Basics of Mechanical Engineering
(BME)

Syllabus

Module -1

- ~~vectors~~
- System of Forces & Structure Mechanics: Addition of forces, Moment of a force, couple, Varignon's theorem, Free body diagram, Equilibrium in 2-D & 3D. Equivalent forces & moment.

Trusses

- Types of trusses, plane & space Trusses, Analysis of plane trusses by: Method of Joints and method of sections

Analysis of frames with Hinged joints

Strength of solid

- Hooke's law of elasticity, Stress & Strain, Relation b/w elastic Constants + Thermal Stresses

~~Thermal stresses~~

Rotation

- Properties of surfaces such as centroid and Moment of Inertia.

Module -1

chapter - (1) vectors

(2) Truss

(3) Strength of Material [Timoshenko & Gere)

(4) Thermal Stresses.

(5) Properties of Surface : centroid & Moment of Inertia.

What is Mechanics?

Mechanics is the science which describes & predicts the conditions of rest or motion of bodies under the action of forces.

Categories of mechanics:

- Rigid bodies
  - Statics
  - Dynamics
- Deformable bodies
- fluids

- Mechanics is an applied science- it is not an abstract or pure science but does not have the empiricism found in other engineering sciences
- Mechanics is the foundation of most engineering sciences & is an indispensable prerequisite to their study.

Fundamental concepts :-

Space- associated with the notion of the position of a point P given in terms of 3 coordinates measured from a reference point or origin

Time - Definition of an event requires specification of the time and position at which it occured

Mass - used to characterize and compare bodies eg. response to earth's gravitational attraction and resistance to changes in translational motion.

Force - represents the action of one body on another A force is characterised by its point of application, magnitude and direction ie. force is a vector quantity.

In Newtonian mechanics, space, time & mass are absolute concepts, independent of each other.

Force, however, is not independent of the other three. The force acting on a body is related to the mass of the body & the variation of its velocity with time.

Idealizations

Particle

i. Mass is considered.

ii. size is neglected.

ie. geometry of the particle won't be involved in the analysis of the problem.

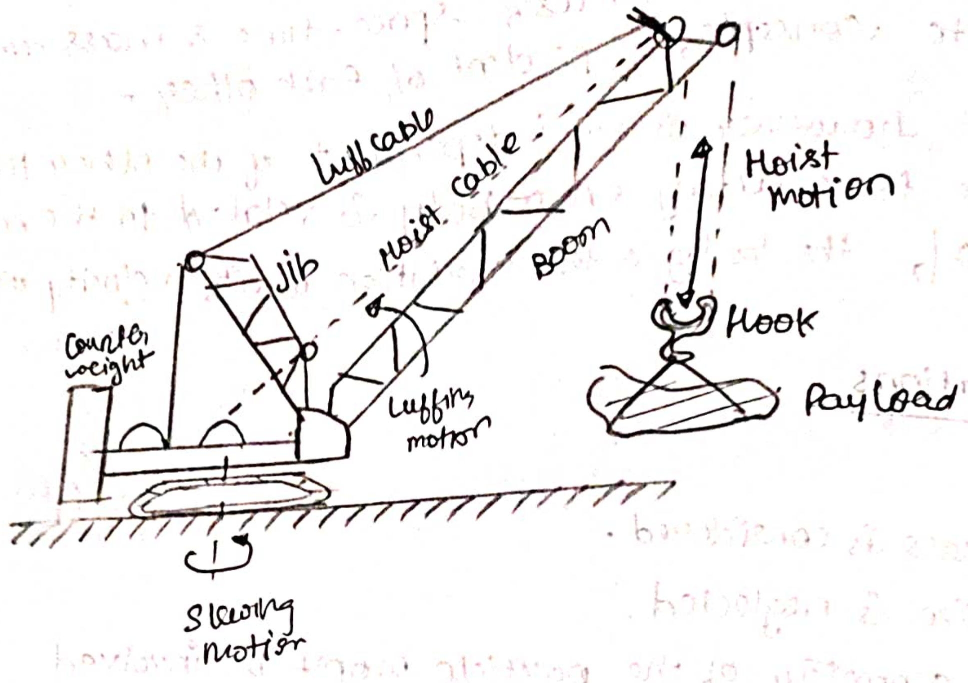
eg: to study orbital motion of the earth (Compared to the size of orbit, Earth size is insignificant)

Rigid Body

i. combination of large no. of particle, at fixed distance from each other.

ii. No deformation.

eg. In large structures deformation is small & to study the mechanism in big machines, the deformation is neglected.



Introduction to vectors.

- **Scalar Quantity :**

  Parameters possessing magnitude but not direction

  eg. length e.g. 93,000,000 miles

  mass e.g. 180 kg

  Speed eg. 186,000 miles/sec

- **Vector Quantity**

  Parameters possessing magnitude & direction which add according to the parallelogram law.

  eg. Force, e.g. 20 N eastward

  velocity e.g. 20 m/s North

  Acceleration e.g. 9.8 m/s² downward

Vector

- An arrow drawn to scale used to represent a vector quantity.

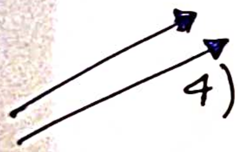
Vector notation $\vec{F}$

Vector Classification :-

1) Fixed or bounded vectors have well defined points of application that cannot be changed without affecting an analysis (only for deformable bodies)

2) Free vectors may be freely moved in space without changing their effect

3) Sliding vectors may be applied anywhere along their line of action without affecting an analysis (transmissibility).



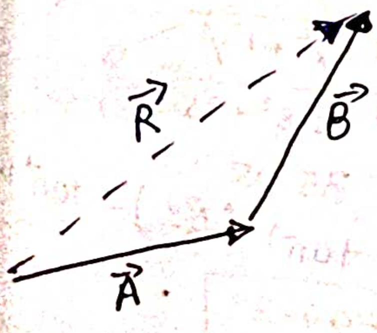
4) Equal vectors have same magnitude and direction.

5) Negative vectors of a given vector has the same magnitude & the opposite direction.

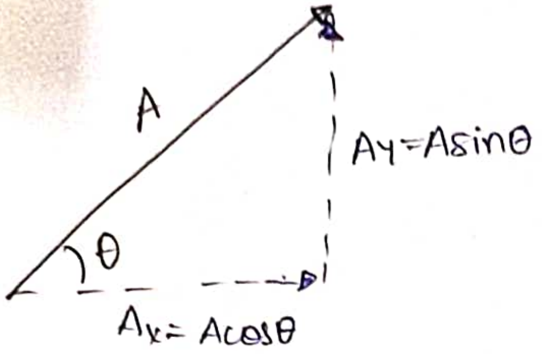
Addition of vector

- Graphical vector addition

$\vec{A} + \vec{B} = \vec{R}$



Components of a vector



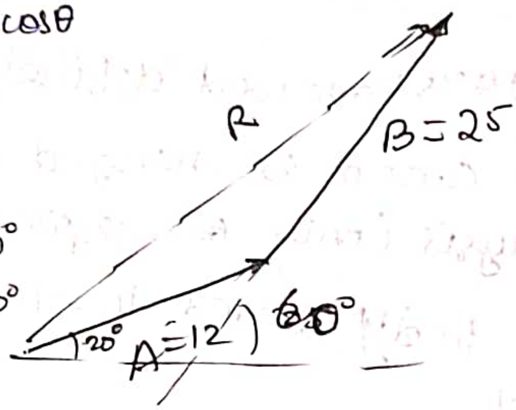
$$A = \sqrt{Ax^2 + Ay^2}$$

$$\theta = \tan^{-1} \frac{Ay}{Ax}$$

eg

A = 12 at 20°

B = 25 at 60°



$Ax = 12\cos 20^\circ = 11.3$

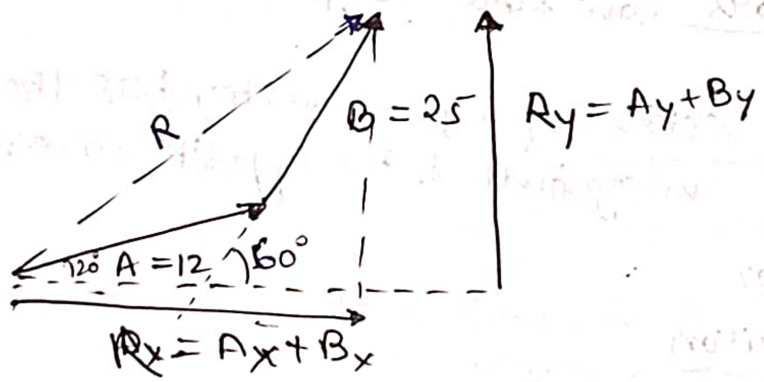
$Ay = 12\sin 20^\circ = 4.1$

$Bx = 25\cos 60^\circ = 12.5$

$By = 25\sin 60^\circ = 21.7$

Addition of vectors

Polar form



$Rx = Ax + Bx = 11.3 + 12.5$

$Rx = 23.8$

$Ry = Ay + By = 4.1 + 21.7$

$Ry = 25.8$

$R = \sqrt{Rx^2 + Ry^2}$

$R = \sqrt{23.8^2 + 25.8}$

$R = 35.05$

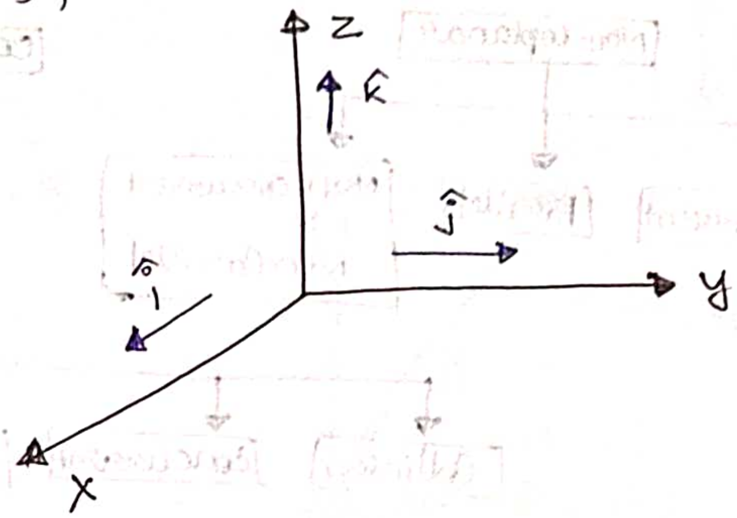
$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right) = \tan^{-1} 1.084$

$\boxed{\theta = 47.3^\circ}$

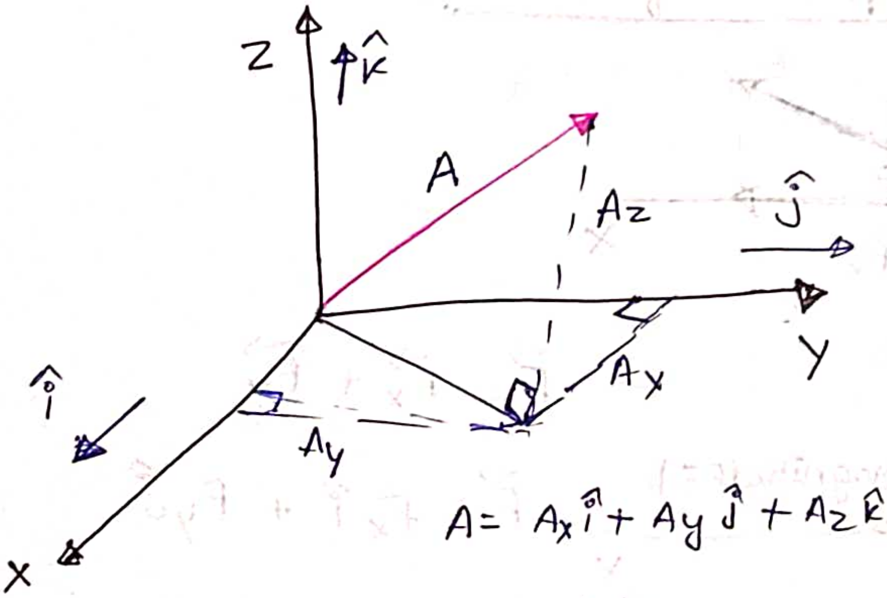
Unit vector :-

Magnitude of 1 and a direction

$\hat{i}, \hat{j}, \hat{k}$



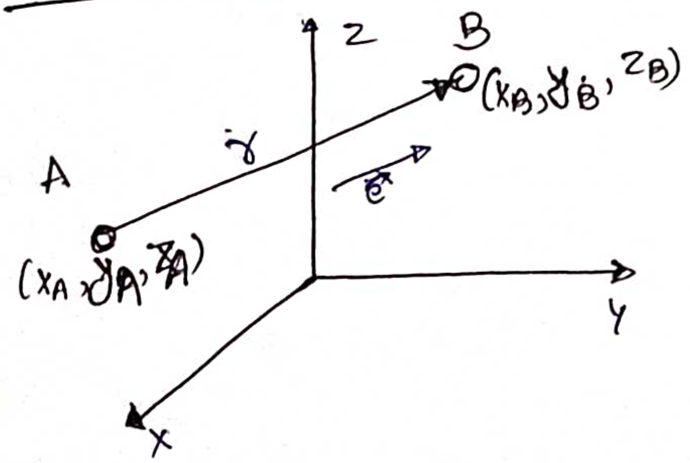
Rectangular coordinates in 3-D



$A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

A vector connecting two points

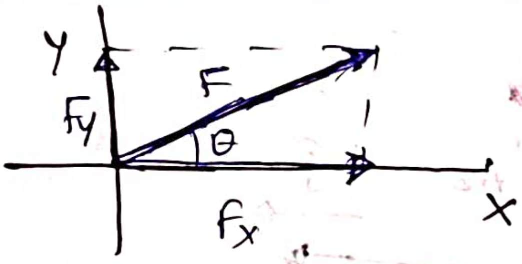
$r = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

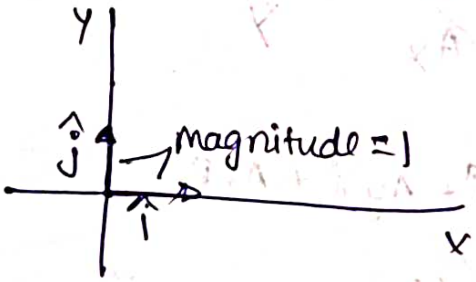


Force system

- Non-coplanar
  - Concurrent
  - Parallel
  - Non-concurrent & Non-Parallel
- Coplanar
  - Collinear
  - Concurrent
  - Parallel
  - Non-concurrent & Non-Parallel

Rectangular components of a force :-

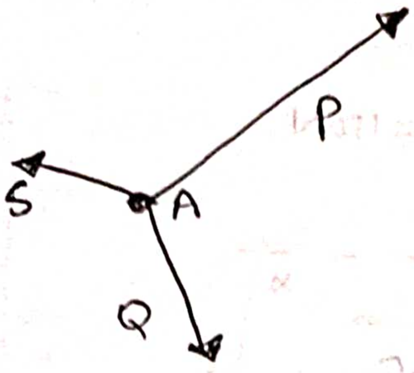


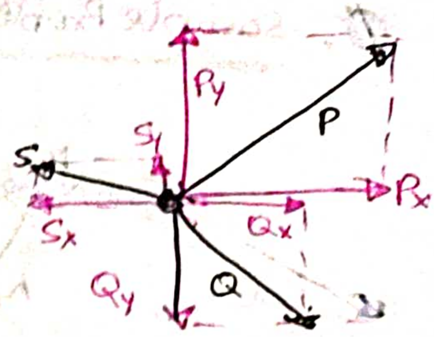


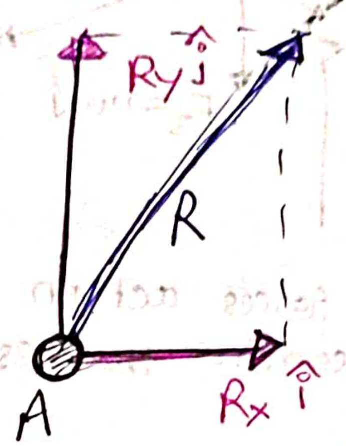
$$\vec{F} = \vec{F_x} + \vec{F_y}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

Addition of Forces







$$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

$$R_x\hat{i} + R_y\hat{j} = P_x\hat{i} + P_y\hat{i} + Q_x\hat{i} + Q_y\hat{j}$$

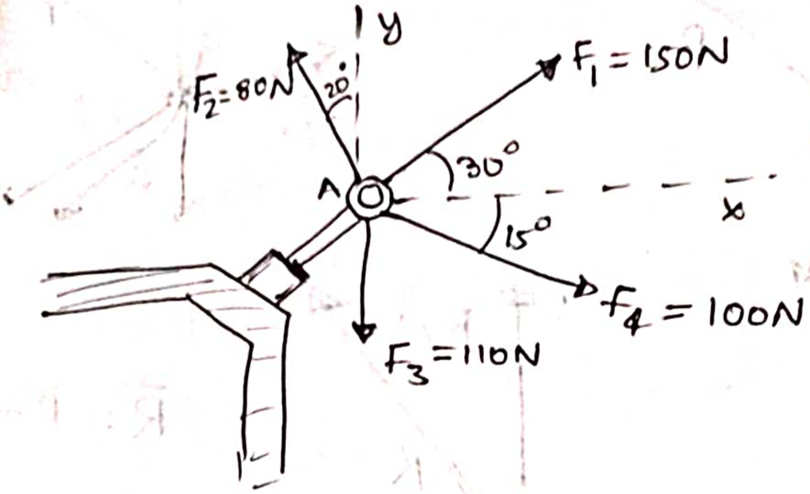
$$+ S_x\hat{i} + S_y\hat{k} .$$

$$= (P_x + Q_x + S_x)\hat{i} + (P_y + Q_y + S_y)\hat{k}$$

$$R_x = \Sigma F_x \, ; R_y = \Sigma F_y$$

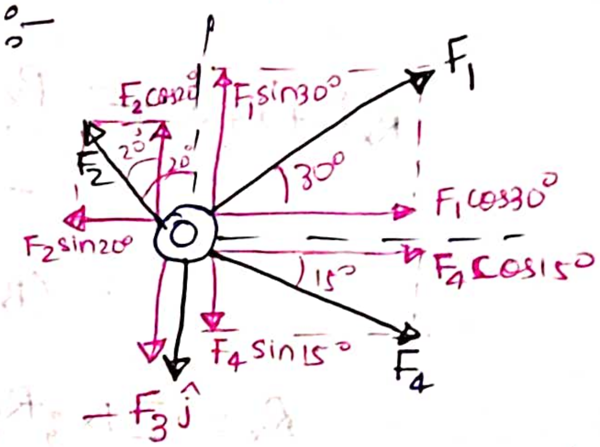
$$R = \sqrt{R_x^2 + R_y^2} \qquad \boxed{\theta = \tan^{-1}\frac{R_y}{R_x}}$$

Q. Sample Problem-1



Four forces act on bolt A as shown
Determine the resultant of the force on the bolt.

Soln :-



force	mag	x-comp.	y-comp.
F_1	150	+129.9	+75.0
F_2	80	−27.4	+75.2
F_3	110	0	−110.0
F_4	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

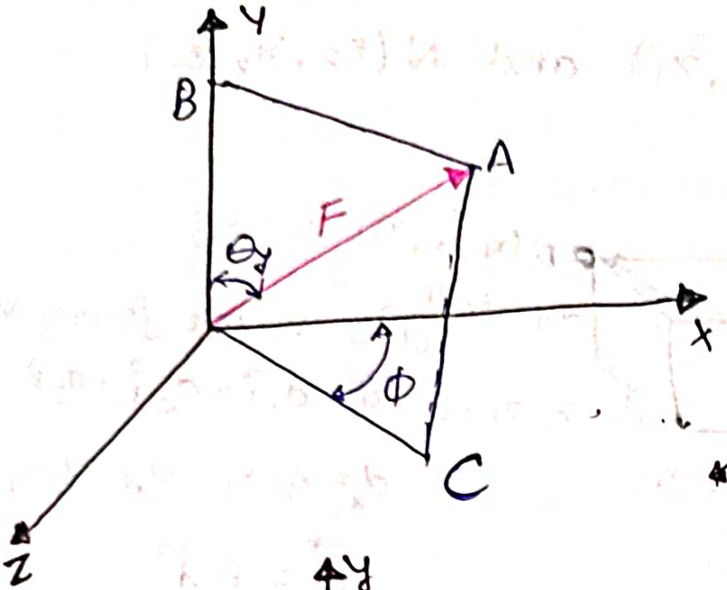
$$R = \sqrt{199.1^2 + 14.3^2}$$

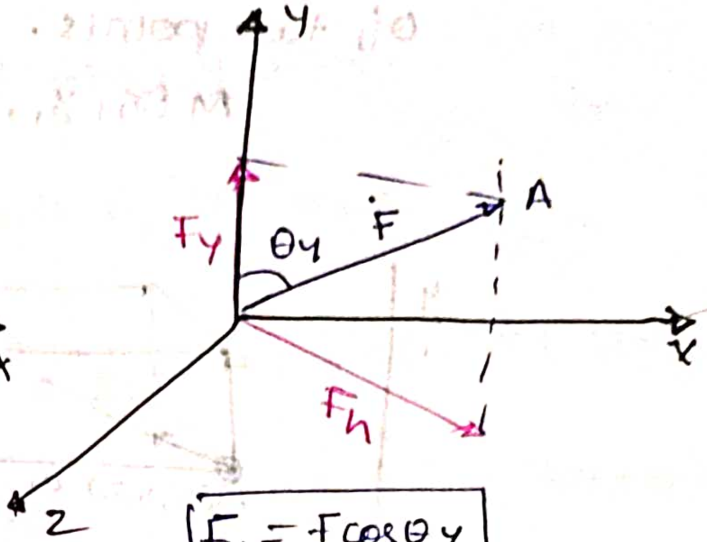
$$\boxed{R = 199.6N}$$

$$\tan\alpha = \frac{R_y}{R_x} \Rightarrow \alpha = \tan^{-1}\left(\frac{14.3}{199.1}\right)$$

$$\boxed{\alpha = 4.1°}$$

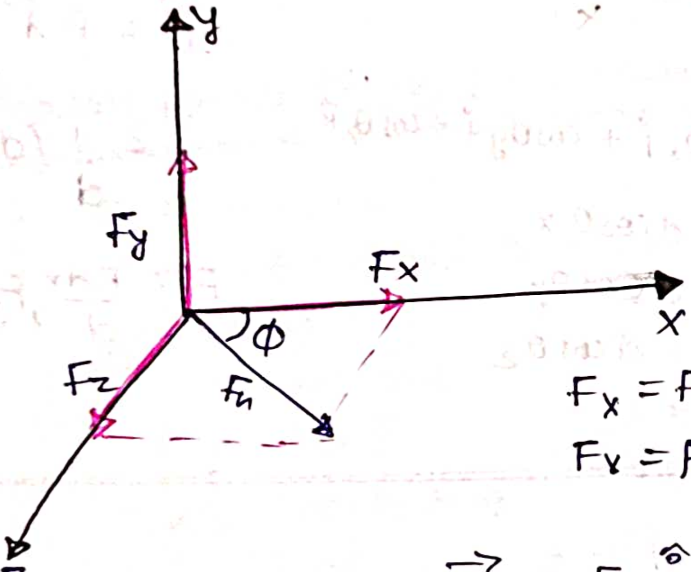
Rectangular components in space :-





$$\boxed{F_y = F\cos\theta_y}$$

$$F_h = F\sin\theta_y$$



$F_x = F_h \cos\phi$ $\qquad$ $F_z = F_h \sin\phi$

$F_x = F\sin\theta_y \cos\phi$ $\qquad$ $F_z = F\sin\theta_y \sin\phi$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{F} = F\sin\theta_y\cos\phi + F\cos\theta_y + F\sin\theta_y\sin\phi$$

$$= \underset{\text{Magnitude}}{|F|}\underbrace{\{\sin\theta_y\cos\phi + \cos\theta_y\,\hat{j} + \sin\theta_y\sin\phi\,\hat{k}\}}_{\text{unit vector}}$$

$$= F(\cos\theta_x\,\hat{i} + \cos\theta_y\,\hat{j} + \cos\theta_z\,\hat{k})$$

$$\boxed{\vec{F} = F\vec{\lambda}}$$

$$\vec{\lambda} = \cos\theta_x\,\hat{i} + \cos\theta_y\,\hat{j} + \cos\theta_z\,\hat{k}$$

$l = \cos\theta_x$

$m = \cos\theta_y$

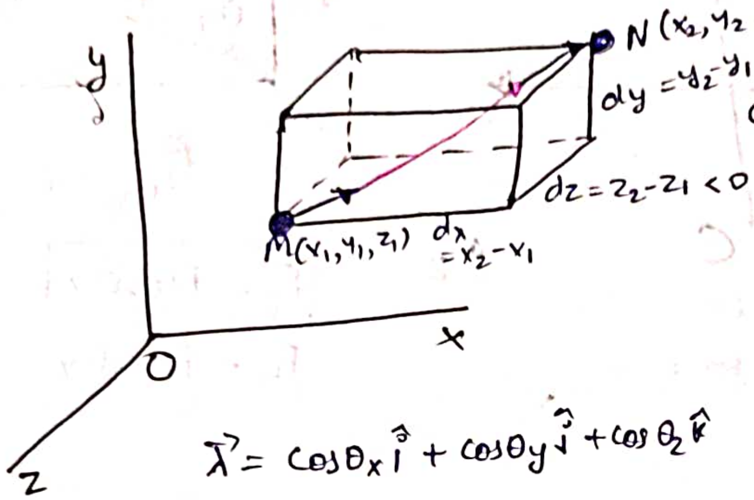
$n = \cos\theta_z$

$$\boxed{l^2 + m^2 + n^2 = 1}$$

$\vec{\lambda}$ is unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are direction cosines for $\vec{F}$

Direction of the force is defined by the location of two points.

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\vec{d}$ = vector joining M & N

$\vec{d} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}$

$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$

$\vec{F} = F \vec{\lambda}$

$\vec{\lambda} = \cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k}$

$\vec{\lambda} = \frac{1}{d}(d_x \hat{i} + d_y \hat{j} + d_z \hat{k})$

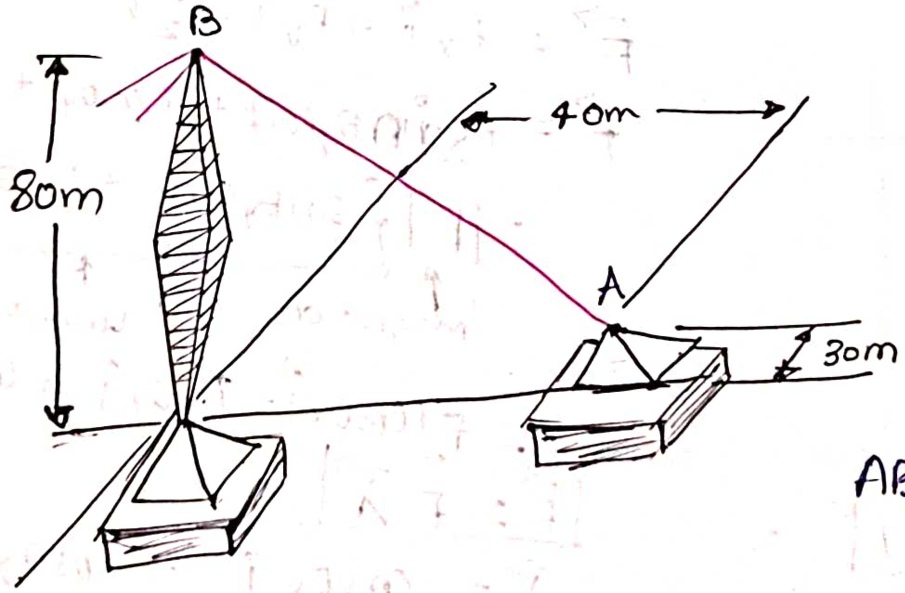
$d_x = d\cos\theta_x$

$d_y = d\cos\theta_y$

$d_z = d\cos\theta_z$

$F_x = \frac{F d_x}{d}, \; F_y = \frac{F d_y}{d}, \; F_z = \frac{F d_z}{d}$

Sample Problem :-



A (40, 0, -30)

B (0, 80, 0)

$AB = (-40\,m)\hat{i} + (80\,m)\hat{j} + (30\,m)\hat{k}$

$AB = \sqrt{(-40)^2 + (80)^2 + (30)^2}$

$\boxed{AB = 94.3\,m}$

$$\vec{\lambda} = \left(-\frac{40}{94.3}\right)\hat{i} + \left(\frac{80}{94.3}\right)\hat{j} + \left(\frac{30}{94.3}\right)\hat{k}$$

$$\vec{\lambda} = -0.424\,\hat{i} + 0.848\,\hat{j} + 0.318\,\hat{k}$$

Determining components of the force.

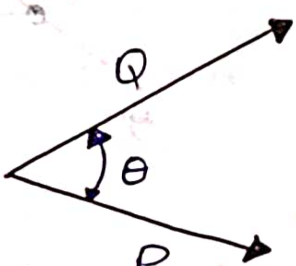
$$\vec{F} = F\vec{\lambda}$$

$$\vec{F} = (2500N)\left(-0.424\,\hat{i} + 0.848\,\hat{j} + 0.318\,\hat{k}\right)$$

$$\vec{F} = (-1060N)\,\hat{i} + (2120N)\,\hat{j} + (795N)\,\hat{k}$$

Scalar Product of two vectors :-

- The scalar Product or dot product b/w two vectors P & Q is defined as $\vec{P}\cdot\vec{Q} = PQ\cos\theta$ (scalar result).



- Scalar Products
  - are commutative
    $\vec{P}\cdot\vec{Q} = \vec{Q}\cdot\vec{P}$
  - are distributive
    $\vec{P}\cdot(\vec{Q_1}+\vec{Q_2}) = \vec{P}\cdot\vec{Q_1} + \vec{P}\cdot\vec{Q_2}$
  - are not associative
    $(\vec{P}\cdot\vec{Q})\cdot\vec{S} =$ undefined

$$\vec{P}\cdot\vec{Q} = (P_x\hat{i} + P_y\hat{j} + P_z\hat{k})(Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k})$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

$$\vec{P}\cdot\vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\vec{P}\cdot\vec{P} = P_x^2 + P_y^2 + P_z^2 = P^2$$

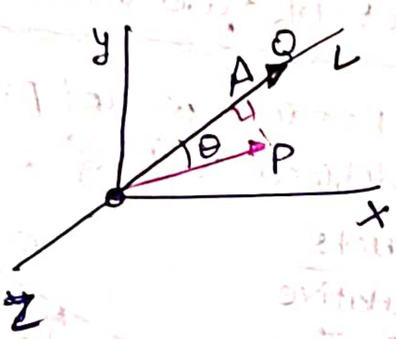
Application: of scalar Pdt

• Angle b/w two vectors.

$$\vec{P}\cdot\vec{Q} = PQ\cos\theta = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ}$$

• Projection of a vector on a given axis

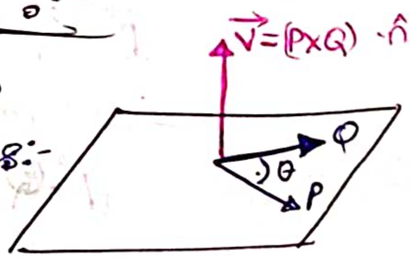


$$P\cdot Q = PQ\cos\theta$$

$$P\cos\theta = \frac{P\cdot\vec{Q}}{Q} = P_{OL}$$

Projection of P along Q

Vector Product of two vectors :-



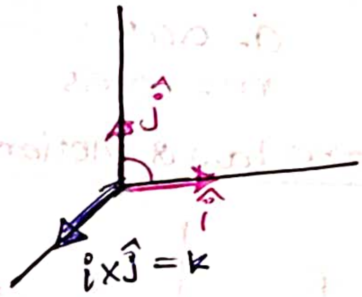
Vector Pdt of two vectors P & Q satisfies following conditions:-

1. Line of action of V is ⊥r to plane containing P and Q.
2. Magnitude of V, $V = PQ\sin\theta$.
3. Dirn of V is obtained from right hand rule.

Vector Products are

- anti-commutative, $Q \times P = -(P \times Q)$
- are distributive $P \times (Q_1 + Q_2)$
$= P \times Q_1 + P \times Q_2$.
- are not associate
$(P \times Q) \times S \neq P \times (Q \times S)$

Vector Products: Rectangular components



$\hat{i} \times \hat{j} = k$

vector pdt of
cartesian unit vectors

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

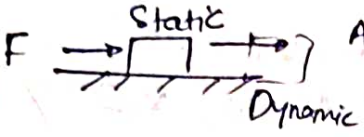
$$V = \vec{P} \times \vec{Q}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

— o —

NLM

1. Newton's 1st law of Motion
   ↳ Law of Inertia



An object tends to be in rest or constant motion until & unless an external Force act upon it.

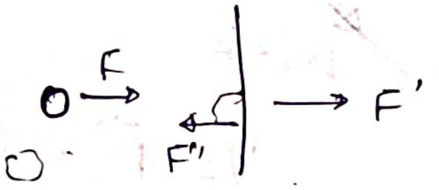
2. Newton's 2nd law of Motion

$$F = ma$$

a = acc^n.

m = mass

3. Newton's 3rd law of Motion

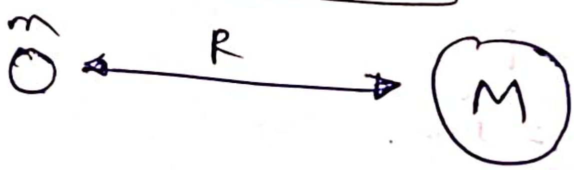


Every action has an equal and opposite reaction

↳ Law of conservation of energy

$$F = F'$$

4. Newton's 4th law of motion

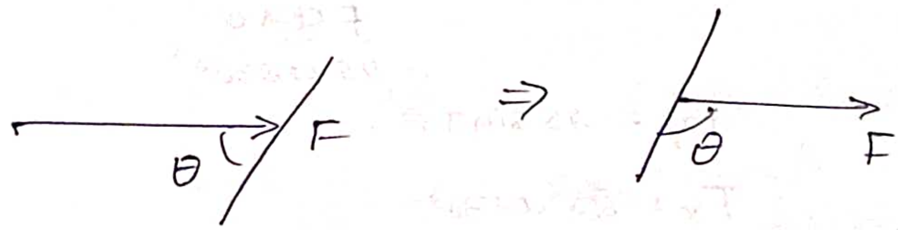


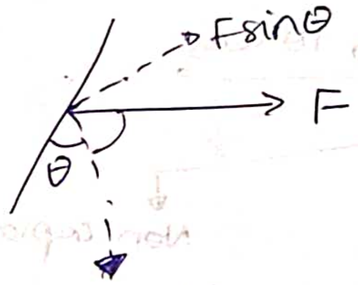
G : Gravitational Force

$$F \propto \frac{mM}{R^2} \Rightarrow F = G\frac{Mm}{R^2}$$

Coplanar forces

Forces which lie (act) in a single plane are called as coplanar forces

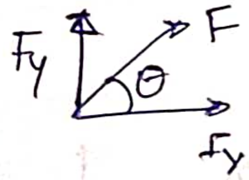




* characteristics ⇒
  - where to apply
  - How to apply
  - what is variation
* effects → After applying what are outputs.

* **Resolution** :

any Forces in coplanar forces are resolute in x-axis & y-axis.

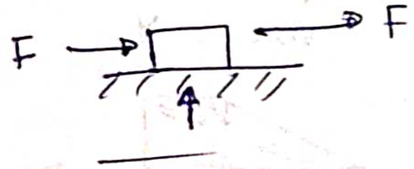


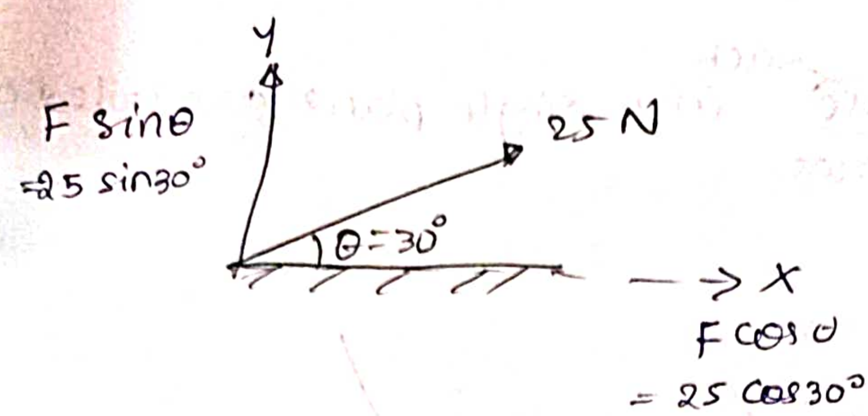
$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

* **Force** (Push or Pull)

Static Dynamic



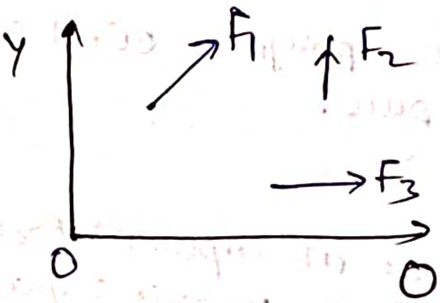


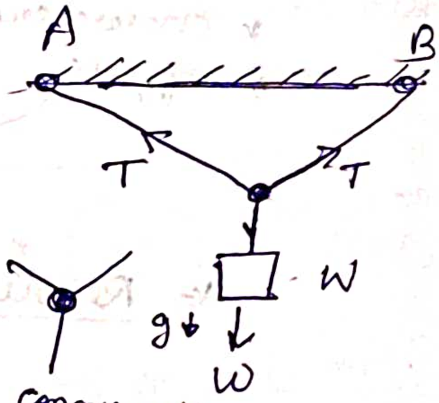
$F_y = 25 \sin 30°$

$F_x = 25 \cos 30°$

System of Forces

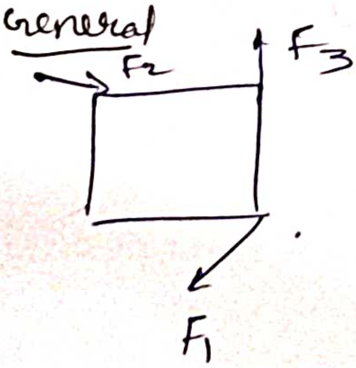
- Coplanar
  - concurrent
  - Parallel
  - General
- Non-coplanar



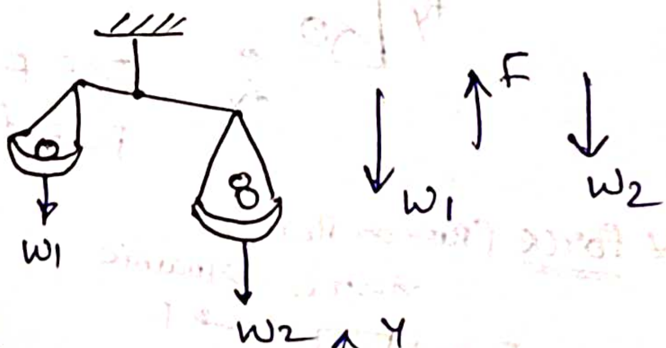


Concurrent

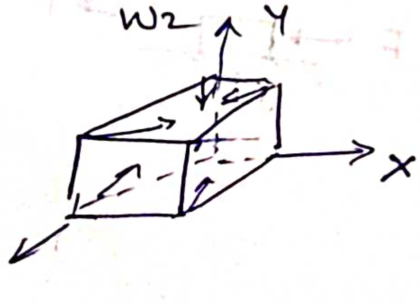
General

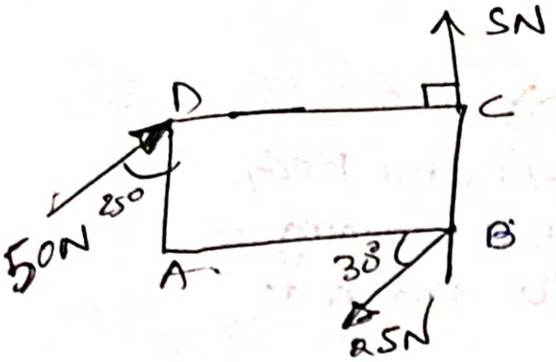


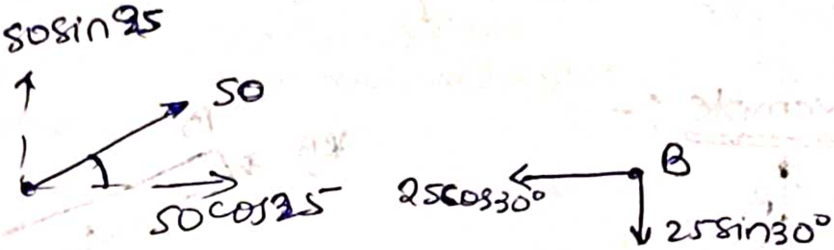
Parallel



Non-coplanar







$C = 5N$

↑ +ve ↓ −ve

← −ve → +ve

$F_y = -25\sin 30 + 50\sin 25 + 5$

$F_x = -25\cos 30 + 50\cos 25$

Coplanar Forces : Equilibrium

Introduction : 3 system of Forces; to find resultant of system" of forces
And if the resultant of system happen to be zero, the system is said to be in state of equilibrium.

Conditions of Equilibrium :-
(COE)

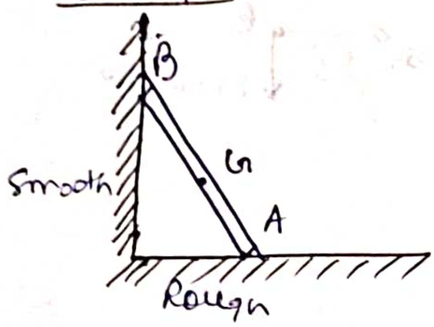
* The sum of all forces should be zero.
  i.e. $\Sigma \vec{F} = 0$
  which means $\Sigma F_y = 0$ & $\Sigma F_x = 0$

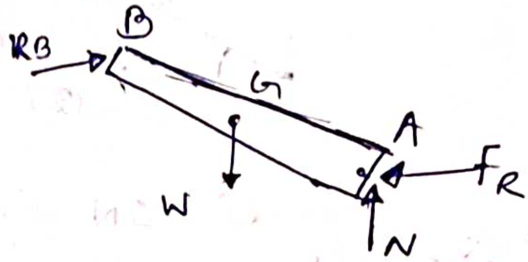
* The sum of all moments of all forces should be zero
  i.e. $\Sigma \vec{M} = 0$.

