assumption of rigidity is usually acceptable.

plane motion :-

A rigid body executes plane motion when all parts of the body move in parallel planes.

For convenience we generally consider the plane of motion to be plane which contains the mass center, & we treat the body as a thin slab whose motion is confined to the plane of the slab.

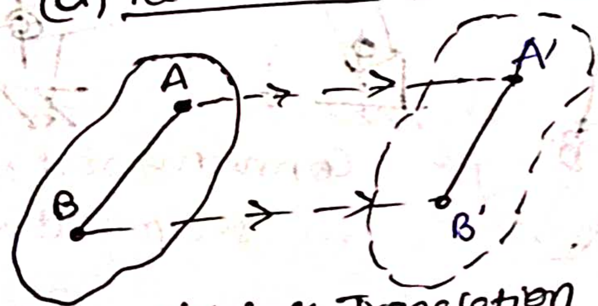
- **Translation** is defined as any motion in which every line in the body remains parallel to its original position at all times. In translation, there is no rotation of any line in the body.

In rectilinear translation, all points in the body

In curvilinear translation, all points move on congruent curves.

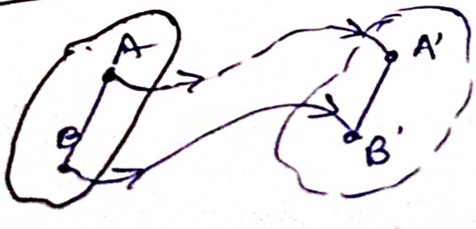
In each of two cases of translation the motion of the body is completely specified by the motion of any point in the body, since all points have the same motion.

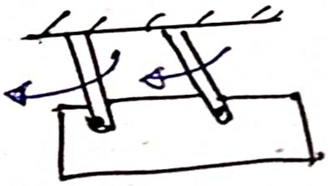
(a) Rectilinear Translation



Rocket test sled.

(b) curvilinear Translation





(c) Rotation about a fixed axis

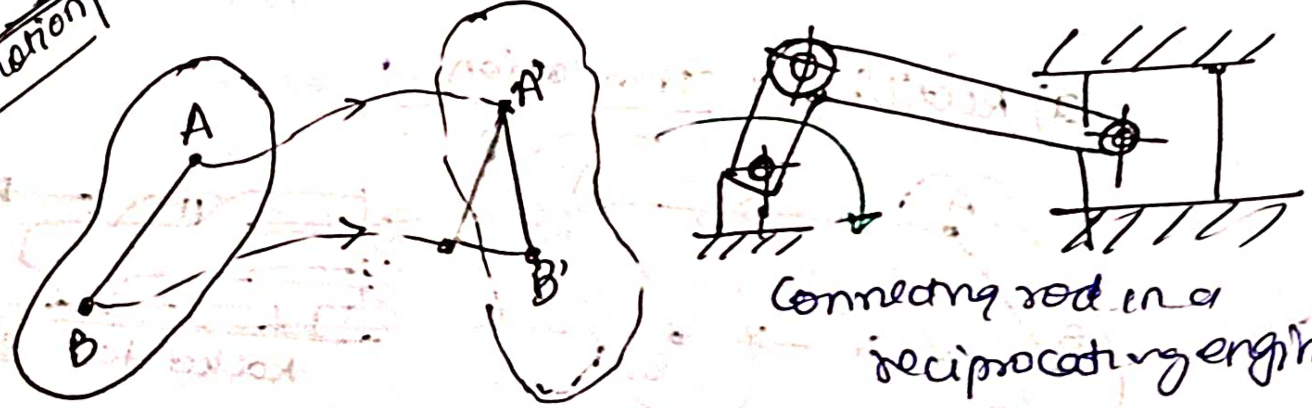
is the angular motion.

It follows that all particles in a rigid body moves in circular paths about the axis of rotation and all lines in the body which are ⊥r to the Axis of rotation (including which do not pass through the axis) rotate through the same angle in the same time.

(d) General plane motion of a rigid body, is a combination of translation and rotation.

Fixed-axis Rotation

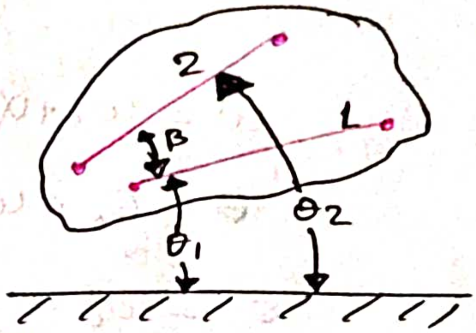
General Plane motion



Rotation

→ The rotation of a rigid body is described by its angular motion.

→ Figure shows a rigid body which is rotating as it undergoes plane motion in the plane of the figure.



* β is invariant
* $\theta_2 = \theta_1 + \beta$.

diff. w.r.t t

$$\boxed{\dot{\theta}_2 = \dot{\theta}_1}$$

again diff. w.r.t t

$$\boxed{\ddot{\theta}_2 = \ddot{\theta}_1}$$

During a finite interval

$$\Delta\theta_2 = \Delta\theta_1$$

Thus all lines on a rigid body in its plane of motion have same angular displacement, the same angular velocity and the same angular acceleration

ω & α & θ

CW → +ve

ACW → -ve

$$\boxed{\omega = \frac{d\theta}{dt} = \dot{\theta}}$$

$$\boxed{\alpha = \frac{d\omega}{dt} = \dot{\omega}} \text{ or } \boxed{\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}}$$

$$\boxed{\omega\, d\omega = \alpha\, d\theta} \text{ or } \boxed{\dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta}$$

$\boxed{\text{Angular Motion Relations}}$

these relation are important to recognize, as they help to demonstrate the symmetry & unity found throughout mechanics.

<u>For rotation with constant angular acceleration:</u>

$\alpha \rightarrow$ Const. $\qquad\qquad a \rightarrow$ const

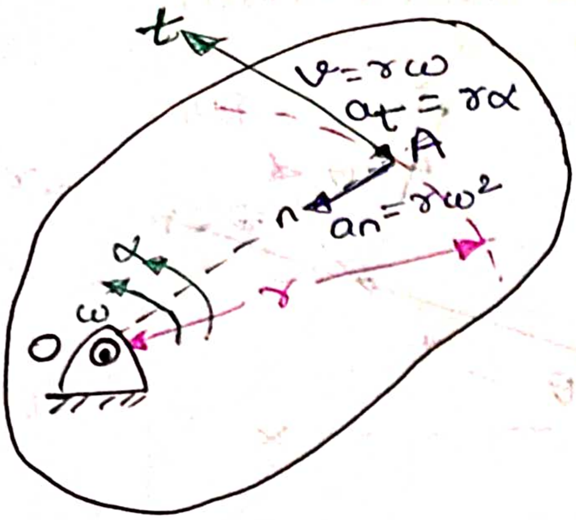
$\omega = \omega_0 + \alpha t \qquad\qquad x = x_0 + (v_0)_x t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad\qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_c t^2$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \qquad\qquad (v_y)^2 = (v_0)_y^2 + 2a(y - y_0)$

θ_0 & ω_0 are calculated at $t = 0$.

Rotation about a fixed Axis :-



when a rigid body rotates about a fixed axis all points other than those on the axis move in concentric circles about the fixed axis.

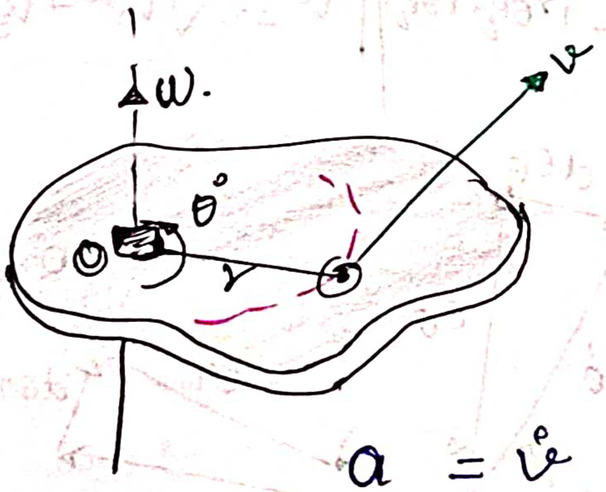
O → fixed pt

A is any point at circle of radius r.

$$\omega = \dot{\theta} \quad \& \quad \alpha = \dot{\omega} = \ddot{\theta}$$

$$v = r\omega$$
$$a_n = \frac{v^2}{r} = r\omega^2 = v\omega$$
$$a_t = r\alpha$$

$$v = r\dot{\theta}$$
$$a_n = \frac{v^2}{r} = r\dot{\theta}^2 = v\dot{\theta}$$
$$a_t = \dot{v} = r\ddot{\theta}$$



$$\boxed{V = \dot{r} = \omega \times r}$$

$$\boxed{a_t = \alpha \times r}$$

$$a_n = \omega \times (\omega \times r)$$

$$\boxed{a_n = \omega \times v}$$

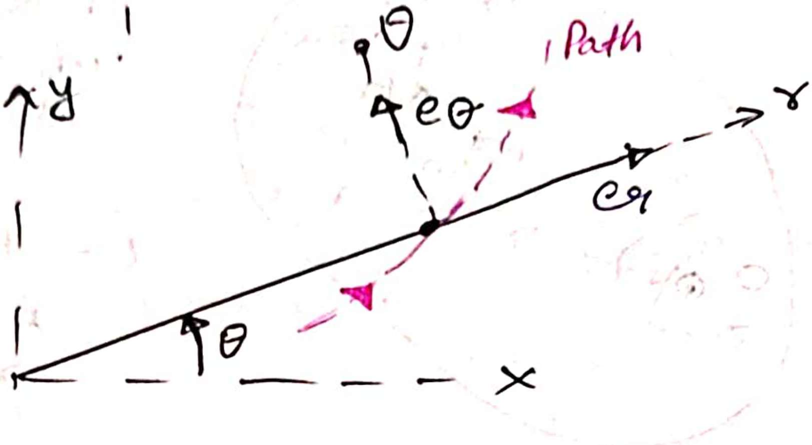
$$a = \dot{v} = \omega \times \dot{r} + \dot{\omega} \times r$$

$$a = \omega \times (\omega \times r) + \dot{\omega} \times r$$

$$\boxed{a = \omega \times v + \alpha \times r}$$

Plane curvilinear Motion

(Polar coordinates → (r, θ))



$$\frac{de_r}{d\theta} = e_\theta$$

$$\frac{de_\theta}{d\theta} = -e_r$$

$$\frac{de_r}{dt} = \left(\frac{d\theta}{dt}\right) e_\theta$$

$$\frac{de_\theta}{dt} = -\left(\frac{d\theta}{dt}\right) e_r$$

$$\boxed{\dot{e}_r = \dot{\theta}\, e_\theta}$$

$$\boxed{\dot{e}_\theta = \dot{\theta}\, e_r}$$

$$r = r e_r$$

$$v_r = \dot{r}$$

$$v_\theta = r\dot{\theta}$$

$$v = \dot{r} = \dot{r} e_r + r\dot{e}_r \rightarrow \boxed{v = \dot{r} e_r + r\dot{\theta} e_\theta}$$

$$\boxed{v = \sqrt{v_r^2 + v_\theta^2}}$$

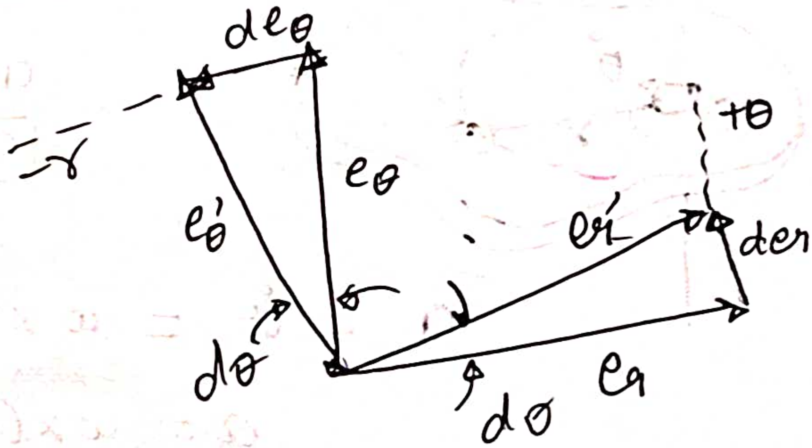
$$a = \dot{v} = (\ddot{r} e_r + \dot{r}\dot{e}_r) + (\dot{r}\dot{\theta} e_\theta + r\ddot{\theta} e_\theta + r\dot{\theta}\dot{e}_\theta)$$

