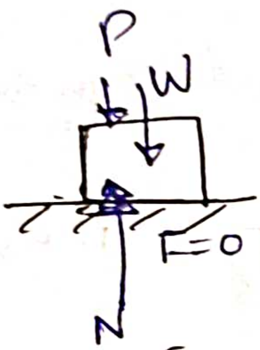
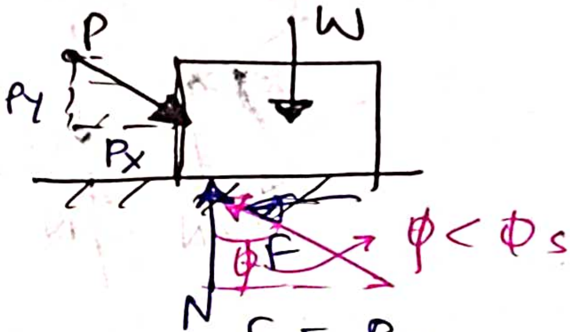
Friction :-

Friction is a force that resists the relative motion between two contacting surfaces.



$N = P + W$

(a) No friction ($P_x = 0$)



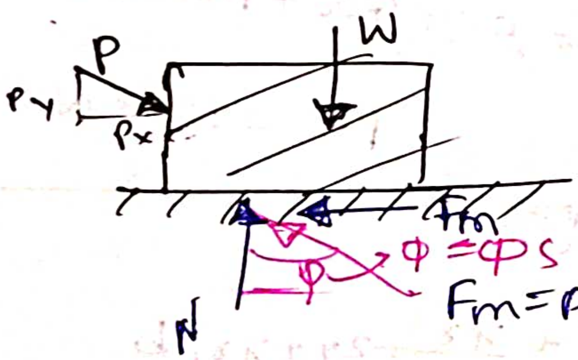
(b) No motion

$P_x < F_m$

$F = P_x$

$F < \mu_s N$

$N = P_y + W$



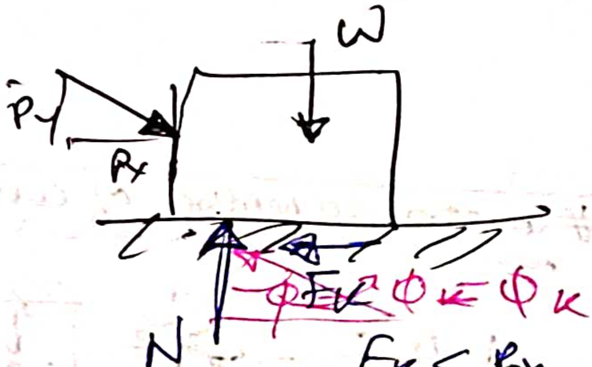
$F_m = P_x$

(c) Motion impending

($P_x = F_m$)