Free Body Diagram (FBD)

A diagram formed by isolating the body from its surroundings and then showing all the forces acting on it known as "Free Body Diagram".

example :-





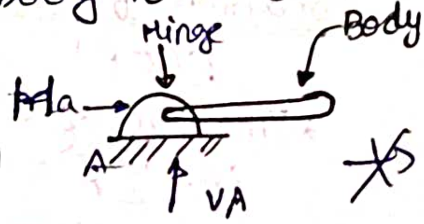
Types of supports and reactions

* whenever a body is supported, the support offers resistance, known as reaction

eg. You are sitting on a chair while reading a book, your weight is being supported by the chair. which offers a force of resistance (reaction) upwards

1. Hinge support :-

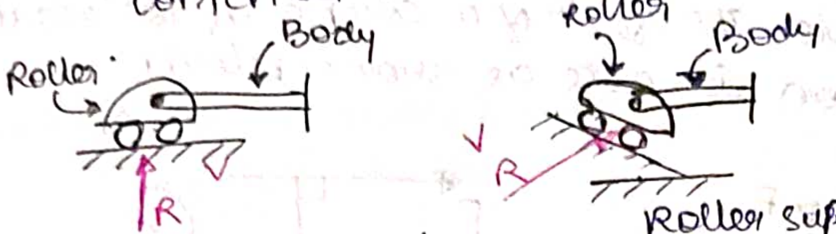
A hinge allows free rotation of the body but doesn't allow the body to have linear motion.



* It therefore offers reaction which can be split into horizontal & vertical component.

2. Roller Support :-

A roller support is free to roll on a surface on which it rests.



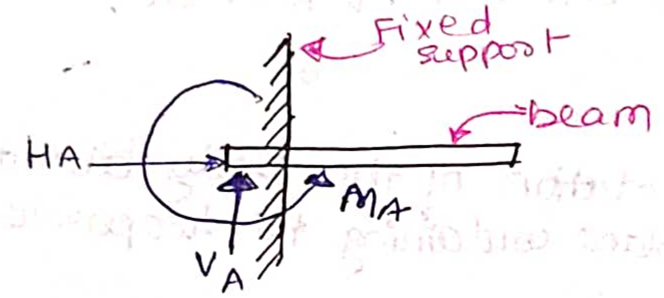
Roller supported on Horizontal surface

Roller support on inclined surface

representation of roller.

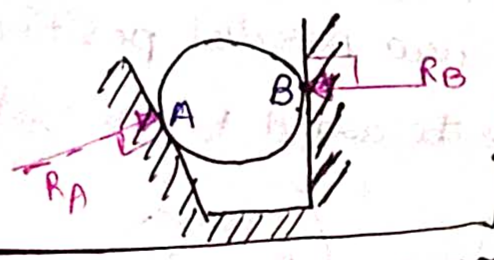
3. Fixed support :-

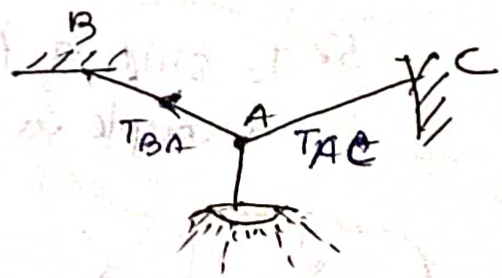
* A fixed support neither allows any linear motion nor allows any rotation. It therefore offers a force reaction which can be split into a horizontal & vertical component and also a moment reaction.



4. Smooth surface support :-

* A smooth surface offers a similar reaction as a roller support ie. a force reaction normal to the smooth surface; as shown.
* Each surfaces offers one force reaction, Normal to the surface at contact points.



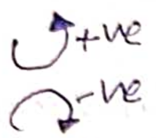


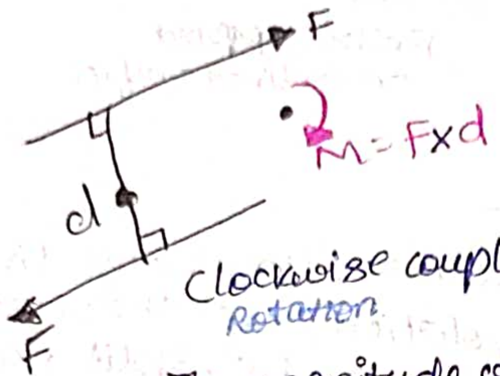
5. Rope / string / cable support :-

* It offers a pull force in a direction away from the body.
* This force is commonly reffered as the tension force, as shown

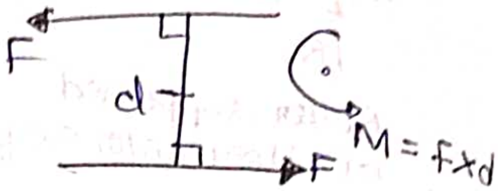
couple :-

Couple is a special case of parallel forces: Two parallel forces "of equal and opposite sense is equal couple. The effect of a couple is to rotate the body on it acts as shown below:





Clockwise couple
Rotation

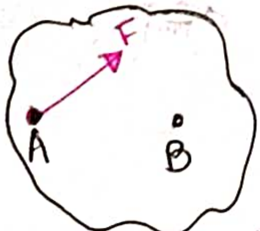


Anticlock-wise couple.

The magnitude of rotation is known as the moment of a couple given as $M = F \times d$ are Nm, kN·m etc.

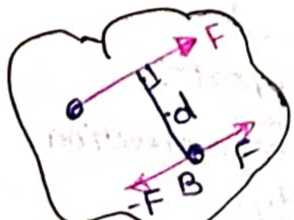
Properties of Couple :

1. Couple tends to cause rotation of the body about an axis $\perp^r$ to the plane containing the two parallel forces.
2. Moment of a couple is equal to the product of one force and the arm of the couple
3. The resultant force of a couple system is zero.
4. To balance a system whose resultant is a couple "another couple" of opposite direction is required
5. To shift a force to a new parallel position a couple is required to be added to the system.



Force originally at A.

=



Two collinear Forces added at B

=

Force shifted to B with a couple added

Resultant of General Force system :-

To find the resultant of "General Force System" Follow the given steps :-

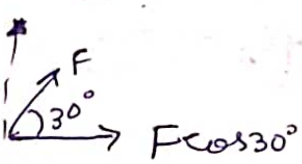
Step 1: Start by resolving forces if there are any, as done in Method of resolution.

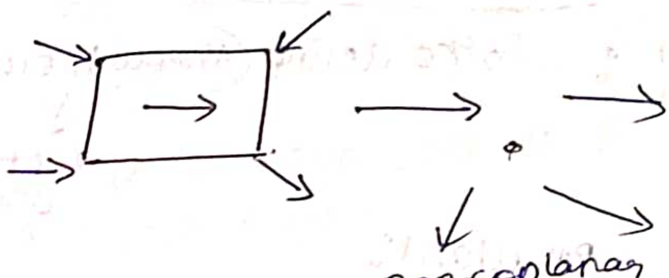
Step 2: Location of the resultant force can be located by taking moment at one point and equate it with the moment of Resultant.

It is expressed as,

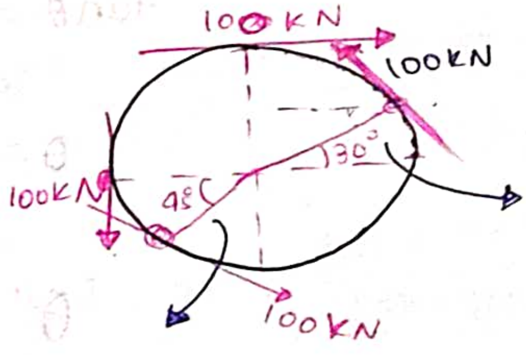
$$\boxed{\Sigma M_A F = M_A^R}$$

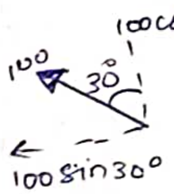
And if the distance 'd' is obtained positive then the assumption was correct, Else the assumption was wrong & it lies to the opposite side to what we assumed.





Q. Determine the resultant of four forces tangential to the circle of radius 4cm. What will be the location of the resultant w.r.t the centre of the circle?





$\Sigma F_x \rightarrow +ve$

$= 100 + 100\cos 45° - 100\sin 30°$

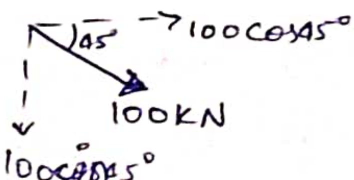
$= 120.7 \rightarrow$

$\Sigma F_y \uparrow +ve$

$= -100 - 100\sin 45° + 100\cos 30°$

$= -84.1$ KN.

$= 84.1$ KN $\downarrow$



$R = \sqrt{120.7^2 + 84.1^2} = 147.1N$

$\tan\theta = \frac{84.1}{\ldots} \Rightarrow \boxed{\theta = 34.86°}$

(R = 147.1N)

Location of resultant force.

Using Varignon's theorem

$\Sigma M_o F = M_o R$

$(-100 \times 4) + 100 \times 4 + 100 \times 4 + 100 \times 4$

$= -147.1 \times d$

$d = -5.44\,m$

$d = 5.44\,m$ (left of O)

Hence resultant force $R = 147.1\,kN$ at $\theta = 34.86^\circ$ located at ⊥r dist of $d = 5.44\,m$ left of O.

Resultant of concurrent system of forces.

using llgm law of forces.

concurrent system : Force acting through one point.

meet at one point

Resultant:

Law states : Mathematically,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

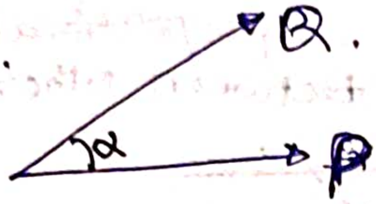
$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

'θ' angle made by "Resultant" with Force "P".

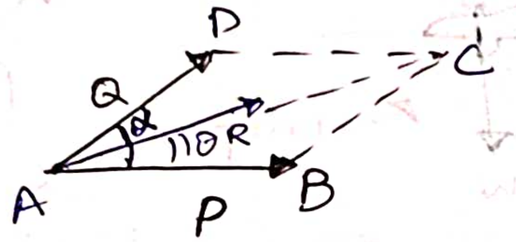
(2.) Which actually means, If '2' forces are acting from one point and separated by angle 'α'

i.e.

if $P > Q$



Then the Resultant will be shown as;

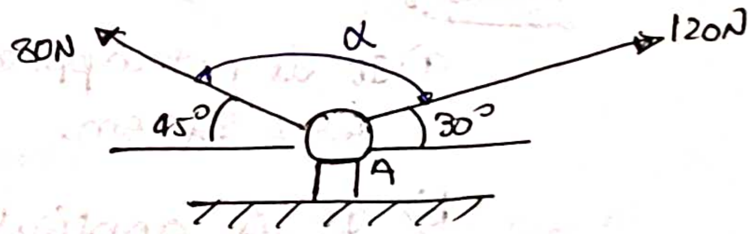


where 'R' is the "Resultant" and "θ" is the angle made by Resultant; with force "P"

Note :- Parallelogram law is for "two forces", it can be used for more forces but considering two at once.

Q. Two forces of 80 & 120N on an eye-bolt at "A" as shown. Determine resultant of two forces.

using llgm law



$P = 120\,N \quad Q = 80\,N$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{80^2 + 120^2 + 2\times 80\times 120\cos 105^\circ}$$

$$\boxed{R = 125.8N}$$

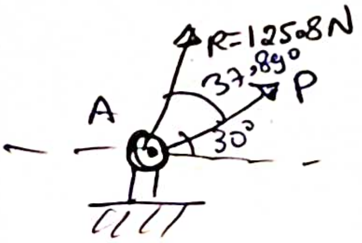
$\alpha = 180 - 45 - 30$

$$\boxed{\alpha = 105^\circ}$$

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\left[\frac{80 \times \sin 105^\circ}{120 + 80\sin 105}\right]$$

$$= \tan^{-1}(0.778)$$

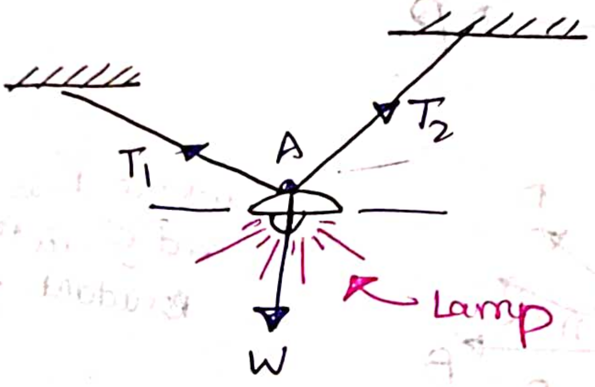
$\theta = 37.89°$ (w.r.t 120N force)



Lami's Theorem

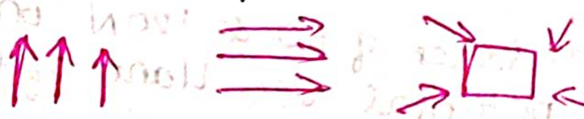
Lami's theorem deals with particular case of equilibrium involves three forces only.

It states: If three concurrent forces act on a body keeping it in equilibrium, then each force is proportional to the sine of the angle between other two forces.



$$\boxed{\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\theta}}$$

Note :-

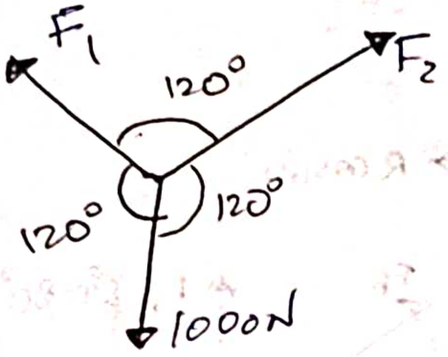


a) It is not applicable for parallel and general force system.

b) It is applicable only when three forces acting at a point are in equilibrium

Example

system is in Equilibrium



By lami's Theorem

$$\frac{F_1}{\sin 120^\circ} = \frac{F_2}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

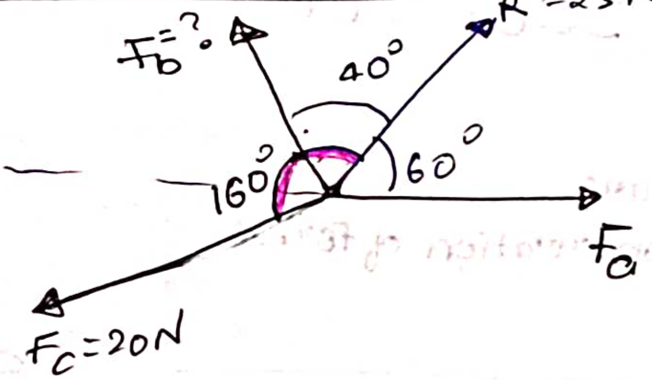
on equating

$$\frac{F_1}{\cancel{\sin 120^\circ}} = \frac{1000}{\cancel{\sin 120^\circ}}$$

$$\boxed{F_1 = 1000\,N}$$

$$F_2 = F_1 \Rightarrow \boxed{F_2 = 1000N}$$

Can be solved this way also



A force $R = 25N$ has components F_a, F_b, and F_c as shown. If $F_c = 20N$. Find F_a & F_b.

soln :- As we can see.

$\xrightarrow{}$ +ve $\quad \Sigma F_x = R\cos 60^\circ = 25\cos 60^\circ = 12.5N$

$\uparrow$ +ve $\quad \Sigma F_y = R\sin 60^\circ = 25\sin 60^\circ = 21.65N$

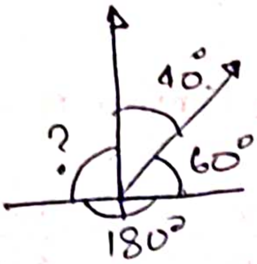
$$\Sigma F_y = 21.65 = F_b \sin 80^\circ - 20\sin 40^\circ$$

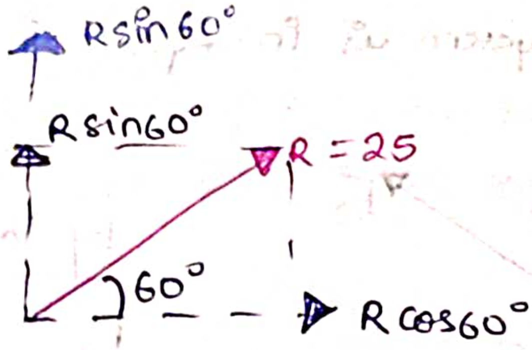
$$F_b = 35.04N$$

$$\Sigma F_x = 12.5 = F_a - F_b\cos 80^\circ - 20\cos 40^\circ$$

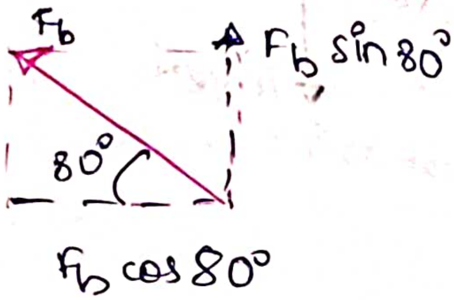
$$= F_a - 35.04\cos 80^\circ - 20\cos 40^\circ$$

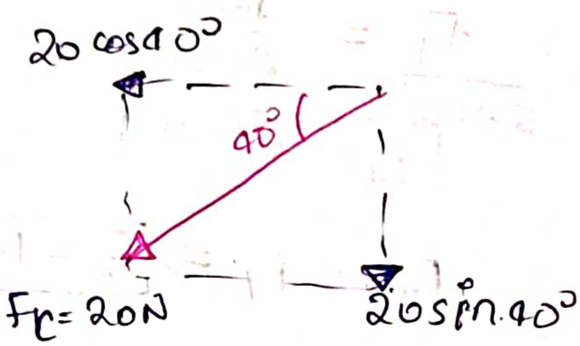
$$F_a = 83.905\,N.$$





$60+40+x=180$
$x=80°$



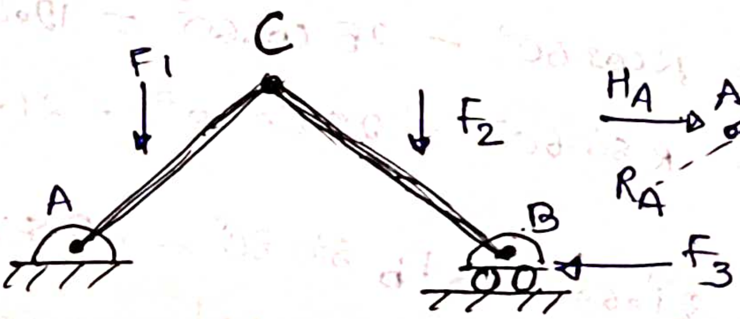


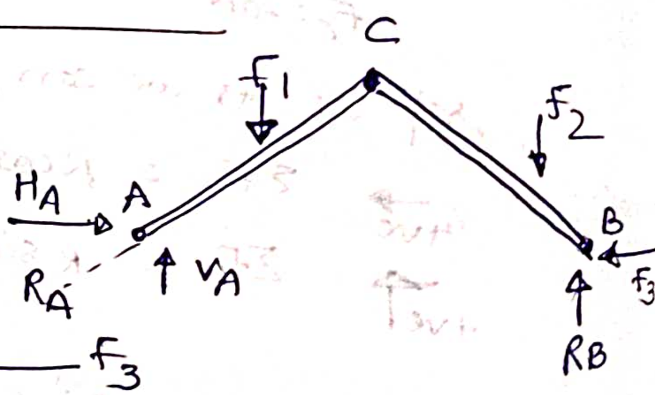
Equilibrium of connected bodies :-

Assuming

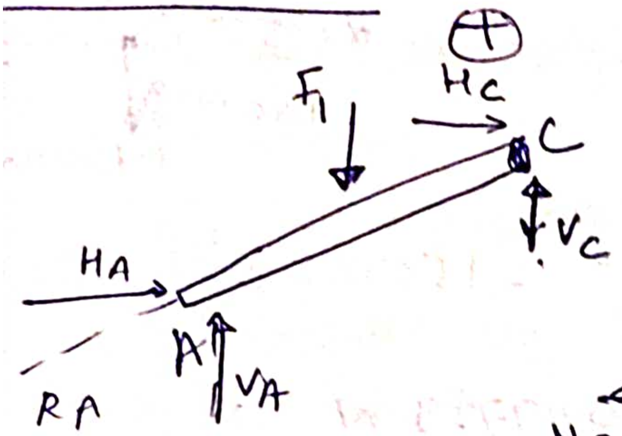
* Rigid structure
* Law of conservation of forces

That's it.

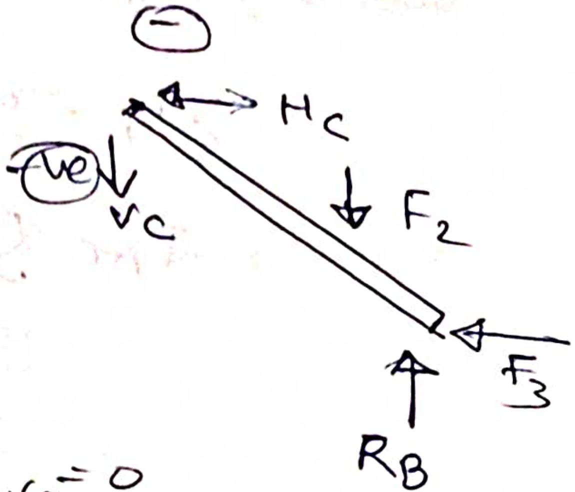




FBD of AC



FBD of CB

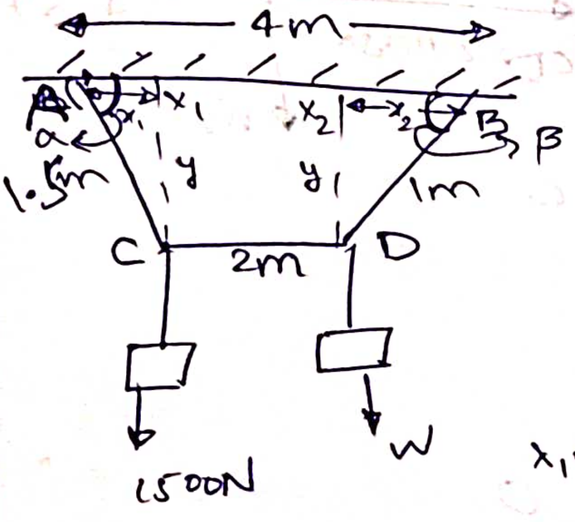


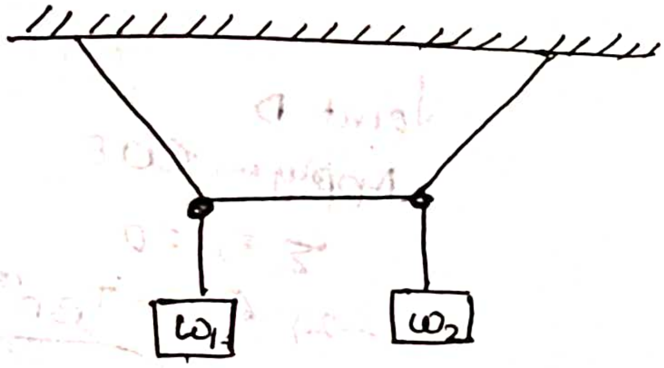
He ← → vo

Pve ↓ $V_C = 0$

- Rope AB 9.5 m long is connected at two points A & B at same level and 4 m apart. Load of $W_1 = 1500N$ is suspended at point C as shown. What load W_2 should be connected at point D to maintain the position.
  AC = 1.5m BD = 1m

Soln :-





$$x_1 + x_2 + 2 = 4$$

$$x_1 + x_2 = 2 \quad \text{--- (1)}$$

$$\cos\alpha = \frac{x_1}{1.5} = \frac{1.3125}{1.5}$$

$$\alpha = 28.96°$$

$$\cos\beta = \frac{x_2}{1} = \frac{0.6875}{1}$$

$$\beta = 46.570$$

$$y^2 = \cancel{AC^2}\,1.5^2 - x_1^2 = 1^2 - x_2^2$$

$$\cancel{y^2}\,x_1^2 - x_2^2 = 1.5^2 - 1$$

$$x_1^2 - x_2^2 = 1.25$$

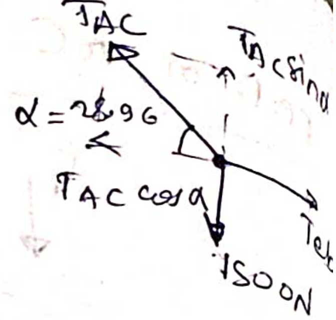
$$x_1 - x_2 = 0.625$$

$$x_1 + x_2 = 2$$

$x_1 = 1.3125$

$x_2 = 0.6875$

Joint C;



$\Sigma F_y = 0 \uparrow +ve$

$T_{AC} \sin 28.96^\circ - 1500 = 0$

$T_{AC} = 3097.9\ N$

$\Sigma F_x = 0 \rightarrow +ve$

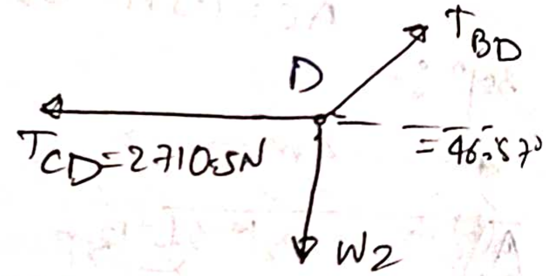
$-T_{AC} \cos 28.96 + T_{CD} = 0$

$-3097.9 \cos 28.96 + T_{CD} = 0$

$T_{CD} = 2710.5\ N$

Joint D

Apply in COE



$\Sigma F_x = 0$

$-2710.5 + T_{BD} \cos 46.57 = 0$

$\boxed{T_{BD} = 3942.8\ N}$

$\Sigma F_y = 0$

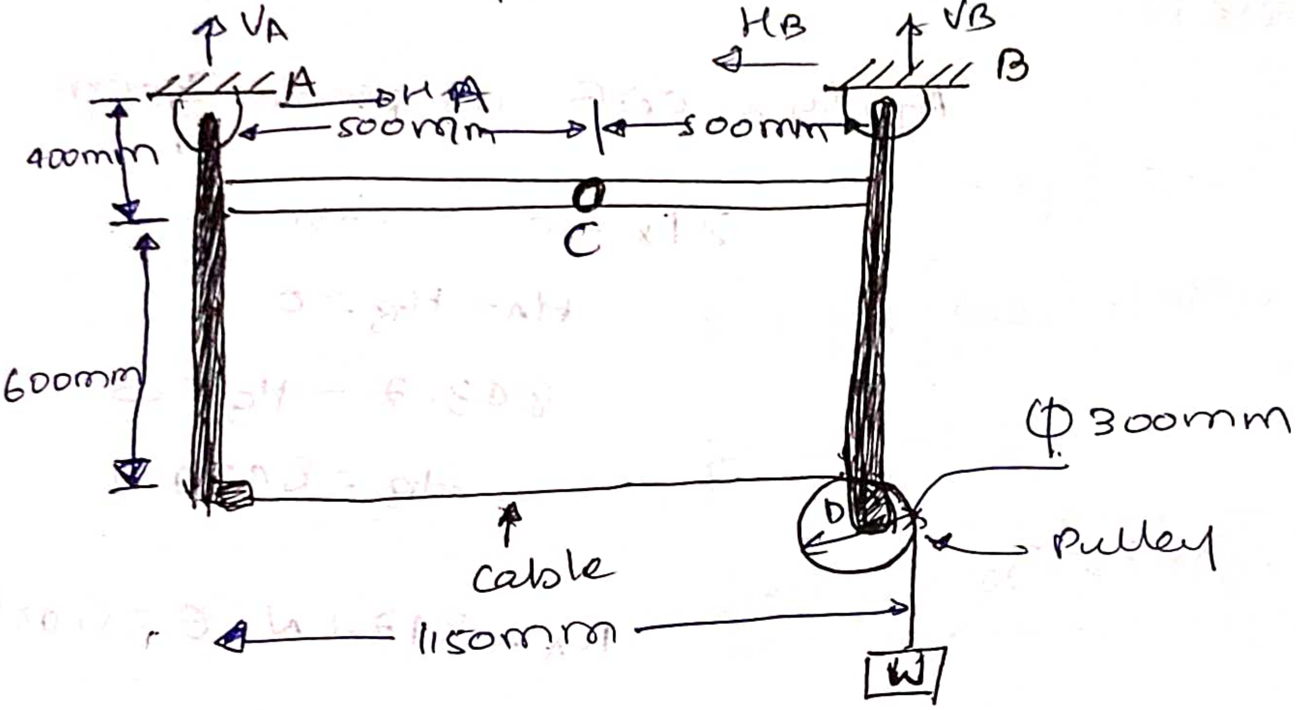
$T_{BD} \sin 46.57^\circ - W_2 = 0$

$\boxed{W_2 = 2868.3\ N}$

A frame supported by hinges at A & B. The pulley of 300mm diameter is pinned at D on the frame and supported a cable carrying a load W = 500N.

(a) The support reaction at A & B.

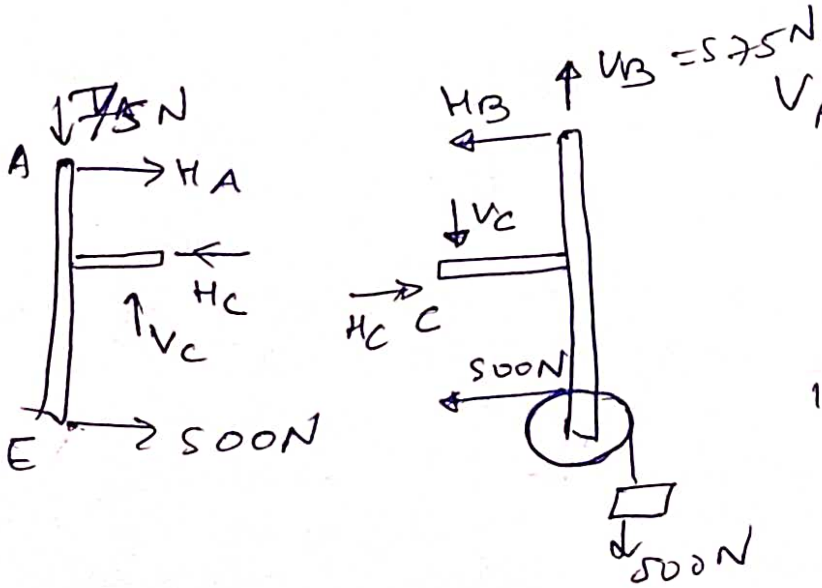
(b) What is force carried by the pin at D as it supports the pulley.



$\Sigma M_A = 0$ ↺ +ve

$$-500 \times 1150 + (V_B \times 1000) = 0$$

$V_B = 575\,N$ ↑



1000

$\Sigma F_y = 0$

$V_A + 575 - 500 = 0$

$V_A = -75\,N$

$V_A = 75\,N$ ↓

$\Sigma M_C = 0$

$(500 \times 600) + (75 \times 500) - (H_A \times 400) = 0$

$H_A = 843.7\ N$

Applying COE to entire system

$\Sigma F_x = 0 \rightarrow +ve$

$H_A - H_B = 0$

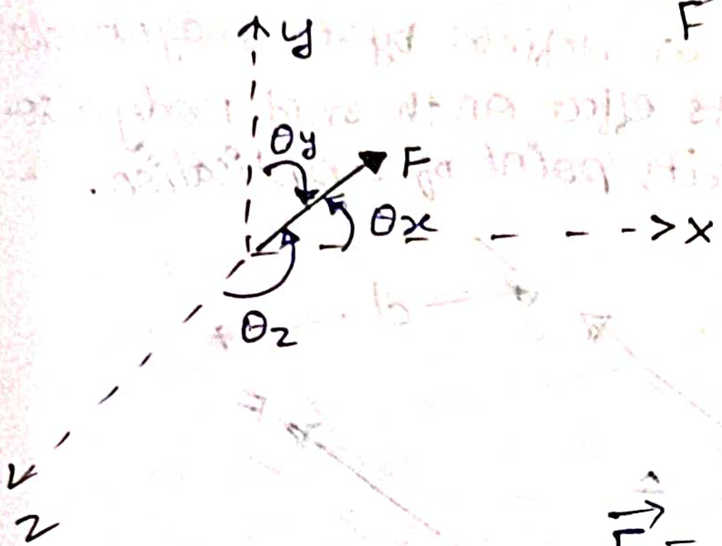
$843.7 - H_B = 0$

$H_B = 843.7\ N$

$R_A = 847.1\ N \quad \theta = 5.08$

$R_B = 1021\ N \quad \theta = 34.2$

Magnitude of Force & dirn.



$$\vec{F} = |F| \lambda_{AB}$$

$$F_x = |F| \cos\theta_x$$

$$F_y = |F| \cos\theta_y$$

$$F_z = |F| \cos\theta_z$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

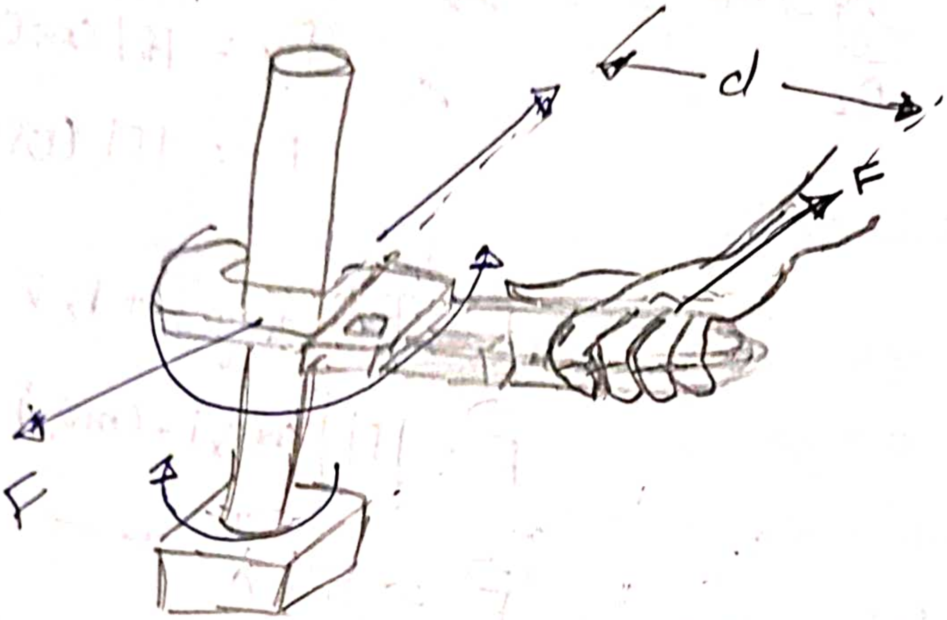
$$\vec{F} = |F| \left[\cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k} \right]$$

$$\vec{F} = |F| \lambda$$

$$\boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$$

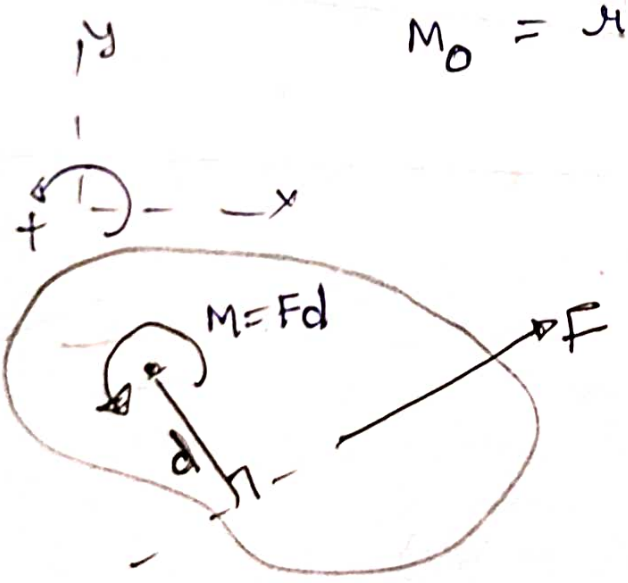
Moment of a Force about a point :

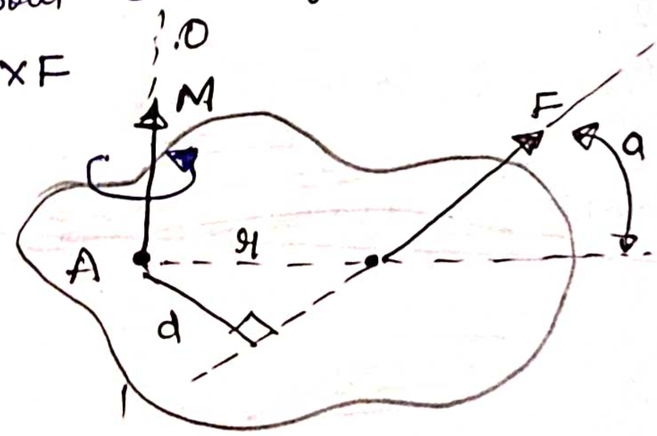
- A Force vector is defined by its magnitude & direction. Its effect on the rigid body also depends on its point of application.



- The moment of F about O is defined as

$$M_O = r \times F$$





- The moment vector M_O is ⊥r to the plane containing O & the force F.

- The sense of the moment may be determined by the right-hand rule.

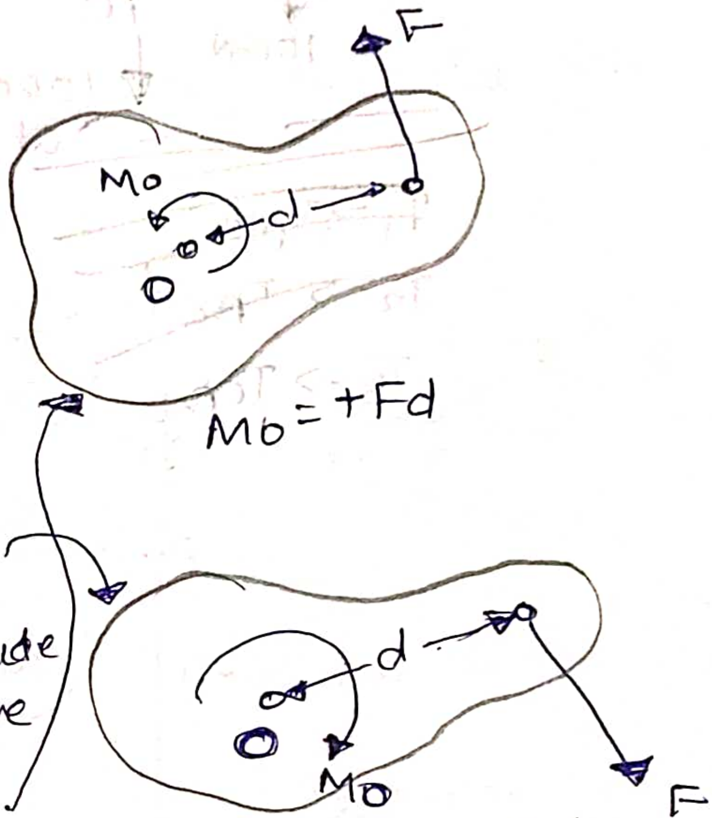
- magnitude of M_O measures the tendency of the force to cause rotation of the body about an axis along M_O

$$M_O = rF\sin\alpha = Fd.$$

Any force F' that has same magnitude and direction as F, is equivalent if it also has the same line of action & therefore produces same moment.

2-D structures have length & breadth but negligible depth & are subjected to forces contained in the plane of the structure.

- The plane of structure contains pt O & the force F. Mo, the moment of the force about O is ⊥r to the plane.

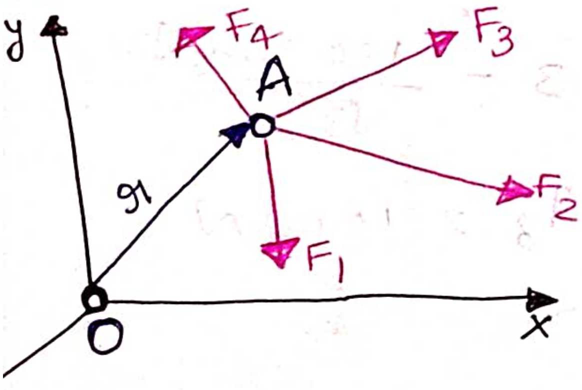


- If force tends to rotate the structure clockwise (CW) the sense of the moment vector is out of the plane of the structure & magnitude of the moment is negative

$M_O = -Fd$

- If the force tends to rotate the structure counterclockwise (CCW) the structure the sense of the moment vector is into the plane of the structure & magnitude of the moment is positive.

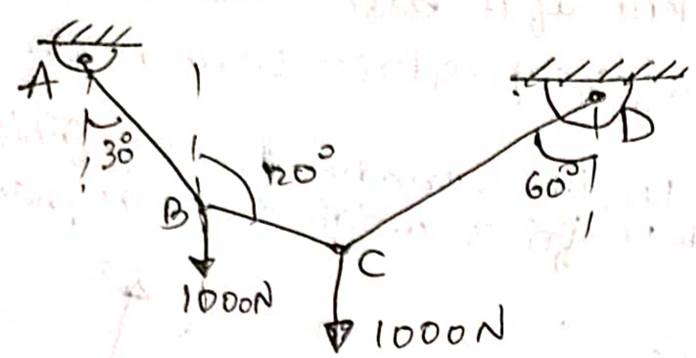
Varignon's Theorem :



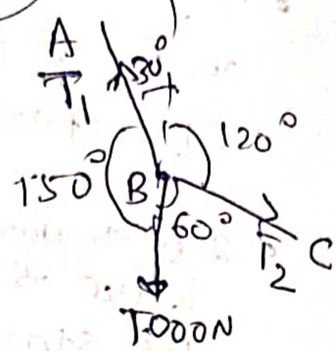
The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point O.

$$\boxed{r \times (F_1 + F_2 + \ldots) = r \times F_1 + r \times F_2 + \ldots}$$

Q.