$$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

$$\boxed{a_r = \ddot{r} - r\dot{\theta}^2}$$

$$\boxed{a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}}$$

$$a = \sqrt{a_r^2 + a_\theta^2}$$

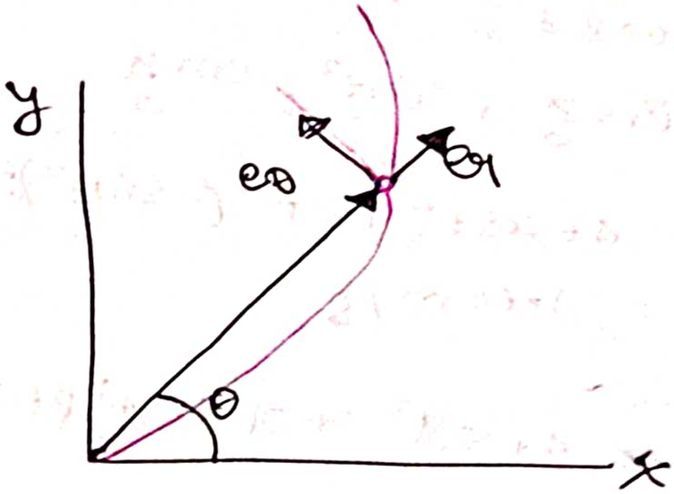


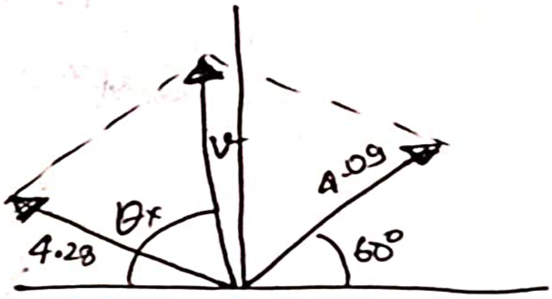
Circular motion

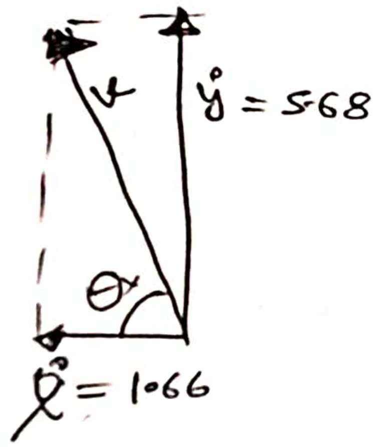
$v_r = 0$ $\quad v_\theta = r\dot{\theta}$

$a_r = -r\dot{\theta}^2$ $\quad a_\theta = r\ddot{\theta}$

Q. A particle moves along path whose eqn is $r = 2\theta$ m. If the angle $\theta = t^2$ rad, determine the velocity of the particle when θ is 60°. Use two methods.







$$\theta = t^2$$

$$r = 2\theta = 2t^2$$

$$\dot{r} = 4t$$

v at $\theta = \frac{\pi}{3}$

$$\theta = \frac{\pi}{3} = t^2$$

$$t = 1.023\ s$$

$$v = \dot{r}e_r + r\dot{\theta}e_\theta$$

$$v = 4(1.023)\,e_r + [2(1.023)^2[2(1.023)]]\,e_\theta$$

$$v = 4.09\,e_r + 4.28\,e_\theta$$

$$v = \sqrt{(4.09)^2 + (4.28)^2}$$

$$v = 5.92\ m/s$$

$$\theta_v = 30^\circ + \tan^{-1}(4.09/4.28) = 73.70$$

(b) 1. cartesian coordinate

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$x = 2\theta\cos\theta$ $\quad$ $y = 2\theta\sin\theta$

$x = 2t^2\cos t^2$ $\quad$ $y = 2t^2\sin t^2$

$t = 1.023\ s$

$\cos t^2 = \cos\frac{\pi}{3}$, $\sin t^2 = \sin\frac{\pi}{3}$

$\dot{x} = 4t\cos t^2 + 2t^2(-\sin t^2)(2t)$

$\dot{x} = -1.66\ m/s$

$\dot{y} = 4t\sin t^2 + 2t^2(\cos t^2)(2t)$

$\dot{y} = 5.68\ m/s$

$v = \sqrt{(-1.66)^2 + (5.68)^2} = 5.92\ m/s$

$\theta_x = \tan^{-1}\left(\frac{5.68}{1.66}\right) = 73.7^\circ$

In the preceding problem determine the magnitude & accn of the particle using the same two methods

(a) Polar coordinates

$a = (\ddot{r} - r\dot{\theta}^2)e_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})e_\theta$

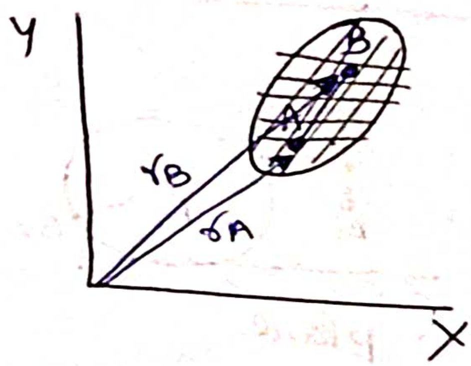
$= -4.77\,e_r + 20.94\,e_\theta$

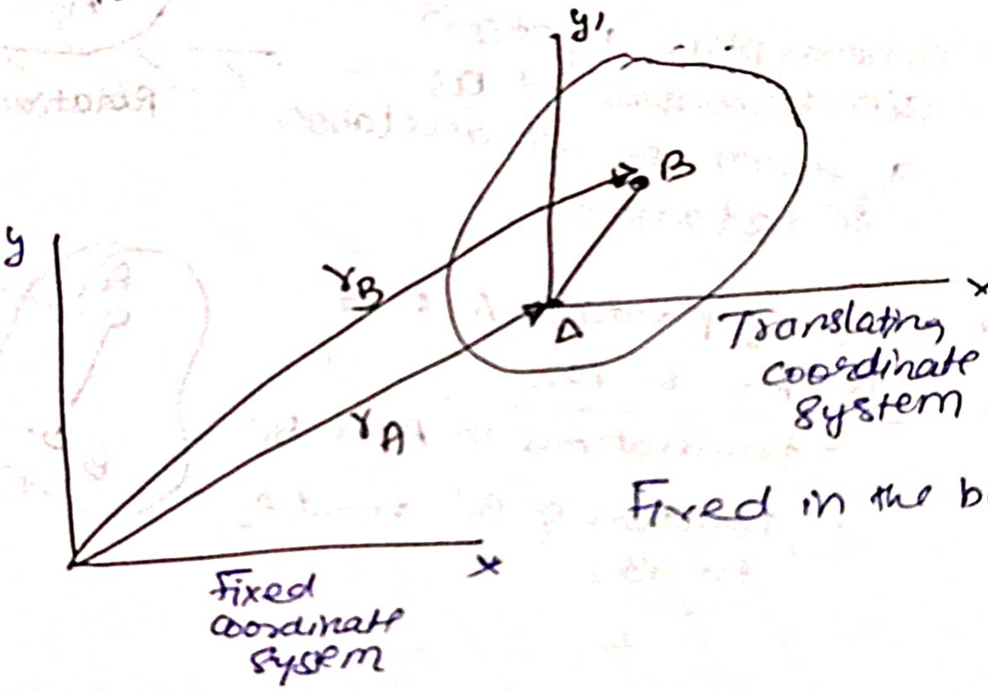
$a = \sqrt{(-4.77)^2 + (20.98)^2} = 21.5\ m/s^2$

$\theta_Y = 30^\circ - 12.8 = 17$

Kinematics of Translation

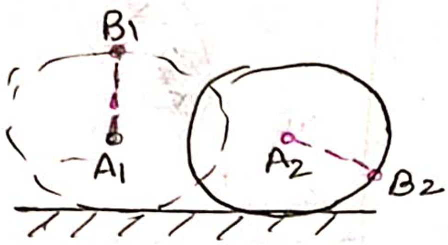
- Kinematics
  - Position
    $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$
  - Velocity
    $\vec{v}_B = \vec{v}_A$
  - Acceleration
    $\vec{a}_B = \vec{a}_A$
- True for all points in a rigid body





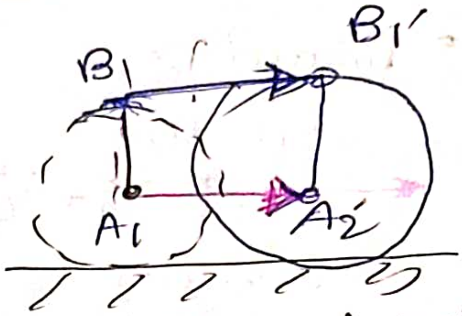
Fixed in the body

General Plane Motion



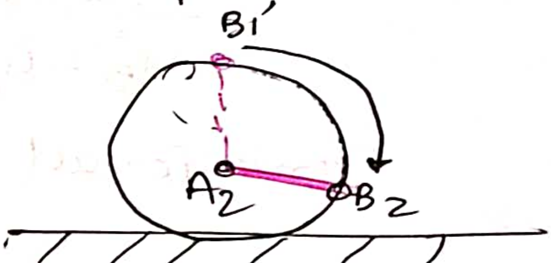
plane

=



Translation with A

+

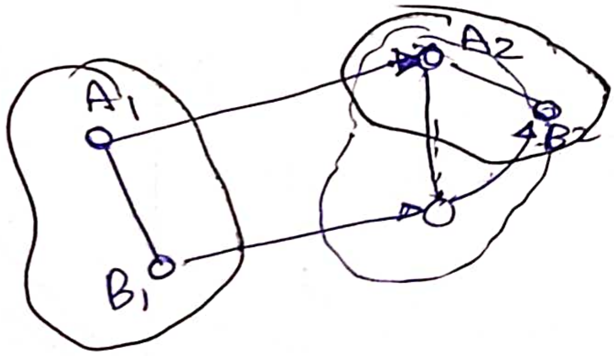


Rotation about A.

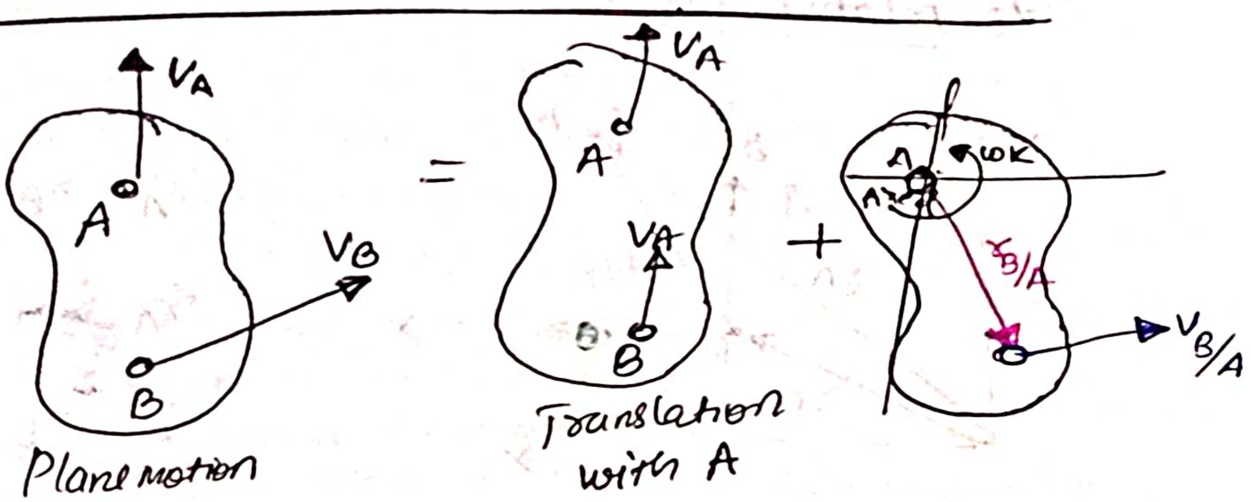
* General plane motion is neither a translation nor a rotation

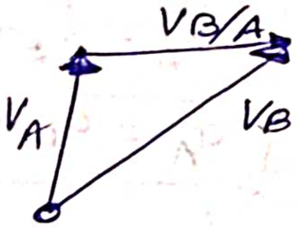
* General plane motion can be considered as a sum of a translation & rotation.