$F_m = \mu_s N$

$N = P_y + W$



$F_k < P_x$

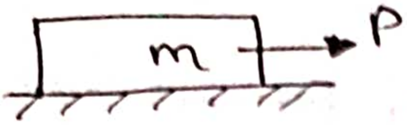
(d) Motion

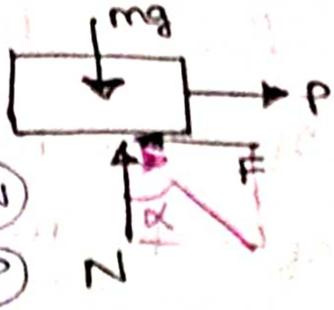
$P_x > F_m$

$F_k = \mu_k N$

$N = P_y + W$

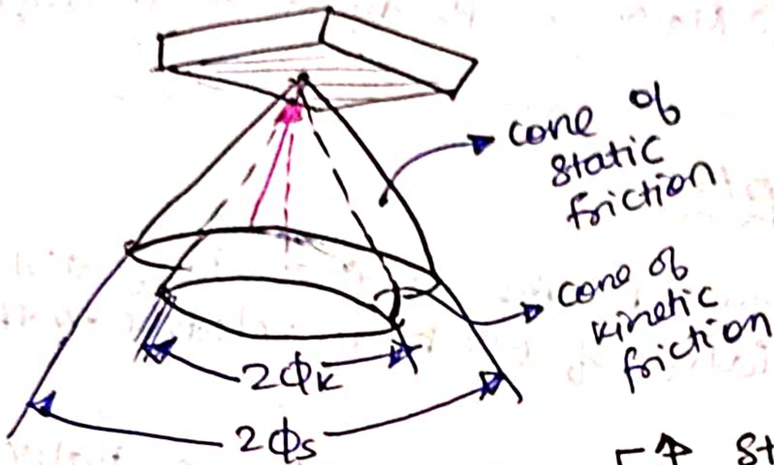
Dry Friction

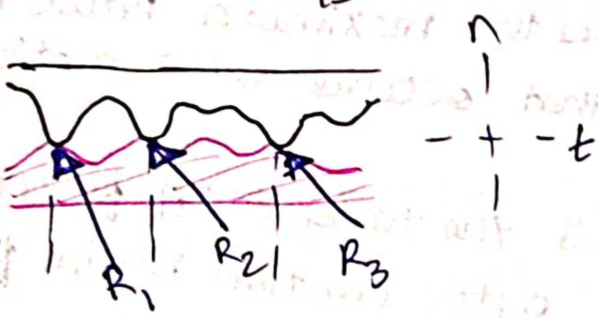


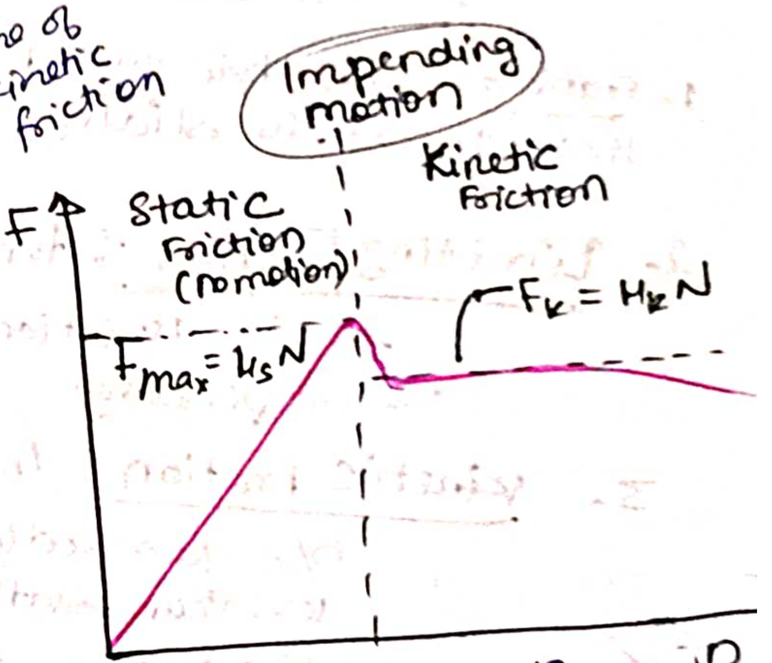


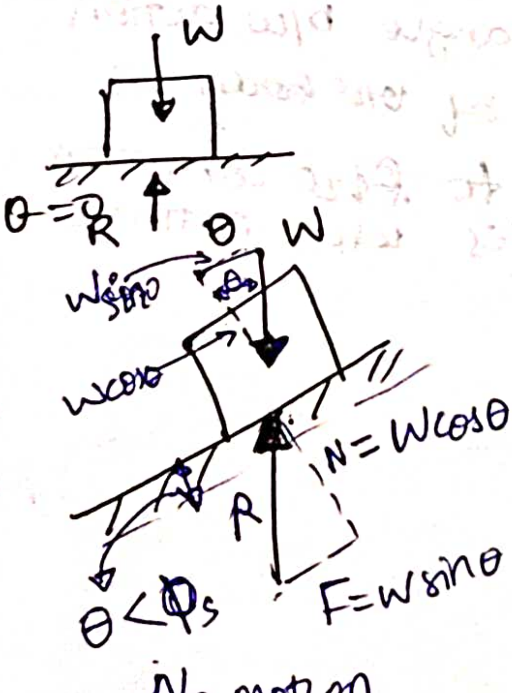
$N = W$

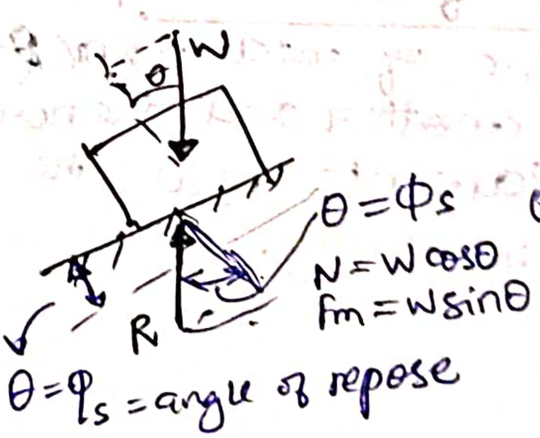
$F = P$

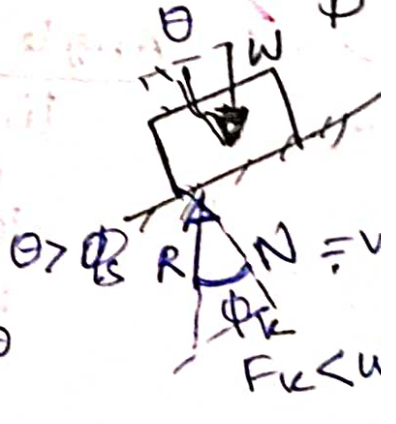












$F_{max} = \mu_s N$ [Maxm static friction Force)

* If $P < \mu_s N$ — Body remains static
* If $P = \mu_s N$ — Body is on the verge of moving (static)
* If $P > \mu_s N$ — Body starts moving.

General Concepts

1. Static friction b/w two bodies is the tangential force F that opposes the sliding of one body relative to the other.

2. Limiting Friction : Fm is the maximum value of static Friction that occurs when motion is impending.

3. Kinetic Friction : Fk is the tangential force b/w two bodies after motion begins It is less than static friction.

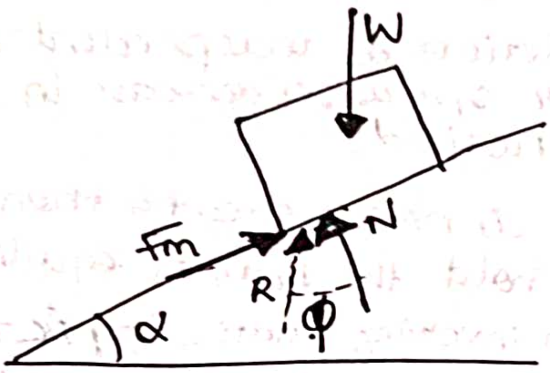
4. Angle of friction ϕ is the angle b/w action line of total Reaction of one body on another and the normal to the common tangent b/w the bodies when motion is impending.