(at B)



at B

$T_1 \rightarrow T_{AB}$

$T_2 \rightarrow T_{BC}$

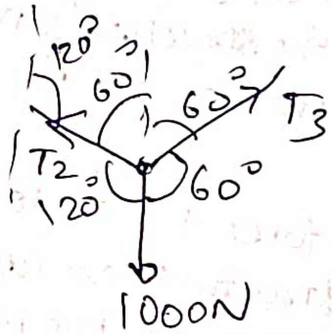
$T_3 \rightarrow T_{CD}$

$$\frac{T_1}{\sin 60^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

$T_1 = 1732$

$T_2 = 1000\,N$

(at C)

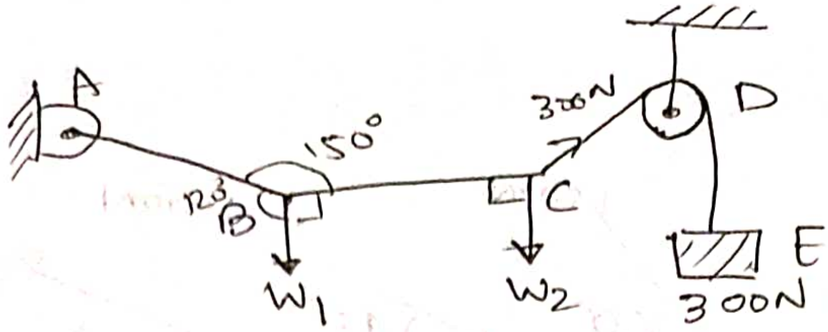


$$\frac{T_2}{\sin 60^\circ} = \frac{T_3}{\sin 60^\circ} = \frac{1000}{\sin 120^\circ}$$

$$T_3 = \frac{1000}{\sqrt{3}/2} \times \sqrt{3}/2$$

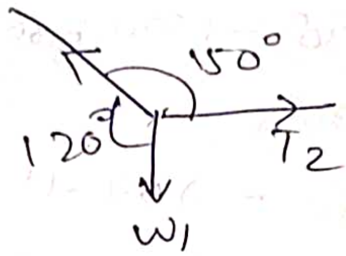
$T_3 = 1000\,N$

(2)



T_1 : T_{AB} T_2 : T_{BC} T_3 : T_{CD}

At B



$$\frac{T_1}{\sin 90^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{W_1}{\sin 150^\circ}$$

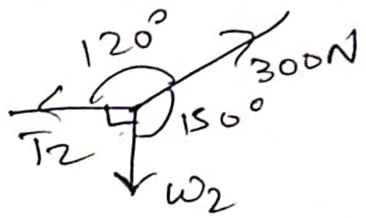
$$T_1 = \frac{150}{\sqrt{3}} \times 2$$

$$\boxed{T_1 = 173.2}$$

$$W_1 = \frac{150 \times 1/\cancel{2}}{\sqrt{3}/\cancel{2}}$$

$$\boxed{W_1 = 86.6\,N}$$

At C



$$\frac{T_2}{\sin 150} = \frac{W_2}{\sin 120} = \frac{300}{\sin 90}$$

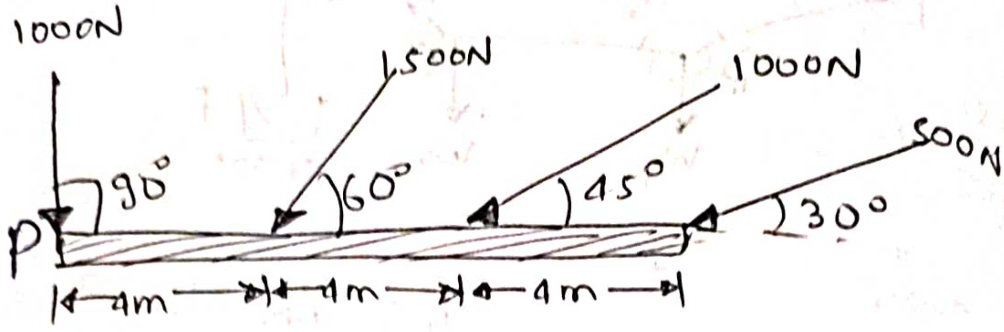
$$T_2 = 300 \times \frac{1}{2}$$

$$\boxed{T_2 = 150N}$$

$$W_2 = 300 \times \frac{\sqrt{3}}{2}$$

$$\boxed{W_2 = 259.8}$$

Q.



$$\Sigma F_x = -1000\cos 90^\circ - 1500\cos 60^\circ - 1000\cos 45^\circ - 500\cos 30^\circ$$

$$\Sigma F_x = -0 - 750 - 707.11 - 433$$

$$\Sigma F_x = -1890\,N$$

$$\Sigma F_y = -1000\sin 90^\circ - 1500\sin 60^\circ - 1000\sin 45^\circ - 500\sin 30^\circ$$

$$\Sigma F_y = -3256\,N$$

$$R = \sqrt{(-1890)^2 + (3256)^2}$$

$$\boxed{R = 3764\,N}$$

$$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

$$\theta = \tan^{-1}\frac{3256}{1890}$$

$$\theta = 59.86^\circ$$

for position of Resultant.

d be the dist b/w P & line of action of resultant force

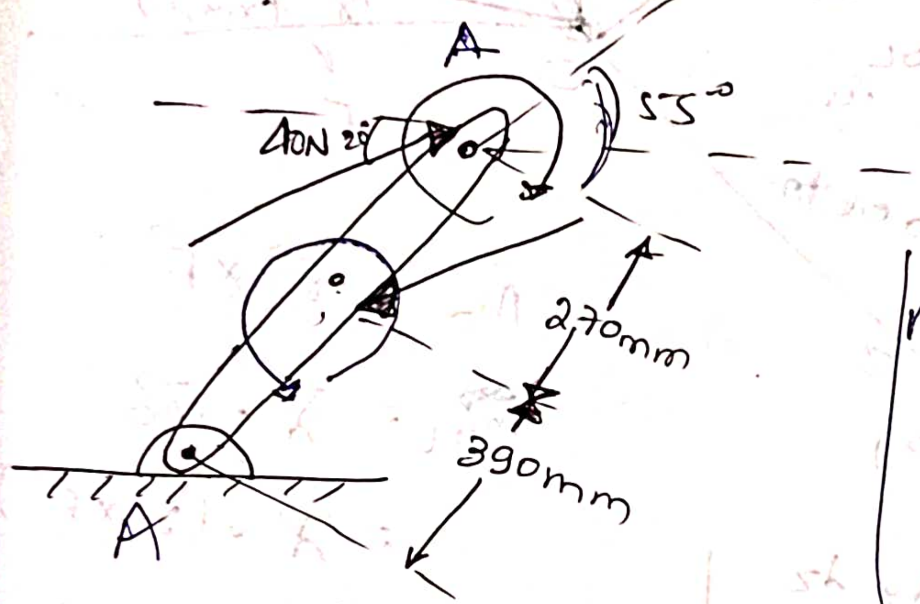
$R_y \cdot d = (1000 \sin 90^\circ \times 0 + 1500 \sin 60^\circ \times 4 + 1000 \sin 45^\circ \times 8 + 500 \sin 30^\circ \times 12)$

$3256 \times d = 13852$

$d = \frac{13852}{3256}$

$\boxed{d = 4.25 \text{ m}}$

Q.



about A

$M = (40 \times \frac{660}{1000} \sin(55-20^\circ) - 40 \times \frac{390}{1000} \sin(55+?)$

$M = 6.19 \text{ Nm}$

(a) Resolve in $\updownarrow$ $\leftrightarrow$

Horizontal components form a couple with moment

$m_h = 40 \cos 20^\circ \times 270 \sin 55^\circ$

vertical component form a couple with moment

$m_v = 40 \sin 20^\circ \times 270 \cos 55^\circ$

$M = M_h - M_v \Rightarrow \boxed{M = 6.19 \text{ Nm}}$

(b) ⊥r dist b/w two force can be calculated as

$d = 270 \sin(55 - 20^\circ) = 270 \sin 35^\circ$

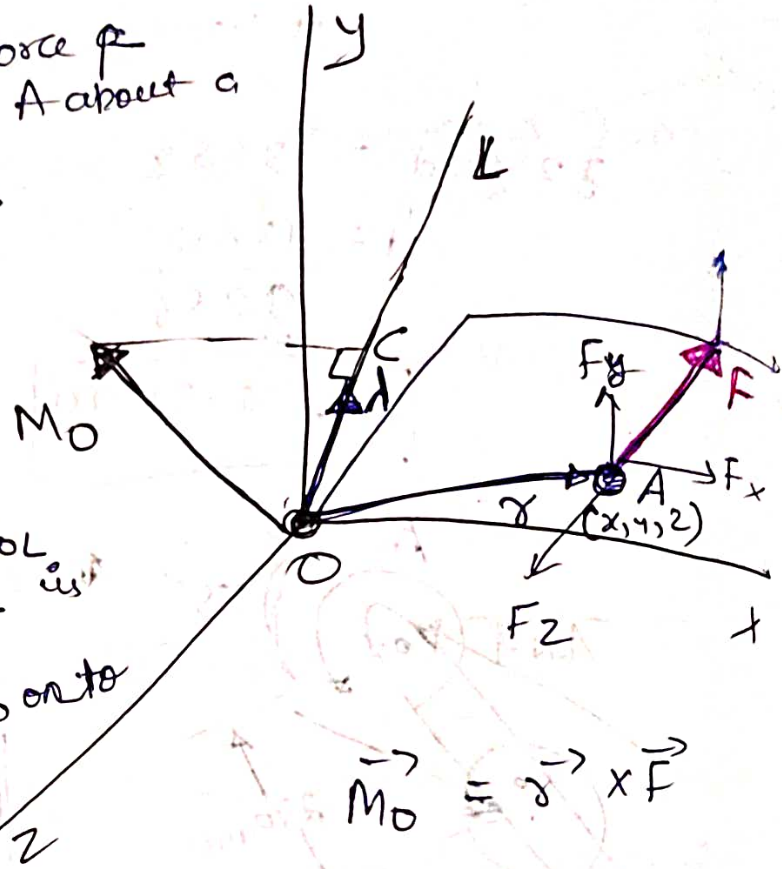
$M = F \times d = 40 \times \frac{270 \sin 35}{1000}$

$= 6.19 \text{ Nm}$

Moment of a force about a given axis

- Moment M_O of a force F applied at pt A about a pt O

$\vec{M_O} = \vec{r} \times \vec{F}$



- Scalar moment M_{OL} about an axis OL is the projection of moment vector M_O onto the axis.

$M_{OL} = \vec{\lambda} \cdot \vec{M_O}$

$M_{OL} = \vec{\lambda} \cdot (\vec{r} \times \vec{F})$

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$\vec{M_O} = \vec{r} \times \vec{F}$

$\vec{M_{OL}} = \vec{\lambda} \cdot \vec{M_O}$

$= \vec{\lambda} \cdot (\vec{r} \times \vec{F})$

$|\vec{M_x}| = \hat{i} \cdot \vec{M_O}$

$|\vec{M_y}| = \hat{j} \cdot \vec{M_O}$

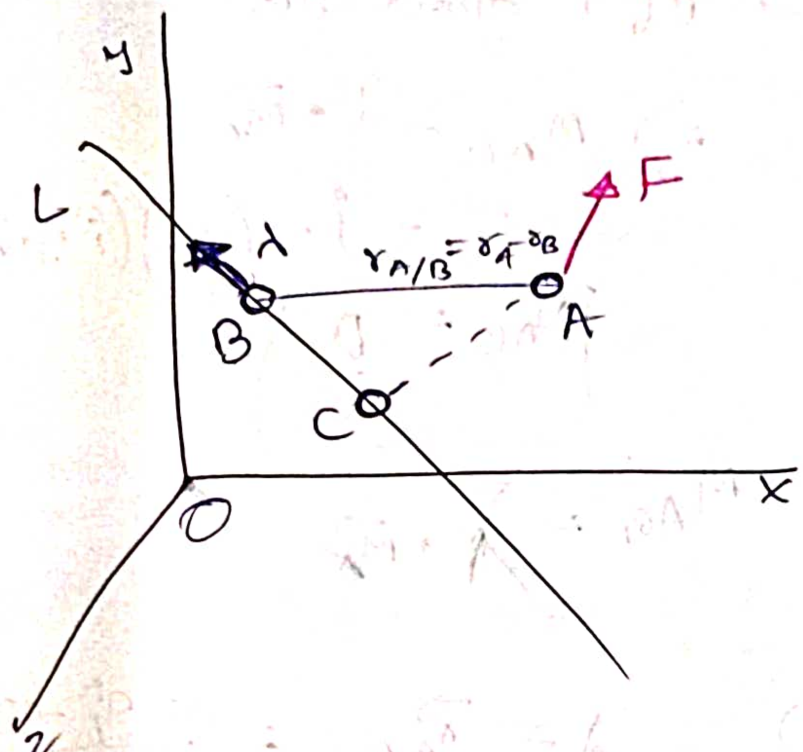
$|\vec{M_z}| = \hat{k} \cdot \vec{M_O}$

- moment of F about coordinate

$M_x = yF_z - zF_y$

$M_y = zF_x - xF_z$

$M_z = xF_y - yF_x$



$M_{BL} = \vec{\lambda} \cdot \vec{M_B}$

$= \vec{\lambda} \cdot (\vec{r}_{A/B} \times F)$

$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$

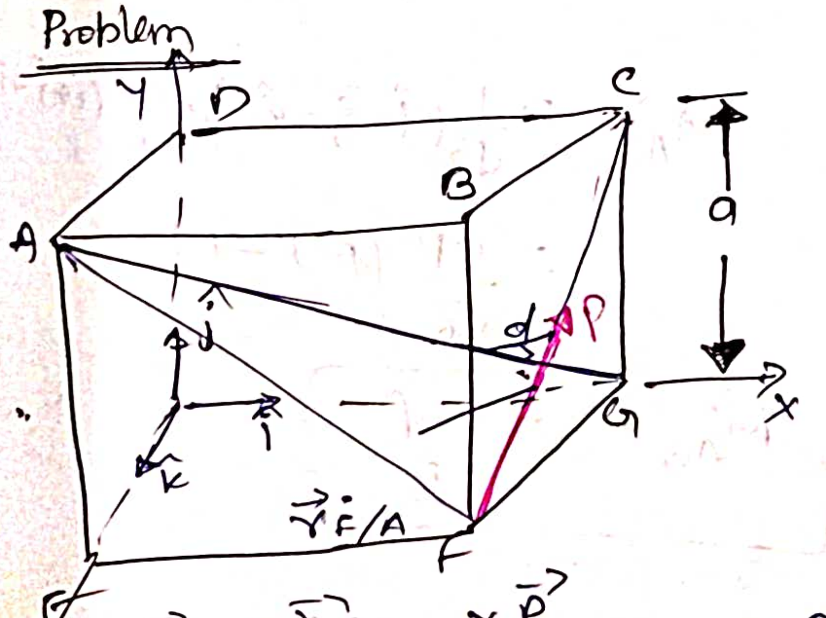
$M_{CL} = \lambda \cdot [(r_A - r_C) \times F]$

$= \lambda \cdot [(r_A - r_B) + \lambda \cdot [\overset{0}{(r_B - r_C) \times F}]$

The resultant is independent of pt B along given axis

vol^m of ||ppiped is zero

Problem



A cube is acted on by a force P as shown determine moment of P

a) about A.

b) about edge AB

c) about diagonal AG of cube

d) Determine ⊥r dist b/w AG & FC.

$\vec{M}_A = \vec{r}_{F/A} \times \vec{P}$

$\vec{r}_{F/A} = a\hat{i} - a\hat{j} = a(\hat{i} - \hat{j})$

$\vec{P} = P\{-\frac{1}{\sqrt{2}}\hat{k} + \frac{1}{\sqrt{2}}\hat{j}) = \frac{P}{\sqrt{2}}(\hat{j} - \hat{k})$

$\vec{M}_A = \frac{aP}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$

Moment of P about AB

$\Rightarrow M_{AB} = \hat{i} \cdot \vec{M_A}$

$= \hat{i} \cdot \left(\frac{aP}{\sqrt{2}}\right)(\hat{i}+\hat{j}+$

$\boxed{M_{AB} = \frac{aP}{\sqrt{2}}}$

Moment of P about diagonal AG.

$M_{AG} = \vec{\lambda} \cdot \vec{M_A}$

$\perp r$ dist b/w AG & FC

$P \cdot \vec{\lambda} = \frac{P}{\sqrt{2}}(\hat{j}-\hat{k}) \cdot \frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

$= \frac{P}{\sqrt{6}}(0-1+1)$

$= 0$

Therefore P is $\perp r$ to AG

$M_{AG} = \frac{aP}{\sqrt{6}} = Pd$

$d = \frac{a}{\sqrt{6}}$

$\lambda = \frac{\vec{r}_{A/G}}{r_{A/G}} = \frac{a\hat{i}-a\hat{j}-a\hat{k}}{a\sqrt{3}}$

$\lambda = \frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

$\vec{M_A} = \frac{aP}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

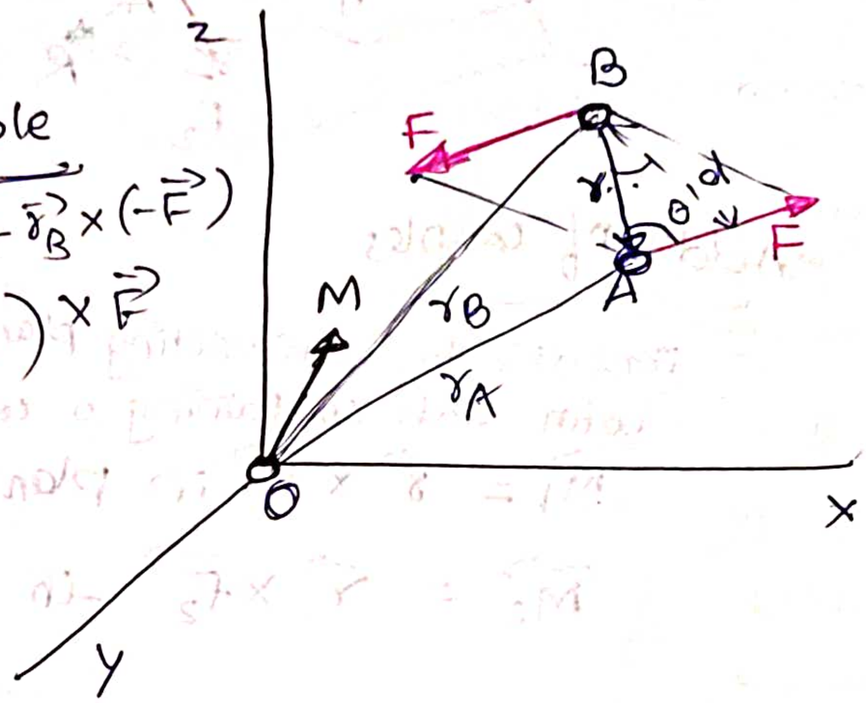
$\vec{M}_{A/G} = \frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k}) \cdot \frac{aP}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

$= \frac{aP}{\sqrt{6}}(\cancel{1}-\cancel{1}-1)$

$\boxed{\vec{M}_{AG} = -\frac{aP}{\sqrt{6}}}$

Moment of a couple

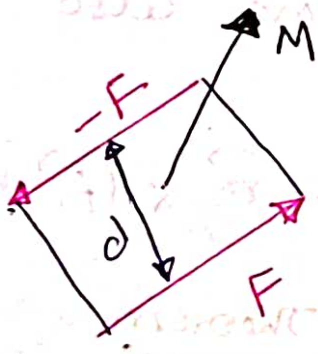
→ Two forces F & −F having the same magnitude || lines of action & opposite sense are said to form a couple.



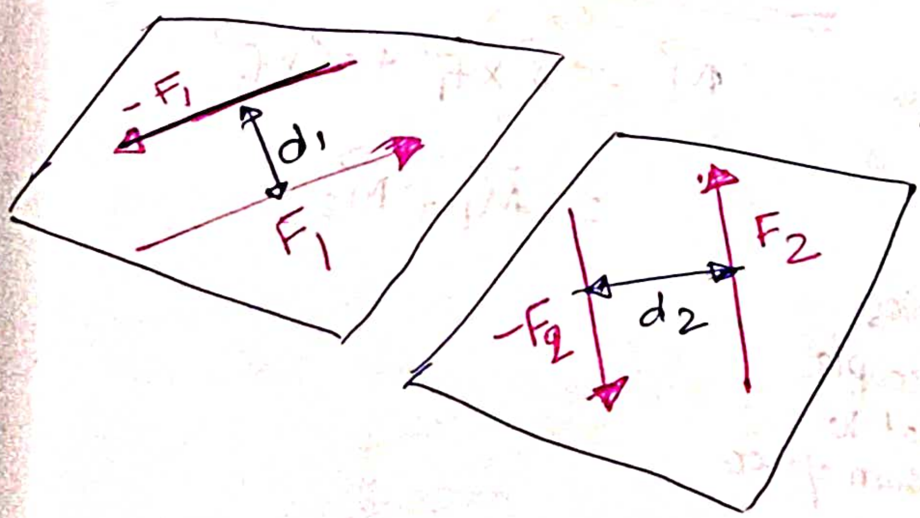
Moment of couple

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F})$$
$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$
$$\vec{M} = \vec{r} \times \vec{F}$$

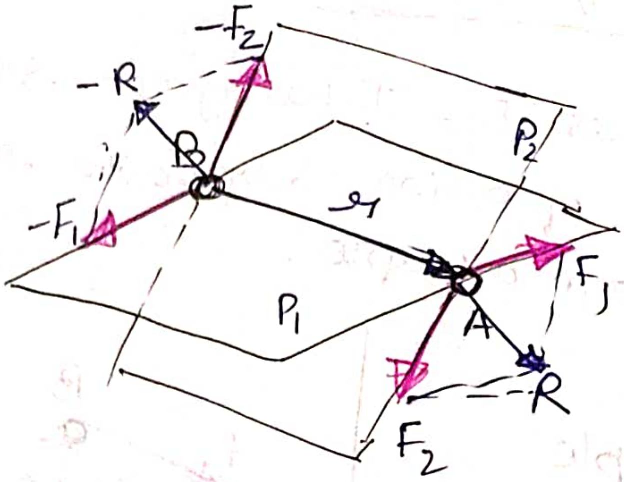
$$M = rF\sin\theta$$
$$= Fd$$



- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e. it is a free vector that can be applied at any point with the same effect.



- Two couples will have equal moments if
  $F_1 d_1 = F_2 d_2$
- Two couples lie in || planes
  &
- Two couples have same sense or the tendency to cause rotation in the same direction



Addⁿ of couples

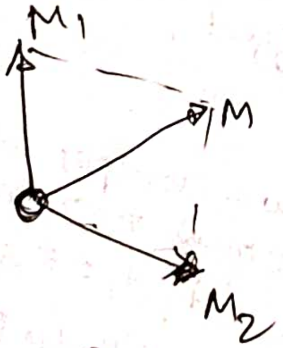
Consider two intersecting planes P_1 & P_2 with each containing a couple

$\vec{M_1} = \vec{r} \times \vec{F_1}$ in plane P_1

$\vec{M_2} = \vec{r} \times \vec{F_2}$ in plane P_2

Resultants of vectors also form a couple

$$\vec{M} = r \times \vec{R} = \vec{r} \times (\vec{F_1} + \vec{F_2})$$



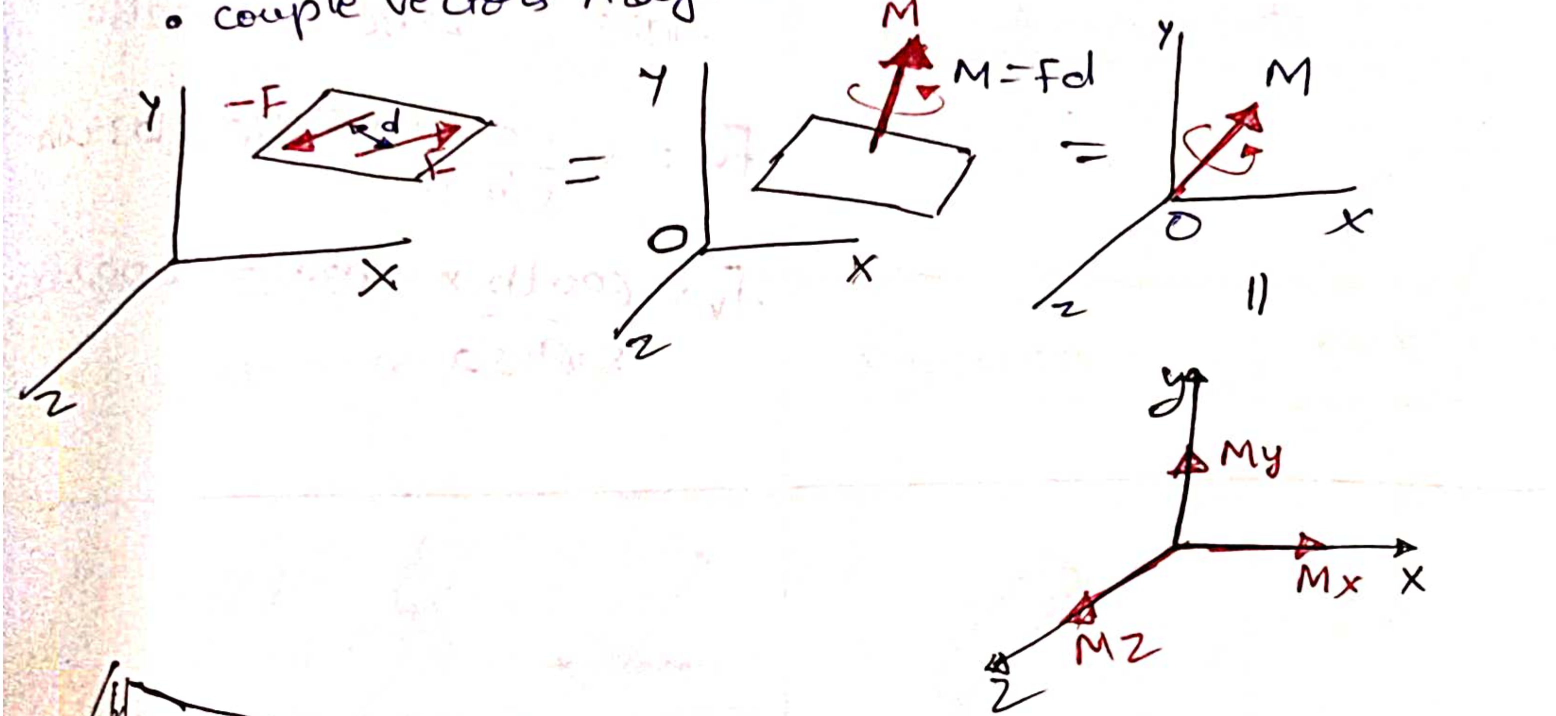
By Varignon's Theorem

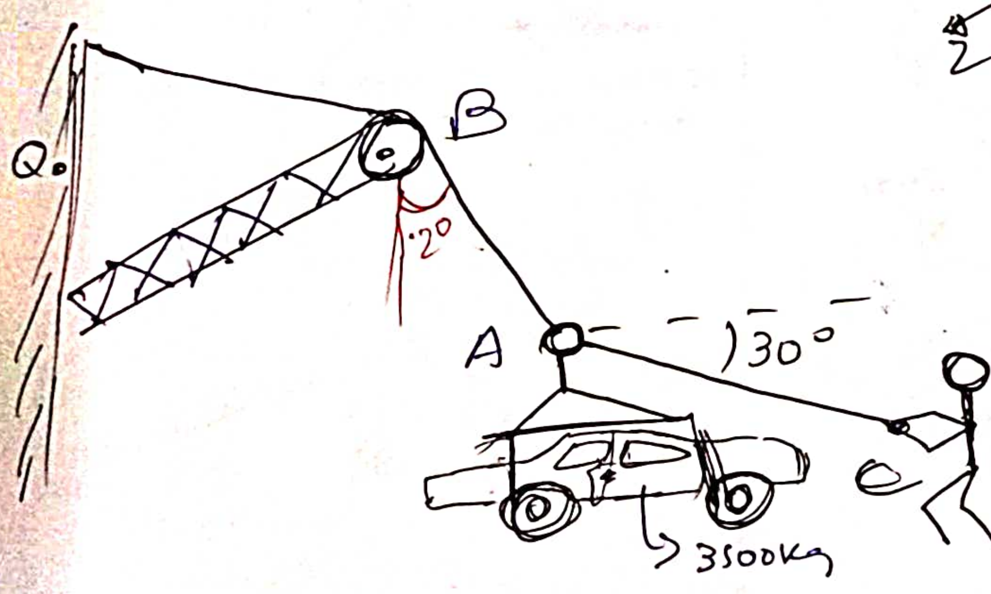
$$\vec{M} = \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2}$$

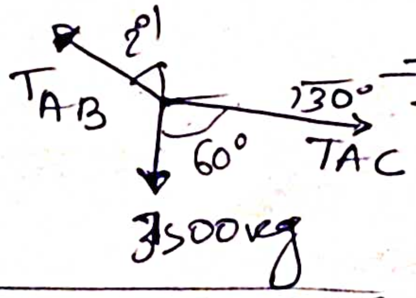
$$= \vec{M_1} + \vec{M_2}$$

- Sum of two couples is also a couple that is equal to the vector sum of two couples.

- Couple can be represented by a vector with magnitude & direction equal to the moment of the couple.
- couple vectors obey the law of addition of vectors
- couple vectors are free vectors i.e. the point of application is not significant.
- couple vectors may be resolved into component vectors







$\frac{T_{AB}}{\sin 60°} = \frac{T_{AC}}{\sin 178°} = \frac{3500 \text{ kg}}{\sin 122°}$ (By Lami's thm

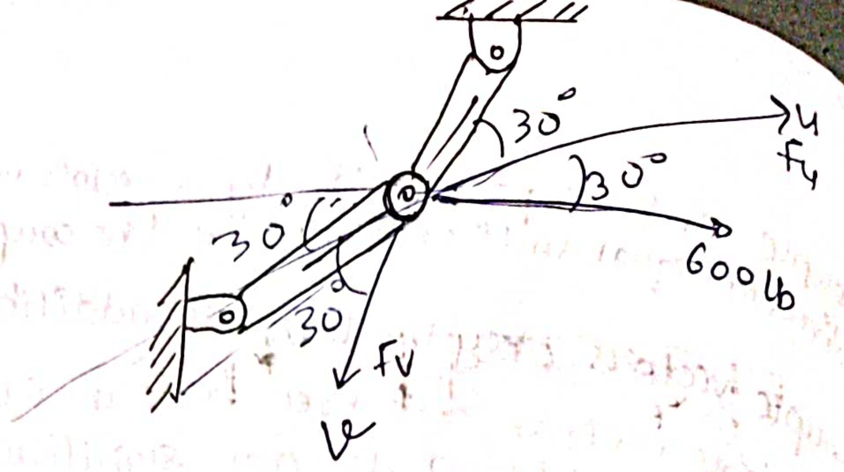
$T_{AC} = 3500 \times \frac{\sin 178}{\sin 122°}$

$= 3500 \times 0.04153$

$\boxed{T_{AC} = 144 \text{ N}}$

$T_{AB} = \frac{3500}{\sin 122} \times \sin 60$

$= 3574.2 \text{ N}$ //



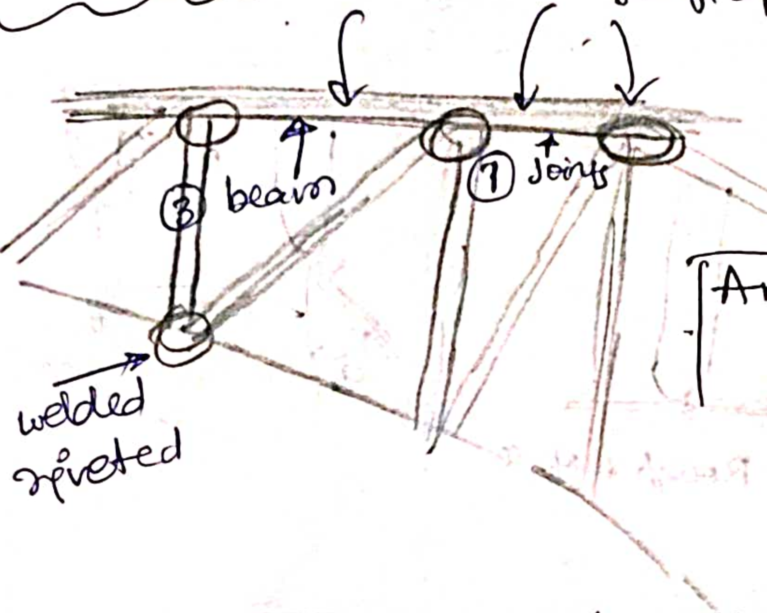
$$\frac{F_u}{\sin 120^\circ} = \frac{F_v}{\sin 30^\circ} = \frac{600\,lb}{\sin 15}$$

$$F_u = \frac{600\,lb}{\sin 150} \times \sin 120 = 103$$

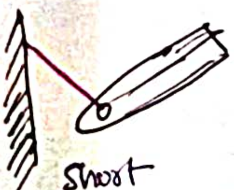
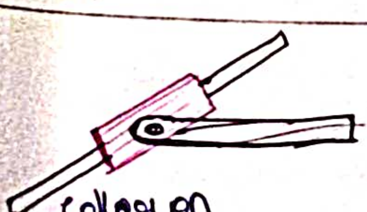
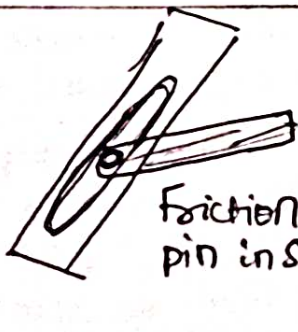
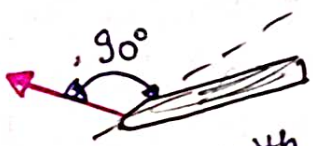
$$F_v = \frac{600\,lb}{\sin 150} \times \sin 30 = 600$$

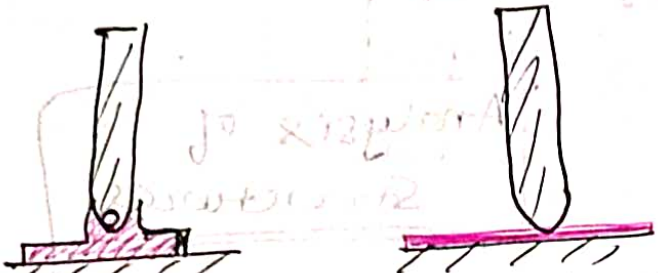
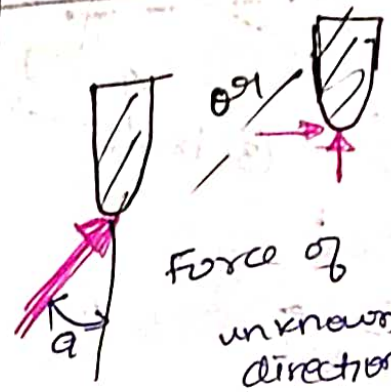
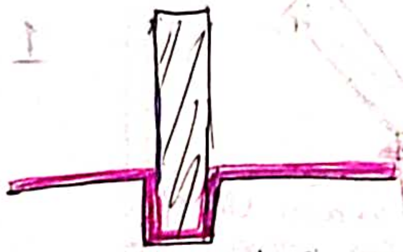
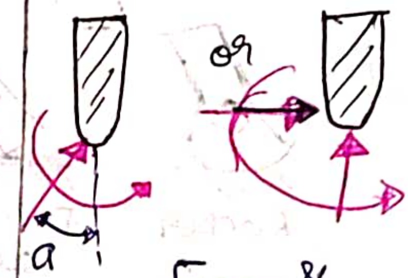
Module - 1

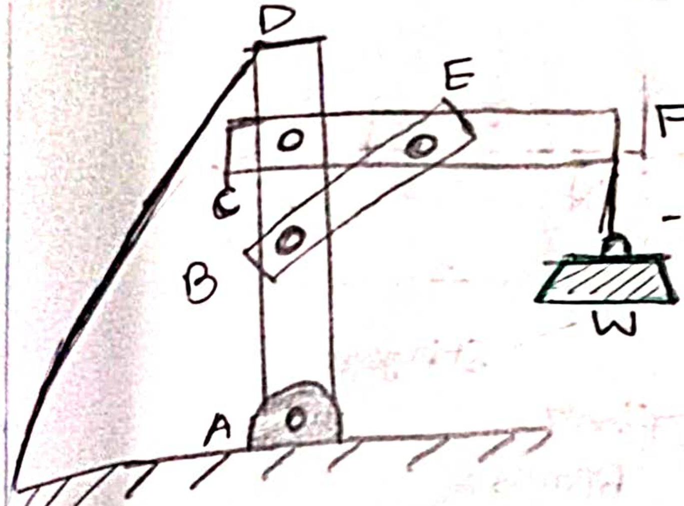
Ch-2 Truss

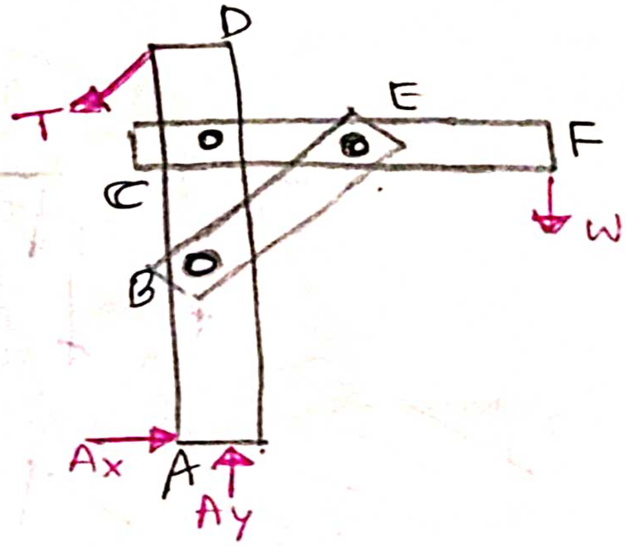


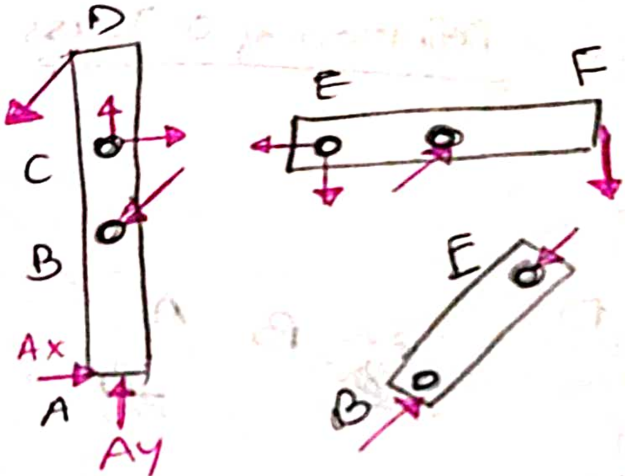
Analysis of Structures

Support or connection	Reaction	No of unknowns
 Rollers, Rocker, Frictionless surface.	 Force with known line of action perpendicular to surface.	1.
 Short cable  Short link..	 Force with known line of action along cable or link	1 links are often used to support suspended span of highway bridge
 collar on frictionless rod  Frictionless pin in slot	 Force with known line of action perpendicular to rod or slot	1

Support or connection	Reaction	No. of unknowns
 Frictionless pin or Hinge; Rough surface	 Force of unknown direction	2
 Fixed support	 Force & couple	3 Cantilever support is fixed at one end.





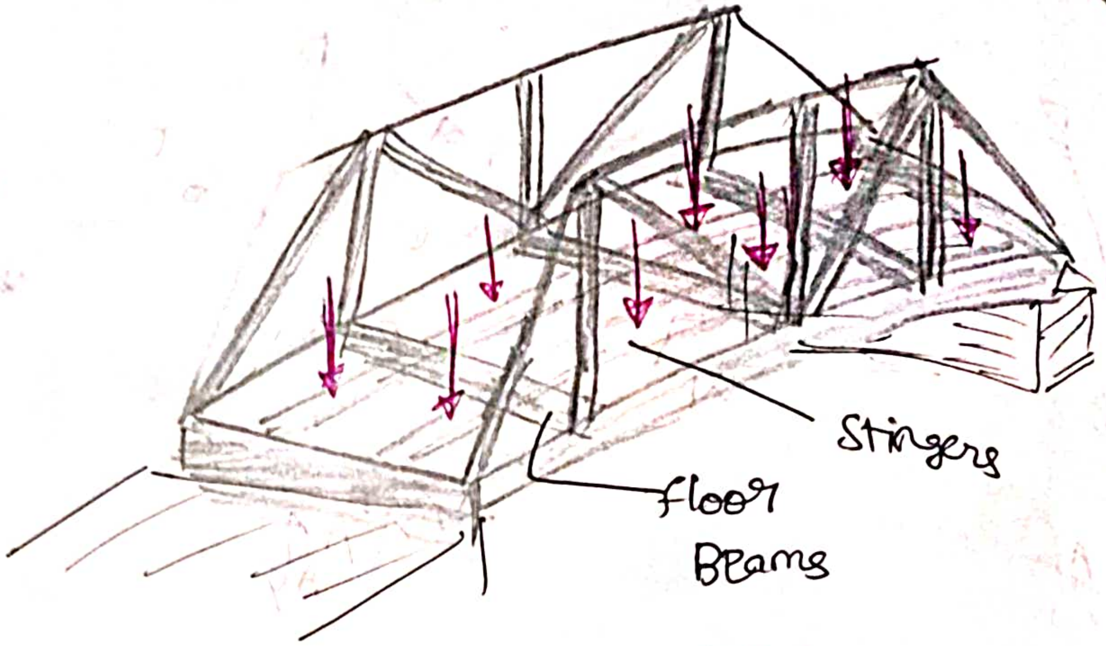


Trusses, which are designed to support loads and are usually stationary, fully constrained structures. Trusses consist exclusively of straight members connected at joints located at ends of each member.