* Disp of particle A & B to A_2 & B_2
  - translations to A_2 & B_1'
  - rotation of B_1' about A_2 to B_2



Absolute & Relative velocity in Plane motion



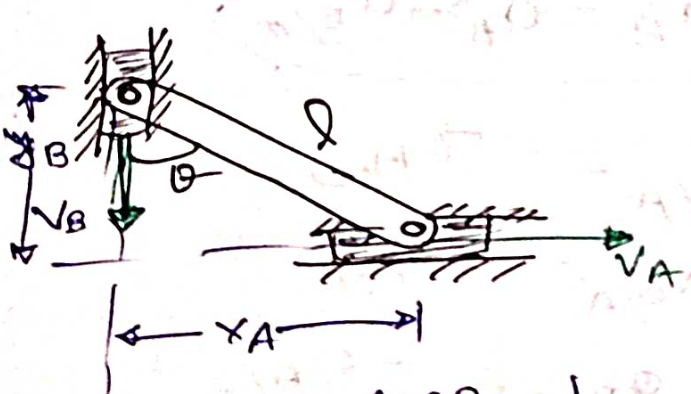


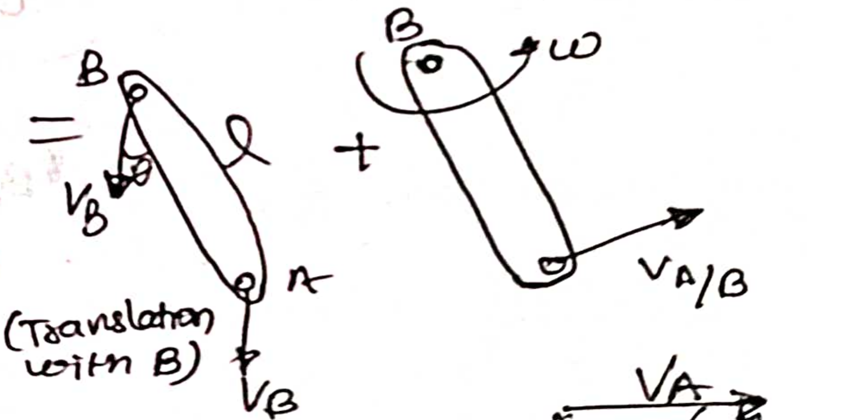
- Any plane motion can be replaced by a translation of an arbitrary reference pt A & a simultaneous rotation about A.

$$\vec{V_B} = \vec{V_A} + \vec{V}_{B/A}$$

$$\vec{V}_{B/A} = \omega \vec{k} \times \vec{r}_{B/A} \qquad V_{B/A} = r\omega$$

$$\vec{V_B} = \vec{V_A} + \omega \vec{k} \times \vec{r}_{B/A}$$



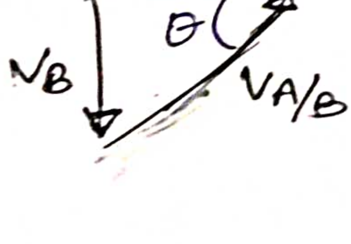


$x_A = l\sin\theta \quad y_B = l\cos\theta$

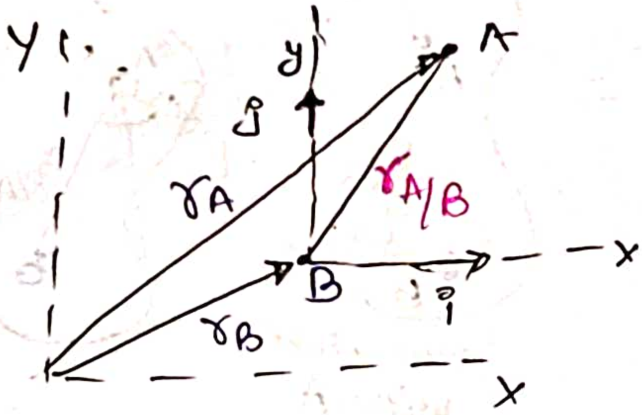
$V_A = \dot{x}_A$
$= l\dot{\theta}\cos\theta$
$\boxed{V_A = l\omega\cos\theta}$

$V_B = \dot{y}_B$
$V_B = -l\dot{\theta}\sin\theta$
$\boxed{V_B = -l\omega\sin\theta}$

$V_A = V_B + V_{A/B}$



Relative Motion - Translating axes



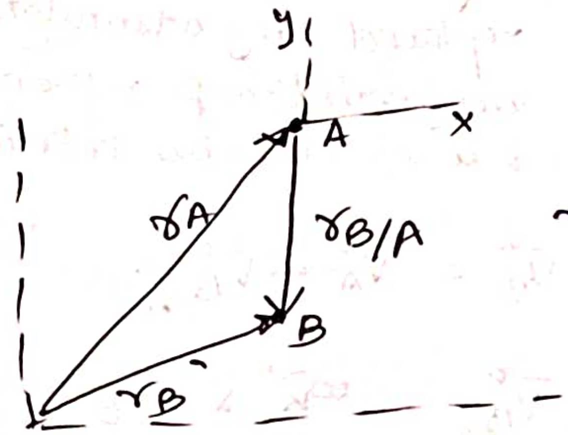
$$r_A = r_B + r_{A/B}$$

$$\dot{r}_A = \dot{r}_B + \dot{r}_{A/B}$$

$$\boxed{V_A = V_B + V_{A/B}}$$

$$\ddot{r}_A = \ddot{r}_B + \ddot{r}_{A/B}$$

$$\boxed{a_A = a_B + a_{A/B}}$$



$$r_B = r_A + r_{B/A}$$

$$\dot{r}_B = \dot{r}_A + \dot{r}_{B/A}$$

$$\boxed{V_B = V_A + V_{B/A}}$$

$$a_B = a_A + a_{B/A}$$

$$r_{B/A} = -r_{A/B}$$

$$V_{B/A} = -V_{A/B}$$

$$a_{B/A} = -a_{A/B}$$

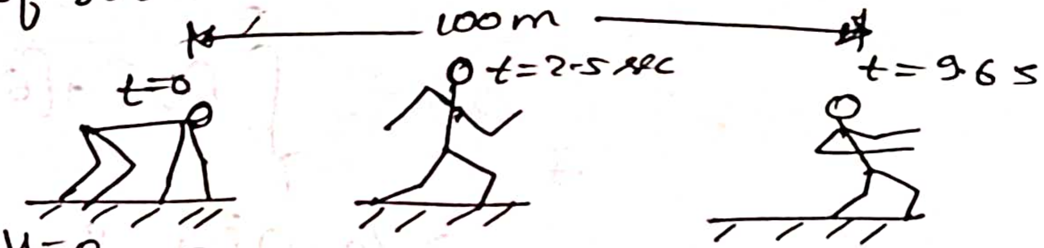
Q. A train starts from rest at a station and travels with a constant accn of $1 m/s^2$. Determin vel of train when $t = 30s$ & dist. travelled during this time.

$V = u + at$ $\qquad$ $S = ut + \frac{1}{2}at^2$

$V = 0 + 1 \times 30$ $\qquad$ $S = 0 + \frac{1}{2} \times 1 \times 30$

$V = 30 m/s$ $\qquad$ $S = 450m$

Q. A sprinter reaches his max speed in 2.5 s from rest with const accn. He then maintains that speed and finishes the 100m in the overall time of 9.6 s. Determine his max speed.



$u=0$. a $\qquad$ V_{max} . $a=0$ $\qquad$ V_{max}

$t_2 = 9.6 - 2.5$ $\qquad$ S_1 $\qquad$ S_2

$t_2 = 7.1$

$S_1 + S_2 = 100m$

$(ut + \frac{1}{2}at^2) + (ut + \frac{1}{2}at^2) = 100$

$(\cancel{0t} + \frac{1}{2}a(2.5)^2) + [V_{max} \times 7.1 + 0) = 100$

$\frac{1}{2} \times a \times (2.5)^2 + 7.1(V_{max}) = 100$

$V = u + at$

$V_{max} = 0 + a \times 2.5$

$a = \frac{V_{max}}{2.5}$

$\frac{1}{2} \times \frac{V_{max}}{\cancel{2.5}} \times (\cancel{2.5})^2 + 7.1(V_{max}) = 100$

$V_{max} = \frac{100}{(7.1 - 1.25)}$

$V_{max} = 11.97 m/s$

Q. The vel. of particle is $v = \{3\hat{i} + (6-2t)\hat{j}\}$ m/s
If position $r = 0$ when $t = 0$.
determine the displacement of particle (in m) during ~~t=~~ 1 s to $t = 3$ s.

$$V = \frac{ds}{dt}$$

$$\int_{S_1}^{S_3} ds = \int_1^3 v\,dt$$

$$\Delta s = \int_1^3 [3\hat{i} + [6-2t]\hat{j}]\,dt$$

$$\Delta s = 3(3-1)\hat{i} + [6(3-1) - (3^2-1^2)]\hat{j}$$

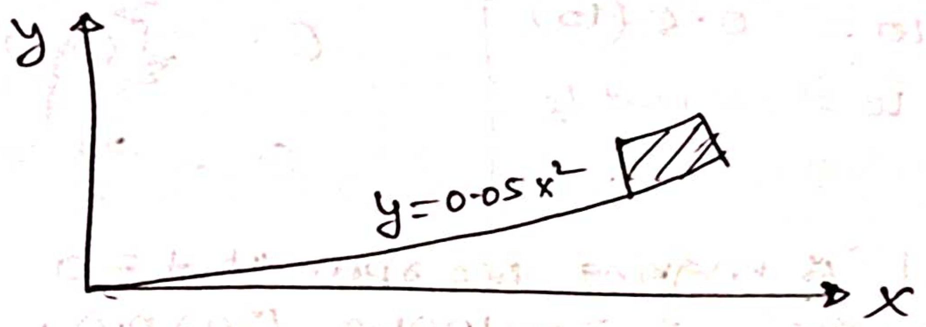
$$\Delta s = 6\hat{i} + [12 - 8]\hat{j}$$

$$\Delta s = 6\hat{i} + 4\hat{j}$$

$$\Delta s = \sqrt{6^2+4^2}$$

$$\boxed{\Delta s = 7.21\text{m}}$$

Q. The box slides down the slope described by the eqn $y = (0.05x^2)$ m where x is in m. If the box has x comp. of vel & accn of $V_x = -3$ m/s & $a_x = -1.5$ m/s² at $x = 5$m, determine the y-comp. of vel. (in m/s) & accn of box at this inst.



$$V_y = \frac{dy}{dt} = \frac{d}{dt}(0.05x^2)$$

$$V_y = 0.05 \times 2x \frac{dx}{dt}$$

$$V_y = 0.1 \times 5 \times (-3)$$

$$V_y = -1.5 \text{ m/s}$$

$$a_y = \frac{dV_y}{dt} = \frac{d}{dt}\left[0.1\, x \frac{dx}{dt}\right]$$

$$= 0.1 \left[x \frac{d^2x}{dt^2} + \frac{dx}{dt}\frac{dx}{dt}\right]$$

$$= 0.1 \left(5 \times (-1.5) + (-3)(-3\right)$$

$$a_y = 0.15 \text{ m/s}^2$$

Q. A flywheel starts from rest & revolves with an accn of 0.5 rad/s what will be its ang. vel & ang disp. after 10 s

$\omega = \cancel{\omega_0}^{0} + \alpha t$

$\omega = 0.5(10)$

$\omega = 5$ rad/s

$\theta = \cancel{\omega_0 t}^{0} + \frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2}(0.5)(10)^2$

$\theta = 25$ rad

Q. A flywheel is making 180 rpm at t = 0 & after 20 sec, it is running 120 rpm. How many revoln will it make & what time will elapse before it stops, if the retardation is uniform

$\alpha = -ve$

$\omega = \omega_0 + \alpha t$

$$\frac{2\pi \times 120}{60} = \frac{2\pi \times 180}{60} + \alpha \times 20$$

$\alpha = -0.314$ rad/s²

2π rad

$565.77 = \frac{1}{2\pi} \times 565.77$

$= 90$ rev.

$\omega = \omega_0 + \alpha t \Rightarrow 0 = \frac{2\pi \times 180}{60} - 0.314 t$

$t = 60$ sec

$\cancel{\omega^2}^{0} = \omega_0^2 + 2\alpha\theta$

$$0 = \left(\frac{2\pi\ 180}{60}\right)^2 - 2(0.314)\theta$$

$\theta = 282.88$ rad.

Q. The eqn for angular displacement of a body moving on a circular path is given by

$$\theta = 2t^3 + 0.5$$

Find ω, θ, α after 2 sec

$$\theta = 2t^3 + 0.5$$

$$\omega = \frac{d\theta}{dt} = 6t^2$$

$$\omega = 6(2)^2$$

$$\omega = 24 \text{ rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 12t$$

$$\alpha = 24 \text{ rad/s}^2$$

$$\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$$

$$\theta = 2 \times 2^3 + 0.5$$

$$\theta = 16.5 \text{ rad/s}$$

Q. $\theta = 18t + 3t^2 - 2t^3$ $[r = 200m]$

$\omega, \alpha = ?$

t (ω_{max})

$\omega_{max} = ?$

$$\omega = \frac{d\theta}{dt} = 18 + 6t - 6t^2$$

$$\alpha = 6 - 12t$$

at $t = 0$

$\omega = 18$

$\alpha = 6$

for ω_{max} $\frac{d\omega}{dt} = 0 \Rightarrow 6 - 12t = 0$

$t = 0.5$ sec

at $t = 0.5$

$$\omega_{max} = 18 + 6 \times 0.5 - 6 \times 0.5^2$$

$$\boxed{\omega_{max} = 19.5 \text{ rad/s}}$$

Q. The angular accn of a shaft is defined by

$\alpha = -0.25\omega$.

at $t=0$ $\omega = 20$ rad/s

determine no. of revolutions the shaft will execute before coming to rest

$\alpha = \frac{d\omega}{dt}$

$= \frac{d\omega}{d\theta}\frac{d\theta}{dt}$

$= \omega\frac{d\omega}{d\theta}$

$\alpha = -\frac{1}{4}\cancel{\omega} = \cancel{\omega}\frac{d\omega}{d\theta}$.