5. Coefficient of static friction μ is ratio of the limiting friction F_m to the normal force N.

$$\mu \text{ or } \mu_s = \frac{F_m}{N}$$

6. Coefficient of kinetic friction μ_k is the ratio of the kinetic friction F_k to the normal force N.

$$\mu_k = \frac{F_k}{N}$$



$F_m = \mu N$

Angle of of repose: α is the angle to which an inclined plane may be raised before an object resting on it will move under the action of force of gravity & the reaction of the plane.

$W = mg$ $\tan\phi = \mu$

$\alpha = \phi$

Laws of Friction

(a) The coefficient of friction is independent of the normal force, however the limiting friction & kinetic friction are proportional to the normal force

(b) The coefficient of friction is independent of the area of contact.

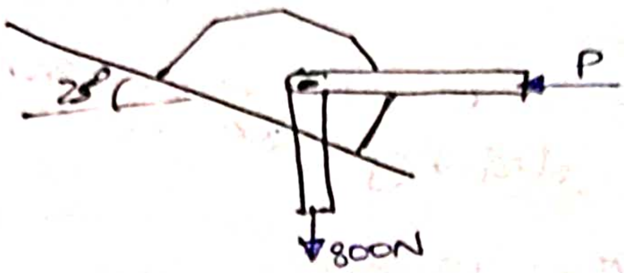
(c) The coefficient of kinetic friction is less than that of static friction.

(d) At low speeds, friction is independent of the speed. At higher speeds, a decrease in friction has been noticed

(e) The frictional force is never greater than that necessary to hold the body in equilibrium

In solving problem involving static ~~fr~~ friction, frictional force should be assumed to be an independent unknown unless the problem clearly states that motion is impending. In the latter case. one may use limiting friction for μN.