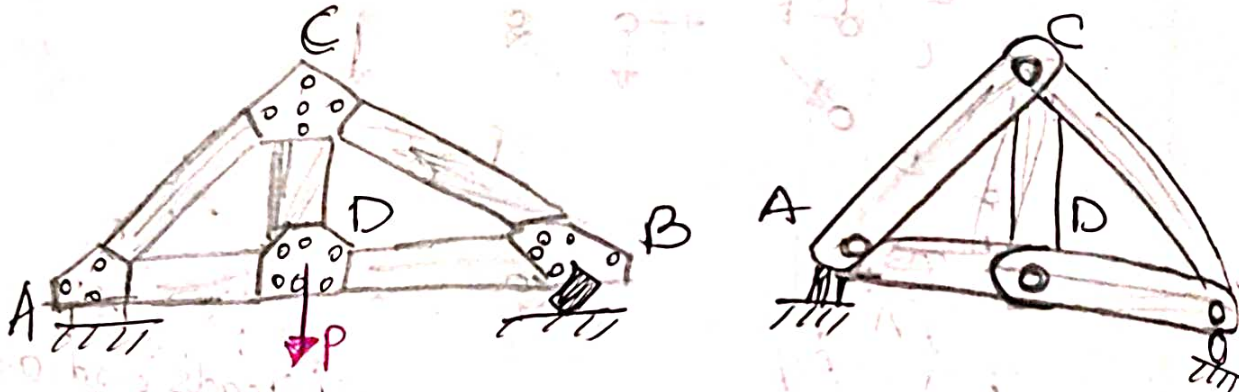
Members of a truss, therefore are two-force members. i.e. members acted upon by two equal & opposite forces directed along the member.

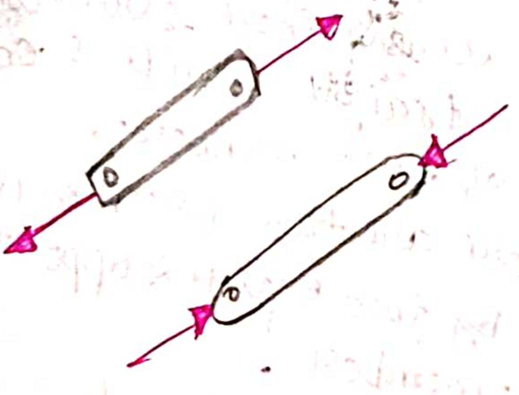
Frames which are also designed to support loads and are also usually stationary, fully constrained structures. However like the fig. of crane given always contain at least one multiforce member i.e. a member acted upon by three or more forces which in general are not directed along the member.

Machines, which are designed to transmit & modify forces & are structures containing moving parts. Machines like frames, always contain atleast one multiforce member.



Definition of a Truss





The no. of members, in a pr
may also be expressed by the
relation :

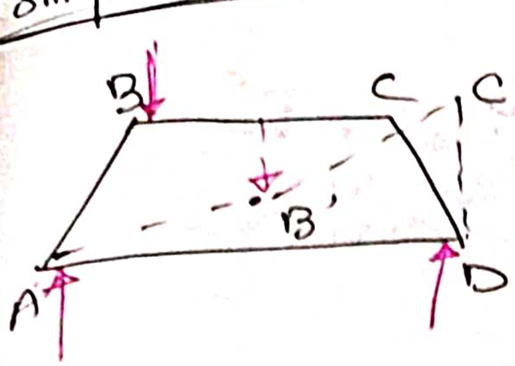
$$n = (2j - 3)$$

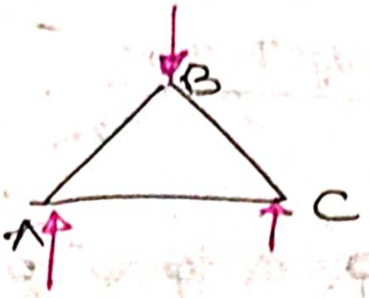
n : no. of members
j : No. of joints.

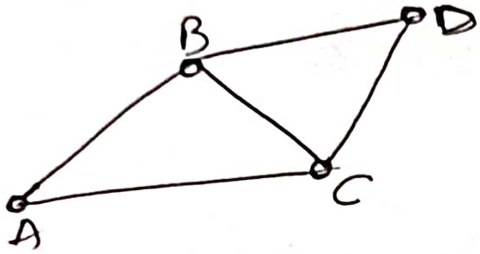
Assumptions

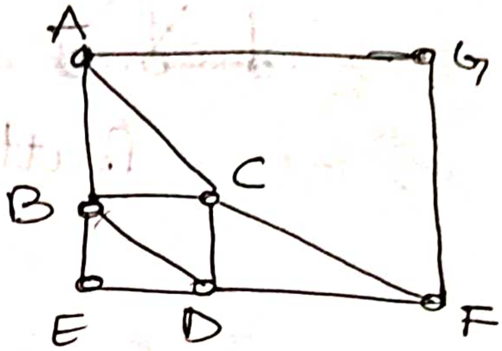
1. Each trusses is assumed to be composed of straight rigid memb
all lying in one plane connected at joints. This means
that coplanar force systems are involved.
2. No members are continuous through a joint.
3. Loads are applied only at joints
4. The weights of the members are neglected b'coz they are
assumed small in comparison with the loads. If they
are included they are split b/w two end pins.
5. Forces are transmitted from one member to another through
smooth pins fitting perfectly in the members. These members
which are called two-force members will be either in Tension
or compression (C)

Simple Trusses :-

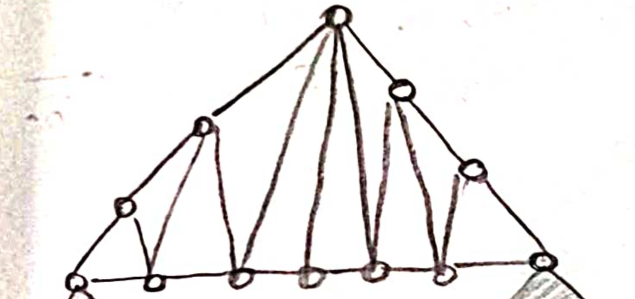




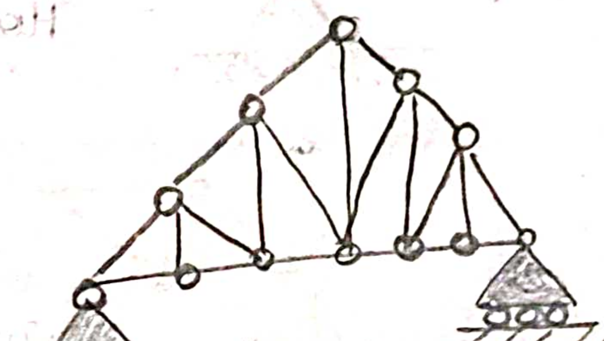




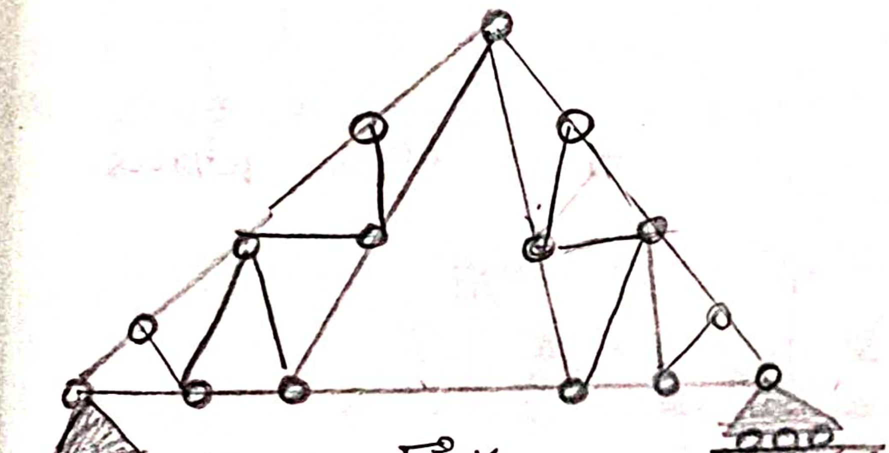
Types of Roof Trusses



Pratt

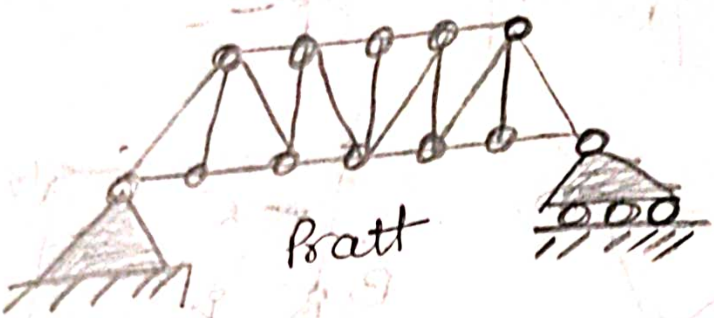


Howe

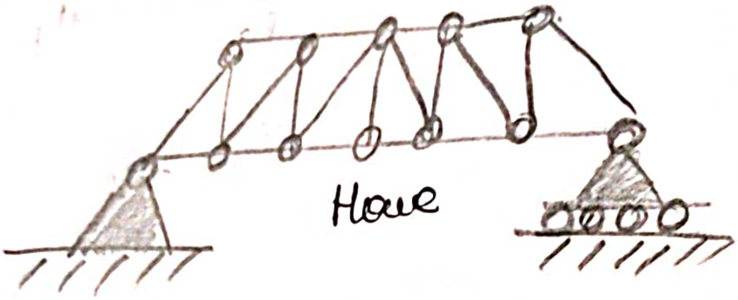


Fink

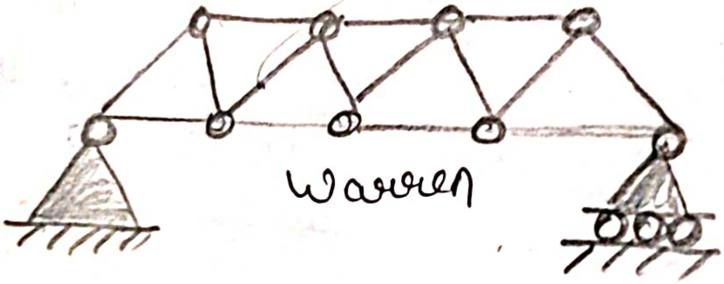
Typical Bridge Trusses



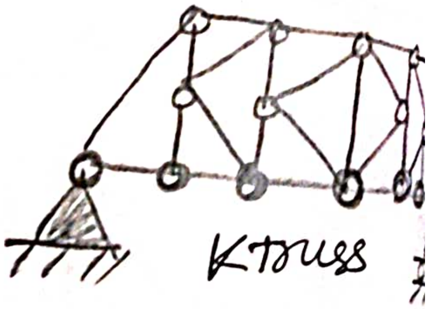
Pratt



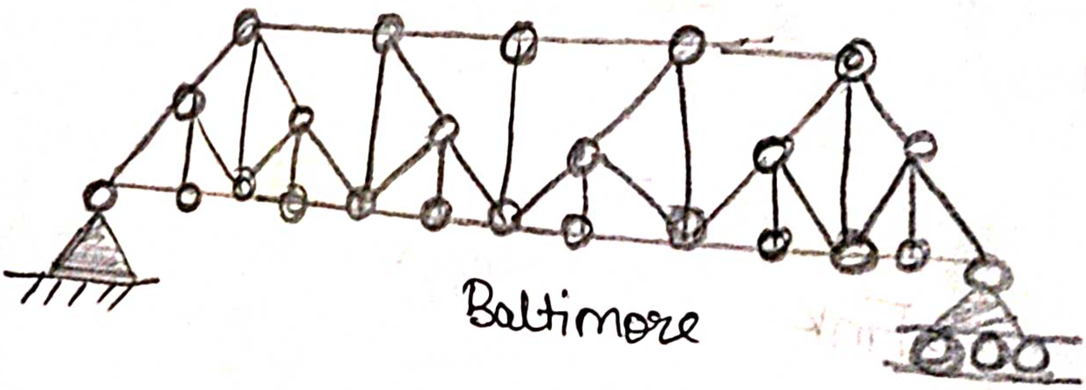
Howe



Warren

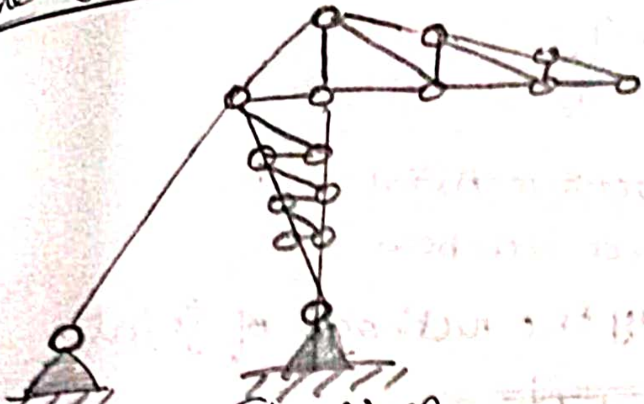


K truss

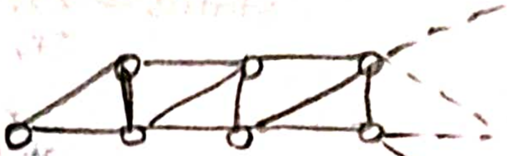


Baltimore

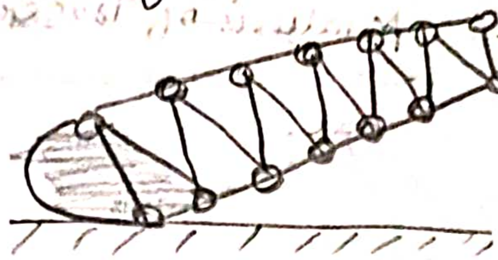
other types of Trusses



Stadium



cantilever Portion of a Truss



Bascule.

Two Objectives of Truss:
1. Load withstand/support
2. share & transfer loads

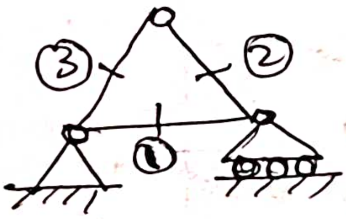
Forces
- external
- $\boxed{\text{Internal}}$ → Internal stresses → Dimension of member for designing of a structure.

Non-collapsable — $m+3=2j$ [Perfect Truss/Rigid Truss]

m = no. of members
j = no. of joints

Non-collapsible — $m+3>2j$ [Over Rigid Truss/Has more members than req. to be rigid.]

collapsible — $m+3<2j$ [Under Rigid Truss/Has less members than req. for be rigid truss.]



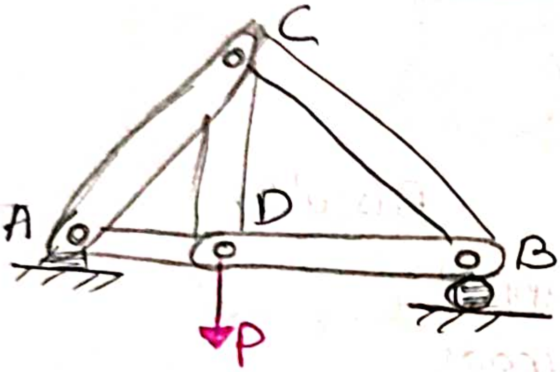
$m=3$
$j=3$

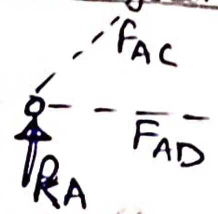
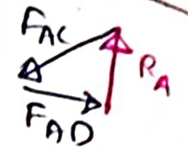
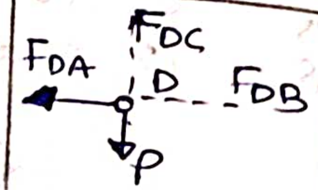
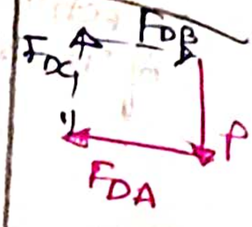
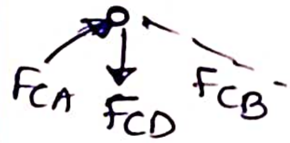
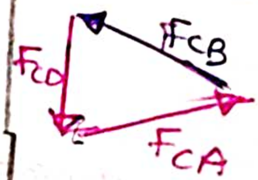
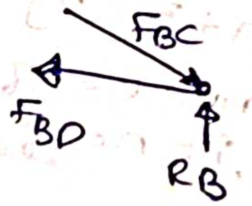
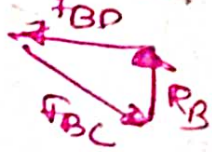
$L.H.S=6=RHS$

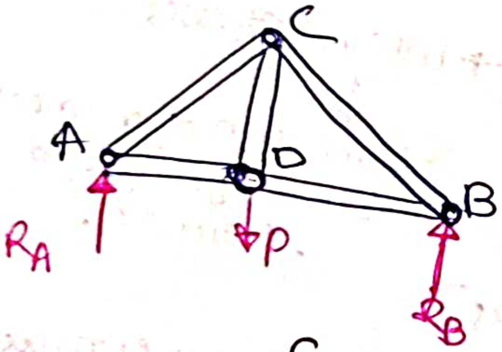
Statically Determinate

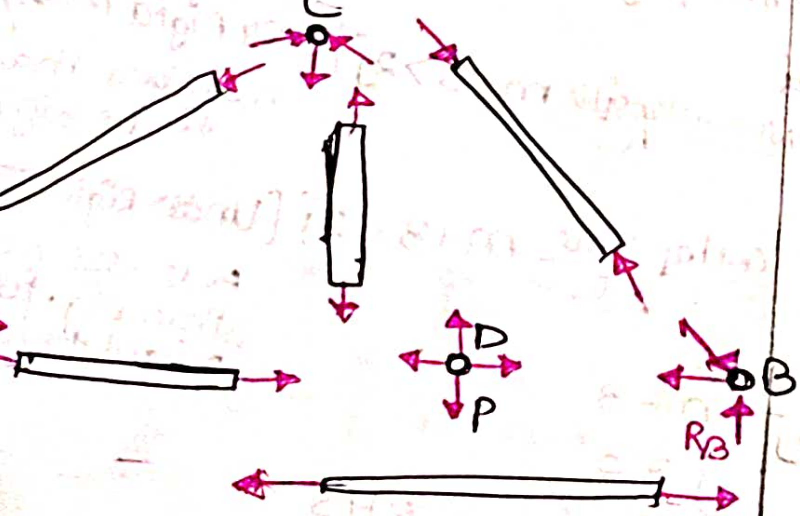
Statics → $\Sigma F_x = 0$, $\Sigma F_y = 0$ → Reactions Axial Force members

Analysis of Trusses by the method of joints

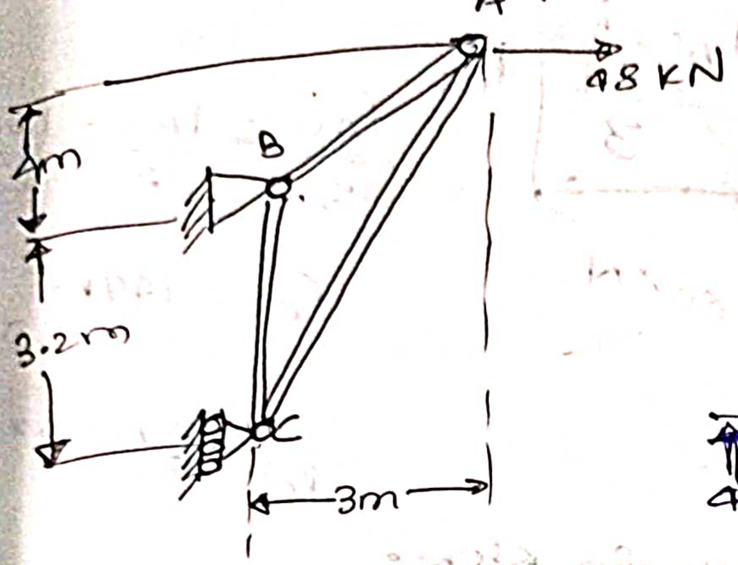


Joints	Free-body Diagram	Force Polygon
joint A.		
joint D		
Joint C		
joint B		

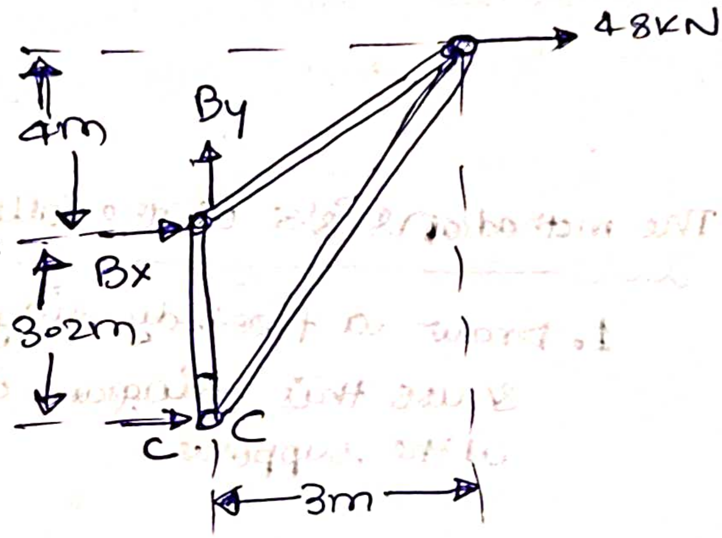




Example : Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.



Sol^n FBD : Entire truss



$+\uparrow \Sigma F_y = 0 : B_y = 0$ ~~By=0~~

$+\circlearrowleft \Sigma M_C = 0$.

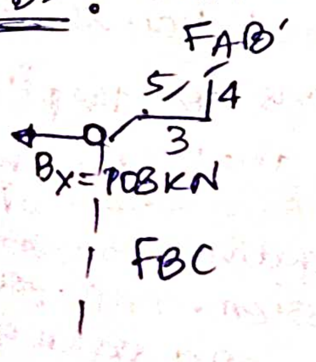
$-B_x(3.2m) - (48\,kN)(7.2m) = 0$.

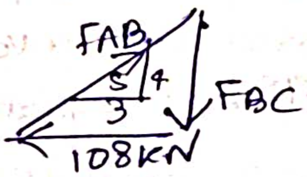
$$\boxed{B_x = -108\,kN \leftarrow}$$

$\xrightarrow{+} \Sigma F_x = 0 \qquad C - 108\,kN + 48\,kN = 0$

$C = 60\,kN \rightarrow$

FBD : Joint B

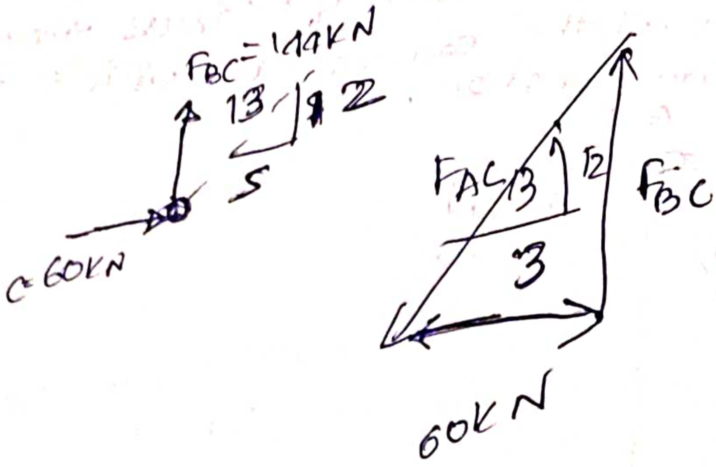




$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{108\,kN}{3}$$

$F_{AB} = 180\,kN$

$F_{BC} = 144\,kN$



$$\frac{F_{AC}}{13} = \frac{F_{BC}}{12} =$$

$F_{BC} = 144\,kN$ (Chec

$F_{AC} = 156$

The method consists of the following steps:

1. Draw a free-body diagram of the entire truss, & use this diagram to determine the reactions at the supports.

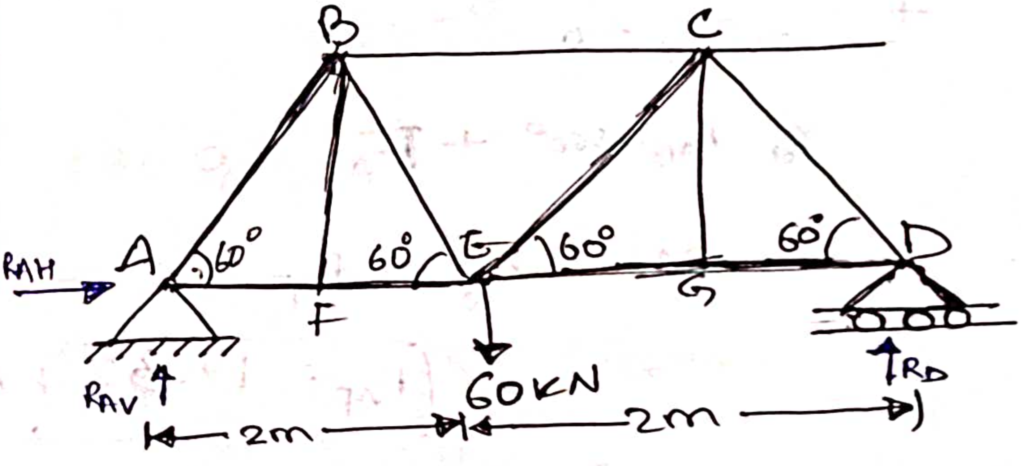
2. Locate a joint connecting only two members and draw the free-body diagram of its pin. Use this FBD to determine the unknown force in each of the two members. If only 3 forces are involved (the two unknown forces & a known force) you will probably find it more convenient to draw and solve the corresponding force triangle. If more than 3 forces are involved, you should write & solve equilibrium equations for the pin, $\Sigma F_x = 0$ & $\Sigma F_y = 0$, assuming that the members are in tension. A positive answer means that member is in tension, a negative answer means that the member is in compression. Once you have found the forces enter their values on a sketch of the

Next, locate a joint where the forces in only two of the connected members are still unknown.
- Draw FBD of the pin and use it as indicated in step 2 to determine the two unknown forces

Repeat this procedure until you have found forces in all the members of the truss
since you previously used the 3 eqm eqn associated with the FBD of the entire truss to determine the rxns at the supports you will end up with 3 extra eqns these eqn can be used to check your computations

5. Note that the choice of the first joint is not unique.

Find the force in all the members of the truss shown by the method of joint



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad R_{AH} = 0$

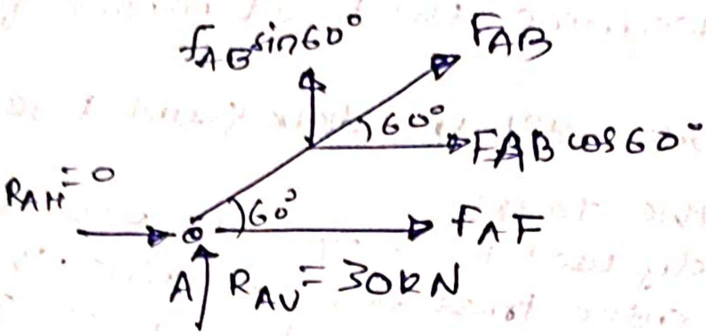
$\circlearrowright^{+} \Sigma M_A = 0 \qquad 60K \times 2 - R_D \times 4 = 0$

$$\boxed{R_D = 30\,KN}$$

$\uparrow \Sigma F_y = 0, \qquad R_{AV} + 30 - 60 = 0$

$$\boxed{R_{AV} = 30\,KN}$$

FBD of jt A



$$+\uparrow \Sigma F_y = 0$$

$$F_{AB}\sin 60 + 30 = 0$$

$$F_{AB}\sin 60 = -30$$

$$\boxed{F_{AB} = -34.641\,kN}\ (C)$$

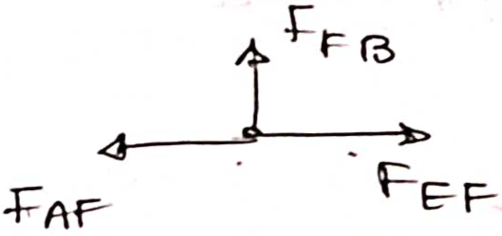
$$\overset{+}{\longrightarrow} \Sigma F_x = 0$$

$$F_{AB}\cos 60° + F_{AF} = 0$$

$$-34.641 \times \frac{1}{2} + F_{AF}$$

$$\boxed{F_{AF} = 17.32\,kN}$$

FBD of jt F



$\xrightarrow{+} \Sigma F_x = 0$

$F_{EF} - F_{AF} = 0$

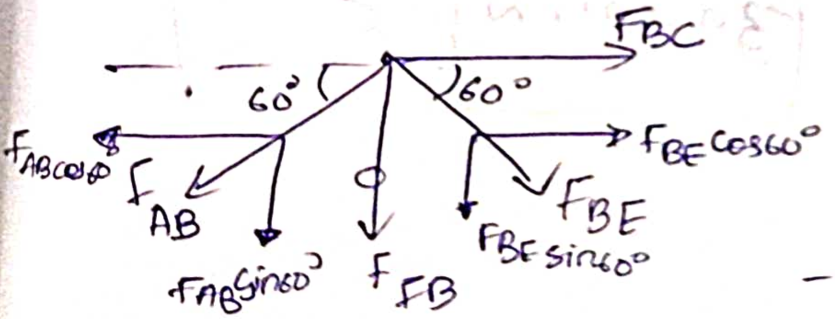
$F_{EF} - 17.32 = 0$

$\boxed{F_{EF} = 17.32\,kN}$ (T)

$+\uparrow \Sigma F_y = 0$

$\boxed{F_{FB} = 0}$

FBD of jt B.



$+\uparrow \Sigma F_y = 0$

$-F_{BE}\sin 60° - F_{AB}\sin 60° - \cancel{F_{FB}}^{\,0} = 0$

$-F_{BE}\sin 60 - (-34.641\sin 60°) - 0 = 0$

$\boxed{F_{BE} = 34.641\,KN}$ (T)

$\xrightarrow{+} \Sigma F_x = 0$

$F_{BC} + F_{BE}\cos 60 - F_{AB}\cos 60 = 0$

$F_{BC} + 34.641\cos 60 + 34.641\cos 60 = 0$

$F_{BC} + 34.641 = 0$

$\boxed{F_{BC} = -34.641}$ (C)

Due to symmetry of structure

$F_{CD} = F_{AB}$

$F_{GD} = F_{AF}$

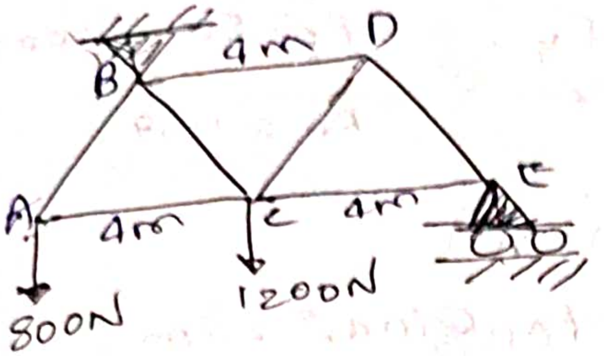
$F_{GE} = F_{EF}$

$F_{CG} = F_{FB}$

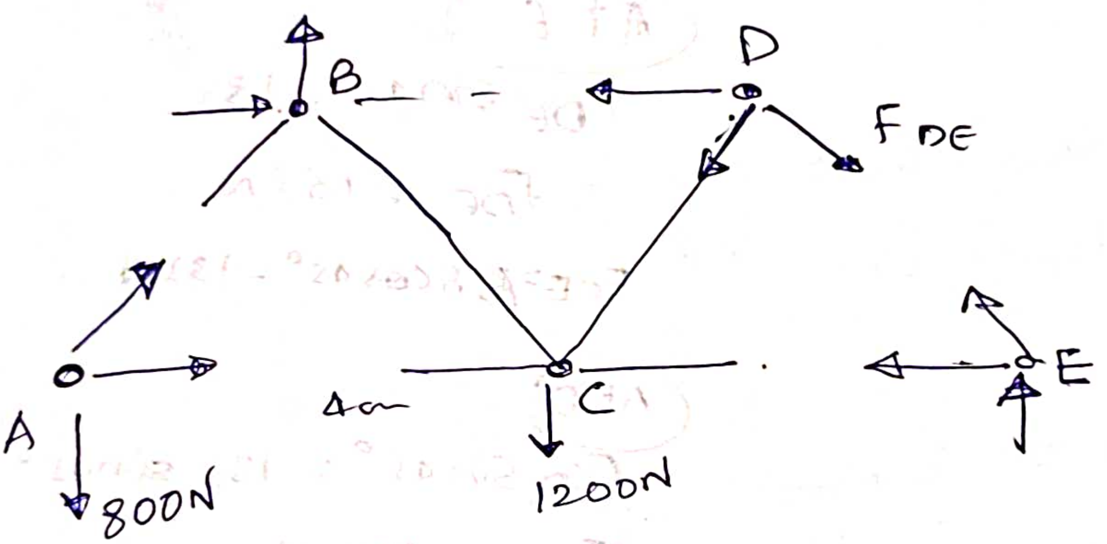
$F_{CE} = F_{BE}$

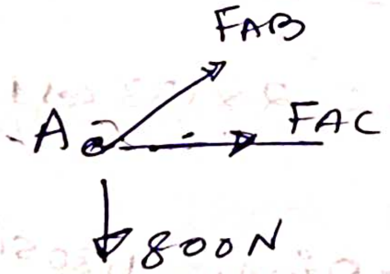
Sr. No.	Force in member	Magnitude	Nature
1.	F_{AB}, F_{CD}	34.641 kN	C
2.	F_{AF}, F_{GD}	17.32 kN	T
3.	F_{FB} F_{CG}	0	—
4.	F_{BE}, F_{CE}	34.641 kN	T
5.	F_{EF}, F_{GE}	17.32 kN	T
6.	F_{BC}	34.641 kN	C

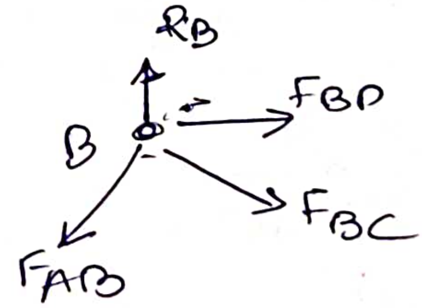
Q. Determine the forces of each member of the truss using the method of joints, the reactions at B & E all 6 acute angles are 45°.

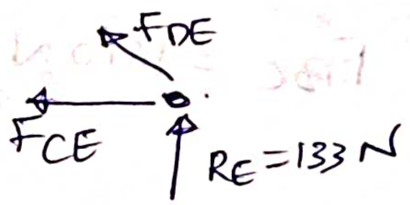


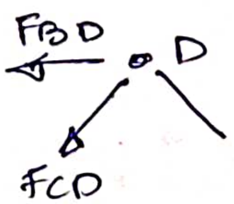
FBD of whole Truss











$\Sigma M_B = 0 = 800 \times 2 - 1200 \times 2 + R_E \times 6$

$R_E = 133\ N$

$\Sigma F_y = 0 = R_B - 800 - 1200 + 133$

$R_B = 1867$

At A

$F_{AB} \sin 45^\circ = 800$

$F_{AB} = 1130\ N$

$F_{AC} = 1130 \cos 45^\circ = 800\ N$

At E

$F_{DE} \sin 45^\circ = 133$

$F_{DE} = 188\ N$

$F_{CE} = 188 \cos 45^\circ = 133\ N$

At D

$F_{CD} \sin 45^\circ = 133 \sin 45^\circ$

$F_{CD} = 133\ N$

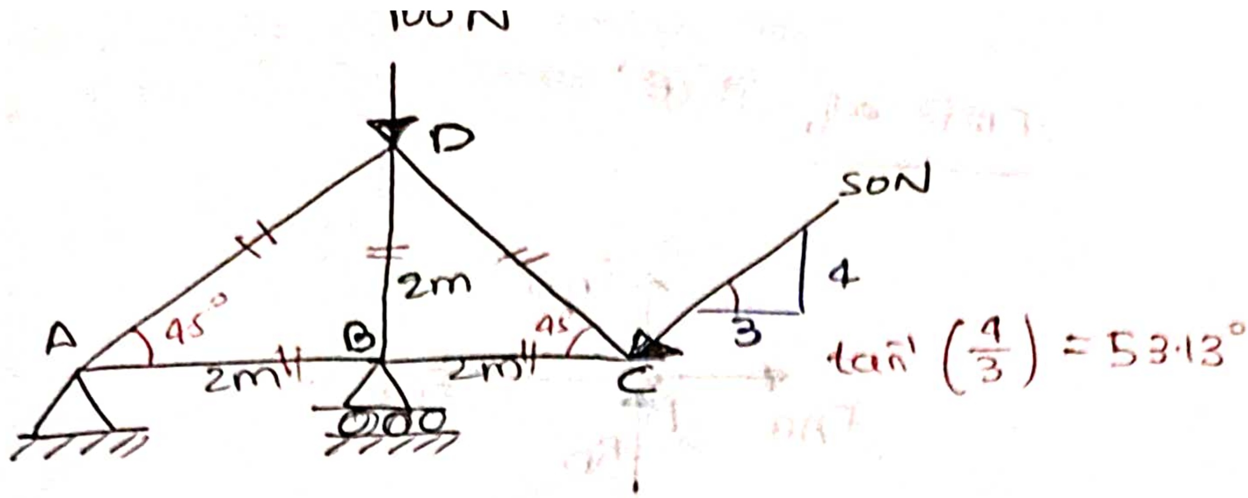
$F_{BD} = 2 \times 133 \cos 45^\circ = 188\ N$

At B

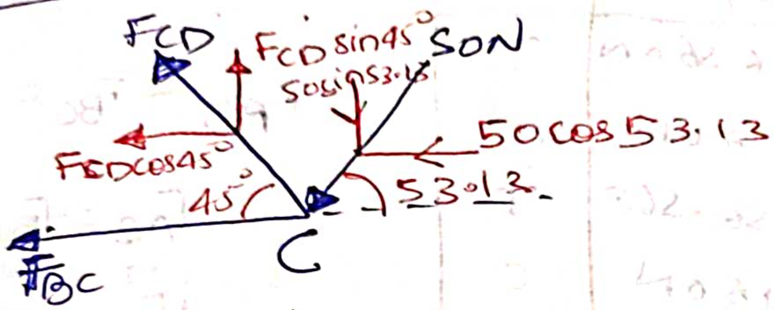
$F_{BC} \sin 45^\circ + 1130 \sin 45^\circ = 1867$

$F_{BC} = 1510\ N$

Q. 2.



FBD of Jt C :



$+\uparrow \Sigma F_y = 0$

$F_{CD} \sin 45 - 50 \sin 53.13 = 0$

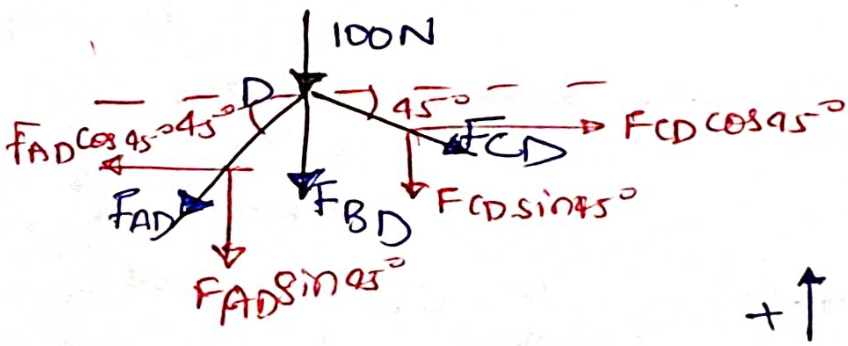
$\boxed{F_{CD} = 56.568}$ N (T)

$\xrightarrow{+} \Sigma F_x = 0$

$- F_{BC} - F_{CD} \cos 45° - 50 \cos 53.13 = 0$

$\boxed{F_{BC} = -70 N}$ (C)

FBD at Jt D



$\xrightarrow{+} \Sigma F_x = 0$

$- F_{AD} \cos 45° + F_{CD} \cos 45° = 0$

$F_{AD} = F_{CD}$

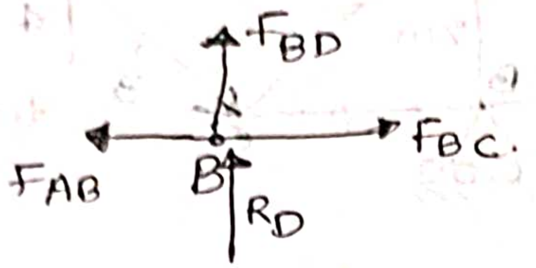
$\boxed{F_{AD} = 56.568 N}$ (T)

$+\uparrow \Sigma F_y = 0$

$- F_{BD} - F_{AD} \sin 45° - F_{CD} \sin 45 - 100 = 0$

$\therefore \boxed{F_{BD} = -180 N}$ (C)

FBD of jt (B)



Sr. No	Force in member	Magnitude	Nature
1.	F_{CD}	56.568 N	T
2.	F_{BC}	70N	C
3.	F_{AD}	56.568	T
4.	F_{BD}	180N	C
5.	F_{AB}	70N	C

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{AB} + F_{BC} = 0$

$F_{AB} = F_{BC} =$

$\boxed{F_{AB} = -70N}$

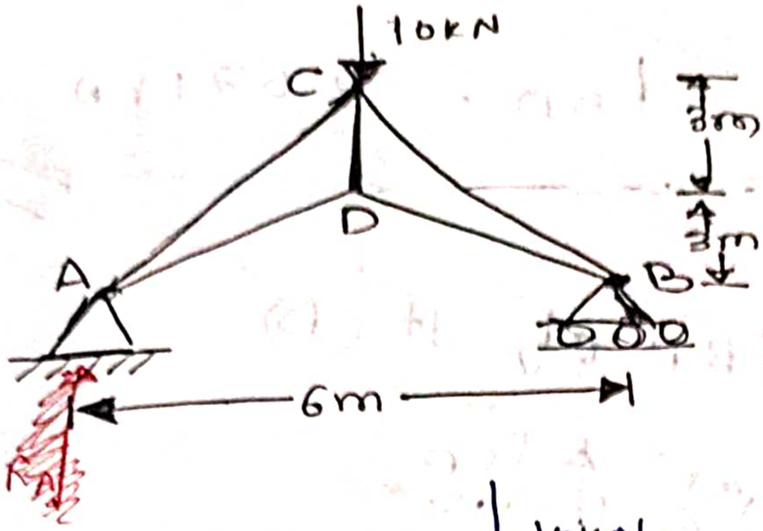
(C)