$\int_{20}^{0} d\omega = \int_{0}^{\theta} -\frac{1}{4} d\theta$

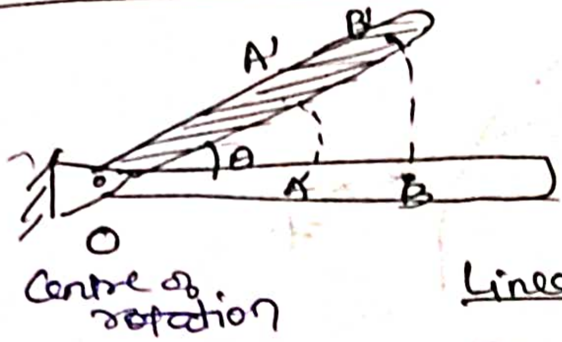
$\theta = 80$ rad.

No. of revolutions $= \frac{\theta}{2\pi}$

$= \frac{80}{2\pi}$

$= 12.73$ rev.

Linear Motion Variables



Linear displacement

$s_A = OA \cdot \theta$

$s_B = OB \cdot \theta$

$$\boxed{S = r\theta}$$

$r \rightarrow$ radial distance from centre of rotation.

$\theta_A = \theta_B$

$S_A \neq S_B$

$V = \frac{ds}{dt} = \frac{d(r\theta)}{dt}$

$V = r\frac{d\theta}{dt}$

$$\boxed{V = r\omega}$$

$\omega_A = \omega_B$

$V_A \neq V_B$

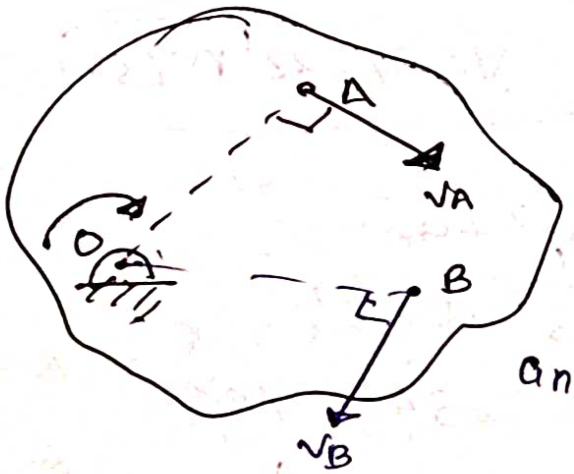
linear acceleration

Tangential accn

$a_t \rightarrow$ changes magnitude of velocity

normal accn

$a_n \rightarrow$ changes direction of velocity



$a_t = \frac{dV}{dt} = \frac{d}{dt}(\omega r)$

$a_t = r\frac{d\omega}{dt}$

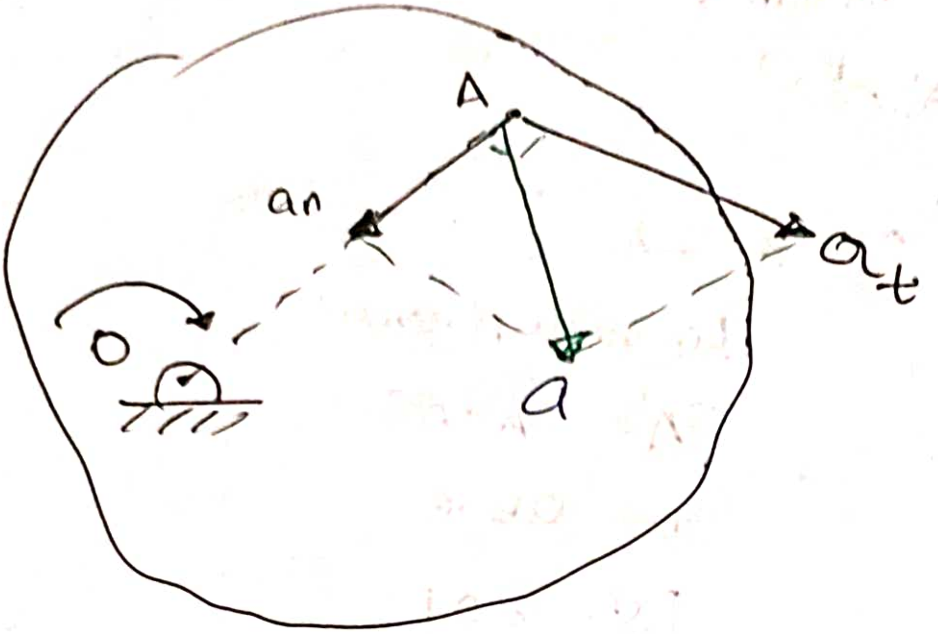
$$\boxed{a_t = \alpha r}$$

$a_n = \frac{V^2}{r}$

$= \frac{(\omega r)^2}{r}$

$$\boxed{a_n = \omega^2 r}$$

$a = \sqrt{a_t^2 + a_n^2} \Rightarrow a = \sqrt{\alpha r + \omega^2 r}$



Q. A wheel of 1m dia. at an initial angular vel of 8 rad/s is accelerated at rate of 6 rad/s². Find velocity (in m/s) & accn of pt on its periphery after wheel turn 2 rev.

$\omega_0 = 8 \text{ rad/s}$
$\alpha = 6 \text{ rad/s}^2$

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$\omega^2 - 8^2 = 2(6)\,2\times 2\pi$

$\omega = 14.65 \text{ rad/s}$

$a_t = \alpha r$
$= 6 \times 0.5$
$a_t = 3 \text{ m/s}^2$

$a_n = \omega^2 r$
$= 14.65^2 \times 0.5$
$a_n = 107.3 \text{ m/s}^2$

$V = \omega r \Rightarrow V = 14.65 \times 0.5$

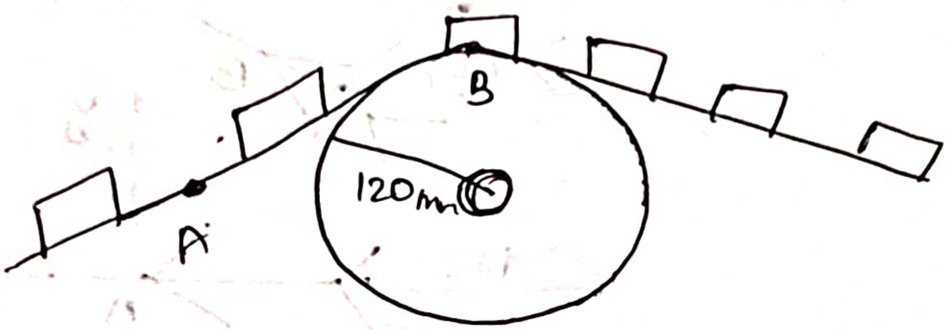
$V = 7.32 \text{ m/s}$

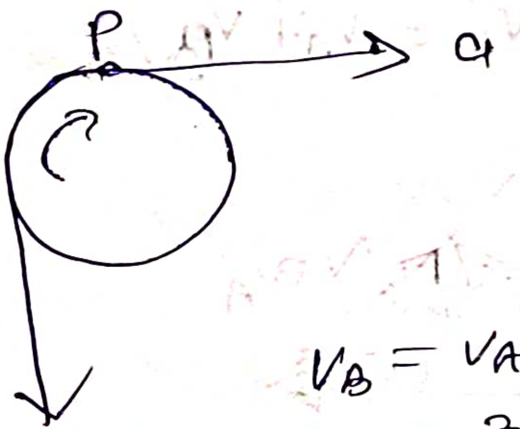
$a = \sqrt{a_t^2 + a_n^2}$
$= \sqrt{3^2 + 107.3^2}$
$a = 107.34 \text{ m/s}^2$

Q. A series of small machine components being moved by a conveyor belt pass over a 120mm radius idler pulley. At instant show vel of pt A is 300mm/s & acc^n is 180 mm/s². Determine

(a) ω, α (No slip

(b) $a_B = ?$





(No slip)

$V_P = V$

$(a_t)_P = a$

$V_B = V_A$

$\omega \cdot r_B = 300$

$\omega = \frac{300}{120}$

$\boxed{\omega = 2.5 \text{ rad/s}}$

$(a_t)_B = a_A$

$\alpha r_B = 180$

$\alpha = \frac{180}{120}$

$\boxed{\alpha = 1.5 \text{ rad/s}^2}$

$(a_n)_B = \omega^2 r_B$

$= 2.5^2 \times 120$

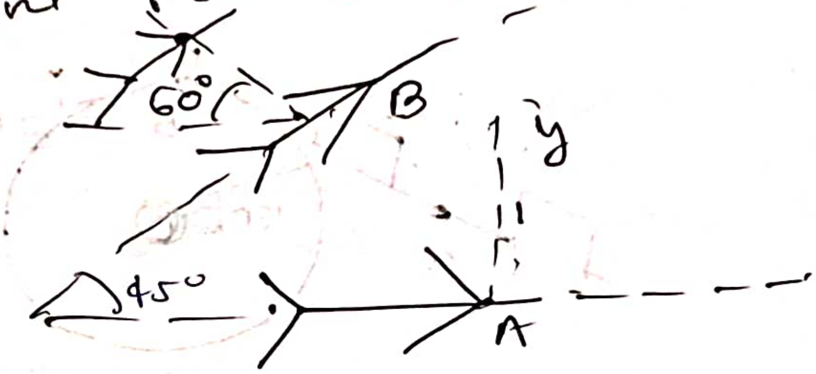
$a_{nB} = 750 \text{ mm/s}^2$

$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2}$

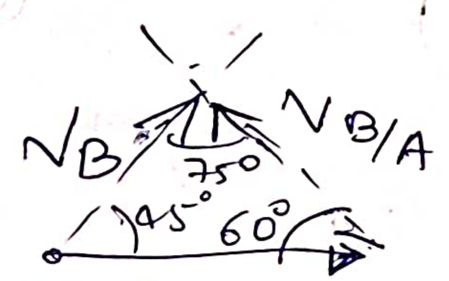
$= \sqrt{180^2 + 750^2}$

$a_B = 771.3 \text{ mm/s}^2$

Q. Passengers in the Jet transport A flying east at a speed of 800 km/h observe a second Jet plane B that passes under the transport in the horizontal flight. Although the nose of B is pointed in the 45° northeast direction, plane B appears to the passengers in A to be moving away from transport at 60° angle. Determine true vel. of B.



$$V_B = V_A + V_{B/A}$$



$V_A = 800\,\hat{i}$ km/h

$V_B = V_B \cos 45^\circ\,\hat{i} + V_B \sin 45^\circ\,\hat{j}$

$V_{B/A} = (V_{B/A} \cos 60^\circ)(-\hat{i}) + (V_{B/A} \sin 60^\circ)\,\hat{j}$

(i-terms) $V_B \cos 45^\circ = 800 - V_{B/A} \cos 60^\circ$

$$\frac{V_B}{\sqrt{2}} = 800 - \frac{V_{B/A}}{2}$$

$$\frac{V_B}{\sqrt{2}} + \frac{V_{B/A}}{2} = 800 \quad — (1)$$

(j-terms) $V_B \sin 45^\circ = V_{B/A} \sin 60^\circ$

$$\frac{V_B}{\sqrt{2}} - V_{B/A} \frac{\sqrt{3}}{2} = 0 \quad — (2)$$

(1) - (2)

$$\frac{V_{B/A}}{1} \frac{\boxed{1+\sqrt{3}}}{2} = 800$$

$$V_{B/A} = \frac{800 \times 2}{2.73}$$

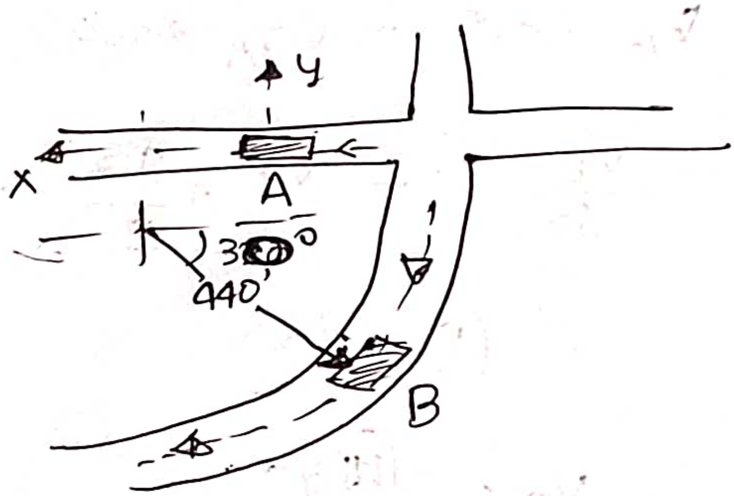
$$V_{B/A} = 586 \text{ km/h}$$

$$V_B = V_{B/A} \frac{\sqrt{3}}{\sqrt{2}}$$

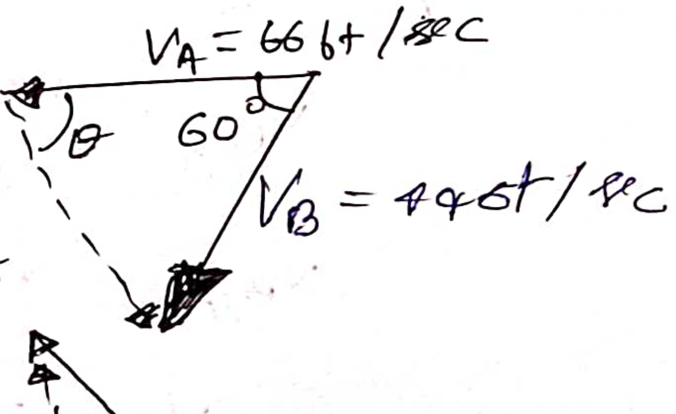
$$= 586 \times \frac{\sqrt{3}}{\sqrt{2}}$$

$$V_B = 717 \text{ km/h}$$

Q. Car A is accelerating in the direction of its motion at the rate of 3 ft/sec^2. Car B is rounding a curve of 440 ft radius at a const speed of 30 mi/hr. Determine the vel & accn which car B appears to have to an observer in car A, if car A has reached a speed of 45 mi/hr for position represented.



$V_B = V_A + V_{B/A}$



$V_A = 45 \times \frac{44}{30}$

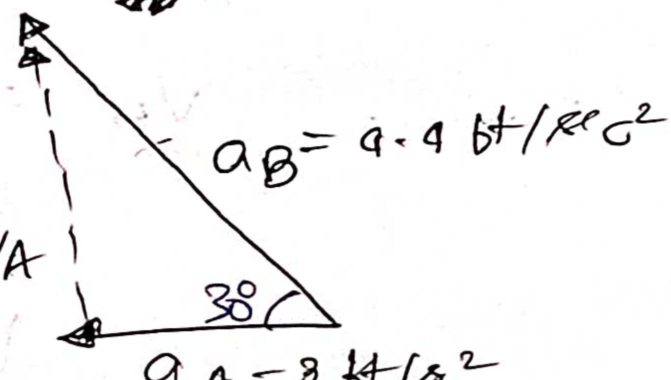
$V_A = 66$ ft/sec

$V_B = 30 \times \frac{44}{30}$

$V_B = 44$ ft/sec

$V_{B/A} = 58.2$ ft/sec

$\theta = 40.9°$



$a_B = a_A + a_{B/A}$

$$a_B = \frac{(4.4)^2}{440} = 4.4 \text{ ft/sec}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$(a_{B/A})_x = 4.4 \cos 30^\circ - 3 = 0.810 \text{ ft/sec}^2$$

$$(a_{B/A})_y = 4.4 \sin 30^\circ = 2.2 \text{ ft/sec}^2$$

$$a_{B/A} = \sqrt{(0.810)^2 + (2.2)} = 2.34 \text{ ft/sec}^2$$

Ans

$$\frac{4.4}{\sin \beta} = \frac{2.34}{\sin 30^\circ}$$

$$\beta = 110.2^\circ$$

Q. A pulley & a two loads are connected by unextensible cords as shown. Load A has a constant accn of 300 mm/s^2 & an initial velocity of 240 mm/s both directed upward.

Determine.

(a) no. of rev. executed by pulley in 3 s.

(b) vel. of load B after 3 s

(c) accn of pt D on rim of pulley at $t = 0$

$V_A = 240$ mm/s

$a_A = 300$ mm/s^2

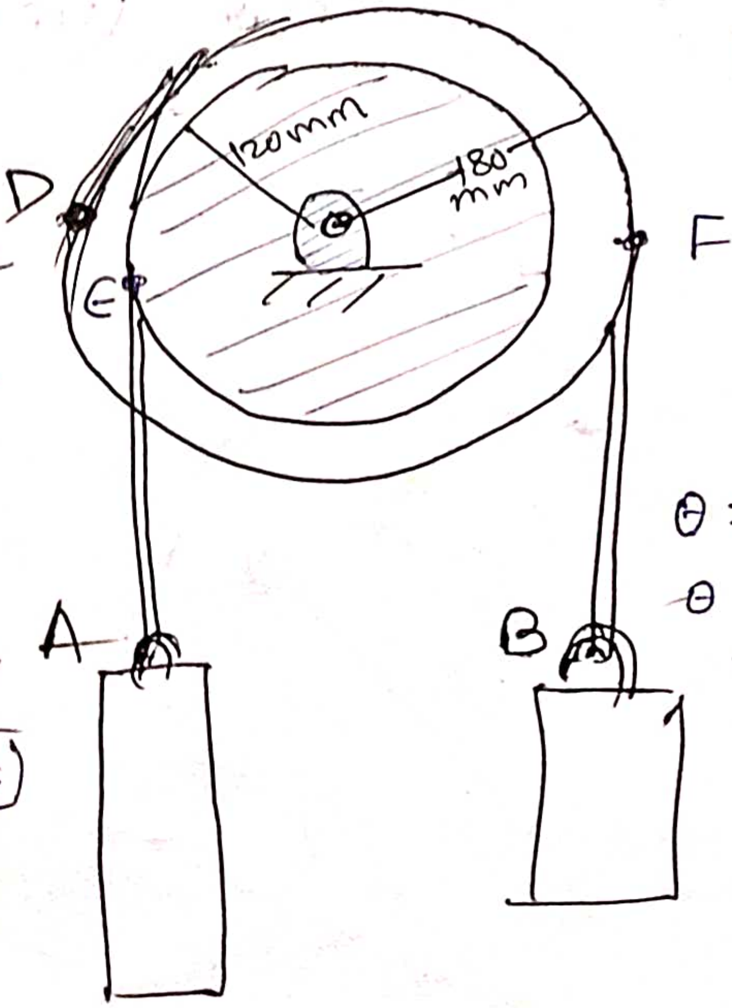
Initial velocity of E

$V_E = V_A$

$\omega_0 r_E = 240$

$\omega_0 = \frac{240}{120}$ rad/s

$\boxed{\omega_0 = 2 \text{ rad/s}}$



$(a_t)_E = a_A$

$\alpha r_E = 300$

$\alpha = \frac{300}{120}$

$\alpha = 2.5$ rad/s^2

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = 2 \times 3 + \frac{1}{2} \times 2.5 \times 3^2$

$\boxed{\theta = 17.025 \text{ rad}}$

No. of rev.

$= \frac{\theta}{2\pi}$

$= \frac{17.25}{2\pi}$

$= 2.74$ rev

$V_B = V_F$

$= \omega \cdot r_F$

$= 9.5 \times 180$

$\boxed{V_B = 1710 \text{ mm/s}}$

at $t = 0$

$(a_n)_D = \omega^2 \times r_D$

$= 2^2 \times 120$

$\boxed{(a_n)_D = 480 \text{ mm/s}}$

$(a_t)_D = \alpha \cdot r_D$

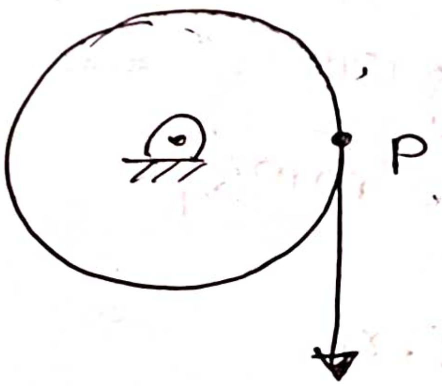
$= 2.5 \times 180$

$\boxed{(a_t)_D = 450 \text{ mm}}$

$a_D = \sqrt{720^2 + 450^2}$

$a_D = 849.06 \text{ mm/s}^2$

Q. A chord is wrapped around a wheel of radius 0.5 m which is initially at rest. If a force is applied to the chord which gives it an acceleration of 2t m/s² find angular velocity of wheel and no. of revolutions ~~m~~made after 5 sec.