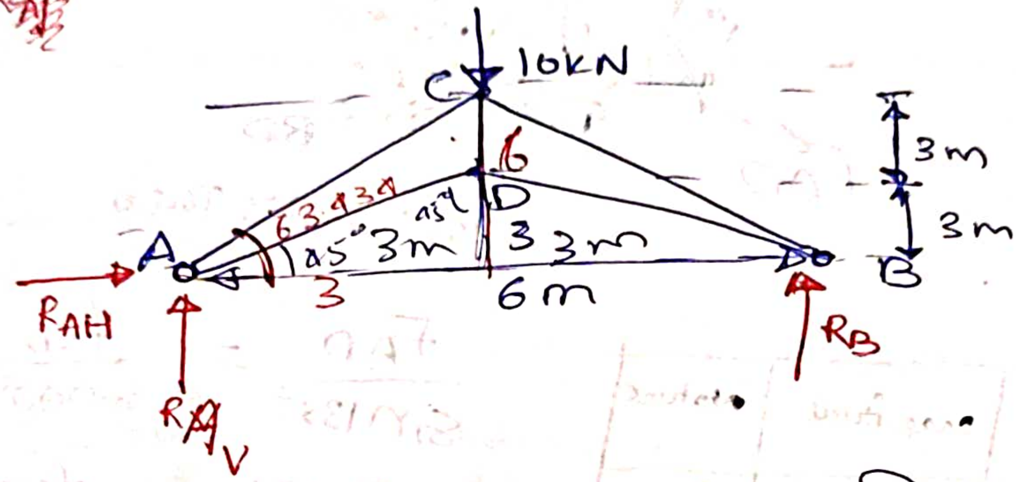
$\uparrow \Sigma F_y = 0$

$F_{BD} + R_B = 0$

$\boxed{R_B = 180N}$

Q.3. Find the force in all the members of the Truss shown by the method of joint





$\tan^{-1}(2) =$ 63.439

$\uparrow \Sigma F_y = 0$

$R_{AV} + R_B - 10 = 0$

$R_{AV} + 5\,kN - 10\,kN = 0$

$\boxed{R_{AV} = 5\,kN}$

$+\circlearrowright \Sigma M_A = 0$

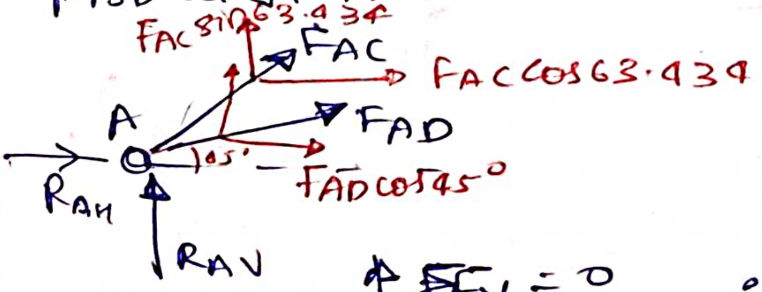
$10 \times 3 - R_B \times 6 = 0$

$\boxed{R_B = 5\,kN}$

$\xrightarrow{+} \Sigma F_x = 0$

$R_{AH} = 0$

FBD at Jt A



$\xrightarrow{+} \Sigma F_x$

$F_{AD} \cos 45° + F_{AC} \cos 63.434 = 0$

$0.707 F_{AD} + 0.447 F_{AC} = 0$

$\uparrow \Sigma F_y = 0$

$F_{AD} \cos 45° + F_{AC} \sin 63.436 + 5 = 0$

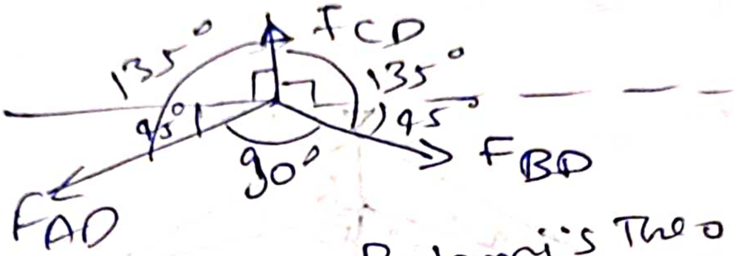
$0.707 F_{AD} \pm 0.89 F_{AC} = \mp 5$

$-0.443 F_{AC} = 5 \Rightarrow \boxed{F_{AC} = -11.2}$

$F_{AC} = -11.23\ kN$ (C)

$F_{AD} = 7.071\ kN$ (T)

FBD of Jt (D)



By Lami's Theo

$$\frac{F_{AD}}{\sin 135^\circ} = \frac{F_{CD}}{\sin 90^\circ} = \frac{F_{BD}}{\sin 135^\circ}$$

$$\frac{7.071}{\sin 135^\circ} = \frac{F_{CD}}{\sin 90^\circ}$$

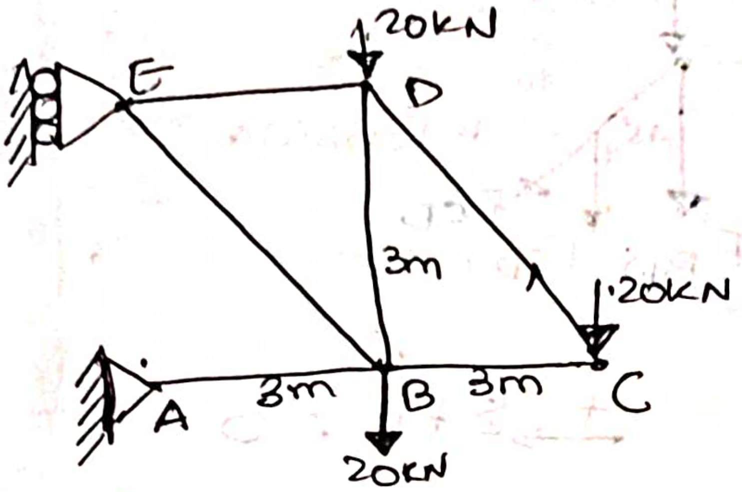
$$\boxed{F_{CD} = 10\ kN}\ T$$

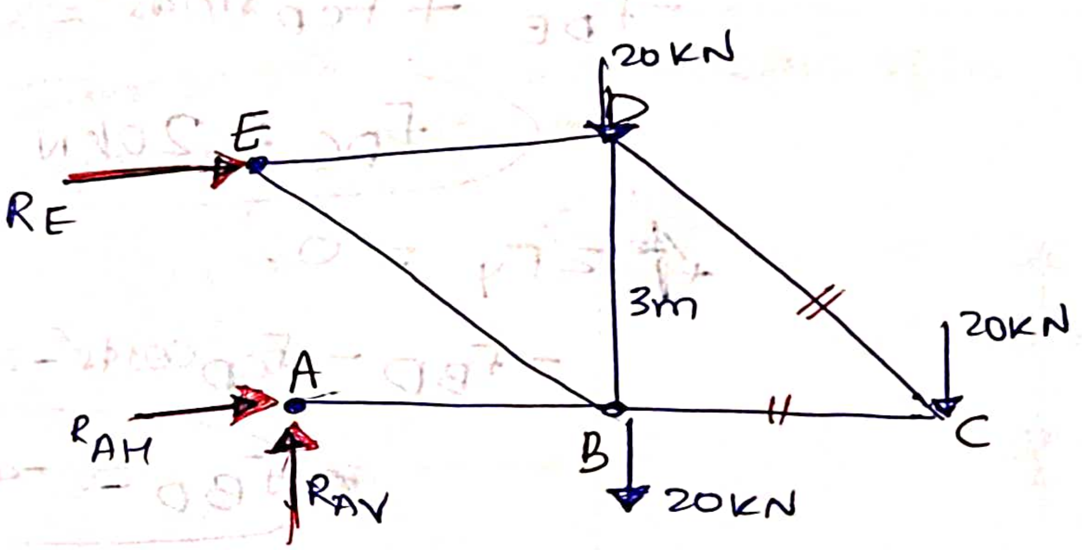
Due to symmetry of structure

$\boxed{F_{BC} = F_{AC}}$, $\boxed{F_{BD} = F_{AD}}$

Sr No	Force in members	Magnitude	Nature
1.	FAD	7.071 kN	T
2.	FAC	11.18 kN	C
3.	FCD	10 kN	T
4.	FBC	11.18 kN	C
5.	FBD	7.071 kN	T

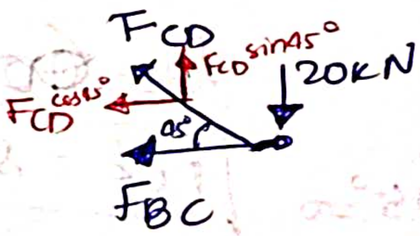
Find the force in all the members of the Truss shown by the method of joint





C → D → B

FBD of jt C



$+\uparrow \Sigma F_y = 0$

$F_{CD}\sin 45^\circ - 20\,KN = 0$

$F_{CD} = 20\sqrt{2}\,KN$

$\boxed{F_{CD} = 28.284\,KN}$ (T)

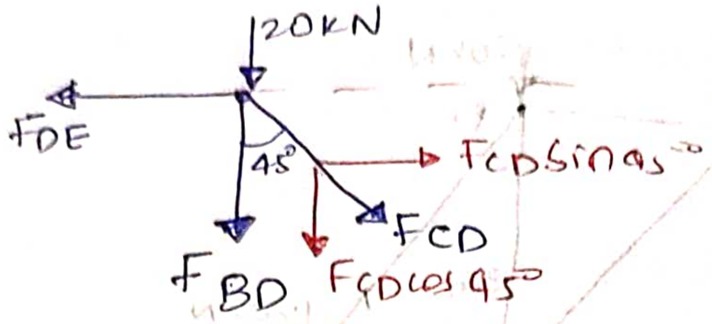
$\xrightarrow{+} \Sigma F_x = 0$

$-F_{BC} - F_{CD}\cos 45^\circ = 0$

$\boxed{F_{BC} = -20\,KN}$ (C)

A purpose so glorious that it feels lik...

FBD at jt (D)



$\xrightarrow{+} \Sigma F_x = 0$

$-F_{DE} + F_{CD}\sin 45° = 0$

$F_{DE} = 20kN$ (T)

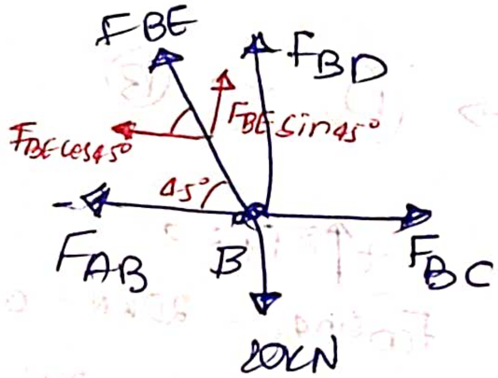
$+\uparrow \Sigma F_y = 0$

$-F_{BD} - F_{CD}\cos 45° - 20kN = 0$

$F_{BD} = -40kN$

(C)

FBD at jt B



$+\uparrow \Sigma F_y = 0$

$F_{BD} + F_{BE}\sin 45° - 20 = 0$

$F_{BE} = 60 \cdot \times \sqrt{2}$

$F_{BE} = 84.852 kN$ (T)

$\xrightarrow{+} \Sigma F_x = 0$

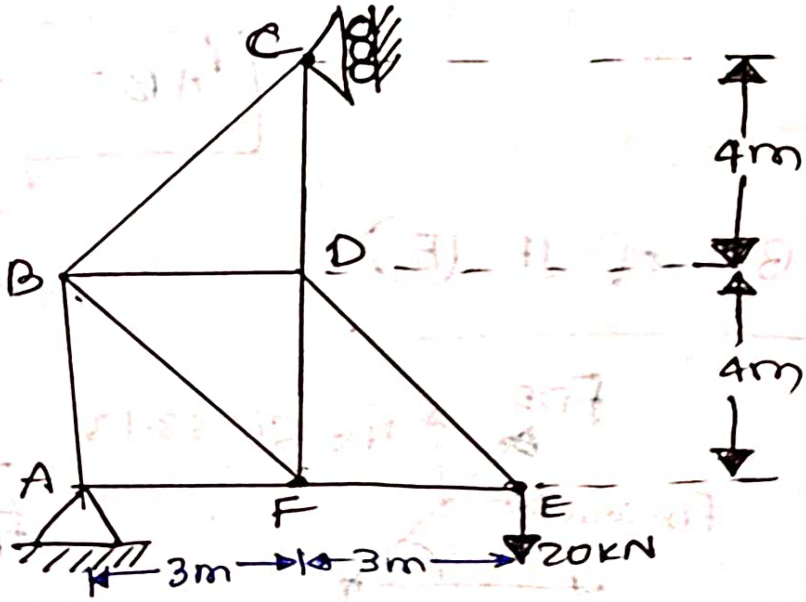
$-F_{AB} + f_{BC} - F_{BE}\cos 45° = 0$

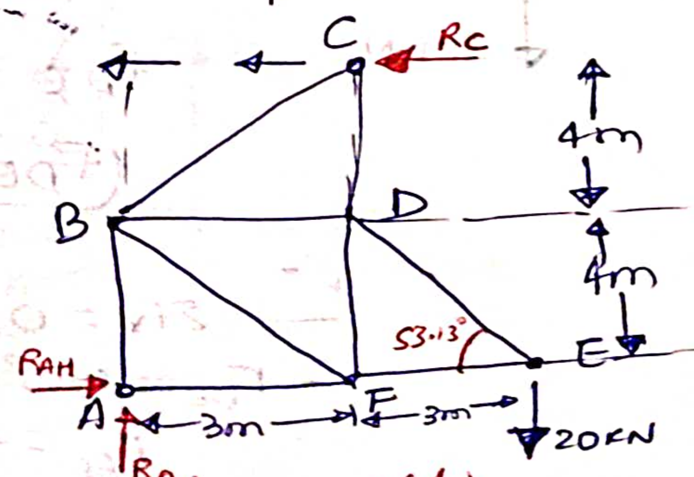
$-F_{AB} + (-20) - 60 = 0$

$F_{AB} = -80kN$ (C)

Sr. No	Force in member	Magnitude	Nature
1.	F_{AB}	80 KN	C
2.	F_{BE}	84.852 KN	T
3.	F_{BD}	40 KN	C
4.	F_{DE}	20 KN	T
5.	F_{CD}	28.284 KN	T
6.	F_{BC}	20 KN	C

Q. Find the force in all the members of the Truss shown by the method of joint . .





$\tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$

A
E
C
F
D

$+\uparrow \Sigma F_y = 0$

$R_{AV} - 20 = 0$

$\boxed{R_{AV} = 20\ KN}$

$\Sigma M_A = 0$

$20 \times 6 - R_C \times 8 = 0$

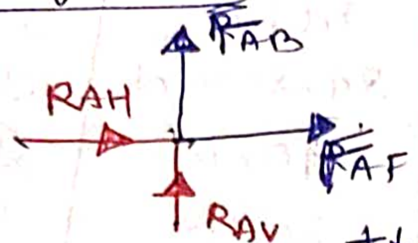
$\boxed{R_C = 15\ KN}$

$\xrightarrow{+} \Sigma F_x = 0$

$-R_C + R_{AH} = 0$

$\boxed{R_{AH} = 15\ KN}$

FBD of Jt (A)



$\xrightarrow{+} \Sigma F_x = 0$

$F_{AF} + R_{AH} = 0$

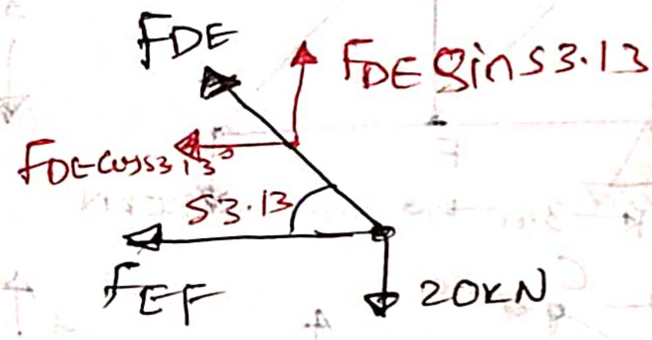
$\boxed{F_{AF} = -15\,KN}$ (C)

$+\uparrow \Sigma F_y = 0$

$F_{AB} + R_{AV} = 0$

$\boxed{F_{AB} = -20\,KN}$ (C)

FBD of Jt (E)



$+\uparrow \Sigma F_y = 0$

$F_{DE} \sin 53.13^\circ = 0$
$= 20KN$

$F_{DE} = 20 \times \frac{5}{4}$

$\boxed{F_{DE} = 25\,KN}$

(T)

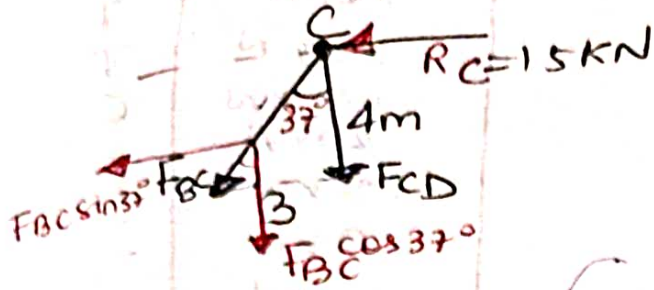
$\xrightarrow{+} \Sigma F_x = 0$

$-F_{DE} \cos 53.13^\circ - F_{EF} = 0$

$F_{EF} = -\cancel{25}^{5} \times \frac{3}{\cancel{5}}$

$\boxed{F_{EF} = -15\,KN}$ (C)

FBD of Jt C



$\tan^{-1}\left(\frac{3}{4}\right) = 37°$

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{BC}\sin 37° - R_C = 0$

$F_{BC} = -15 \times \frac{5}{3}$

$\boxed{F_{BC} = -25\,KN}$ C

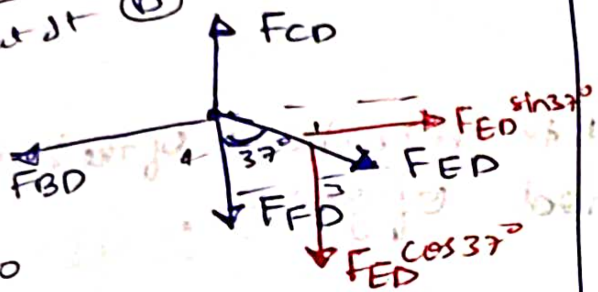
$+\uparrow \Sigma F_y = 0$

$-F_{BC}\cos 37° + F_{CD} = 0$

$F_{CD} = 25 \times \frac{4}{5}$

$\boxed{F_{CD} = 20\,KN}$ T

FBD at Jt D



$\Sigma F_x = 0$

$-F_{BD} + F_{ED}\sin 37° = 0$

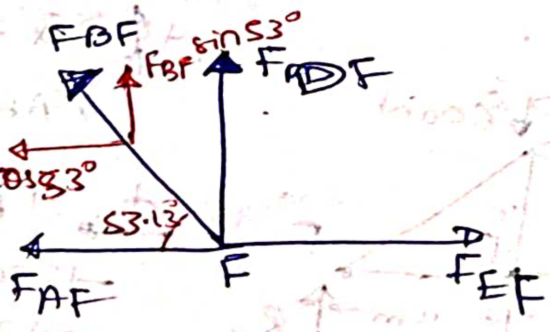
$F_{BD} = \cancel{25}^{5} \times \frac{3}{\cancel{5}} = 15\,KN$

$\boxed{F_{BD} = 15\,KN}$ T

$\Sigma F_y = 0$

$F_{CD} = F_{ED}\cos 37° + \cancel{F_{FD}}^{0}$

FBD of Jt F



$\xrightarrow{+} \Sigma F_x = 0$

$-F_{AF} + F_{EF} - F_{BF}\cos 53° = 0$

$-\cancel{(-15)} - \cancel{15} - F_{BF}\cos 53°$

$\boxed{F_{BF} = 0}$

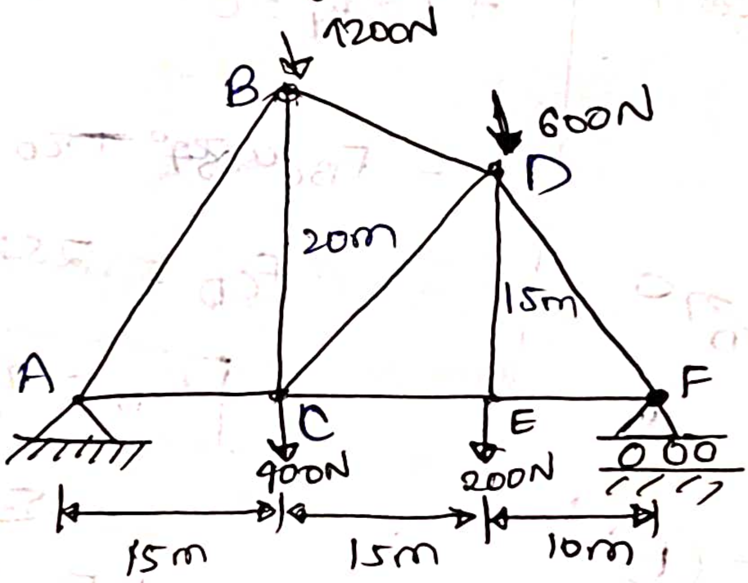
$+\uparrow \Sigma F_y = 0$

$F_{DF} + F_{BF}\sin 53° = 0$

$F_{DF} = 0$

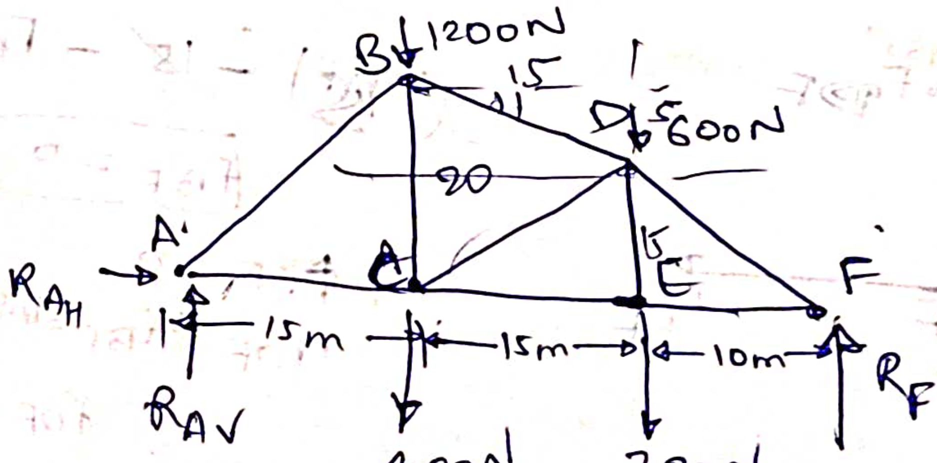
S.No.	Force in members	magnitude	Nature
1	F_{BC}	25 KN	C
2	F_{CD}	20 KN	T
3	F_{DE}	25 KN	T
4	F_{EF}	15 KN	C
5	F_{AB}	20 KN	C
6	F_{AF}	15 KN	C
7	F_{AB}	0 KN	–
8	F_{DF}	0 KN	–
9	F_{BD}	15 KN	T

Q. Find the force in all the members of the truss shown by the method of joint.



$\tan^{-1}\left(\frac{20}{15}\right) = \left(\frac{4}{3}\right)$

$\tan^{-1}\left(\frac{5}{15}\right) = 0.4\!\!\!\!$



$+ \rightarrow \Sigma F_x = 0$

$R_{AH} = 0$

$+\uparrow \Sigma F_y = 0$

$-1200 - 600$
$-400 - 200$
$+ R_{AV} + R_F = 0$

$\boxed{R_{AV} + R_F = 2400N}$ — (1)

$(+\circlearrowleft)\ \Sigma M_A = 0$

$(400\times15 + 200\times30 + 1200\times15 + 600\times30) - R_F\times40 = 0$

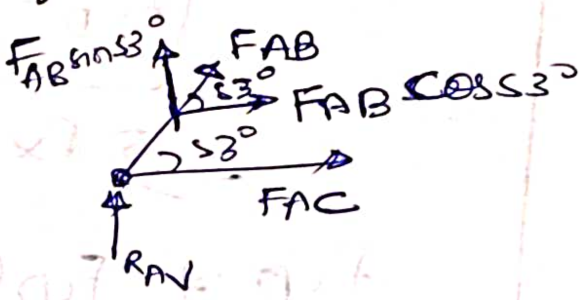
$48000 - R_F\times40 = 0$

$\boxed{R_F = 1200N}$

from eqn

$\boxed{R_{AV} = 1200N}$

FBD of Jt A



$+\uparrow \Sigma F_y = 0$

$F_{AB}\sin53^\circ = -R_{AV}$

$F_{AB} = -\cancel{1200}^{\,300}\times\frac{5}{\cancel{4}}$

$\boxed{F_{AB} = -1500N}$ (C)

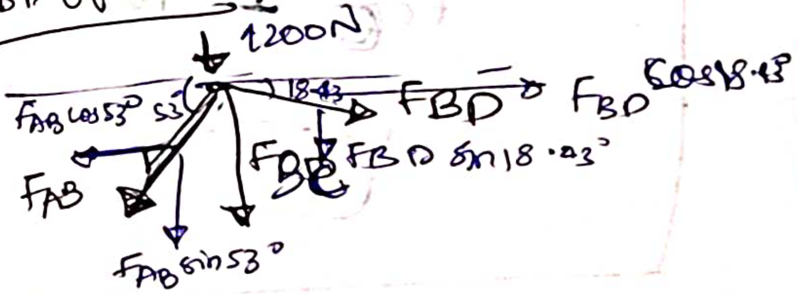
$\xrightarrow{+} \Sigma F_x = 0$

$F_{AB}\cos53 + F_{AC} = 0$

$F_{AC} = +\cancel{1500}^{\,300}\times\frac{3}{\cancel{5}}$

$\boxed{F_{AC} = +900N}$ (T)

FBD of Jt (B)



$\xrightarrow{+} \Sigma F_x = 0$

$F_{BD}\cos18.43 - F_{AB}\cos53.13 = 0$

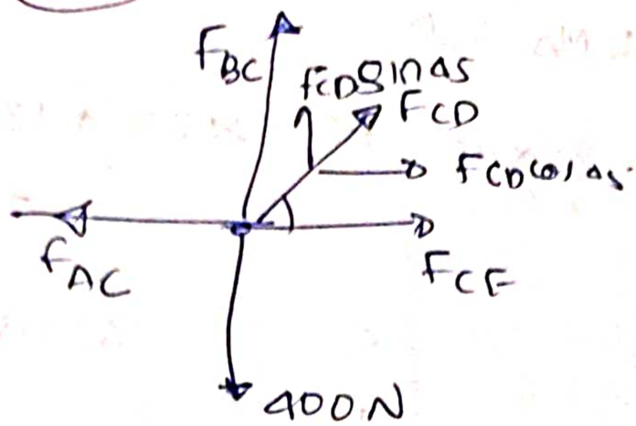
$F_{BD} = \dfrac{-1500\times\frac{4}{5}}{\cos18.4^\circ}$

$\boxed{F_{BD} = -948.65}$ N(C)

$+\uparrow \Sigma F_y \quad -F_{BC} - F_{AB}\sin53.13^\circ - F_{BD}\sin18.434 - 1200 = 0$

$-F_{BC} + \cancel{1200} + 300 - \cancel{1200} = 0$; $\boxed{F_{BC} = 300N}$ (T) at C

FBD at C



$+\uparrow \sum F_y = 0$

$F_{CD}\sin 45° + F_{BC} - 400 = 0$

$F_{CD}\sin 45° + 300 - 400 = 0$

$F_{CD} = 100\sqrt{2}$

$\boxed{F_{CD} = 141.4N\ (T)}$

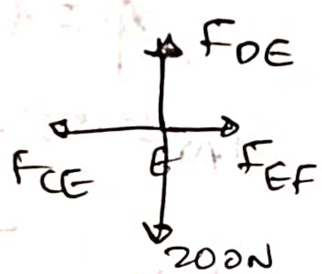
$\xrightarrow{+} \sum F_x = 0$

$F_{CE} + F_{CD}\cos 45 - F_{AC} = 0$

$F_{CE} = 900 - 100$

$\boxed{F_{CE} = 800N}$ (T)

at J & E

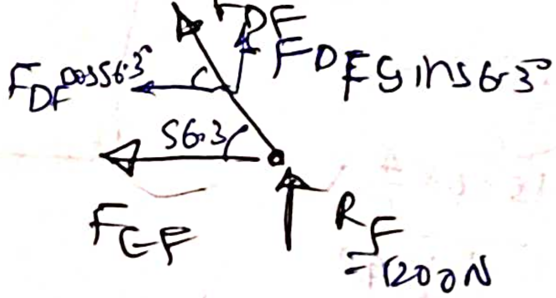


$F_{DE} = 200N$ (T)

$F_{CE} = F_{EF}$

$F_{EF} = 800N$ (T)

at F



$\tan^{-1}\left(\frac{15}{10}\right) = 56.3°$

$\cancel{F_{DF} = \frac{1200}{\sin 56.3°}}$

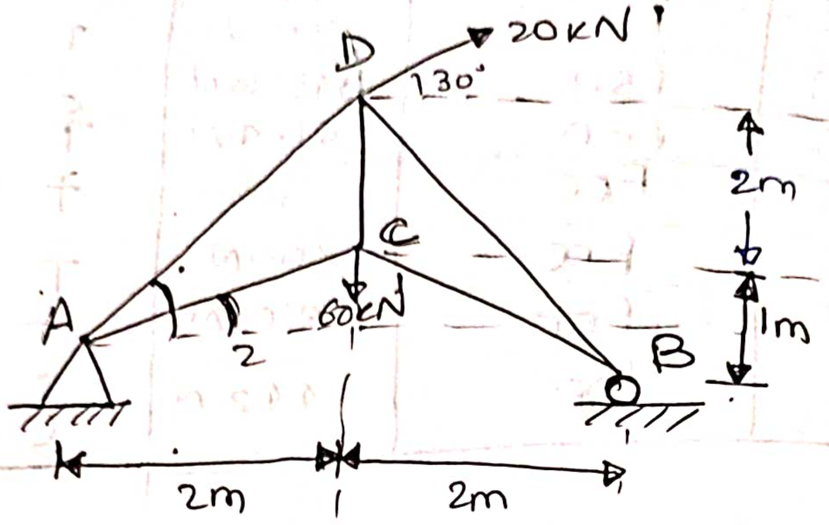
$F_{DF}\cos 56.3° = F_{EF}$

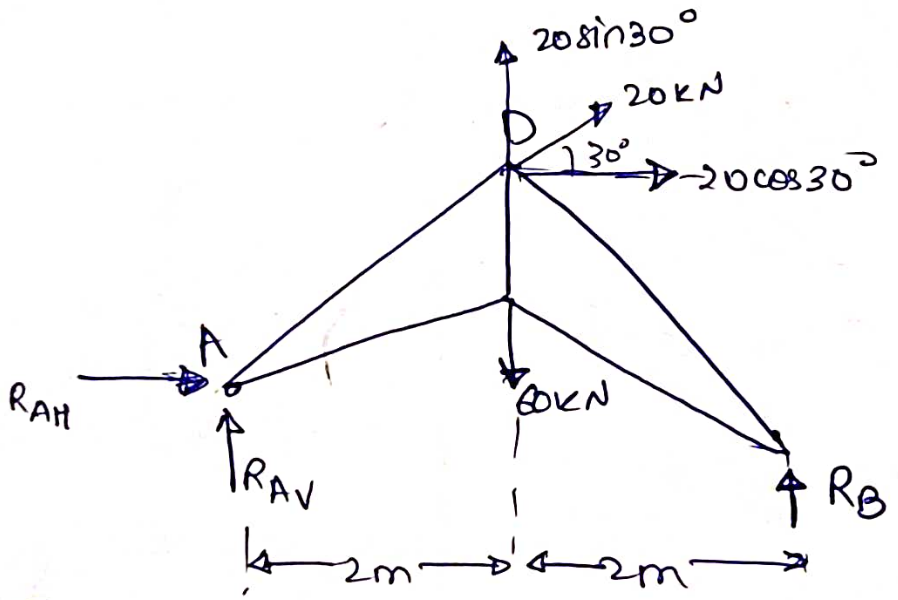
$F_{DF} = \frac{800}{\cos 56.3}$

$F_{DF} = 1442N$

Sl.No.	Force in members	Magnitude	Nature
1	F_{AC}	900N	T
2	F_{AB}	1800N	C
3	F_{BC}	300N	T
4	F_{BD}	348.65N	C
5	F_{CD}	141.4N	T
6	F_{CE}	800N	T
7	F_{DE}	200N	T
8	F_{EF}	800N	T
9	F_{DF}	1442N	C

Q. Find the force in all the members of Truss shown





$\overset{+}{\rightarrow} \Sigma F_x = 0$

$20\cos 30° + R_{AH} = 0$

$R_{AH} = -\overset{10}{\cancel{20}} \times \frac{\sqrt{3}}{\cancel{2}}$

$\boxed{R_{AH} = -17.32 KN}$

$\overset{+}{\circlearrowleft}$ ~~$\Sigma M_A = 0 \Rightarrow (-60 \times 2 + 20 \times 2) + R_B \times 4 = 0$~~

~~$R_B \times = 40 KN$~~

$+\circlearrowleft \sum M_A = 0$

$(60 \times 2 - 20\sin 30^\circ \times 2 + 20\cos 30^\circ \times 3) - R_B \times 4 = 0$

$120 \quad 20 + 51.96 = R_B \times 4$

$\boxed{R_B = 37.99\,kN}$

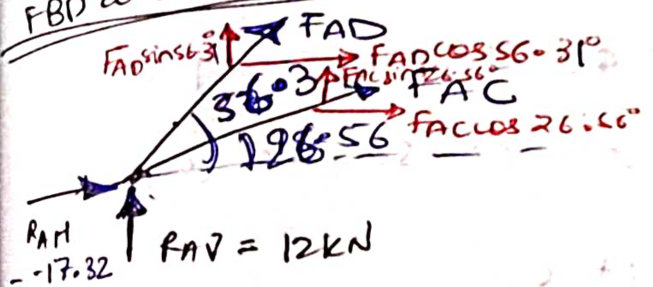
$+\uparrow \sum F_y$

$R_{AV} + R_B + 20\sin 30 - 60 = 0$

$R_{AV} + 38 + 20\sin 30 - 60 = 0$

$\boxed{R_{AV} = 12\,KN}$

FBD at Jt A



$\tan^{-1}\left(\frac{1}{2}\right) = 26.56$

$\tan^{-1}\left(\frac{3}{2}\right) = 56.309^\circ$

$\xrightarrow{+} \sum F_x = 0$

$F_{AC}\cos 26.56 + F_{AD}\cos 56.31 + R_{AH} = 0$

$F_{AC}\cos 26.56 + F_{AD}\cos 56.31 = 17.32\,KN$ — (1)

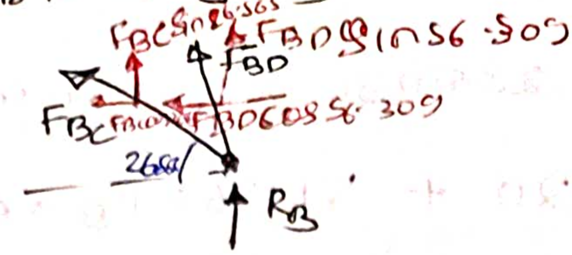
$+\uparrow \sum F_y = 0$

$F_{AD}\sin 56.31^\circ + F_{AC}\sin 26.56^\circ + 12\,KN = 0$ — (2)

$F_{AC} = 42.464\,kN$ (T)

$F_{AD} = -37.246\,kN$ (C)

FBD at Jt B



$\xrightarrow{+} \Sigma F_x = 0$

$-F_{BC}\cos 26.565 - F_{BD}\cos 56.309 = 0$ — (3)

$\uparrow \Sigma F_y = 0$

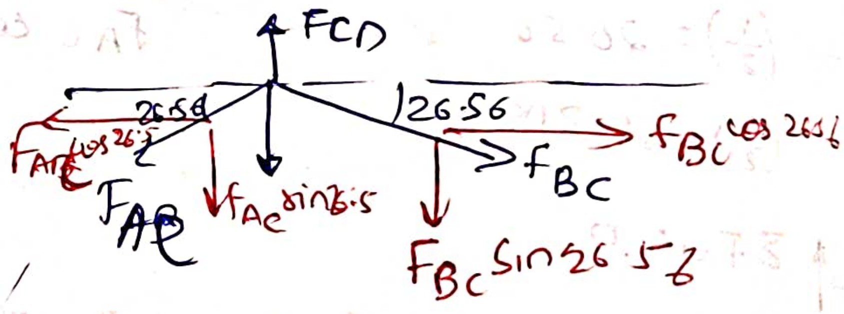
$F_{BC}\sin 26.565 + F_{BD}\sin 56.309 + 39.9$ — (4)

from (5) & (4)

$F_{BC} = 42.487$ kN (T)

$F_{BD} = -68.507$ kN (C)

FBD at Jt C



$+\uparrow \Sigma F_y \quad F_{CD} - 60 - F_{AC}\sin 26.565 - F_{BC}\sin 26.565 = 0$

$\boxed{F_{CD} = 97.991\text{ kN}}$

$\boxed{F_{CD} = 98\text{ KN}}$ T

~~ΣF_x~~

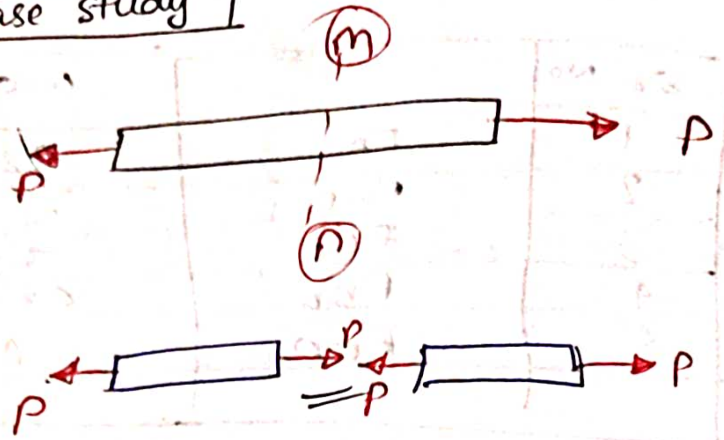
~~ΣD~~

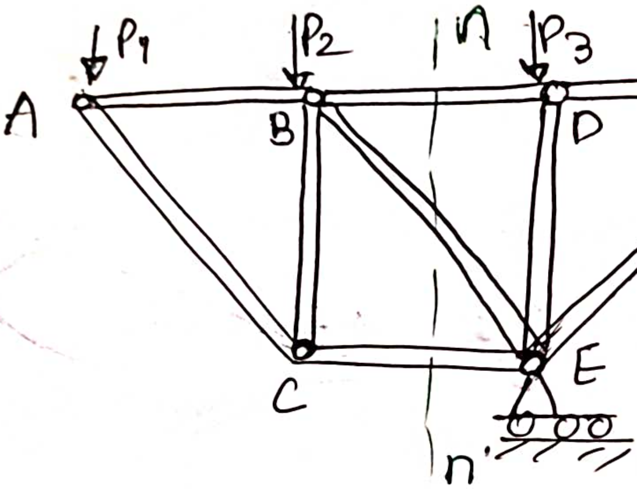
Sr. No		Magnitude & Nature	
1	FAC	42.464 kN	T
2	FAD	37.246 kN	C
3	FBC	42.487 kN	T
4	FBD	68.507 kN	C
5	FCD	98 kN	T

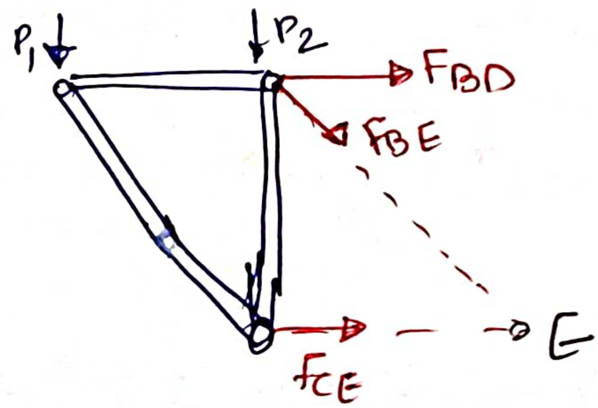
Truss

method of section

Case study



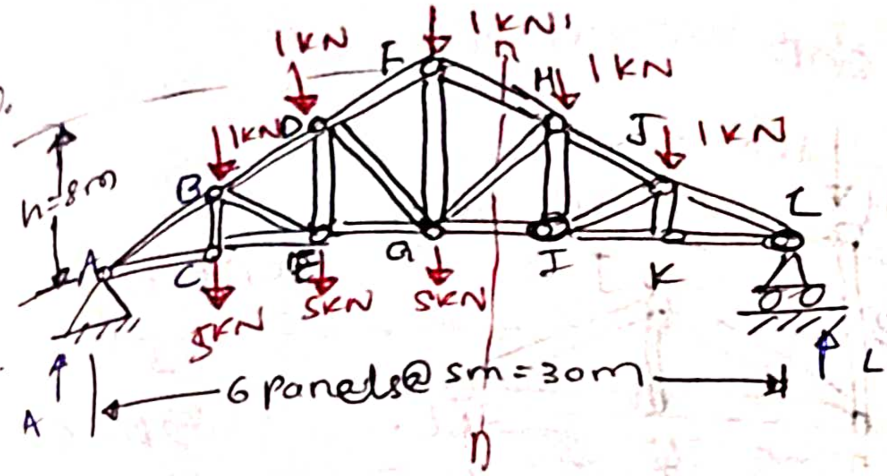




Note

* Not more than 3 members whose forces are unknown should be cut in a single section since we have only 3 independent equilibrium equations

Q.