$a = 2t\ m/s^2$

$(a_t)_P = a = 2t$

$\alpha r_P = 2t$

$\alpha = \frac{2t}{0.5}$

$\alpha = 4t\ rad/s^2$

$\alpha = \frac{d\omega}{dt}$

$\int_0^{\omega} d\omega = \int_0^5 \alpha\, dt$

$\omega = \int_0^5 4t\, dt$

$\omega = 2(t^2)_0^5$

$\omega = 50\ rad/s$

$\omega = \frac{d\theta}{dt}$

$\int_0^{\theta} d\theta = \int_0^5 \omega\, dt = \int_0^5 2t^2\, dt$

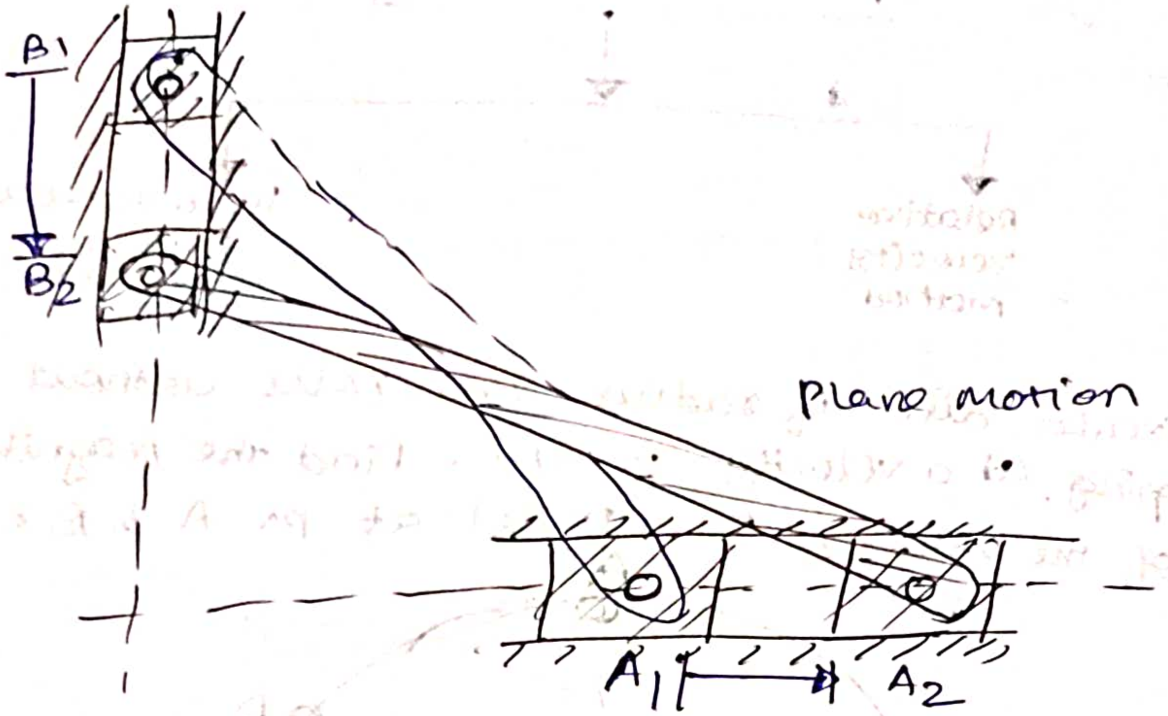
$\theta = \frac{2}{3}(t^3)_0^5 =$

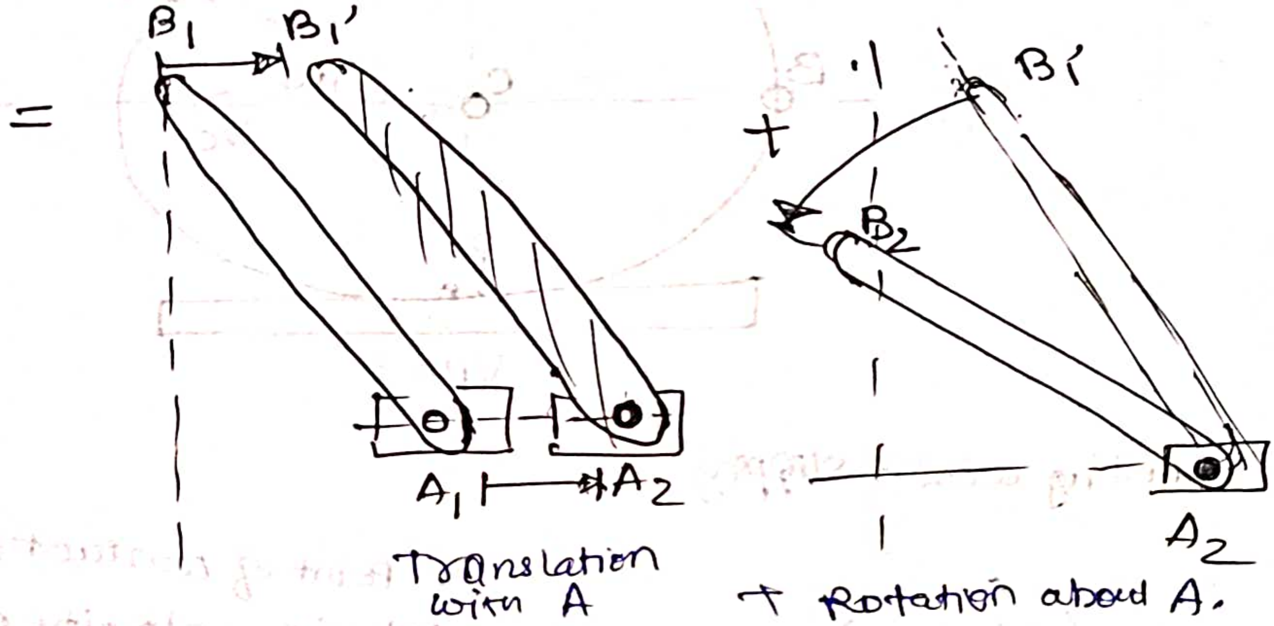
$\theta = 83.33\ rad$

No. of rev $= \frac{83.33}{2\pi}$

$\boxed{\text{No. of rev.} = 13.26\ \text{rev}}$

· General Motion

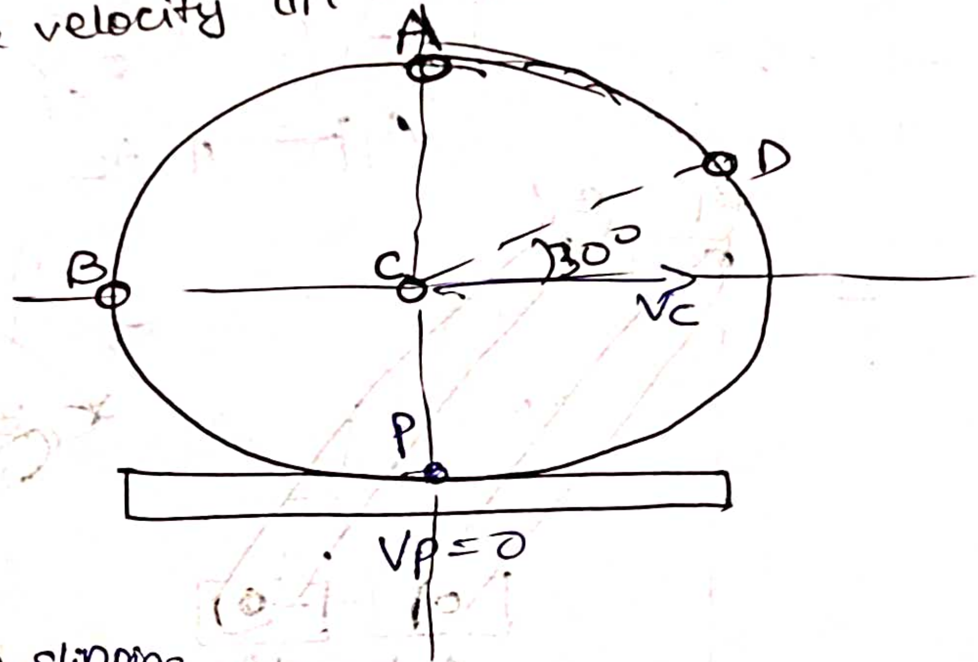




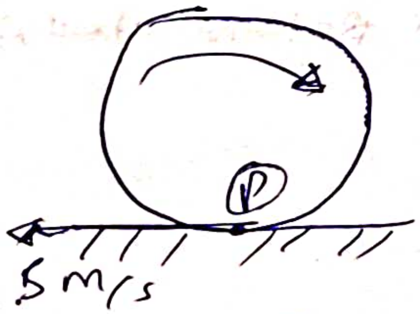
Kinematics of General Motion

- Relative velocity method
- Instantaneous Center Method

Q. A circular disk of radius 2m rolls without slipping at a velocity 10 m/s. Find the magnitude of the velocity (in m/s) at pt A & B & D.



Rolling without slipping



At Point of contact (P) relative velocity of wheel & surface must be zero

$$V_A = V_C + V_{A/C}$$

$$V_A = \vec{V}_P + V_{A/P}$$

$V_P = 5$ m/s

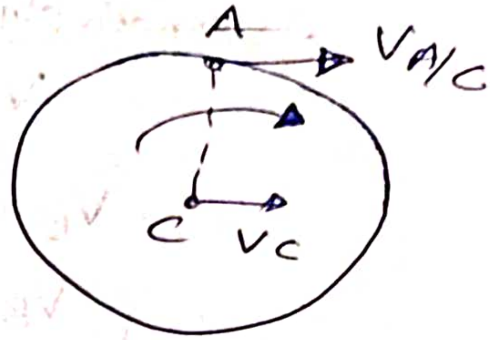
$$\vec{V}_C = \vec{V}_P + \vec{V}_{C/P}$$

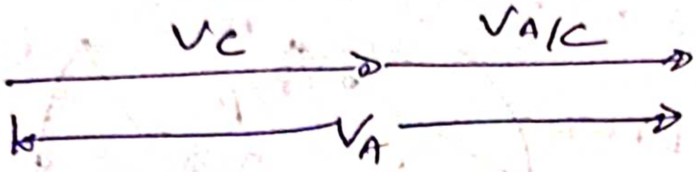
$$10 = 0 + \omega \times CP$$

$$10 = \omega \times 2$$

$$\omega = 5 \text{ rad/s}$$

$$v_A = V_C + v_{A/C}$$

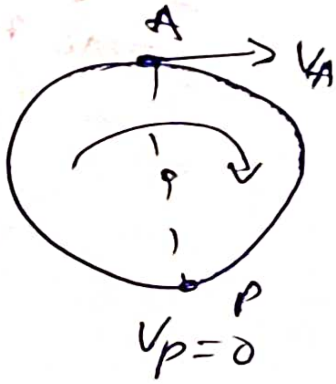




$$V_A = 10 + \omega \times AC$$

$$= 10 + 5 \times 2$$

$$\boxed{V_A = 20\ m/s}$$

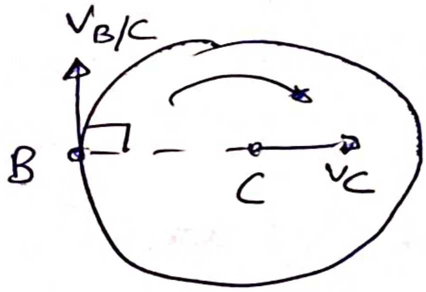


$$\vec{V_A} = \cancel{V_P}^{\,0} + \vec{V}_{A/P}$$

$$V_A = 0 + \omega \times AP$$

$$V_A = 5 \times 4$$

$$\boxed{V_A = 20 m/s}$$



$$V_B = V_C + V_{B/C}$$

$$V_B = \sqrt{V_C^2 + V_{B/C}^2}$$

$$= \sqrt{10^2 + (\omega \times BC)^2}$$

$$= \sqrt{10^2 + (5 \times 2)^2}$$

$$V_B = 14.14\ m/s$$

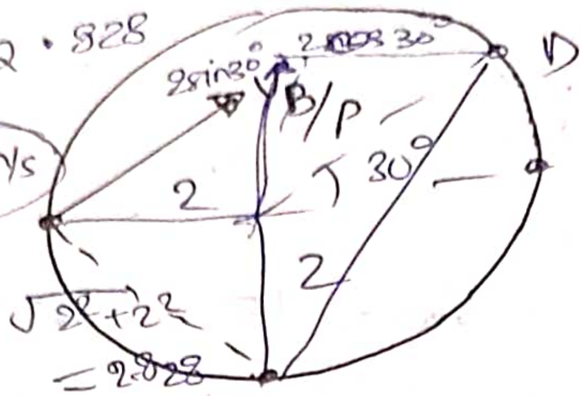
$\cancel{V_B} =$

Alternatively:

$V_B = \vec{V_P} + \vec{V}_{B/P}$

$V_B = 0 + 5 \times 2.828$

$\boxed{V_B = 14.14 \text{ m/s}}$



$DP = \sqrt{3^2 + (\sqrt{3})^2}$

$= \sqrt{12}$

$= 3.464$

$V_D = V_P + \vec{V}_{D/P}$

$V_D = 0 + \omega \times DP$

$V_D = 5 \times 3.464$

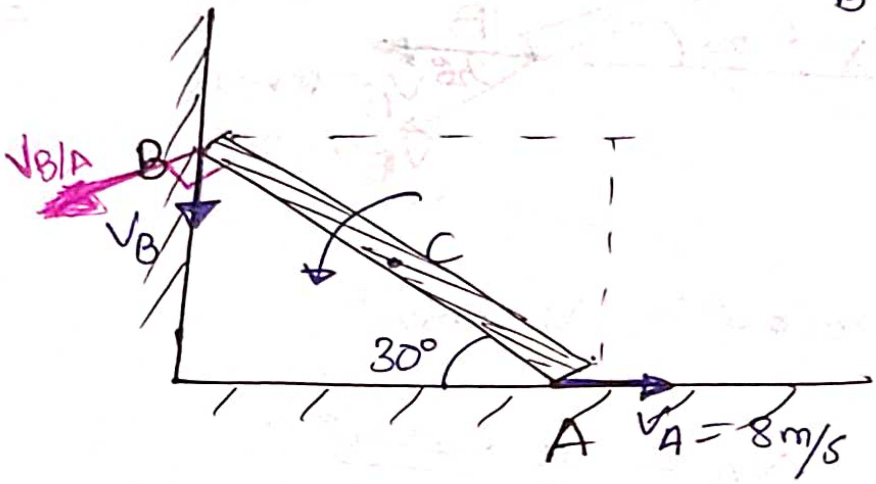
$\boxed{V_D = 17.32 \text{ m/s}}$

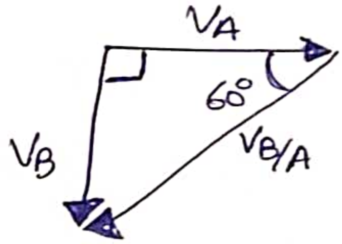
Q. A rod of length 2 m is sliding in a corner as shown. At an instant when the rod makes an angle of 30 degrees with the horizontal plane, the velocity of point A on the rod is 8 m/s. Find ~~velocity~~

(a) velocity of end B (in m/s)

b) velocity of the mid point (in m/s)

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$





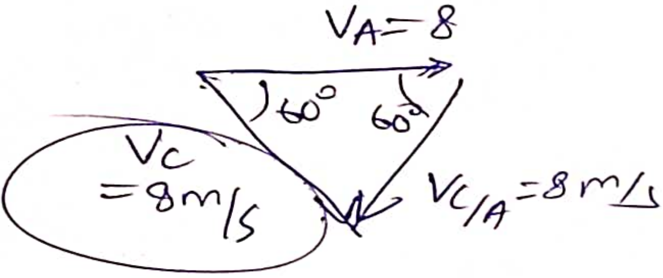
$$\tan 60^\circ = \frac{V_B}{V_A}$$

$$V_B = V_A \tan 60^\circ$$

$$V_B = 8 \times \sqrt{3}$$

$$\boxed{V_B = 13.85 \text{ m/s}}$$

$$V_C = \vec{V}_A + V_{C/A}$$



$$V_{C/A} = \omega \times CA$$

$$= 8 \times 1$$

$$V_{C/A} = 8 \text{ m/s}$$

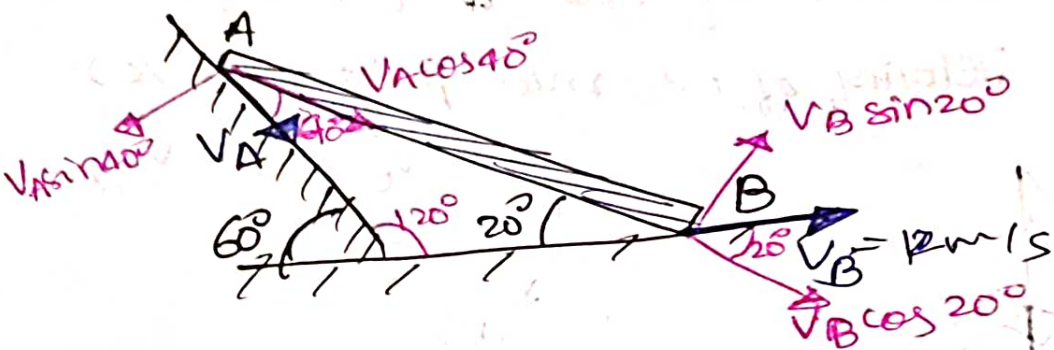
$$\omega \times BA \quad V_{B/A} = \sqrt{V_A^2 + V_B^2}$$

$$\omega \times BA = V_{B/A} = \sqrt{8^2 + 13.85^2}$$

$$\omega \times 2 = 16$$

$$\omega = 8 \text{ rad/s}$$

Q. A rod AB of length 4.5m is sliding in a corner as shown. At an instant when the rod makes an angle of 20° with Horizontal plane, the vel of pt B on rod is 12m/s to the right find vel. of end A. (in m/s