Find out Internal forces in members FH, GH & GI.

Find out reactions

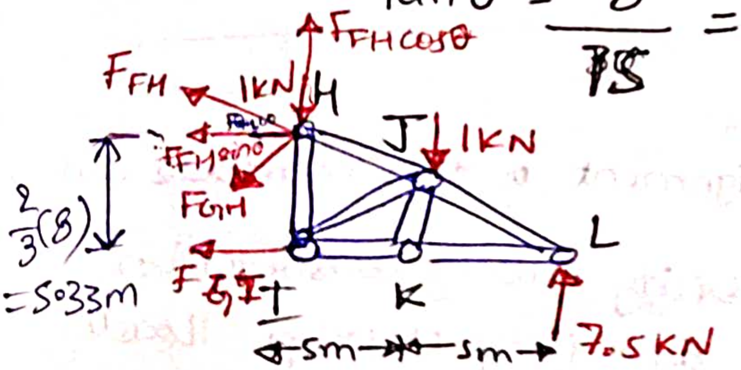
$$\Sigma M_A = 0 = -(5\ m)\,6kN - (10)\,6kN - (15m)(6kN)$$
$$-20)(1kN) - (25)(1kN) + 30L$$

$$\Rightarrow L = 7.5\,kN \uparrow$$

$$\Sigma F_y = 0 = -20kN + L + A$$

$$A = 12.5\,kN \uparrow$$

$$\tan\theta = \frac{8}{15} \Rightarrow \theta = 28.07^\circ$$



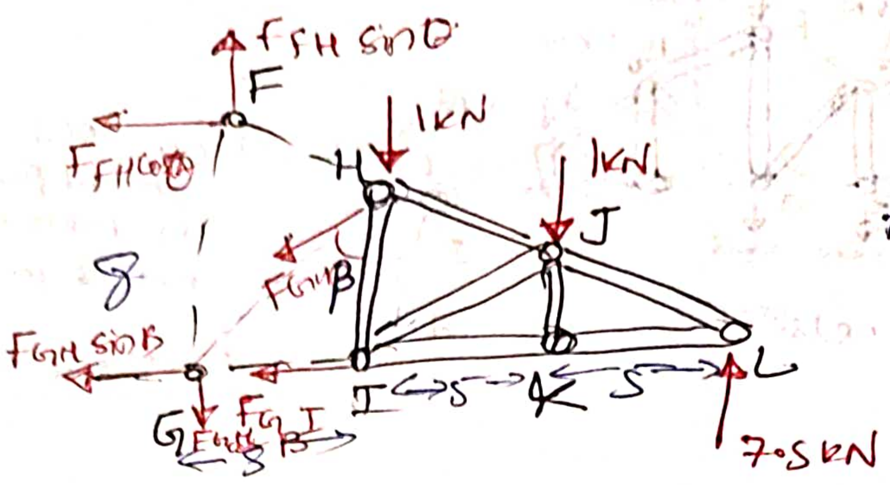
$$\Sigma M_H = 0$$

$$(7.5)(10) - (1kN)\times(5) - F_{GI}(5.33) = 0$$

$$\boxed{F_{GI} = 13.13\,kN}$$

(T)

~~ΣM_G = 0~~



$\tan\beta = \frac{5}{\frac{2}{3}(8)}$

$\Rightarrow \beta = 43.15$

$\Sigma M_L = 0$

$(1kN)(10) + (1kN)5 + F_{GH}\cos\beta)15 = 0$

$F_{GH} = -1.371\ KN$

$\Sigma M_G = 0$

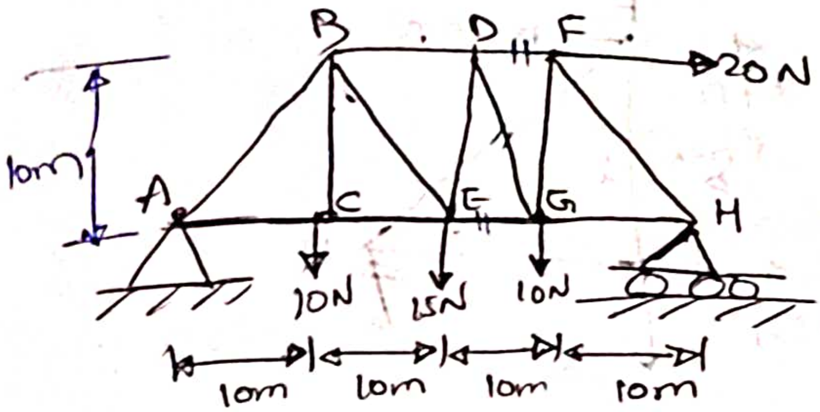
$(7.5K)(15) - (1kN)(10) - (1kN)(5) + F_{FH}\cos\theta\ 8 = 0$

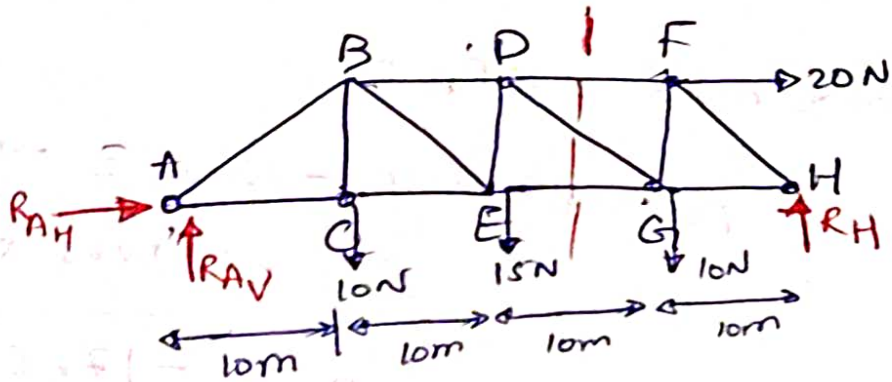
$\boxed{F_{FH} = -13.82\,kN}$ (C)

Redundant members :-

- To maintain alignment of two members during construction.
- to increase stability during construction
- To maintain stability during loading (eg. to prevent buckling of compression members)
- To provide support if the applied load is changed.
- To act as backup members if some members fail or require strengthening
- Analysis is difficult often Castigliano's Theorem is used to solve this type of problem.

Q. Find force in members DF, DG, EG





$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{AH} + 20 = 0$

$\boxed{R_{AH} = -20N}$

$+\uparrow \Sigma F_y = 0$

$R_{AV} - 10 - 15 - 10 - R_H = 0$

$\boxed{R_{AV} + R_H = 35N}$

$+\circlearrowright \Sigma M_A = 0$

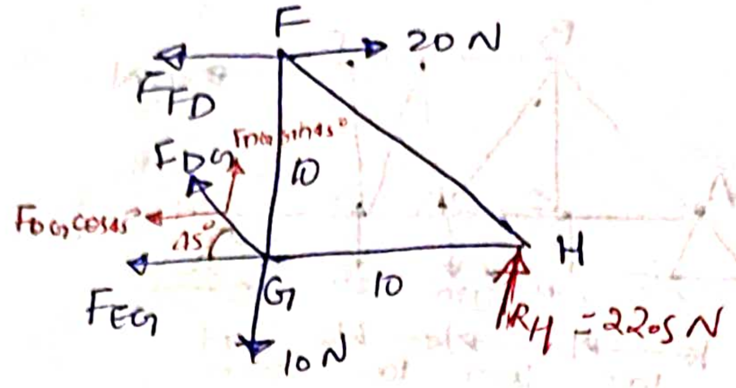
$(10 \times 10 + 15 \times 20 + 10 \times 30 + 20 \times 10) - R_H \times 40 = 0$

$R_H = \frac{90}{4}$

$\boxed{R_H = 22.5N}$

$R_{AV} = 35 - 22.5$

$R_{AV} = 12.5 N$



$\uparrow \Sigma F_y = 0$

$F_{DG}\sin 45^\circ - 10 + R_H = 0$

$F_{DG}\sin 45^\circ = 10 - 22.5 = -12.5$

$F_{DG} = -12.5 \times \sqrt{2}$

$\boxed{F_{DG} = -17.677 N}$

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{DF} - F_{DG}\cos 45^\circ + 20 N = 0$

$-F_{EG} - F_{DF} + F_{DG}\frac{1}{\sqrt{2}} = 20$

$F_{EG} = (20 + 12.5 + 2.5)$

$\boxed{F_{EG} = 35 N}$

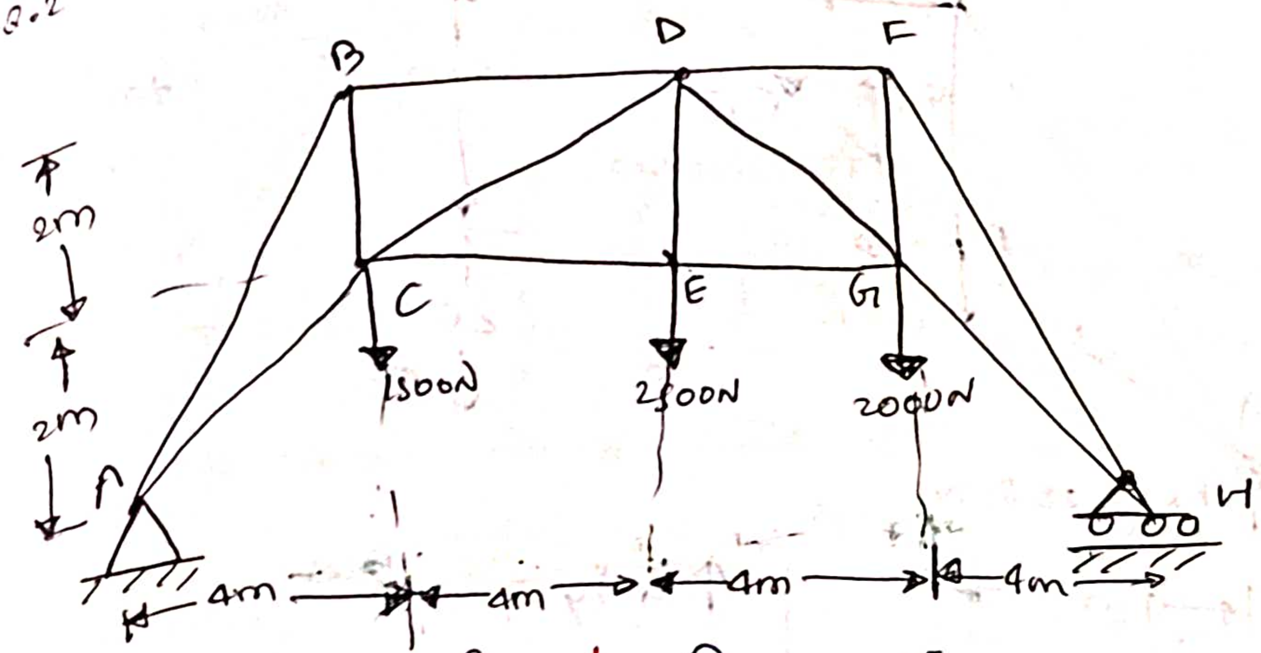
$\circlearrowleft \Sigma M_G = 0$

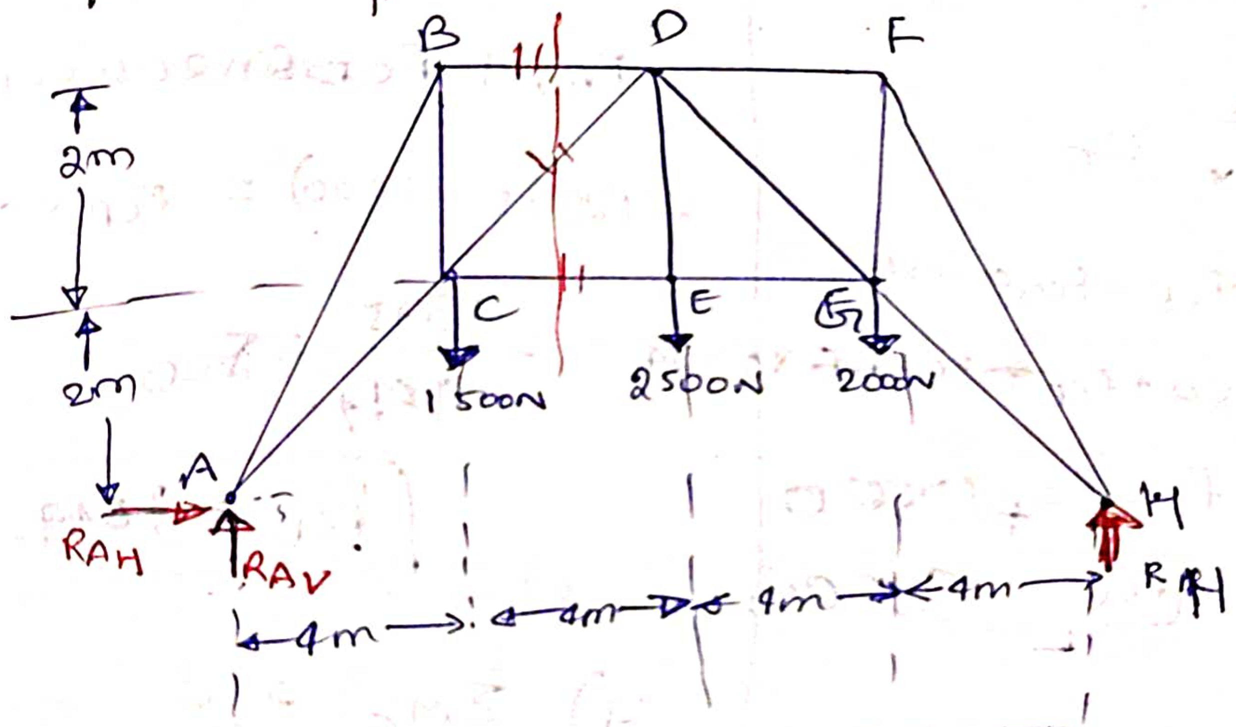
$-F_{DF} \times 10 + 20 \times 10 - 22.5 \times 10$

$\boxed{F_{DF} = -2.5 N}$

Sr No.	Forces	Magnitude	Nature
1	F_{DF}	2.5 N	C
2	F_{DG}	17.677 N	C
3	F_{EG}	35 N	T

Q.2 Find the force in members BD, CD & CE.





$\xrightarrow{+} \Sigma F_x = 0$

$\boxed{R_{AH} = 0}$

$\Sigma F_y = 0$

$R_{AV} + R_H - 1500 - 2500 - 2000 = 0$

$R_{AV} + R_H = 6000$

$\boxed{R_{AV} = 2875N}$

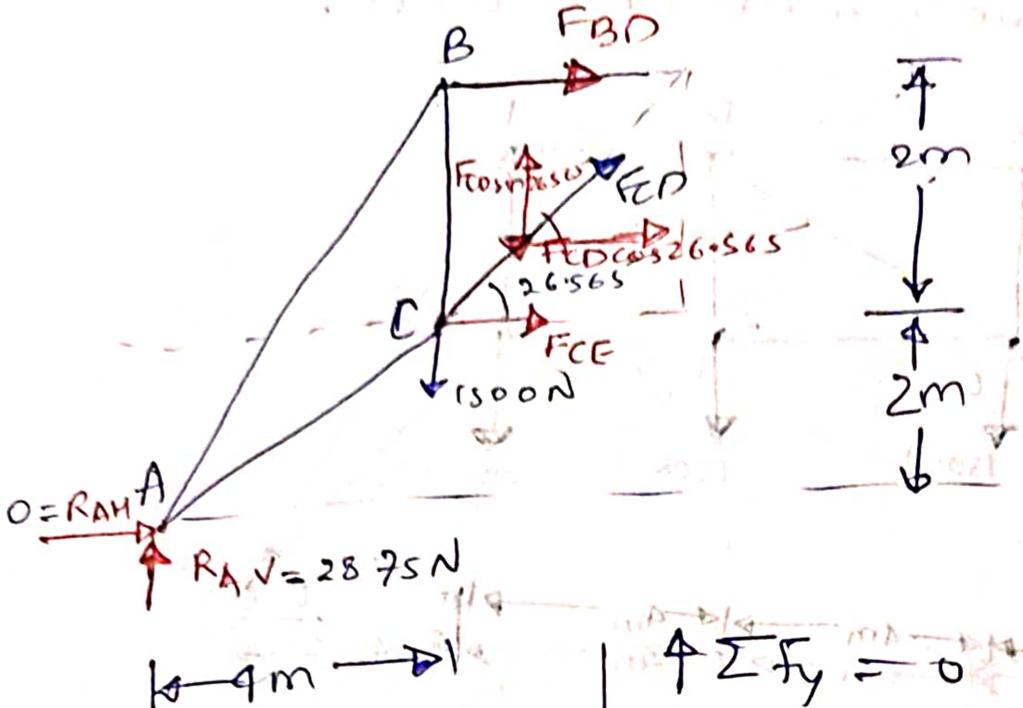
$\circlearrowright + \Sigma M_A = 0$

$1500 \times 4 + 2500 \times 8 + 2000 \times 12 - R_H \times 16 = 0$

$$R_H = \frac{50000}{16}$$

$\boxed{R_H = 3125N}$

FBD of LHS part of section



$\tan^{-1}\left(\frac{2}{4}\right) = 26.5$

$\xrightarrow{+} \sum F_x = 0$

$F_{BD} + F_{CE} + F_{CD}\cos 26.565 = 0$

$-5750 + F_{CE} + 3074.6 \times 0.894$

$F_{CE} = +8800$

$\boxed{F_{CE} = 8800\ N}$

$+\uparrow \sum F_y = 0$

$R_{AV} + F_{CD}\sin 26.565 - 1500 = 0$

$-(2875 - 1500) = F_{CD} \times 0.4473$

$\frac{1375}{0.4472} = F_{CD}$

$\boxed{F_{CD} = -3074.598}$

$+\circlearrowleft \sum M_C = 0$

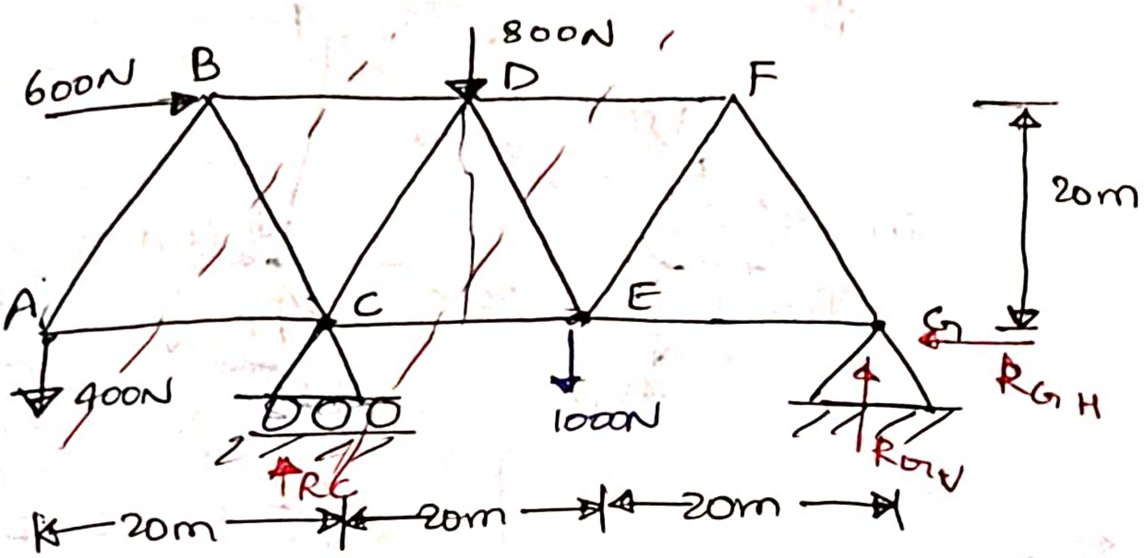
$F_{BD} \times 2 + 2875 \times 4 = 0$

$\boxed{F_{BD} = -5750\ N}$

(C)

SNo	Force	Magnitude (N) & Nature	
	FCD	3074.598 N	(T)
	FBD	5750 N	(C)
	FCE	8800 N	(T)

Q. Find Force members DF; BC, CE

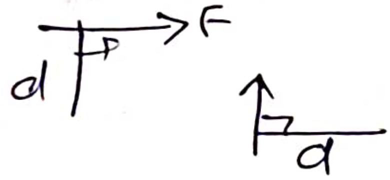


$\rightarrow \Sigma F_x = 0$

$600 - R_{GH} = 0$

$\boxed{R_{GH} = 600\text{ N}}$

$\circlearrowright + \Sigma M_C$



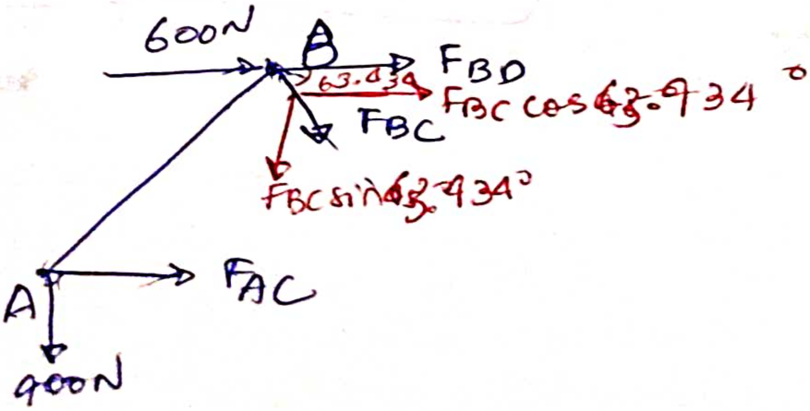
$600 \times 20 + 800 \times 10 + 1000 \times 20 - 400 \times 10 - R_{GV} \times 40 = 0$

$\Rightarrow \boxed{R_{GV} = 800\text{ N}}$

$\uparrow \Sigma F_y$

$R_C + R_{GV} - 400 - \cancel{800} - 1000 = 0$

$\boxed{R_C = 1400\text{ N}}$

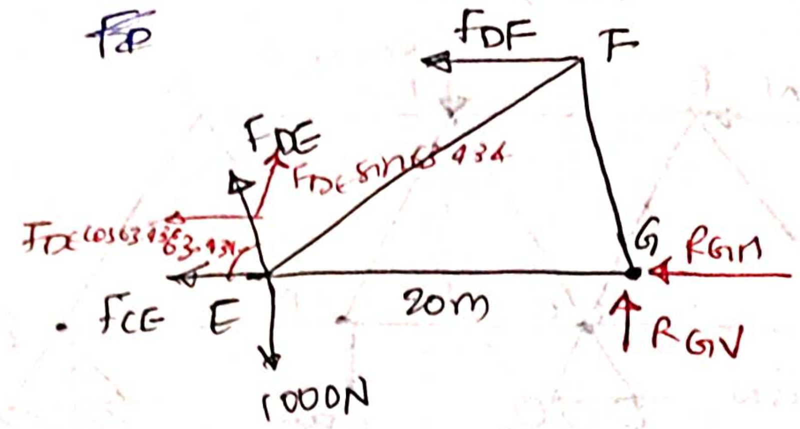


$+\uparrow \Sigma F_y = 0$

$-400 - F_{BC} \sin 63.434 = 0$

$F_{BC} = -\dfrac{400}{\sin 63.434}$

$\boxed{F_{BC} = -447.22\text{ N}}$



$+\circlearrowleft \Sigma M_E = 0$

$-800 \times 20 - F_{DF} \times 20 = 0$

$$\boxed{F_{DF} = -800N}$$

(C)

$+\uparrow \Sigma F_y = 0$

$F_{DE} \sin 63.434 + \overset{800}{R_{GV}} - 1000 = 0$

$$F_{DE} = \frac{200}{\sin 63.434}$$

$$\boxed{F_{DE} = 223.608N}$$

(T)

$\xrightarrow{+} \Sigma F_x = 0$

$-R_{GH} - F_{DE} \cos 63.43 - F_{CE} - F_{DF} = 0$

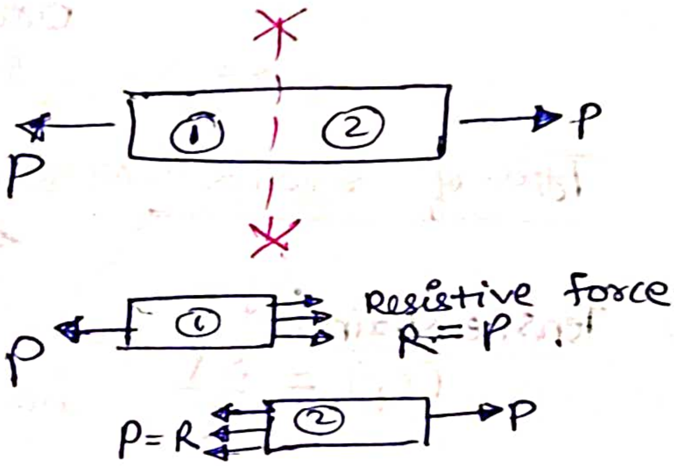
$-600 - 223 \cos 63.43 - F_{CE} + 800 = 0$

$$\boxed{F_{CE} = 99.996 N}$$

(T)

Strength of Material
(Stress & Strain)

Stress :- When a body is acted upon by an external force, body undergoes some deformation (change in shape/size). But the molecules of body setup some resistance to deformation. This resistance force per unit area is called stress (σ).



Mathematically,

$$\text{stress} = \frac{\text{Resistive force}}{\text{Area}}$$

$$\boxed{\sigma = \frac{P}{A}}$$

Unit : $\frac{N}{m^2}$ or Pa

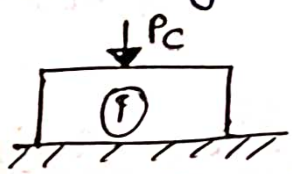
$(1\,\frac{N}{m^2} = 1\,Pa)$

Types of stress

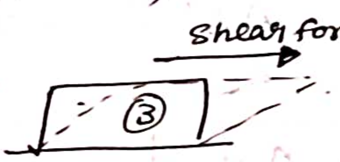
→ Depending upon types of load stress may be classified as ① compressive stress (σ_c)

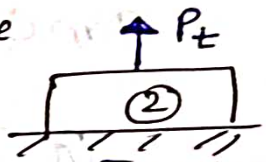
② Tensile stress (σ_t)

③ Shear stress (τ)



Compressive ⇒ Direct stress





Tensile

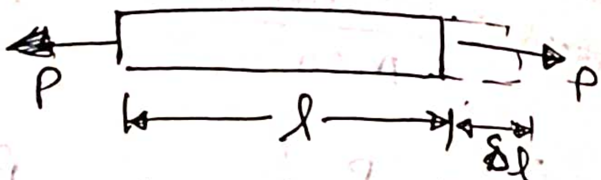
Strain :- Deformation produced due to stress.

⇒ unitless.

Imp ⇒ The ratio of change in dimension (length, breadth, Height, Volm) to the original dimension.

$$e = \frac{\text{change in dimension}}{\text{original dimension}} \quad (\text{unitless})$$

Types of strain



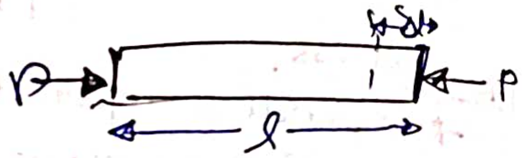
① Tensile strain (e_t)

$$(e_t) = \frac{\delta l}{l}$$

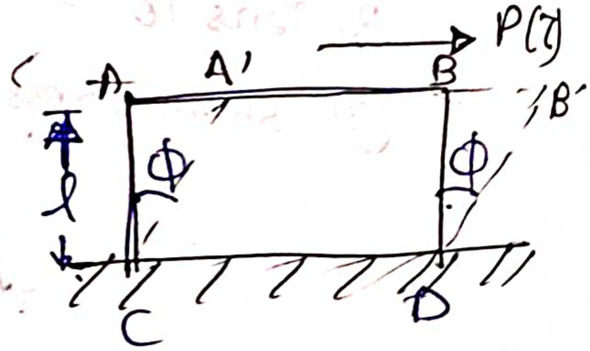
where δl is change in length
l is original length

② compressive strain (e_c)

$$(e_c) = \frac{\delta l}{l}$$



③ Shear strain, (ϕ)



$$\tan\phi = \frac{AA'}{AC}$$

ϕ is very small

$$\boxed{\phi = \frac{AA'}{l}}$$

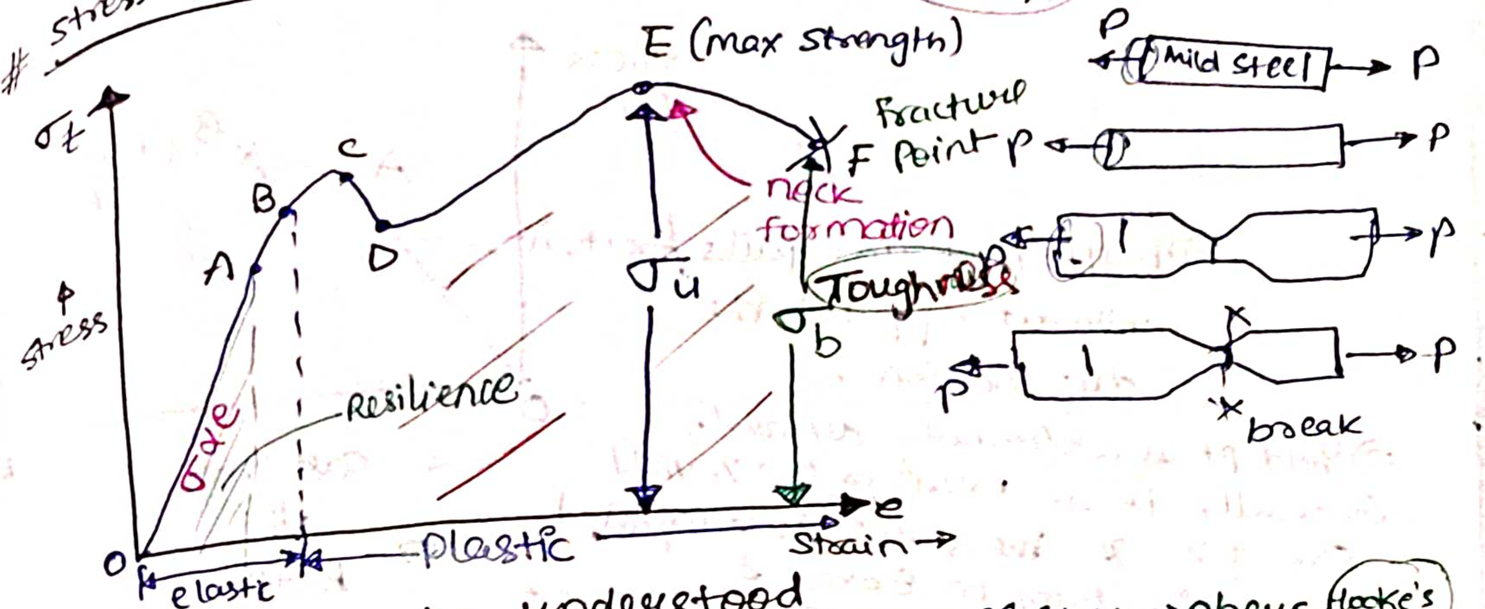
④ Volumetric strain (e_v)

$$e_v = \frac{\text{change in Volume}}{\text{Original volume}} \quad \text{or} \quad e_v = \frac{\Delta V}{V}$$

stress-strain Diagram (for ductile material) ⇒ Mild steel

$\sigma = \frac{P}{A}$

Tensile test



The given can be understood by following points :

(i) A → Proportionality limit → stress ∝ strain ⇒ obeys Hooke's law

(ii) B → elastic limit → when load is removed material regain its original position

(iii) C → upper yield point

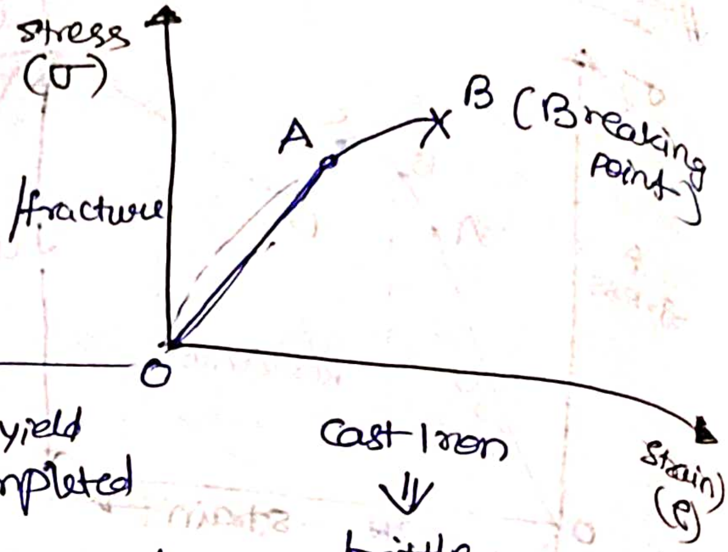
(iv) D → lower yield point

(v) E → ultimate stress

(vi) F → Breaking point

C–D ⇒ strain ↑es rapidly.

Stress- Strain diagram for brittle material



⇒ brittle material fails / fracture without appreciable deformation.

⇒ Yield pt is artificially defined. Generally it is taken as 0.2 % yield strength & the value is completed by taking 0.002 on E axis & drawn a line from this pt ∥ to tangent to σ-E curve at origin.

Cast Iron
⇓
brittle material

A ⇒ Proportionality limit

B ⇒ Breaking Point

Topics to be covered
(i) Hooke's law
(ii) Modulus of Elasticity (E)
(iii) Modulus of Rigidity (G)
(iv) Bulk Modulus (K)

Hooke's law :-

"within elastic limit stress is directly proportional to strain."

$\boxed{\sigma \propto e}$ or $\sigma = Ee$

where σ = stress

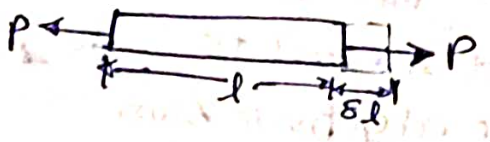
e = strain

E = modulus of elasticity or Young's modulus

Modulus of elasticity (E) :-

$$E = \frac{\text{stress}}{\text{strain}}$$ (Tensile or compressive)

$$E = \frac{\sigma}{e} = \frac{\frac{P}{A}}{\frac{\delta l}{l}}$$

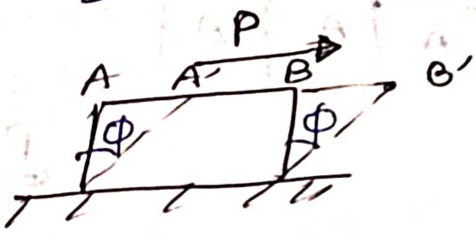


$$\boxed{E = \frac{Pl}{A \cdot \delta l}} \quad -(1)$$

l : length of bar
δl : change in length of bar
A : Area of cross section
P : Pulling force.

Modulus of Rigidity (G) or (C)

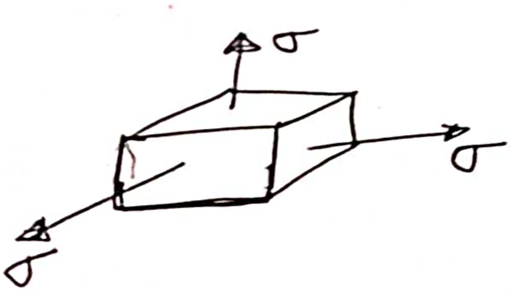
$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$



⇒ Hence it is defined as the ratio of shear stress to the shear strain

Bulk modulus (K)

⇒ It is the ratio of direct stress to the volumetric strain
It is denoted by K.

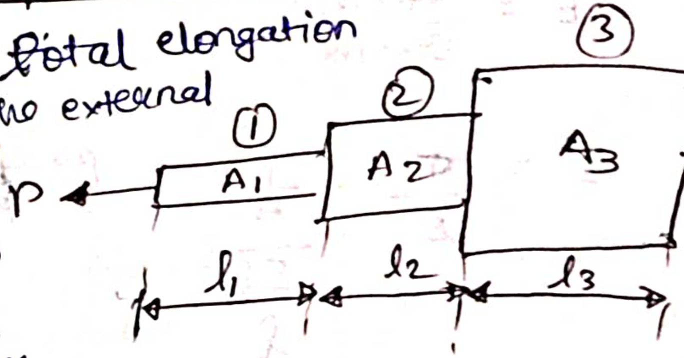


$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$K = \frac{\sigma}{e_v} \quad \text{where, } e_v = \frac{\Delta V}{V}$$

Principal of Superposition :-

"It states that the total elongation of a bar under the external load is equal to the algebraic sum of the elongation of each part of bar.



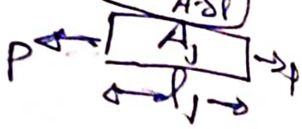
$E = \frac{\sigma}{e}$

$= \frac{P/A}{\delta l / l}$

$E = \frac{P \cdot l}{A \cdot \delta l}$

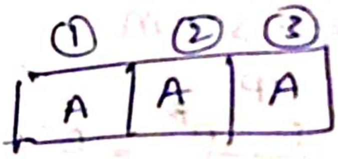
$$\boxed{(\delta l)_{bar} = \delta l_1 + \delta l_2 + \delta l_3}$$

$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

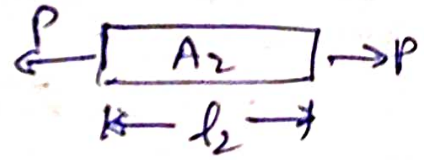


$$\delta l_{bar} = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

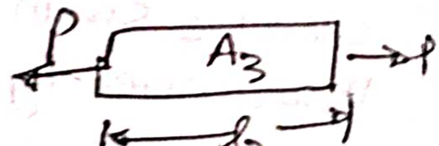
$\delta l = \frac{P \cdot l}{AE}$



$$\delta l_1 = \frac{Pl_1}{A_1 E}$$



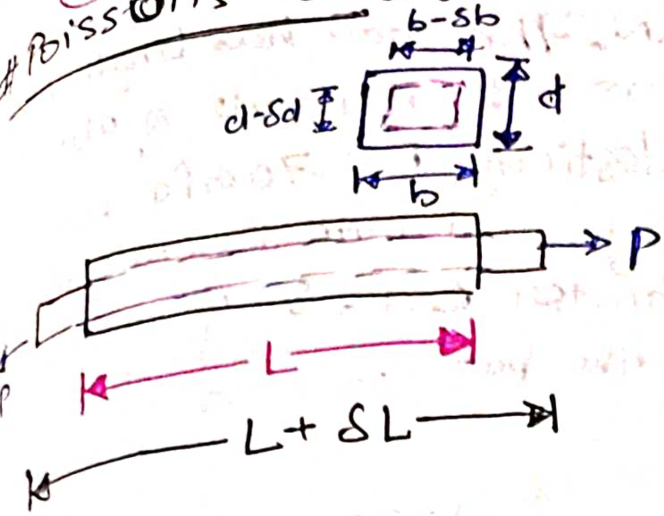
$$\delta l_2 = \frac{Pl_2}{A_2 E}.$$



$$\delta l_3 = \frac{Pl_3}{A_3 E}$$

(Imp)

Poisson's Ratio :-



① Linear strain (axial)

Deformation per unit length in the direction of load.

$$e = \frac{\delta L}{L}$$

② lateral strain

The strain at right angles (⊥) to the direction of applied load is called lateral strain.

$$e = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$

Poisson's Ratio

when a material is loaded within the elastic limit, the ratio of lateral strain to the linear strain is called Poisson's Ratio (μ or $\frac{1}{m}$)

Mathematically,

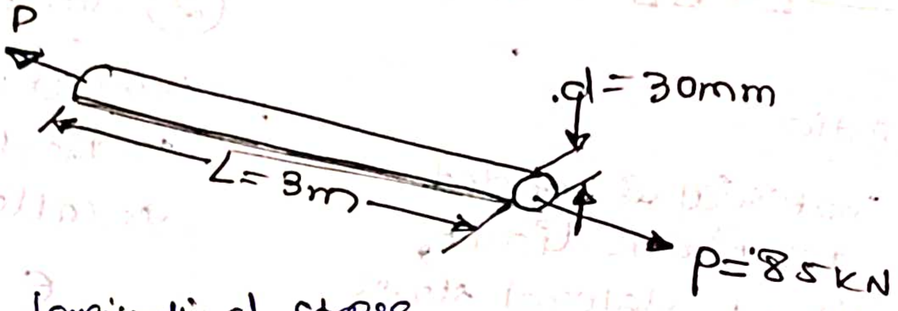
$$\boxed{\mu = \frac{\text{Lateral strain}}{\text{Linear strain}}} = \frac{1}{m}$$

$\Rightarrow$ $\boxed{\text{Lateral strain} = \mu \times \text{Linear strain}}$

Note:- μ varies b/w 0 to 0.5.

$$\mu = \frac{\delta d/d}{\delta L/L}$$

Q. A prismatic bar of circular CS is loaded by tensile forces P = 85 kN. The bar has length L = 3 m & diameter d = 30 mm. It is made of aluminium with modulus of elasticity E = 70 GPa and Poisson's ratio $\nu = \frac{1}{3}$. Calculate the elongation δ, the decrease in diameter Δd, & the increase in volume ΔV of the bar.



Longitudinal stress

$$\sigma = \frac{P}{A} = \frac{85\,kN}{\frac{\pi(30)^2}{4}} = 120.3\,MPa$$

$E = 70\,GPa$

Linear strain

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E} = \frac{120.3\,MPa}{70\,GPa}$$

$$\boxed{e = 0.001718}$$

Lateral strain from Poisson's Ratio

$$\mu = \frac{\text{lateral strain}}{\text{Linear strain}}$$

$$e_{lateral} = -\mu \times e$$

$$= \frac{-1}{3} \times 0.00178$$

$$= -0.0005726$$

$$\frac{\delta l}{l} = e$$

$$\delta l = e\,l$$

$$= (0.01718)(3)$$

$$\boxed{\delta l = 5.15\,mm}$$

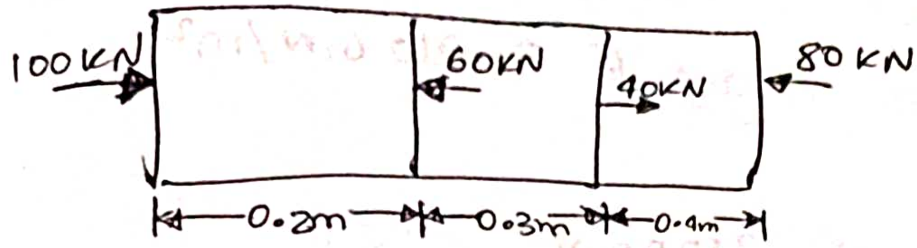
$$\Delta d = e_{lateral} \times d = (0.0005726)\,30\,mm$$

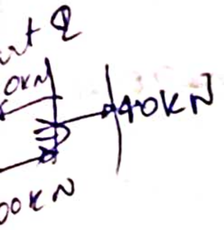
$$= 0.01717\,mm$$

$$\Delta V = V_0\,e(1 - 2\nu)$$

$$= \left(\frac{\pi}{4}\right)(30)^2\,(3)\,(0.001718)\left(1 - \frac{2}{3}\right) = 1210\,mm^3$$

Determine the change in length of bars.
The bar is 200mm in diameter. Take $E = 200 GPa$





(On the line + left side)

$$\delta L = \frac{PL}{AE}$$

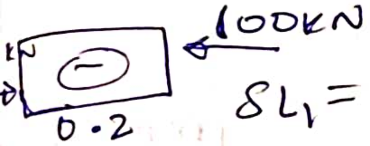
$D = 0.2m$

$A_1 = A_2 = A_3 = A$

$$Ar.\ of\ CS,\ A = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4}(0.2)^2 \Rightarrow A = (0.0314)\,m^2$$

$$\boxed{A = 0.0314\,m^2}$$



$$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{100 \times 0.2}{(0.0314)(200 \times 10^6)}$$

$$\boxed{\delta L_1 = 3.18 \times 10^{-6}\,m}$$

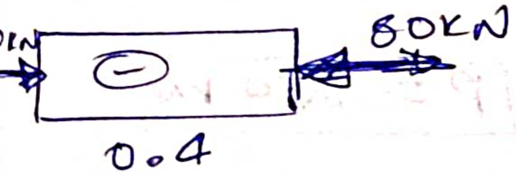
$E = 200 GPa$

$$E = 200 \times 10^9 \frac{N}{m^2}$$

$$E = 200 \times 10^6 \frac{KN}{m^2}$$

$$\delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{40 \times 0.3}{(0.0314)\,200 \times 10^6}$$

$$\boxed{\delta L_2 = 1.91 \times 10^{-6}\,m}$$



$$\delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{80 \times 0.4}{(0.0314) \times 200 \times 10^6}$$

$$= 5.09 \times 10^{-6}\,m$$

Total elongation of bar,

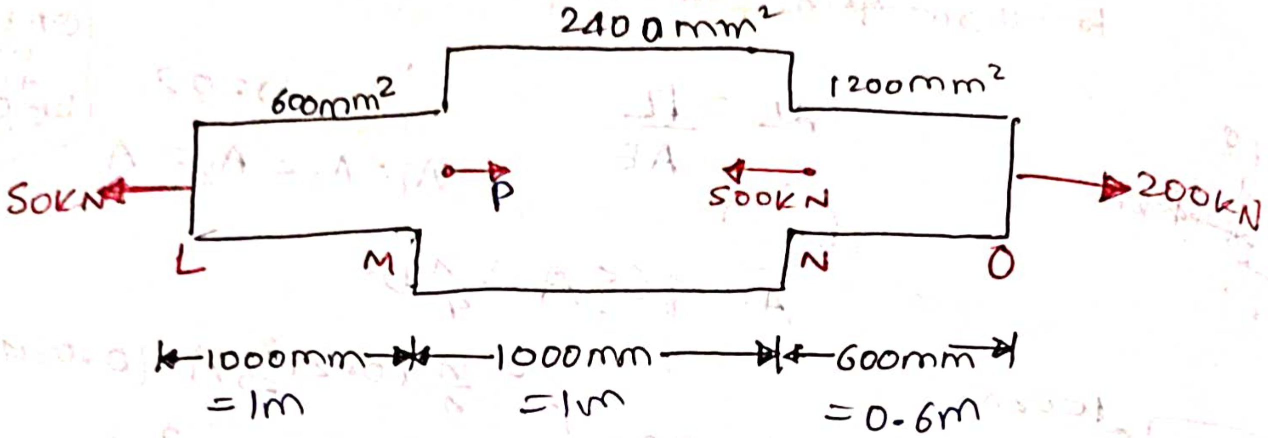
$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$\boxed{\delta L = -10.18 \times 10^{-6}\,m} = -0.01018\,mm$$

sign represents that bar

Q. A bar is subjected to loads as shown in figure. calculate (i) Force P for equilibrium
(ii) Total elongation of bar

Take $E = 210\ GN/m^2$



$A_1 = 600\,mm^2 = 600 \times 10^{-6} m^2$

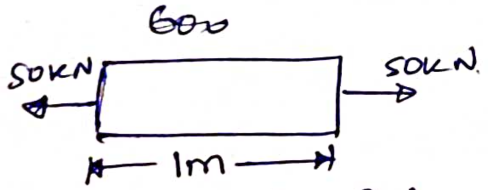
$A_2 = 2400\,mm^2 = 2400 \times 10^{-6} m^2$

$A_3 = 1200\,mm^2 = 1200 \times 10^{-6} m^2$

$1mm = 10^{-3} m$

$1mm = 10^{-6} m^2$

$E = 200\,G\frac{N}{m^2} = 200 \times 10^{6} \frac{kN}{m^2}$



(i) For eqm of bar

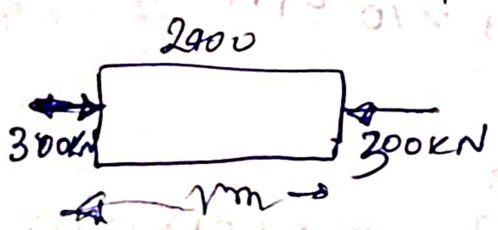
$\Sigma F_x = 0$

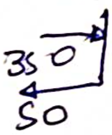
$200 - 500 + P - 50 = 0$

$\boxed{P = 350\ kN}$

$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{50 \times 1}{600 \times 10^{-6} \times 200 \times 10^{6}}$

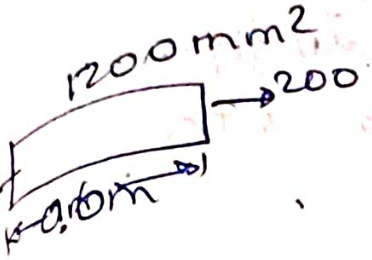
$\boxed{\delta l_1 = 3.968 \times 10^{-4} m}$





$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{300 \times 1}{2400 \times 210 \times 10^{6}}$

$\delta l_2 = 5.95 \times 10^{-4} m$



$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{200 \times 0.6}{1200 \times 10^{-6} \times 210 \times 10^{6}}$$

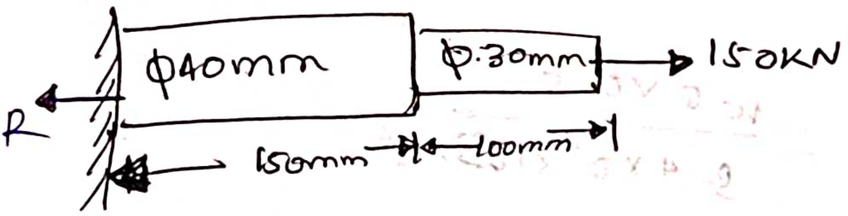
$$\delta l_3 = 4.762 \times 10^{-4}\ m$$

$$(\delta l)_{bar} = \delta l_1 - \delta l_2 + \delta l_3$$

$$= (3.968 - 5.95 + 4.762) \times 10^{-4}$$

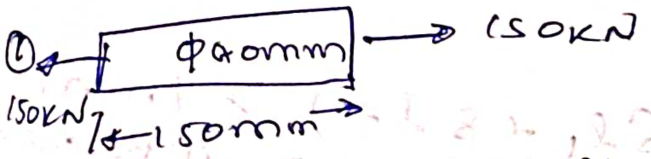
$$\boxed{(\delta l)_{bar} = 2.078 \times 10^{-4}\ m}$$

Determine total extension of bar.
Take $E = 2.5 \times 10^{5}\ N/mm^2$.



$\Sigma F_x = 0 \quad R - 150 = 0$

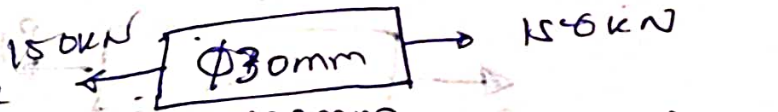
$\boxed{R = 150KN}$



$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{150 \times 10^{3} \times 150}{1256.64 \times 2.5 \times 10^{5}}$$

$\delta l_1 = 0.0716\ mm$

$A_1 = \frac{\pi}{4}(40)^2 = 1256.64\ mm^2$



$A_2 = \frac{\pi}{4}(30)^2 = 706.86\ mm^2$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{150 \times 10^{3} \times 100}{706.86\ mm^2 \times 2.5 \times 10^{5}}$$

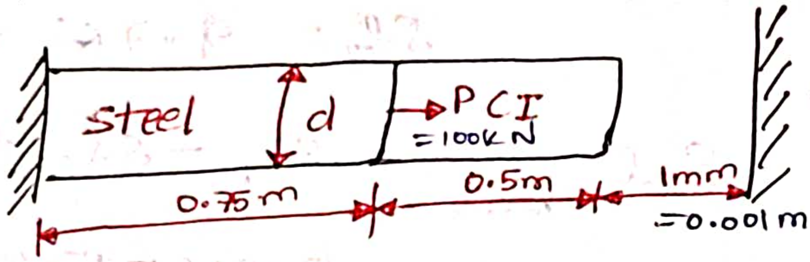
$= 0.0849\ mm$

$(\delta l)_{bar} = \delta l_1 + \delta l_2$

$= 0.0716 + 0.0849$

$\boxed{\delta l = 0.1565\ mm}$

Q. A compound bar is show in fig has a gap of 1mm. If $E_s = 200\,GPa$ & $E_{CI} = 105\,GPa$. Find stresses in the bar.



$P = 100kN$

$d = 20mm = 0.020m$

$\sigma_s = ?$

$\sigma_{CI} = ?$

Area, $A = \frac{\pi d^2}{4} = \frac{\pi}{4}(20)^2$

$= 3.14 \times 10^{-4}\,m^2$

$\delta l_1 = \frac{P_1 l_1}{A_1 E}$

$= \frac{100 \times 0.75}{(3.14 \times 10^{-4} \times 200 \times 10^6)}$

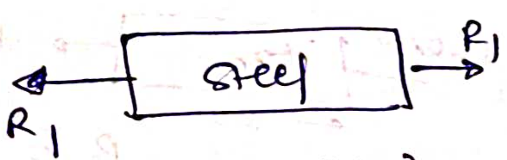
$\delta l_1 = 1.19 \times 10^{-3}\,m$

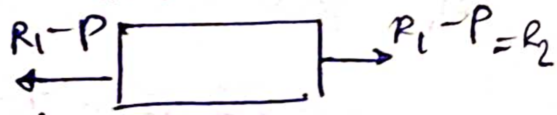
$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{100 \times 0.5}{3.14 \times 10^{-4} \times 105 \times 10^6}$

$\delta l_2 = 1.52 \times 10^{-3}\,m$

$(\delta l_1 + \delta l_2) > 1mm$

∴ Reaction will be developed in the bar due to wall.





$(\delta l)_{steel} + (\delta l)_{CI} = 0.001$

$\frac{R_1 \times 0.75}{3.14 \times 10^{-4} \times 200 \times 10^6} + \frac{(R_1 - 100) \times 0.5}{3.14 \times 10^{-4} \times 105 \times 10^6} = 0.001$

$$\frac{0.75 R_1}{200} + \frac{0.5 R_1}{105} - \frac{50}{105} = 0.001 \times 3.14 \times 10^{-4} \times 10^6$$

$$R_1 = 92.83 kN$$

① Stress in Steel bar

$$\sigma_s = \frac{R_1}{A} = \frac{92.83}{3.14 \times 10^{-4}}$$

$$\sigma_s = 295636.94 \; kN/m^2$$

$$\sigma_s = 295.636 \; MPa$$

② stress in C. I. bar

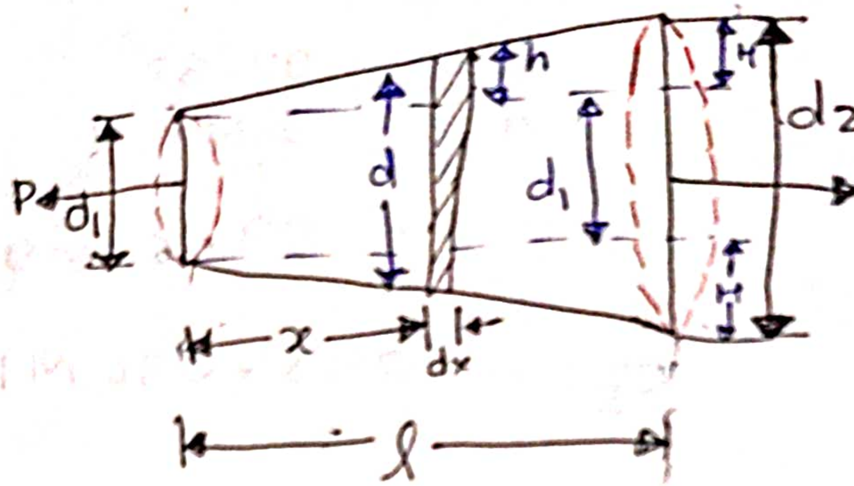
$$\sigma_{CI} = \frac{R_1 - P}{A} = \frac{(92.83 - 100)}{3.14 \times 10^{-4}}$$

$= -22083.4$ mPa ~~(comp~~

$= 22.834$ mPa (compressive)

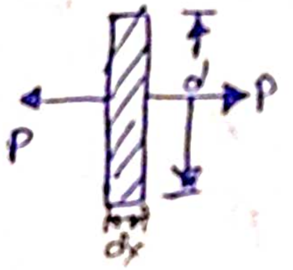
Q. Derive an expression for elongation of circular tapered bar.

Soln :- Elongation of a tapered bar (circular)



$d_2 = d_1 + 2H$

$H = \frac{d_2 - d_1}{2}$



From Similar Δ

$\frac{h}{x} = \frac{H}{l} = \frac{d_2 - d_1}{2l}$

$d = d_1 + 2h$

$\boxed{h = \frac{(d_2 - d_1)x}{2l}}$

or $d = d_1 + \frac{(d_2 - d_1)}{l}x$ —①

$d = d_1 + kx$ → where $k = \frac{(d_2 - d_1)}{l}$

Area of strip $= \frac{\pi}{4} d^2 = \frac{\pi}{4}(d_1 + kx)^2$

stress in strip $= \frac{P}{A} = \frac{P}{\frac{\pi}{4}(d_1 + kx)^2}$

$\sigma = \frac{4P}{\pi(d_1 + kx)^2}$

strain in strip (e) $= \frac{\sigma_s}{E}$.

$$\frac{\delta l}{dx} = \frac{4P}{\pi E (d_1+kx)^2}$$

$$\delta l_s = \frac{4P}{\pi E (d_1+kx)^2} dx$$

$$(\delta l)_{bar} = \int_0^l (\delta l)_{strip} = \int_0^l \frac{4P}{\pi E (d_1+kx)^2} dx$$

$$= \frac{4P}{\pi E} \int_0^l (d_1+kx)^{-2} dx$$

$$= \frac{4P}{\pi E} \left[\frac{(d_1+kx)^{-1}}{-1k} \right]_0^l$$

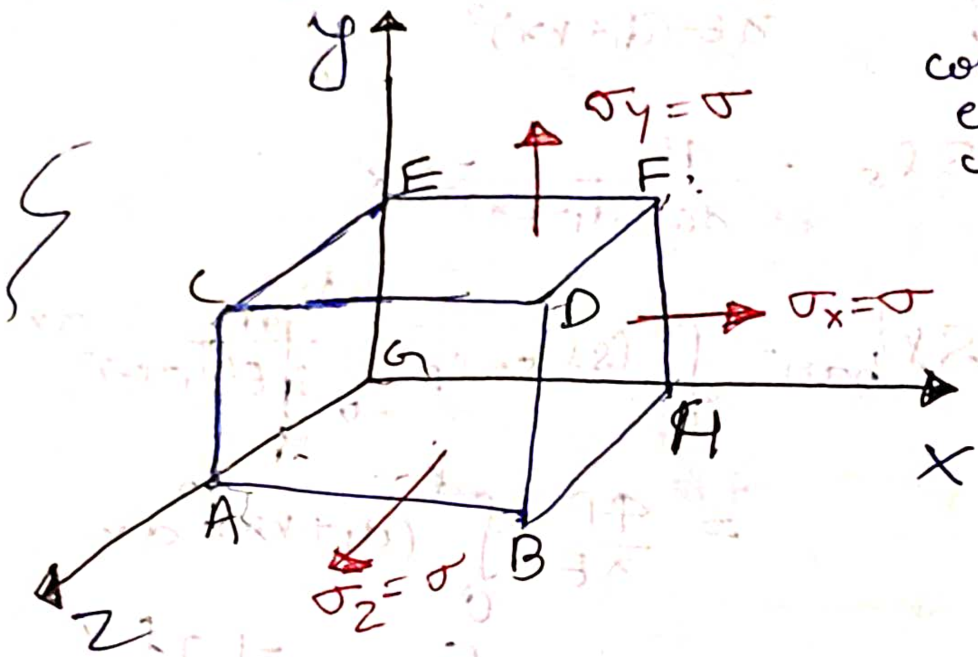
$$= -\frac{4P}{\pi E k} \left[\frac{1}{(d_1+kl)} - \frac{1}{d_1} \right]$$

$$= -\frac{4P \cdot l}{\pi E (d_2-d_1)} \left[\frac{1}{d_1 + \frac{(d_2-d_1)}{l} \times l} - \frac{1}{d_1} \right]$$

$$= \frac{4P \cdot l}{\pi E (d_2-d_1)} \left[\frac{1}{d_1} - \frac{1}{d_2} \right]$$

$$\boxed{(\delta l)_{bar} = \frac{4Pl}{\pi E d_1 d_2}}$$

Q. Find Relation b/w Young's Modulus (E) & Bulk modulus (K).



consider a cubical element as shown where $\sigma_x = \sigma_y = \sigma_z = \sigma$

Let Poisson's Ratio $= \mu$

$$K = \frac{\text{Direct stress } (\sigma)}{e_v}$$

$$e_v = e_x + e_y + e_z$$

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E}$$

∴ strain in x-dirn

lateral strain in y dirn $= \frac{\sigma_y}{E} = \mu\frac{\sigma}{E}$ & Linear strain

lateral strain in z-dirn $\frac{\sigma_z}{E} = \mu\frac{\sigma}{E}$

$$e_x = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\boxed{e_x = \frac{\sigma}{E}(1-2\mu)} \quad -(1)$$

strain in y-dirn

$$e_y = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\boxed{e_y = \frac{\sigma}{E}(1-2\mu)} \quad -(2)$$

Strain in z-dirn

$$e_z = \frac{\sigma}{E} - \mu\frac{\sigma}{G} - \mu\frac{\sigma}{E}$$

$$\boxed{e_z = \frac{\sigma}{E}(1-2\mu)} \quad -(3)$$

$$e_v = 3\frac{\sigma}{E}(1-2\mu) \quad \left[\text{Adding (1), (2) \& (3)}\right.$$

$$E = 3\frac{\sigma}{e_v}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

or $\boxed{K = \frac{E}{3(1-2\mu)}}$

Q. Find relation b/w Three elastic constants [Young's modulus (E), modulus of Rigidity (G) & Bulk modulus (K)]

Prove that $\boxed{E = \dfrac{9KG}{3K+G}}$

Soln :- We know that,

$$E = 2G(1+\mu) \quad \text{—(1)}$$

$$E = 3K(1-2\mu) \quad \text{—(2)}$$

from (1) $\mu = \dfrac{E}{2G} - 1$

substitute value of μ in eqn (2)

$$E = 3K\left(1 - \frac{E}{G} + 2\right)$$

$$= 3K\left(3 - \frac{E}{G}\right)$$

$$E = 3K\left(\frac{3G-E}{G}\right)$$

$$E = \frac{9KG - 3KE}{G}$$

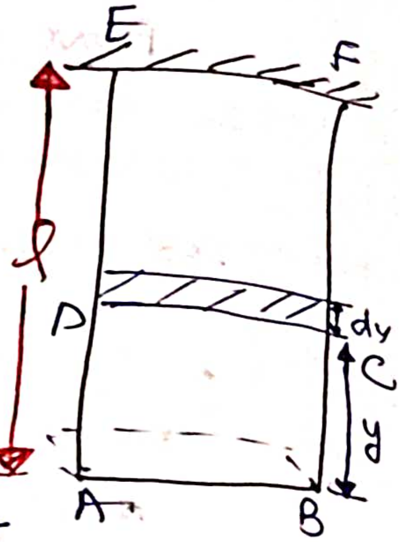
$$EG = 9KG - 3KE$$

$$E(3K+G) = 9KG$$

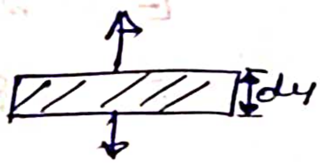
$$\boxed{E = \frac{9KG}{3K+G}}$$

Elongation of a uniform bar due to its self weight :-

Soln :- consider a uniform bar of length l, cross-sectional area A and density ρ



$P =$ wt of Part ABCD
$= A \cdot y \cdot \rho g$



$P =$ wt of Part ABCD

Elongation in Strip $(\delta l) = \dfrac{Pl}{AE}$

$$(\delta l) = \frac{\cancel{A} y \rho g \, dy}{\cancel{A} E}$$

$$(\delta l)_{strip} = \frac{\rho g y \, dy}{E}$$

Total elongation of bar

$$(\delta l)_{bar} = \int_0^l (\delta l)_{strip}$$

$$= \frac{\rho g}{E} \int_0^l y \, dy$$

$$= \frac{\rho g}{E} \left[\frac{y^2}{2}\right]_0^l$$

$$\boxed{(\delta l)_{bar} = \frac{\rho g l^2}{2E}}$$

In terms of wt of bar

wt of bar $W = \rho A l g$

$$\rho = \frac{W}{Alg}$$

$$(\delta l)_{bar} = \frac{W}{A\cancel{l}\cancel{g}} \times \frac{\cancel{g} l^{\cancel{2}}}{2E}$$

$$\boxed{(\delta l)_{bar} = \frac{Wl}{2AE}}$$

Expression for strain energy when load is applied gradually :-

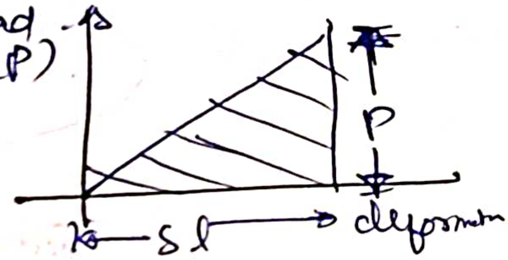
- The energy which is stored in a body due to straining effect is known as strain energy (U)
- This energy is equal to the work done by the applied load in stretching the body.

strain energy = work done

$= \frac{1}{2} \times \delta l \times P$

$= \frac{1}{2} \times \left(\frac{Pl}{AE}\right) \times \sigma A)$ P = 0 ---- 100KN

$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times A \times l$



$$\boxed{U = \frac{1}{2} \frac{\sigma^2}{E} \times V}$$

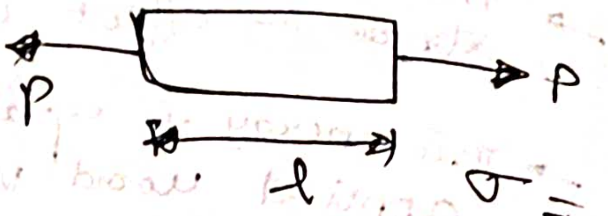
V : volm of body

σ = stress

E = modulus of elasticity (young's modulus)

Expression for stress when load is applied suddenly.

Strain energy = work done = Area under curve



$$\frac{\sigma^2}{2E}\times V = P\times\delta l \qquad \sigma = \frac{P}{A}$$

$$\frac{\sigma^2}{2E}\times A\times l = P\times\frac{Pl}{AE}$$

$$\frac{\sigma^{\cancel{2}}}{2\cancel{E}}\times A\times\cancel{l} = P\times\cancel{\frac{\sigma}{E}}\,\cancel{l}$$

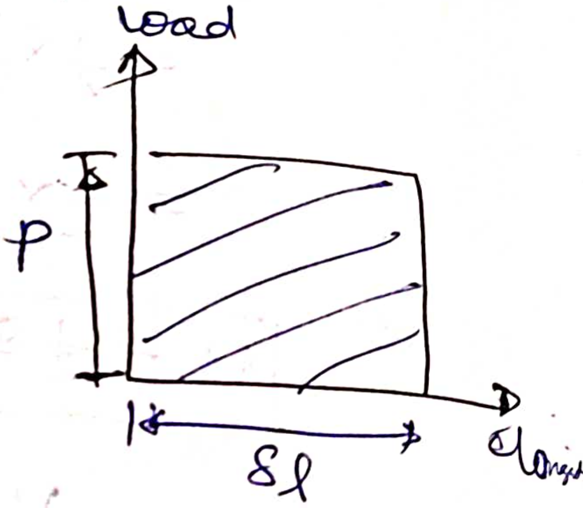
$$\boxed{\sigma = \frac{2P}{A}}$$

Stress will be doubled when

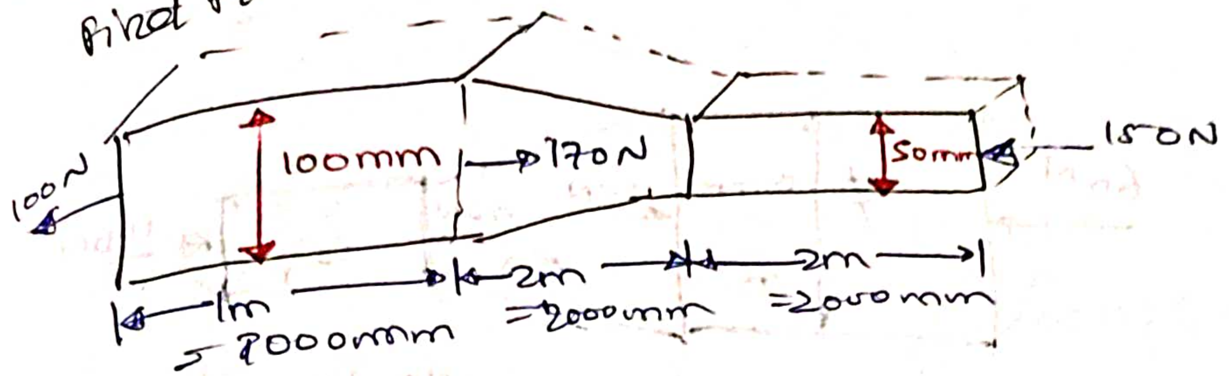
when load is applied gradually

P = 0 - - - 100 kN

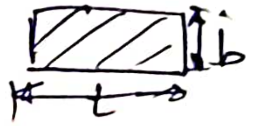
P = 100 kN



Q. A brass bar having uniform thickness 25mm is subjected to axial forces as shown in figure. Find total elongation of bar. Take $E = 1\times10^5\ N/mm^2$



$(\delta l)_{bar} = ?$

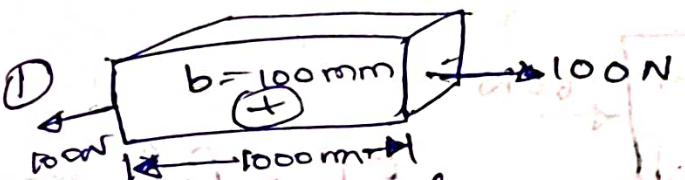


$t = 25mm$

$b_1 = 100mm$

$b_2 = 50mm$

$A = b \times t$

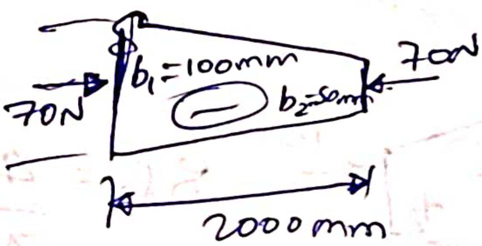
① 

$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{100 \times 1000}{100\times25\times1\times10^5}$$

$$\boxed{\delta l_1 = 4\times10^{-4}\ mm}$$

$b_1 > b_2$

②
170N
100N



$$\boxed{\delta l = \frac{Pl}{t(b_1-b_2)E}\ln\frac{b_1}{b_2}}$$

Elongation of rectangular taper bar

$$\delta l_2 = \frac{70\times2000}{25(50)\times1\times10^5}\ln\left(\frac{2}{1}\right)$$

$$\boxed{\delta l_2 = 7.76\times10^{-4}\ mm}$$

③ 

$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{150\times2000}{50\times25\times1\times10^5}$$

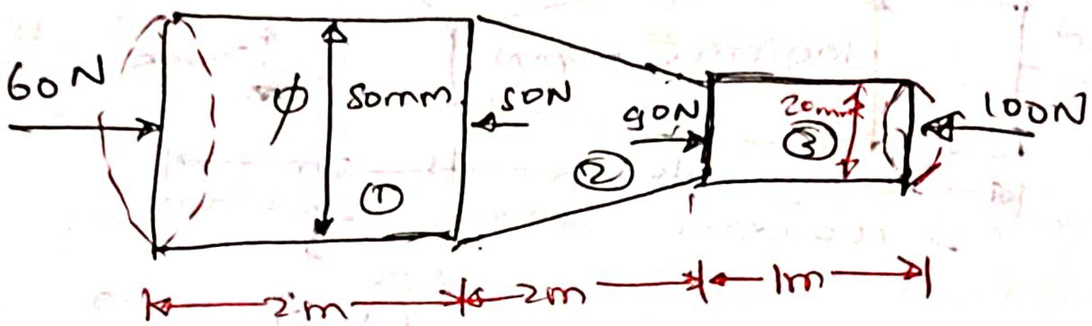
$$\boxed{\delta l_3 = 2.4\times10^{-3}\ mm}$$

$$(\delta l)_{bar} = \delta l_1 - \delta l_2 - \delta l_3$$
$$= 4\times10^{-4} - 7.76\times10^{-4} - 2.4\times10^{-3}$$

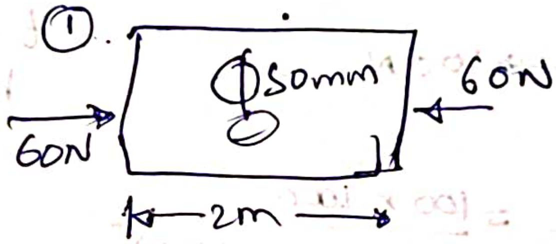
$$\boxed{(\delta l)_{bar} = -2.776\times10^{-3}\ mm}$$

-ve sign represents bar will compressed.

Q. Find total elongation of bar shown below take $E = 200\ GPa$



①.



$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{60 \times 2}{\frac{\pi}{4}(0.050)^2 \times 200 \times 10^9}$$

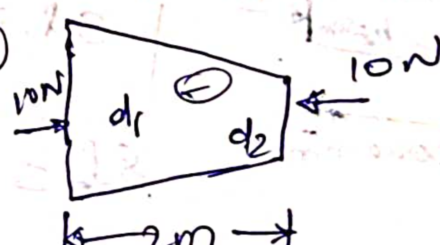
$d_1 = 50mm = 0.050\ m$

$d_2 = 20mm = 0.020\ m$

$E = 200\ GPa \times 10^9 \frac{N}{m^2}$

$$\delta l_1 = 3.055 \times 10^{-7} m$$

②

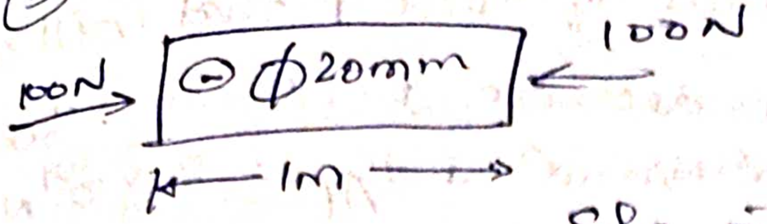


$$\delta l_2 = \frac{4Pl}{\pi E d_1 d_2}$$

$$= \frac{4 \times 10 \times 2}{\pi \times 200 \times 10^9 \times 0.05 \times 0.02}$$

$$\delta l_2 = 1.273 \times 10^{-7} m$$

(3)



$$\delta l_3 = \frac{P_3 l_3}{A_3 \cdot E}$$

$$= \frac{100 \times 1}{\frac{\pi}{4}(0.020)^2 \times 200 \times 10^9}$$

$$\delta l_3 = 1.59 \times 1 \times 10^{-6} m$$

$$\delta l = -\delta l_1 - \delta l_2 - \delta l_3$$

$$= 3.055 \times 10^{-7} - 1.273 \times 10^{-7}$$

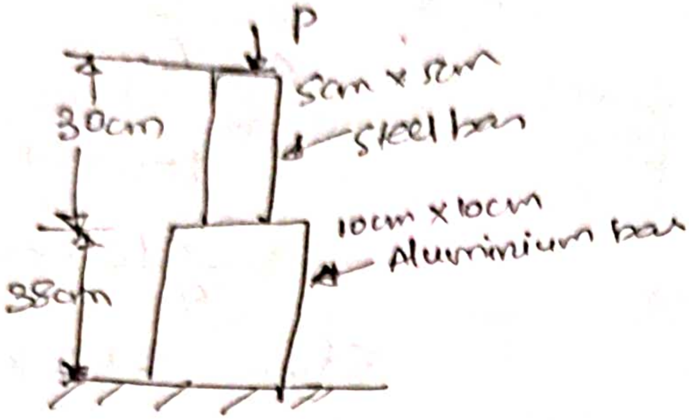
$$- 15.91 \times 10^{-7} m$$

$$= -2.02 \times 10^{-6} m$$

$$\delta l = 2.02 \times 10^{-3} mm$$

$$\boxed{\delta l = 0.00202 mm}$$

Q. Find magnitude of force P that will cause total length decrease 0.25 mm. take $E_s = 2.1 \times 10^5$ N/mm² & $E_{Al} = 7 \times 10^4$ N/mm²



given $\delta l = 0.25$ mm (Reduction)

$$\delta l = \delta l_1 + \delta l_2$$

$$0.25 = \frac{P_1 l_1}{A_1 E_s} + \frac{P_2 l_2}{A_2 E_{Al}}$$

$$0.25 = \frac{P \times 30 \times 10}{(50 \times 50) \times 2.1 \times 10^5} + \frac{P \times 38 \times 10}{(100 \times 100) \times 7 \times 10^4}$$

$$0.25 = P\left[5.714 \times 10^{-7} + 5.42 \times 10^{-7}\right]$$

$$P = \frac{0.25}{11.142 \times 10^{-7}}$$

$$P = 2.2437 \times 10^5 \text{ N}$$

$$P = 224.37 \text{ KN}$$

Q. A bar of 24 mm diameter & 400 mm length is acted upon an axial load of 38 kN. The elongation of bar & change in diameter are measured as 0.165 mm & 0.031 mm respectively
Determine (a) Poisson's ratio
(b) Value of 3 moduli

Soln:

$$A = \frac{\pi}{4}(24)^2 = 144\pi \, mm^2$$

$$\sigma = \frac{P}{A} = \frac{38000}{144\pi}$$

$$\boxed{\sigma = 84 \, MPa}$$

Poisson's Ratio, $\mu = \frac{\text{linear strain}}{\text{lateral strain}}$

$$E = \frac{\sigma}{e} = \frac{84}{0.165/400}$$

$$= 203.636 \, MPa$$

$$E = 2G(1+\mu) = 3K(1-2\mu)$$

$$G = \frac{E}{2(1+\mu)} \qquad K = \frac{E}{3(1-2\mu)}$$

$$G = \frac{203636}{2(1+0.313)} \qquad K = \frac{203636}{3(1-2\times 0.313)}$$

$$G = 77.546 \, MPa \qquad K = 181.490 \, MPa$$

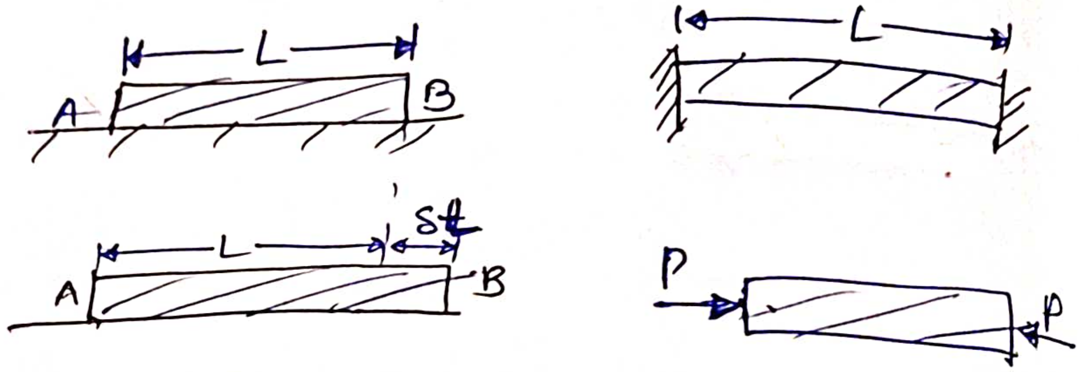
$$\mu = \frac{\partial L/L}{\partial d/d}$$

$$\mu = \frac{0.165/400}{0.031/24} =$$

$$\mu = 0.313$$

Thermal Stress

↳ when temperature of a material is changed, its dimensions change. If this change in dimension is prevented, then a stress is setup in the material, which is called a Thermal stress.



change in length: $\Delta L = \alpha \Delta T L$

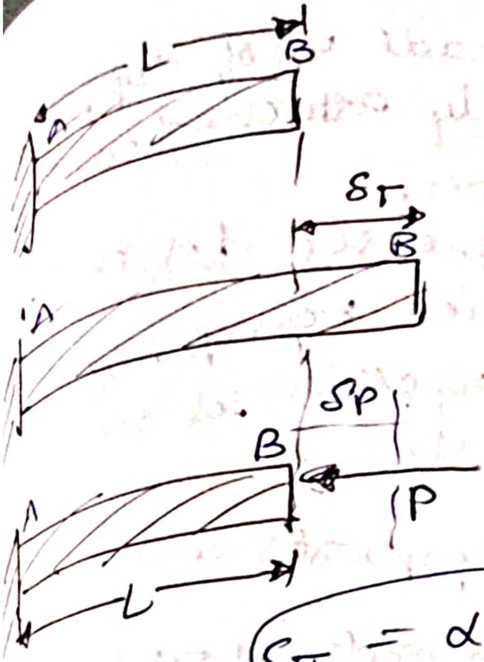
strain, $e = \dfrac{\Delta L}{L} = \alpha \Delta T$

Stress, $\sigma_T = e E$

$$\boxed{\sigma_T = E \alpha \Delta T}$$

→ Thermal stress

↳ compressive in nature in this case.



$$\delta_T = \alpha(\Delta T)L$$

Thermal stress

$$\delta_P = \frac{PL}{AE}$$

Due to redundant

Total deformation = 0

$$\delta = \delta_T + \delta_P$$

$$\delta = \alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$P = -AE\alpha(\Delta T)$$

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

Q. A composite bar made up of copper, steel & brass is rigidly attached to end supports as shown. Determine the stresses in three portions when temp. of composite system is raised by 70°C if

(i) the supports are rigid

(ii) the supports yield by 0.6mm

$E_c = 100\,GPa$

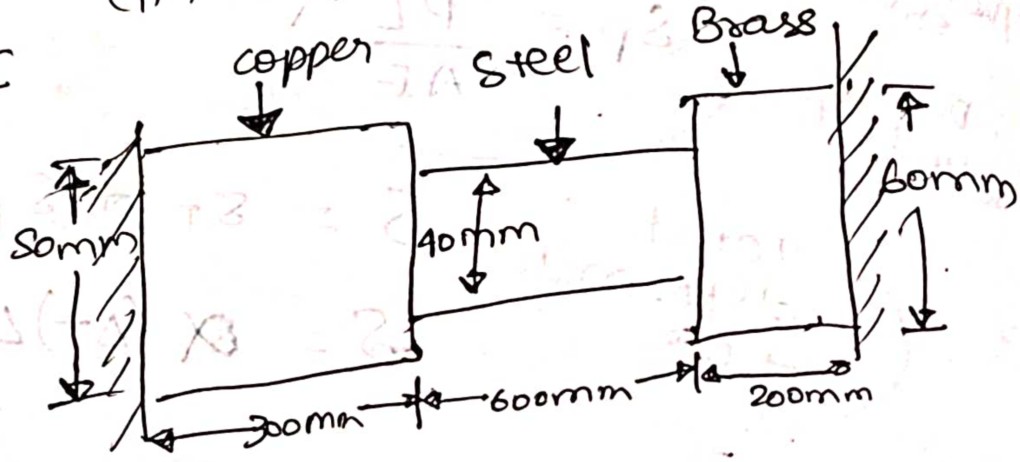
$E_s = 200\,GPa$

$E_b = 95\,GPa$

$\alpha_c = 18\times10^{-6}/°C$

$\alpha_s = 11\times10^{-6}/°C$

$\alpha_b = 19\times10^{-6}/°C$



Sol^n $\Delta T = 70°C$

(1) if supports are rigid

$$A_c = \frac{\pi}{4}(50)^2 = 625\pi\ mm^2$$

$$A_s = \frac{\pi}{4}(40)^2 = 400\pi\ mm^2$$

$$A_b = \frac{\pi}{4}(60)^2 = 900\pi\ mm^2$$

$\sigma_c \cdot A_c = \sigma_s \cdot A_s = \sigma_b \cdot A_b$

$$\sigma_c = \frac{A_b}{A_c}\sigma_b = \frac{900\pi}{625\pi} \times \sigma_b = 1.44\sigma_b$$

$$\sigma_s = \frac{A_b}{A_s}\sigma_b = \frac{900\pi}{400\pi}\sigma_b = 2.25\sigma_b.$$

Elongation in the absence of supports

$$\Delta = \Delta_c + \Delta_s + \Delta_b$$

$$= \alpha_c L_c \Delta t_c + \alpha_s L_s \Delta t_s + \alpha_b L_b \Delta t_b$$

$$= 18 \times 10^{-6} \times 800 \times 70 + 11 \times 10^{-6} \times 600 \times 70 + 19 \times 10^{-6} \times 200 \times 70$$

$$= 1.106\,mm$$

$$\Delta = \frac{\sigma_c L_c}{E_c} + \frac{\sigma_s L_s}{E_s} + \frac{\sigma_b L_b}{E_b}$$

$$\Delta = \frac{1.44\,\sigma_b \times 300}{100000} + \frac{2.25\,\sigma_b \times 600}{205000} + \frac{\sigma_b \times 200}{95000} = 1.106$$

$$\Delta = (0.01302)\,\sigma_b = 1.106$$

$$\sigma_b = 84.95\,MPa$$

$$\sigma_c = 122.33\,MPa$$

$$\sigma_s = 191.73\,MPa$$

When support yields by 0.6 mm

$0.0132\,\sigma_b = 1.106 - 0.6$

$= 0.506$

$\sigma_b = 38.33\,MPa$

$\sigma_c = 55.2\,MPa$

$\sigma_s = 86.24\,MPa$

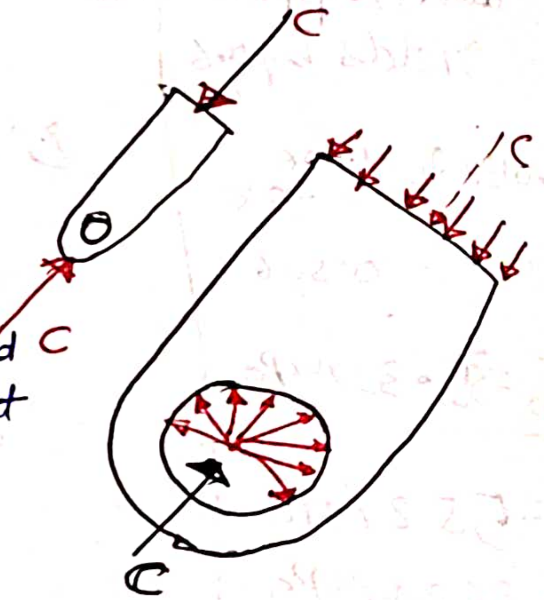
Center of Mass, Centroids & Center of Gravity

Concentrated forces

↳ If dimensions of the contact area is negligible compared to other dimensions of the body → The contact forces may be treated as concentrated forces

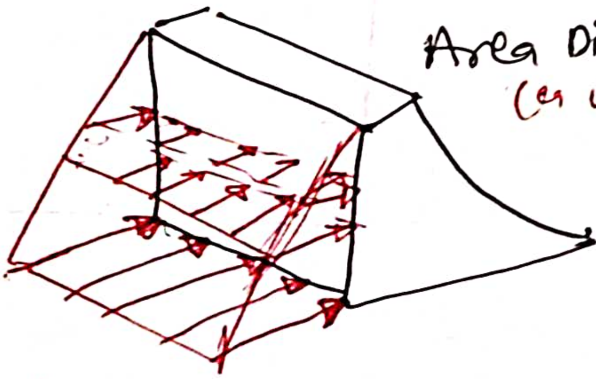
Distributed Forces

If forces are applied over a region whose dimension is not negligible compared with other pertient dimensions
→ Proper distribution of contact forces must be accounted for to know intensity of force at any location.