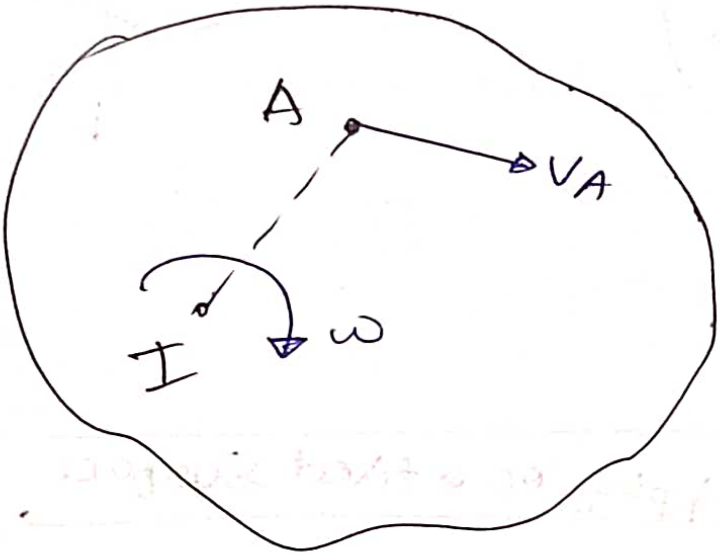
$$V_A \cos 40° = V_B \cos 20°$$

$$V_A = \frac{12 \times \cos 20°}{\cos 40°}$$

$$V_A = 14.72 \text{ m/s}$$

Instantaneous Center method (I center method)

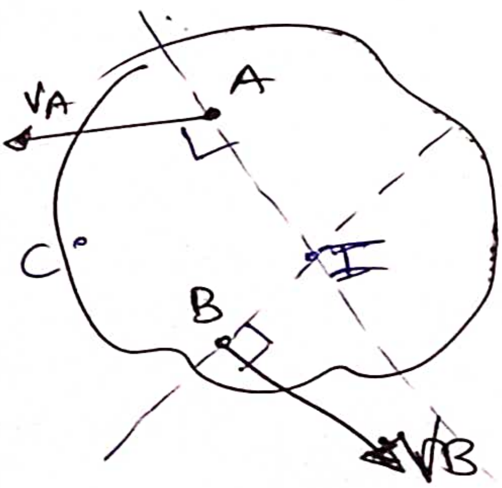
- Combined motion of rotation & translation may be assumed to be a motion of pure rotation about some imaginary center known as instantaneous center.
- As the position of body goes changing, therefore instantaneous center also goes on changing.



$$V_A = \omega \times IA$$

Location of I-center

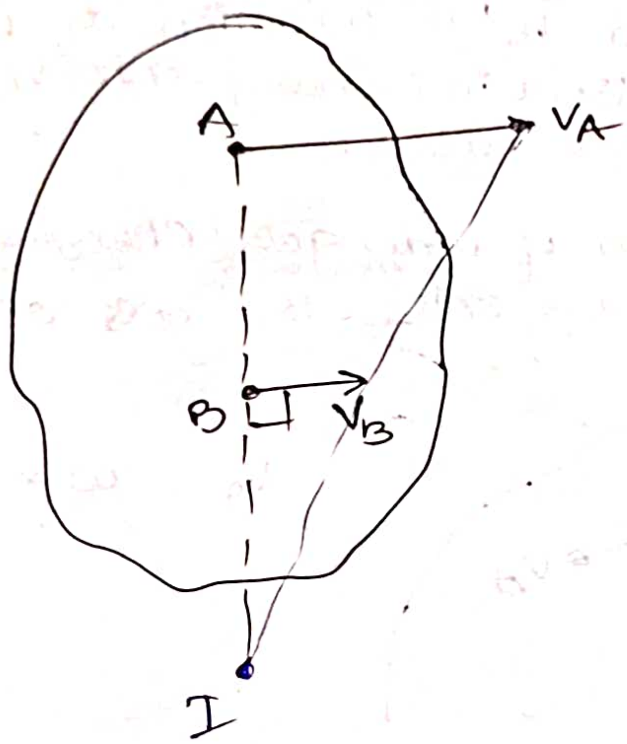
Case-1. when the dirn of velocities of two particles A & B are known and V_A is not parallel to V_B.

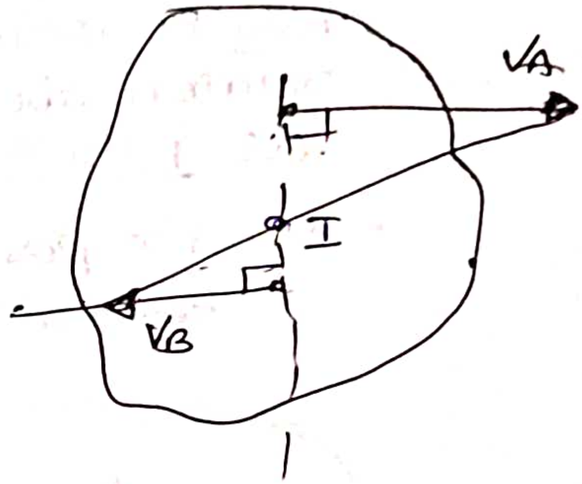


$$V_A = \omega IA$$

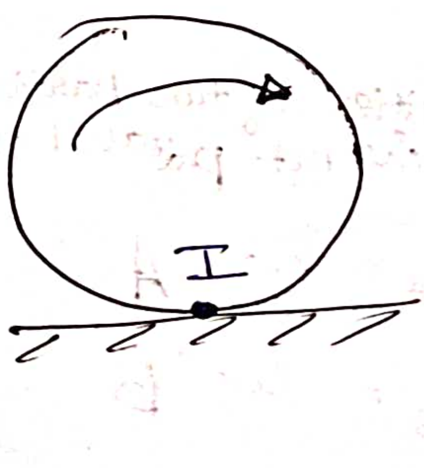
$$V_B = \omega \cdot IB$$

Case-2 when the dirn & magnitude of velocities of two particles A & B are known and V_A is parallel to V_B.

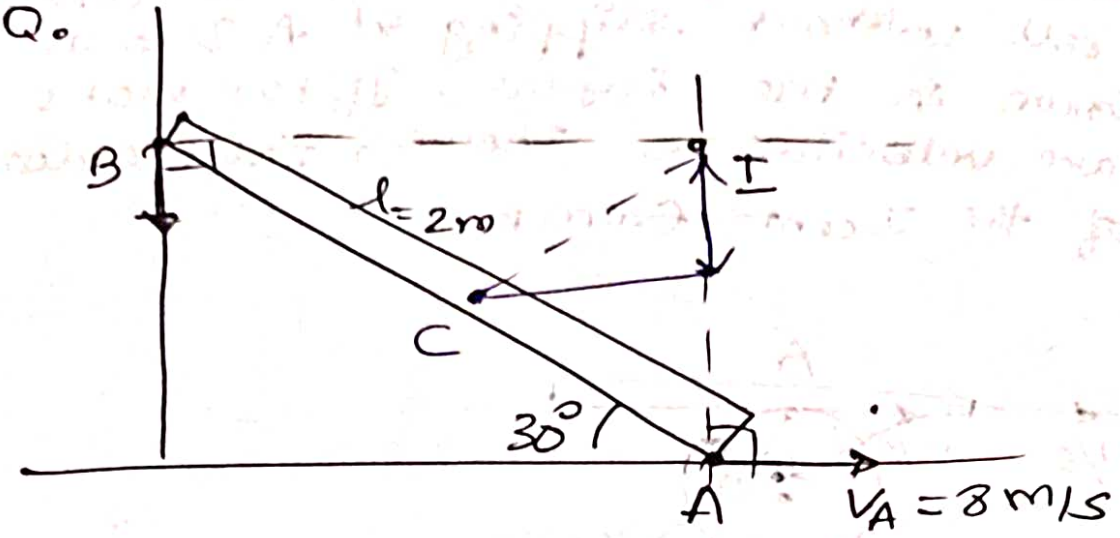




Case-3
Rolling without slipping on a fixed surface



Q.



$$V_A = \omega \times I_A$$

$$8 = \omega \times 2 \sin 30^\circ$$

$$\omega = 8 \text{ rad/s}$$

$$V_B = \omega \times IB$$

$$= 8 \times 2 \cos 30^\circ$$

$$\boxed{V_B = 13.85 \text{ m/s}}$$

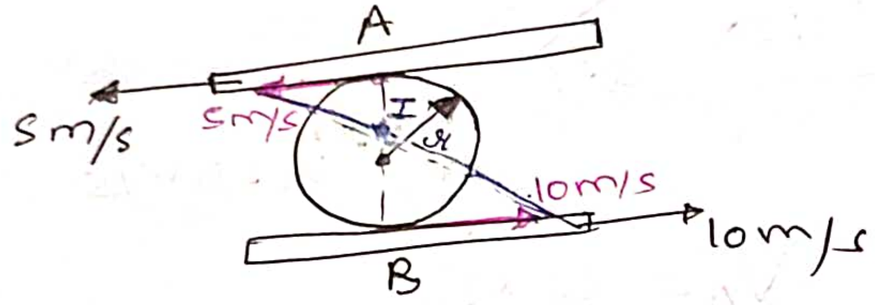
$$IC = \sqrt{(1\cos 30^\circ)^2 + (1\sin 30^\circ)^2} = 1\text{m}$$

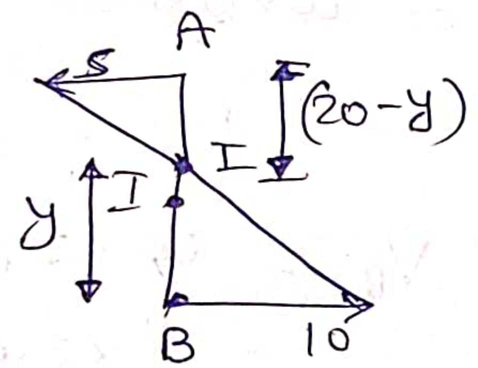
$$V_c = 8\omega \times I_c$$

$$= 8 \times 1$$

$$V_c = 8 \text{m/s}$$

Q. A circular disk of radius 10m is confined to roll without slipping at A & B as shown in the figure. If the plates have velocities as shown, find the dist. of the I center from B.