Line distribution (UDL on beam)

Area Distribution
(ex water pressure)

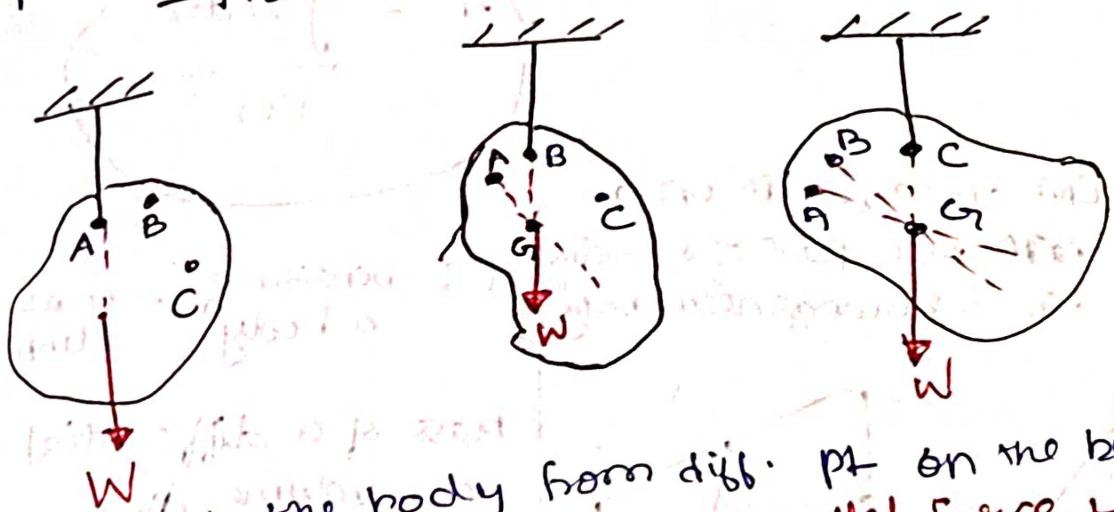


Body Distribution
(ex self weight

Center of Mass

A body of mass m in equilibrium under the action of tension in the cord & resultant W of the gravitational forces acting on all particles of the body

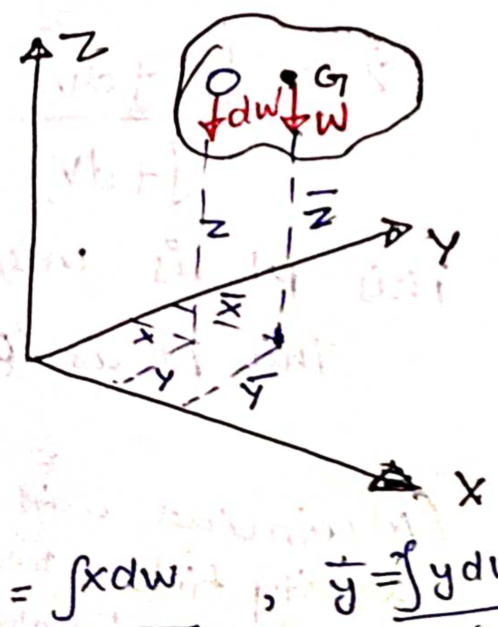
- The resultant is collinear with the cord



suspend the body from diff. pt on the body.

we assume uniform and parallel force field due to the Gravitational attraction of the earth.

The unique Point G is called the Center of Gravity of body (CG)



$$W = \int dW$$

Moment of wt of an element (dW) @ x-axis = y dW

From Principal of Moments

$$\int y\,dW = \bar{y}W$$

$$\bar{x} = \frac{\int x\,dW}{W}, \quad \bar{y} = \frac{\int y\,dW}{W} \quad \bar{z} = \frac{\int z\,dW}{W}$$

$$W = mg$$

$$dW = g\,dm$$

$$\Rightarrow \bar{x} = \frac{\int x\,dm}{m} \Big| \bar{y} = \frac{\int y\,dm}{m} \Big| \bar{z} = \frac{\int z\,dm}{m}$$

In vector notations

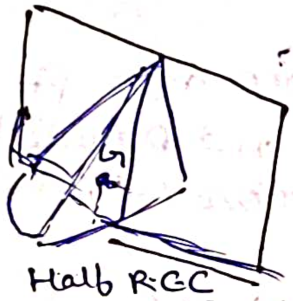
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r}_g = \bar{x}\hat{i} + \bar{y}\hat{j} + \bar{z}\hat{k}.$$

$$\bar{r} = \frac{\int r\,dm}{m}$$

* CM always lie on a line or a plane of symmetry in a homogeneous body.

Right circular cone
CM on central axis



Half RCC
CM on VP of symmetry

ρ : Density of a body = mass per unit volume.

Mass of a differential element of volume dV.

$$dm = \rho\, dV$$

ρ may not be const. throughout the body

$$\bar{x} = \frac{\int x \rho\, dV}{\int \rho\, dV} \cdot \bar{y} = \frac{\int y \rho\, dV}{\int \rho\, dV}$$

$$\bar{z} = \frac{\int z \rho\, dV}{\int \rho\, dV}$$

This pt is independent of g

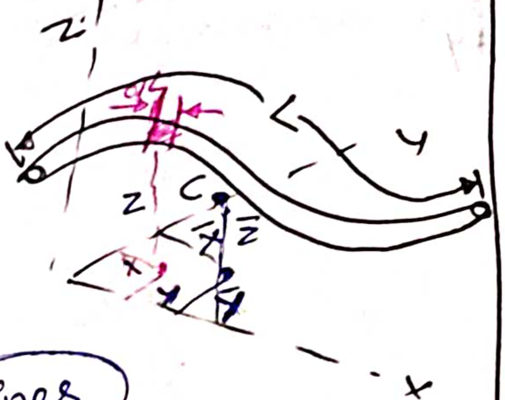
This pt is center of Mass (CM)

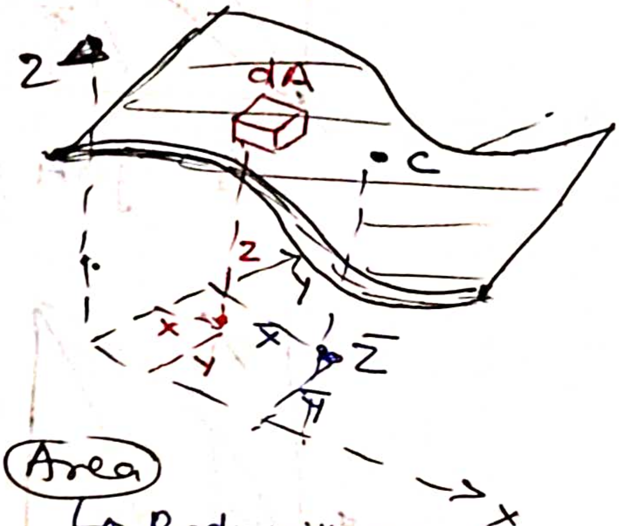
CM coincides with CG as long as gravity field is treated as uniform and parallel.

* CM or CG may lie outside the body.

Centroids of Lines, Areas & Volumes

→ centroid is a geometrical property of a body
when density of a body is uniform throughout
centroid & CM coincide.





Lines

→ slender rod, wire

CS Area = A

ρ & A are const. over L

$dm = \rho A dL$

centroid = CM

$$\bar{x} = \frac{\int x\,dL}{L}$$

$$\bar{y} = \frac{\int y\,dL}{L}$$

$$\bar{z} = \frac{\int z\,dL}{L}$$

Area

→ Body with small but thickness t.

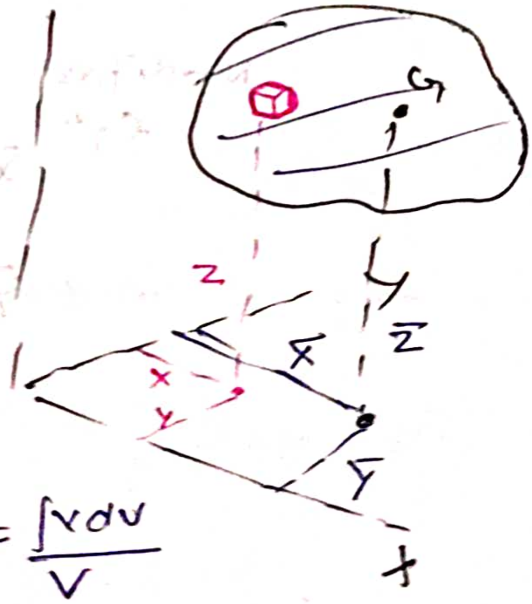
CS Area = A

ρ & A are const over A

$dm = \rho\, t\, dA$. Centroid = CM

$$\bar{x} = \frac{\int x\,dA}{A} \qquad \bar{y} = \frac{\int y\,dA}{A}.$$

$$\bar{z} = \frac{\int z\,dA}{A}$$

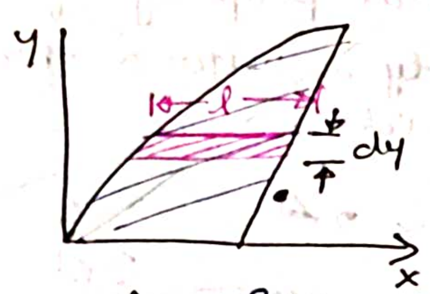


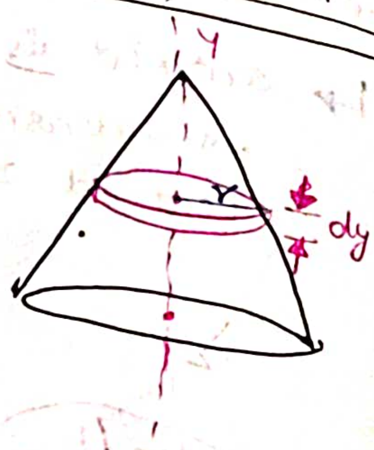
$$\bar{x} = \frac{\int x\,dV}{V}$$

$$\bar{y} = \frac{\int y\,dV}{V}$$

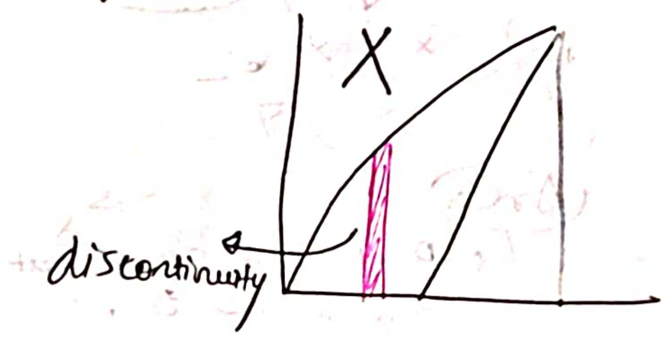
$$\bar{z} = \frac{\int z\,dV}{V}$$

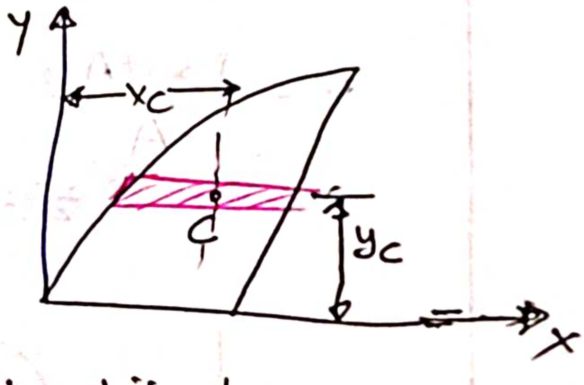
Order of element selected for integration

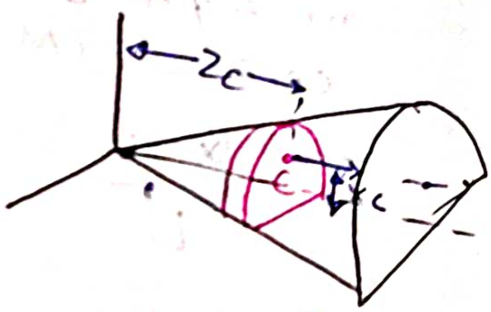




$$A = \int dA = \int l\,dy$$







Modified eq^n

$$\bar{x} = \frac{\int x_c\,dA}{A}$$

$$\bar{y} = \frac{\int y_c\,dA}{A}$$

$$\bar{z} = \frac{\int z_c\,dA}{A}$$

$$\bar{x} = \frac{\int x_c\,dV}{V}$$

$$\bar{y} = \frac{\int y_c\,dV}{V}$$

$$\bar{z} = \frac{\int z_c\,dV}{V}$$

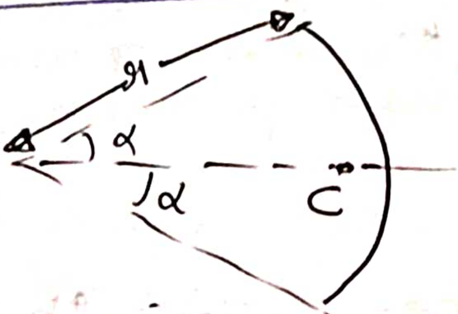
Guidelines for choice of elements for integrations

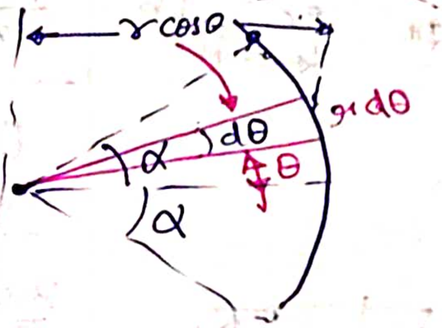
(1) order of element selected for integration
(2) continuity
(3) Discarding higher order terms
(4) choice of coordinates.
(5) centroidal coordinate of differential elements

Centroid of circular arc

$dL = r\,d\theta$

$L = 2\alpha r$





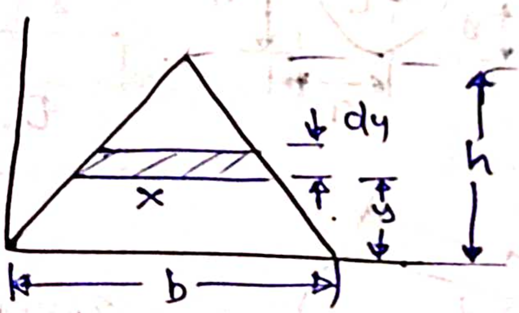
$L x = \int x\,dL$

$2\alpha r x = \int (r\cos\theta)\, r\,d\theta$

$2\alpha r x = 2r^2 \sin\alpha$

$$\boxed{x = \frac{r\sin\alpha}{\alpha}}$$

Centroid of Δ along h from base



$dA = x\,dy$.

$\Rightarrow \frac{x}{(h-y)} = \frac{b}{h}$

total Area $A = \frac{1}{2} bh$ $\quad y = y_c$

$\bar{x} = \frac{\int x_c\,dA}{A}$.

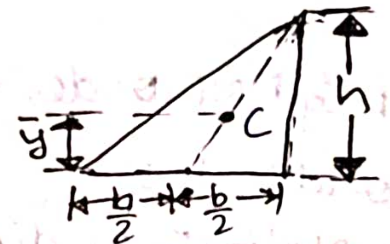
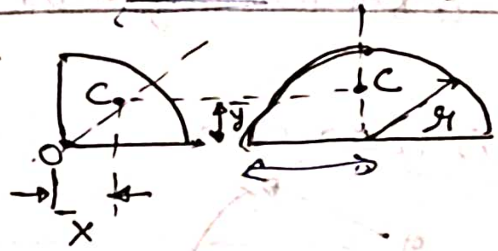
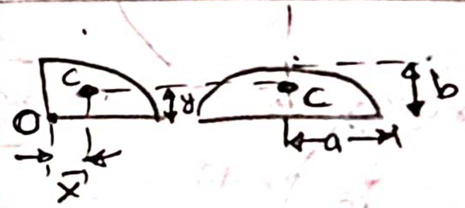
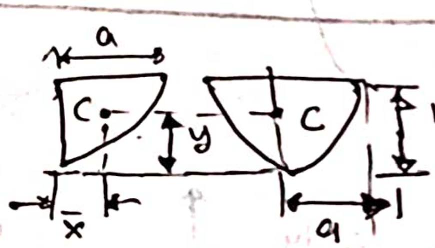
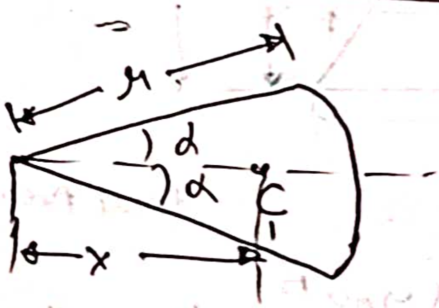
$\bar{y} = \frac{\int y_c\,dA}{A}$

$\bar{z} = \frac{\int z_c\,dA}{A}$

$A\bar{y} = \int y_c\,dA$

$\frac{bh}{2}\bar{y} = \int_0^h y\,\frac{b(h-y)}{y}\,dy = \frac{bh^2}{6}$

$$\boxed{y = \frac{h}{3}}$$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular Area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
semi-circular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
semi elliptical area		0	$\frac{4b}{3\pi}$	$\pi\frac{ab}{2}$
semi Parabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

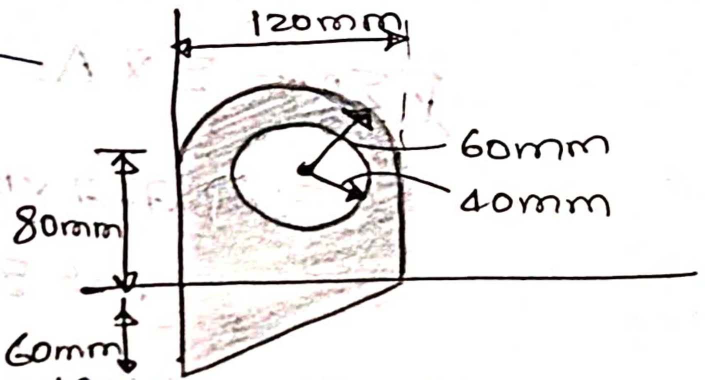
composit...

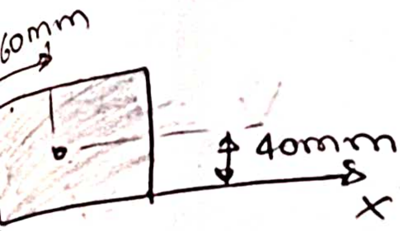
$$\bar{x} = \frac{\Sigma m \bar{x}}{\Sigma m}$$

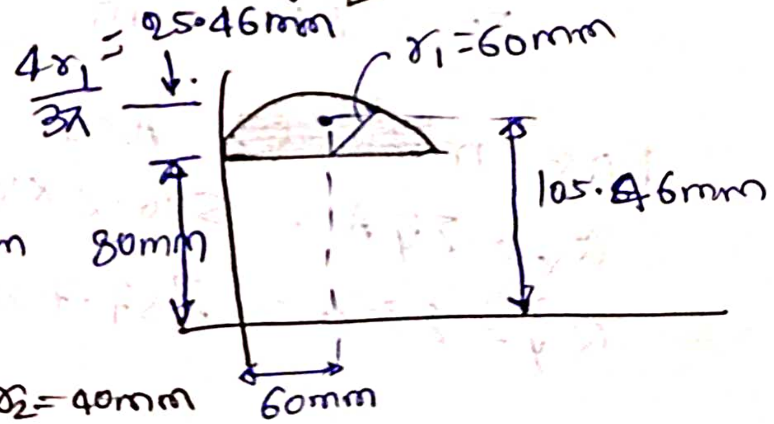
$$\bar{y} = \frac{\Sigma m \bar{y}}{\Sigma m}$$

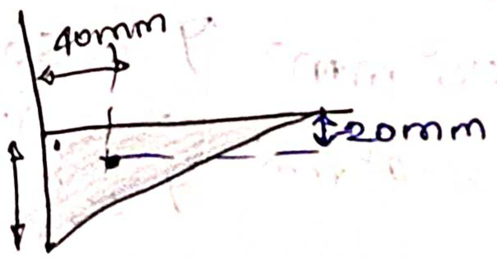
$$\bar{z} = \frac{\Sigma m \bar{z}}{\Sigma m}$$

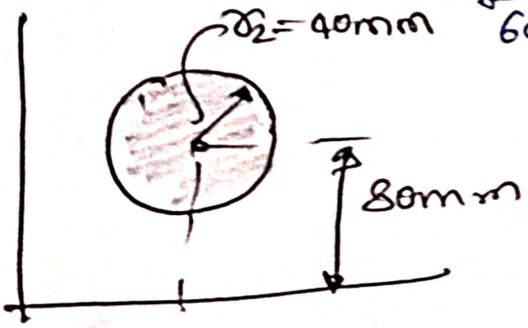
Q. For plane area shown, determine.
(a) The first moments with respect to the x & y axes.
(b) The location of centroid.











Component	$A\ (mm^2)$	$\bar{x}$ mm	$\bar{y}$ mm	$\bar{x}A\ (\times 10^3)$	$\bar{y}A\ (\times 10^3)$
Rectangle	$(120)(80) = 9.6\times10^3$	60	40	+576	+384
Triangle	$\frac{1}{2}(120)(60) = 3.6\times10^3$	40	−20	+144	−72
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655\times10^3$	60	105.46	+339.3	+596.4
Circle	$-\pi(40)^2 = -5.027\times10^3$	60	80	−301.6	402.2

$\Sigma A = 13.828 \times 10^3$

$\Sigma \bar{x} A = +757.7 \times 10^3$

$\Sigma \bar{y} A = +506.2 \times 10^3$

$\bar{X}\Sigma A = \Sigma \bar{x} A$

$$\bar{X} = \frac{757.7\times10^3}{13.828\times10^3}$$

$\bar{X} = 54.8$ mm

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

First moments of Area

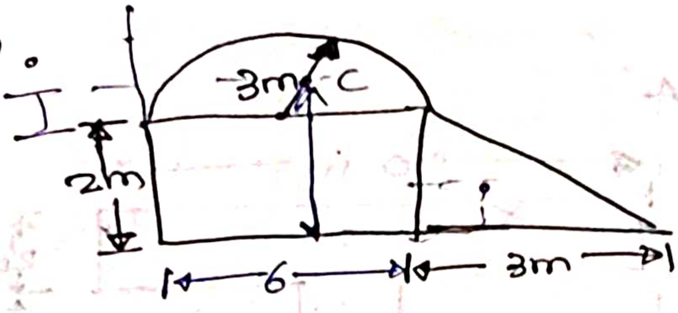
$Q_x = \Sigma \bar{y} A = 506.2\times10^3\ mm^2$

$Q_y = \Sigma \bar{x} A = 757.7 \times 10^3\ mm^2$

$$\bar{Y} = \frac{506.2\times10^3}{13.828\times10^3}$$

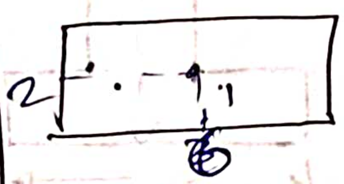
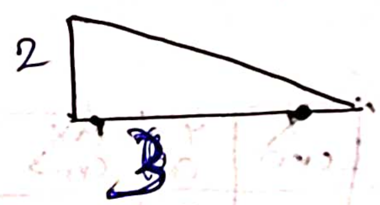
$\bar{Y} = 36.6$ mm

Q.



Centroid $C(\bar{x}, \bar{y})$

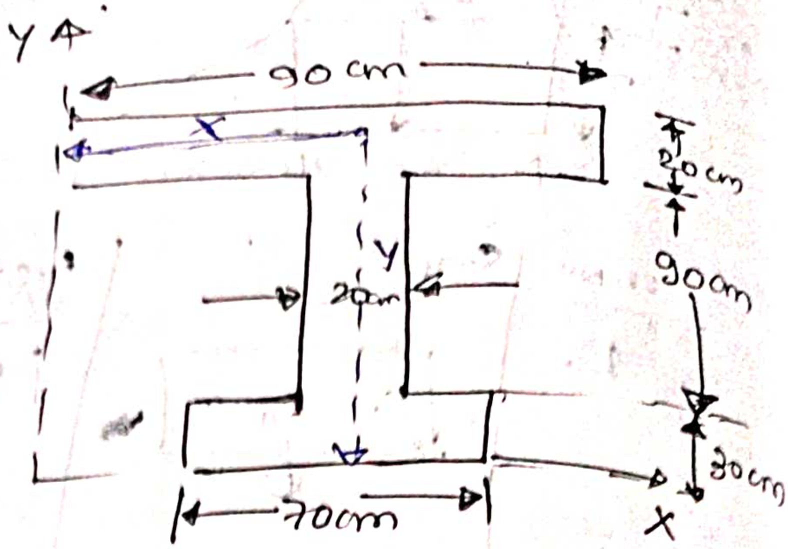
$\bar{x} = \frac{b}{3}, \quad \bar{y} = \frac{h}{3}$

Sr. No.	Shape	Area (m^2)	x (m)	y (m)	Ax (m^3)	Ay (m^3)
1.		12	3	1	36	12
2.		3	7	0.666	21	1.998
3.		14.137	3	3.273	42.411	46.2
		$\Sigma A = 29.137$			$\Sigma Ax = 49.411$	ΣAy 60.2

$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = 3.411$ m

$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 2.068$ m

Q.



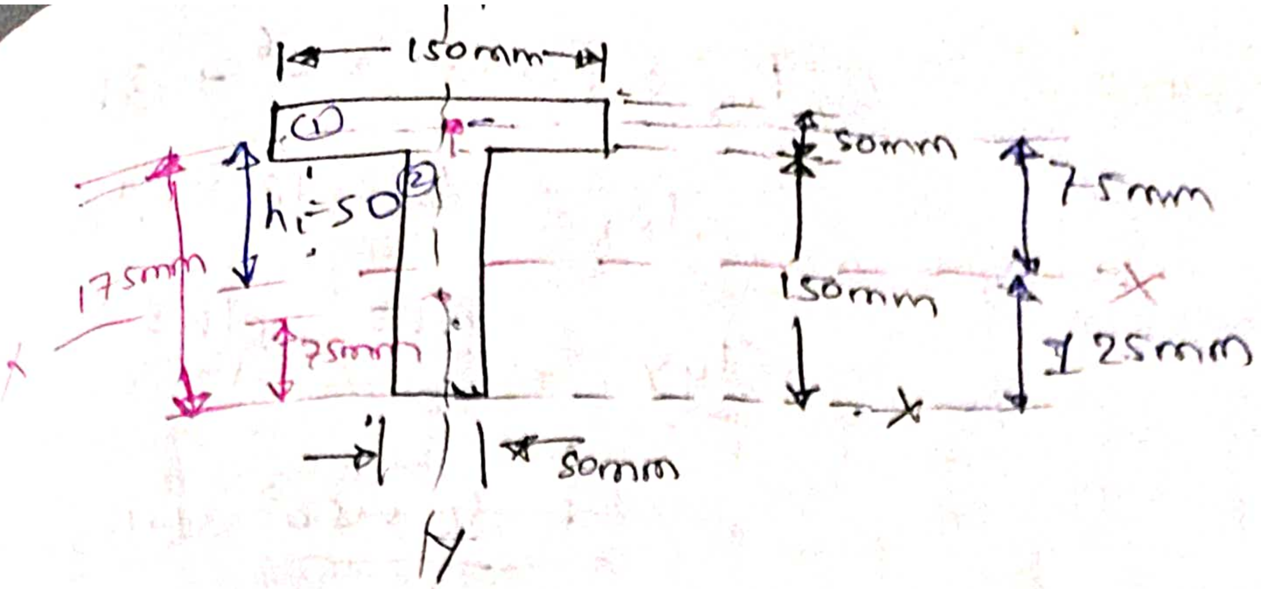
Sl.No	Shape ~~Area~~ cm	Area cm2	x cm	y cm	Ax cm3	Ay cm3
1.	20, 90	1800	45	130	81000	234000
2.	90, 20	1800	45	75	81000	135000
3.	30, 70	2100	45	15	94500	31500

$\Sigma A = 5700$ $\Sigma Ax = 256500$ $\Sigma Ay = 400500$

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{256500}{5700} = 45\,cm$$

$$\bar{Y} = \frac{\Sigma Ay}{\Sigma A} = \frac{400500}{5700} = 70.263\,cm$$

$C(45, 70.263)$



① $A_1 = 150 \times 50$ $7500\,mm^2$

$y_1 = \frac{50}{2} + 150 = 175\,mm$

~~$x_1 = \frac{150}{2} = 75$~~

② $A_2 = 150 \times 50 = 7500\,mm^2$

~~$x_2 = 25\,mm$~~ $\quad y_2 = \frac{150}{2} = 75\,mm$

$I_{xx} = I_{xx_1} + I_{xx_2}$

$I_{xx_1} = I_{G_1} + A_1 h_1^2$

$= \frac{bd^3}{12} + A_1 h_1^2$

$= \frac{150 \times 50^3}{12} + (7500 \times (175 - 125)^2$ 50

$I_{xx_1} = 20.31 \times 10^6\,mm^4$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$\bar{y} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500}$

$\boxed{\bar{y} = 125\,mm}$

from bottom

$I_{xx_2} = I_{G_2} + A_2 h_2^2$

$= \frac{bd^3}{13} + A_2 (50)^2$

$= \frac{50 \times 150^3}{12} + (150 \times 50 \times 50^2) = I_{xx_2} = 32.81 \times 10^6\,mm^4$

$$I_{xx} = (20.31 \times 10^6 + 32.81 \times 10^6$$

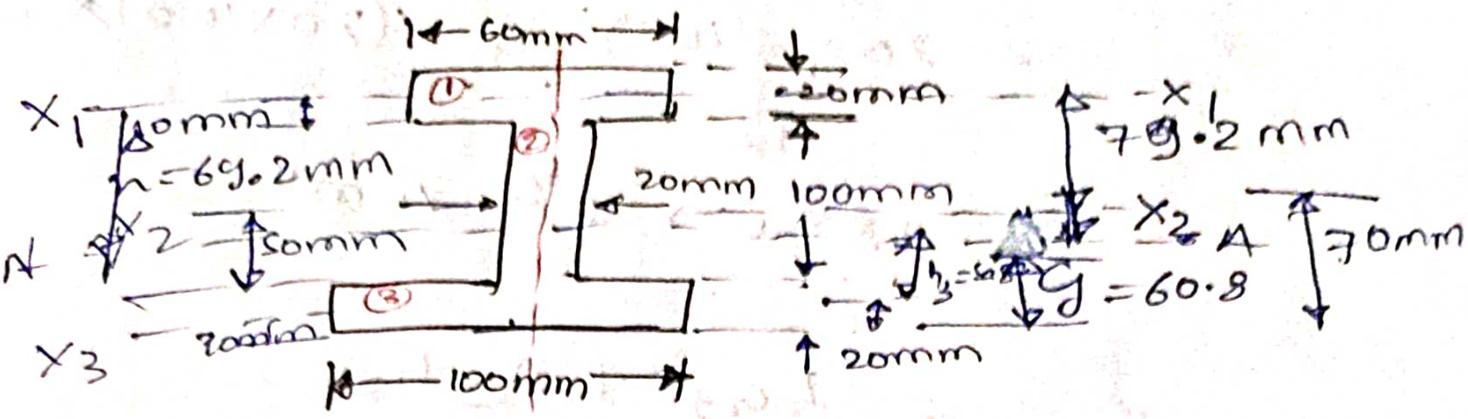
$$I_{xx} = 53.12 \times 10^6 \, mm^4$$

$$I_{yy} = \frac{db^3}{12} + \frac{db^3}{12}$$

$$= \frac{50 \times 150^3}{12} + \frac{150 \times 50^3}{12}$$

$$\boxed{I_{yy} = 15.62 \times 10^6 \, mm^4}$$

Determine Moment of inertia of I section



① $A_1 = 20 \times 60 = 1200 mm^2$

$y_1 = 10mm + 100 + 20 = 130mm$

② $A_2 = 100 \times 20 = 2000 \ mm^2$

$y_2 = 50mm + 20 = 70mm$

③ $\Rightarrow A_3 = 100 \times 20 = 2000$

$y_3 = 10mm$

$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$

$I_{xx_1} = I_{G_1} + A_1 h_1^2$

$= \frac{bd^3}{12} + (20 \times 60) \times (69.2)^2$

$= \frac{60 \times 20^3}{12} + (60 \times 20) \times (69.2)^2$

$\boxed{I_{xx_1} = 57.86 \times 10^3}$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$

$\bar{y} = \frac{(120 \times 130) + (2000 \times 70) + (2000 \times 10)}{(120 + 2000 + 2000}$

$\bar{y} = 60.8 mm$

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= \frac{20 \times (100)^3}{12} + (20 \times 100) \times (9.2)^2$$

$$= 1836 \times 10^3 \text{ mm}^4$$

$$I_{xx_3} = I_{G_3} + A_3 h_3^2$$

$$= \frac{100 \times 20^3}{12} + (100 \times 20) \times (50.8)^2$$

$$I_{xx_3} = 5228 \times 10^3$$

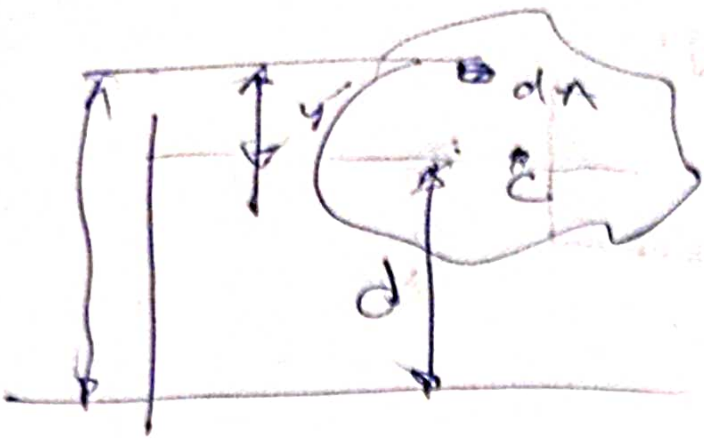
$$I_{xx} = 12850 \times 10^3$$

$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$$= \frac{db^3}{12} + \frac{db^3}{12} + \frac{db^3}{12}$$

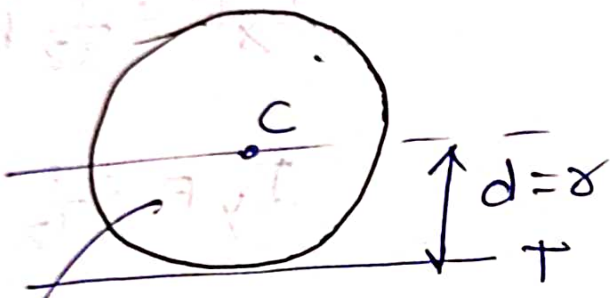
$$= \frac{20 \times 60^3}{12} + \frac{100 \times 20^3}{12} + \frac{20 \times 100^3}{12}$$

$$I_{yy} = 2.093 \times 10^6 \text{ mm}^4$$



Parallel Axis Theorem

$$I = \bar{I} + Ad^2$$



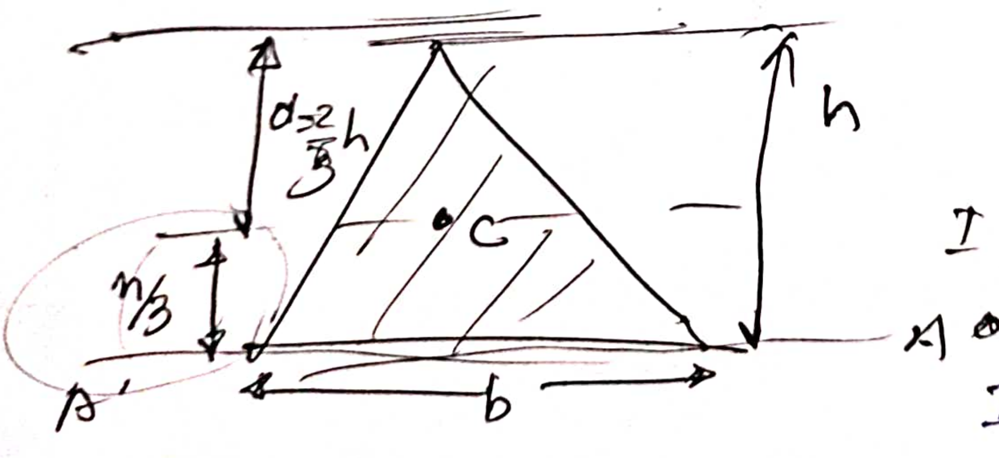
MOI $\Rightarrow \frac{1}{4}\pi r^4$

MOI w.r.t tangent

$$I_T = \bar{I} + Ad^2$$

$$= \frac{\pi r^4}{4} + (\pi r^2)^2$$

$$= \frac{5\pi r^4}{4}$$

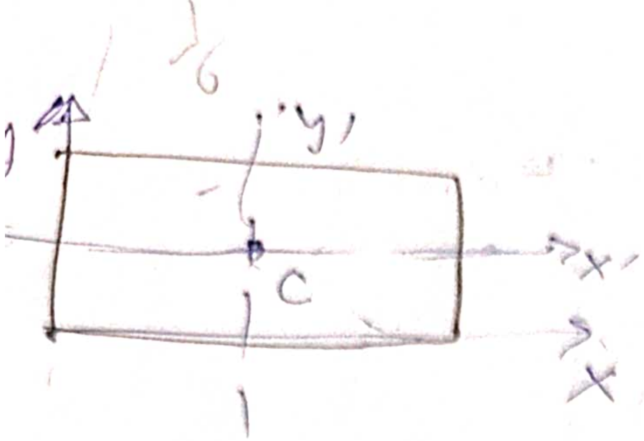


$$I_{AA'} = I_C + Ad^2$$

$$I_C = I_{AA'} - Ad^2$$

$$= \frac{1}{12}bh^3 - \frac{1}{2}bh(\ldots$$

$$I_C = \frac{1}{36}bh^3$$



$$I_x = \frac{bh^3}{3}$$

$$I_y = \frac{b^3h}{3}$$

$$I_{xF} = \frac{1}{12} bh^3$$

$$I_{yF} = \frac{1}{12} b^3 h$$

$$I_C = \frac{1}{12} bh(b^2+h^2)$$