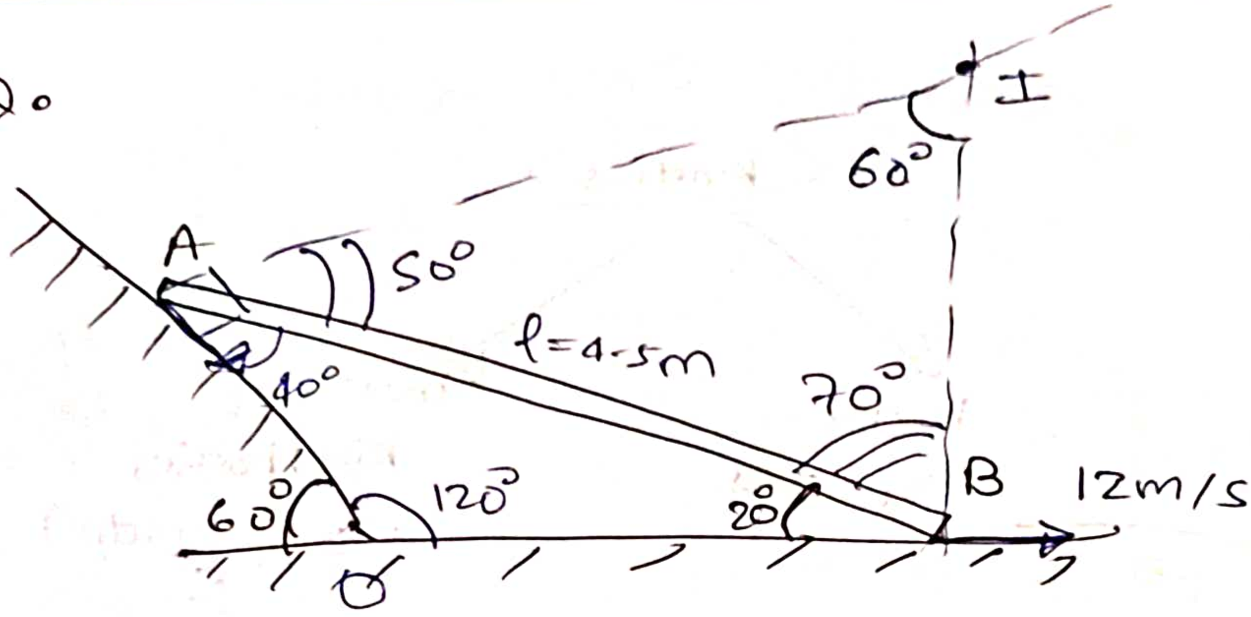
$$\frac{y}{10} = \frac{(20-y)}{5}$$

$$5y = 200 - 10y$$

$$15y = 200$$

$$\boxed{y = 13.33\ m}$$

Q.



$$\frac{IA}{\sin 70^\circ} = \frac{IB}{\sin 50^\circ}$$

$$\frac{IA}{IB} = \frac{\sin 70}{\sin 50}$$

$$V_A = \omega \times IA$$

$$V_B = \omega \times IB$$

$$\frac{V_A}{V_B} = \frac{IA}{IB}$$

$$\frac{V_A}{12} = \frac{\sin 70}{\sin 50}$$

$$\boxed{V_A = 14.72 \text{ m/s}}$$

Kinetics

Kinetics of particles
(Mass)

Kinetics of Rigid bodies
(Mass, Inertia)

Rel^n b/w Forces & motion variables
(s, v, a)

D' Alembert's Principle :-

It states "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction & line of action may be found out by the methods of graphic statics

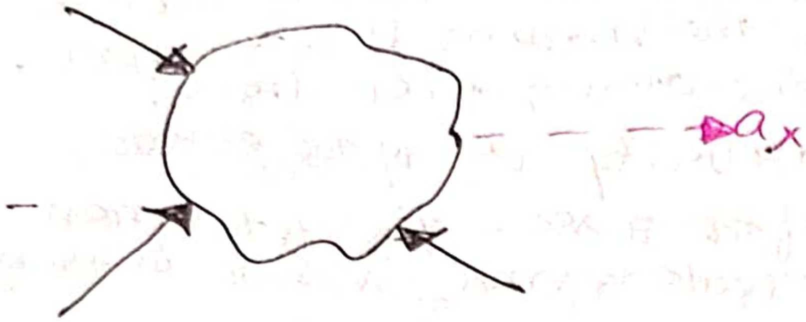
$P = ma$ —— (I) ⇒ eqn of Dynamics

m = mass of the body

a = acc^n of the body

$P - ma$ —— (II) eqn of statics or known as eqn of dynamic eqm under the action of real force P.

Newton's law of Motion :-

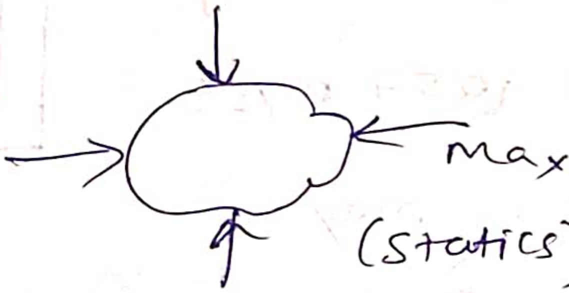


$\boxed{\Sigma F_x = ma_x}$ $a_x \neq 0$

$\Sigma F_y = 0$ $a_y = 0$

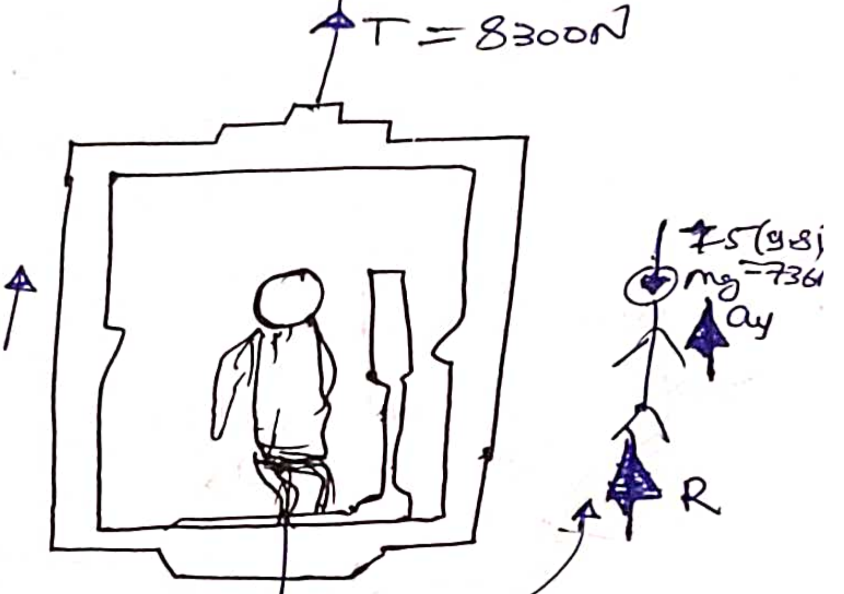
$$\underbrace{\Sigma F_x}_{\text{Total force in } x} - ma_x = 0$$

Kinetics



(statics)

Q. A 75kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval & upward velocity v of the elevator at the end of the 3 sec. The total mass of the elevator, man, & scale is 750kg.



$\Sigma F_y = m a_y$

$8300 - 7360 = 750 a_y$

$a_y = 1.257 \ m/s^2$

$750(9.8) = 7360 N$

$\Sigma F_y = m a_y$

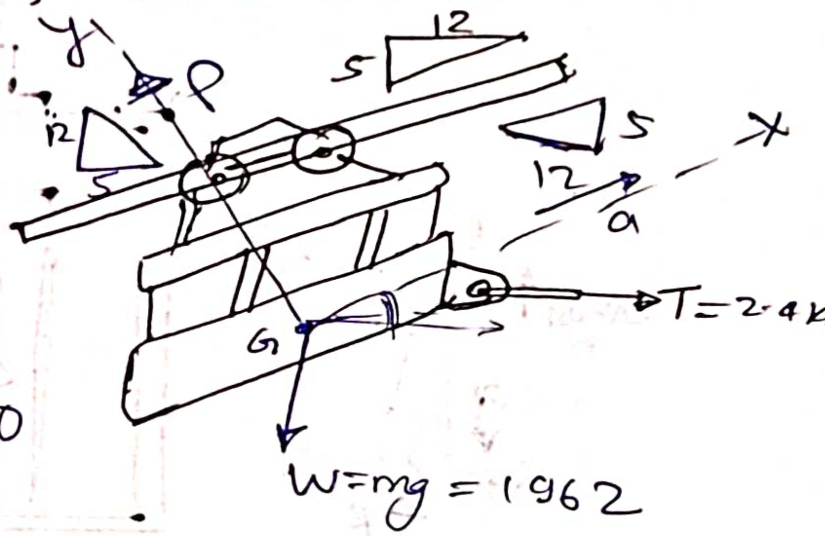
$R - 736 = 75(1.257)$

$R = 830 N$

$$\int dv = \int a\,dt \Rightarrow v = \int_0^3 1.257\,dt$$

$v = 3.77 \ m/s$

Q. A small inspection car with mass of 200 kg runs along the fixed overhead cable & is controlled by the attached cable at A. Determine the accn of the car when the control cable is horizontal & under a tension T = 2.4 kN. Also find the total force P exerted by the supporting cable on the wheel.



ΣF_y

$$P - 2.4 \times 10^3 \times \left(\frac{5}{13}\right) - 1.962\left(\frac{12}{13}\right) = 0$$

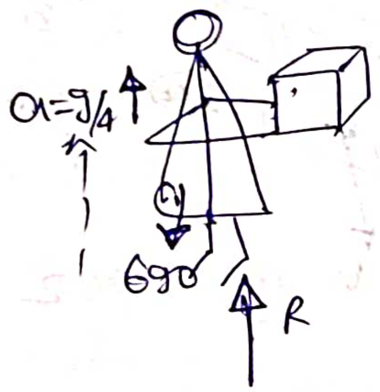
$$P = 2.73\ kN$$

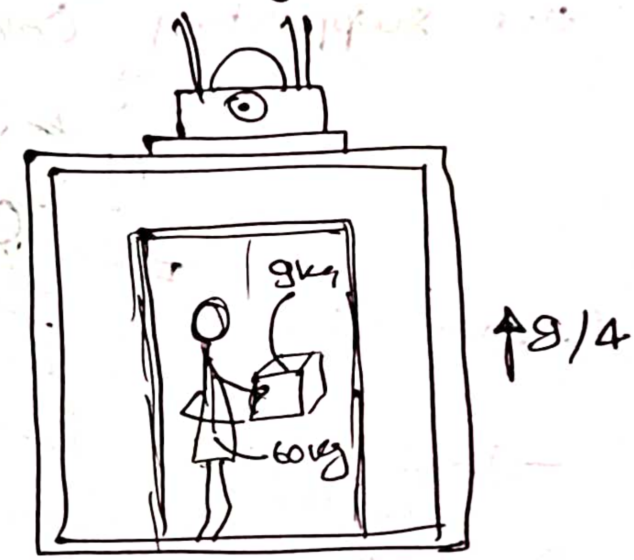
$$\Sigma F_x = m a_x$$

$$2400\left(\frac{12}{13}\right) - 1962\left(\frac{5}{13}\right) = 200a$$

$$a = 7.3\ m/s^2$$

Q. A 60kg woman holds a 9kg package as she stands within an elevator which briefly accelerates upward at a rate of $g/4$. Determine the force which elevator floor exerts on her feet during the acceleration. Assume $g = 10 m/s^2$



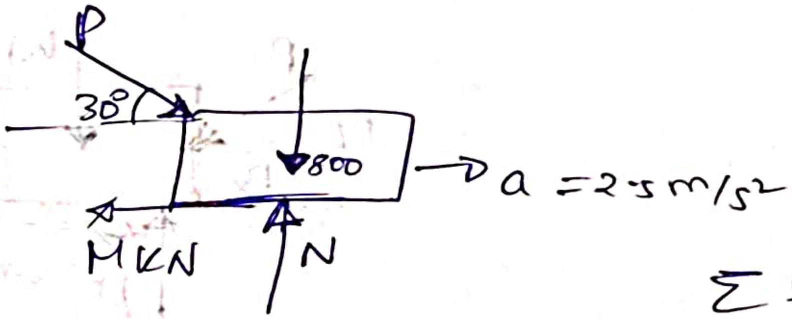


$$\Sigma F_y = ma_y$$

$$R - 690 = 69 \times \frac{g}{4}$$

$$R = 862.5 \, N$$

A 50 kg block rests on a horizontal plane. Find magnitude of force P (in N) required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction b/w block & plane is 0.25.



$\Sigma F_x = m a_x$

$\Sigma F_y = 0$

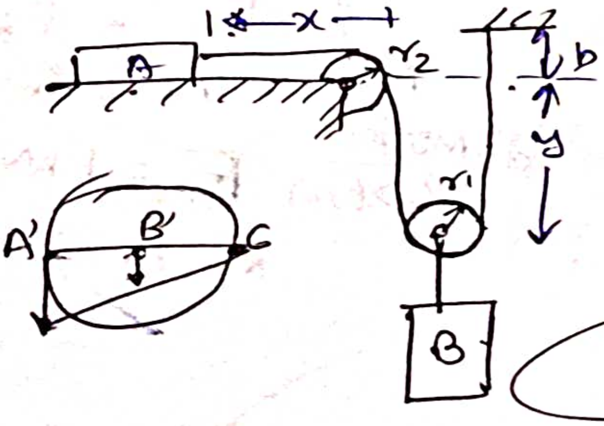
$N = 800 + P\sin 30^\circ$

$(P\cos 30^\circ - \mu_k N) = 80 \times 2.5$

$P\cos 30^\circ - 0.25(800 + P\sin 30^\circ) = 80 \times 2.5$

$P = 539.8\ N$

constrained motion of connected particles



$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 b$

$0 = \dot{x} + 2\dot{y} \Rightarrow V_A + 2V_B = 0$

$0 = \ddot{x} + 2\ddot{y} \Rightarrow a_A + 2a_B = 0$

T·a = constant