Q.



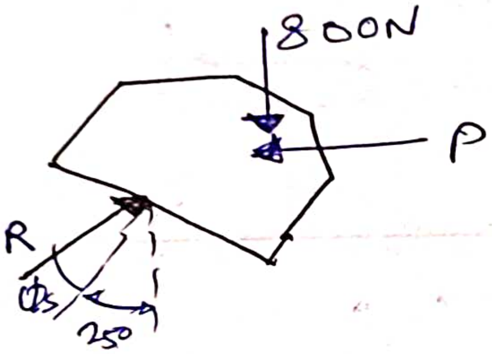
$m_s = 0.35$

$m_k = 0.25$

(a) determine force P req(a) to start the block moving up the incline.

(b) to keep it moving up

(C) prevent it from sliding down

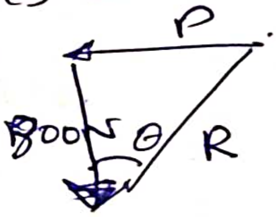


$\tan\phi_s = \mu_s = 0.35$

$\phi_s = 19.29°$

$\theta = 25° + 19.29°$

$\theta = 44.29°$



$P = (800)\tan 44.29°$

$P = 780\,N$

(b) force P to keep block moving up

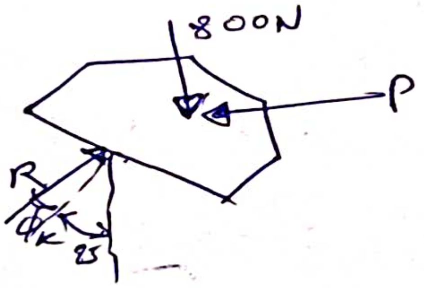
$\tan\phi_k = \mu_k$

$= 0.25$

$\phi_k = 14.04°$

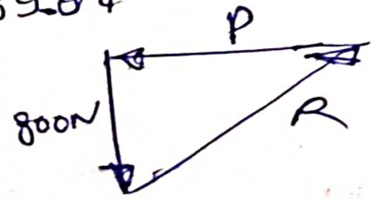
$\theta = 25° + 14.04°$

$\theta = 39.04°$



$P = 800\tan 39.04$

$P = 649\,N$



c. Force P to prevent block from sliding down.

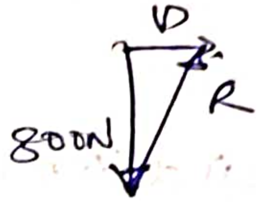
$\phi_s = 19.29°$

$\theta = 25 - 19.29°$

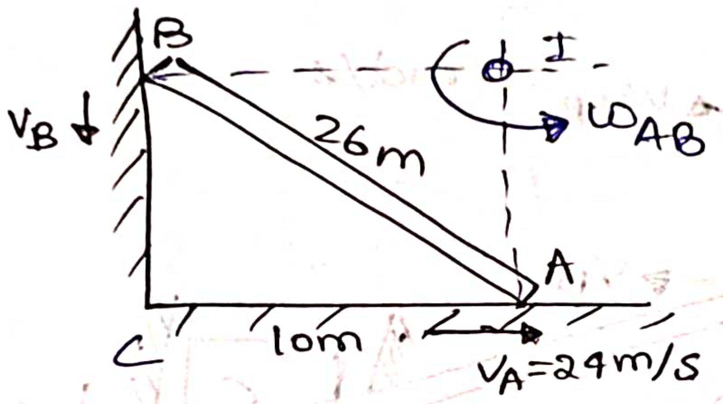
$\theta = 5.71°$

$P = (800) \tan 5.71°$

$P = 80 N$



Q. A rod AB 26m long leans against a vertical wall. the end A on the floor is moving away from the wall at rate of 24 m/sec when the end A of the rod is 10 m from the wall, determine the velocity of end B sliding down vertically and the angular velocity of the rod AB.



ΔACB

$AB^2 = AC^2 + BC^2$

$BC = 24m$

$AC = BI = 10m$

$BC = AI = 24m$

$AI = 24m$

$BI = 10m$

Link AB — Point I is an ICR.

$V_B = BI \times W_{AB}$

$V_B = 10 \times 1$

$\boxed{V_B = 10m/sec}$

$V_A = AI \times W_{AB}$

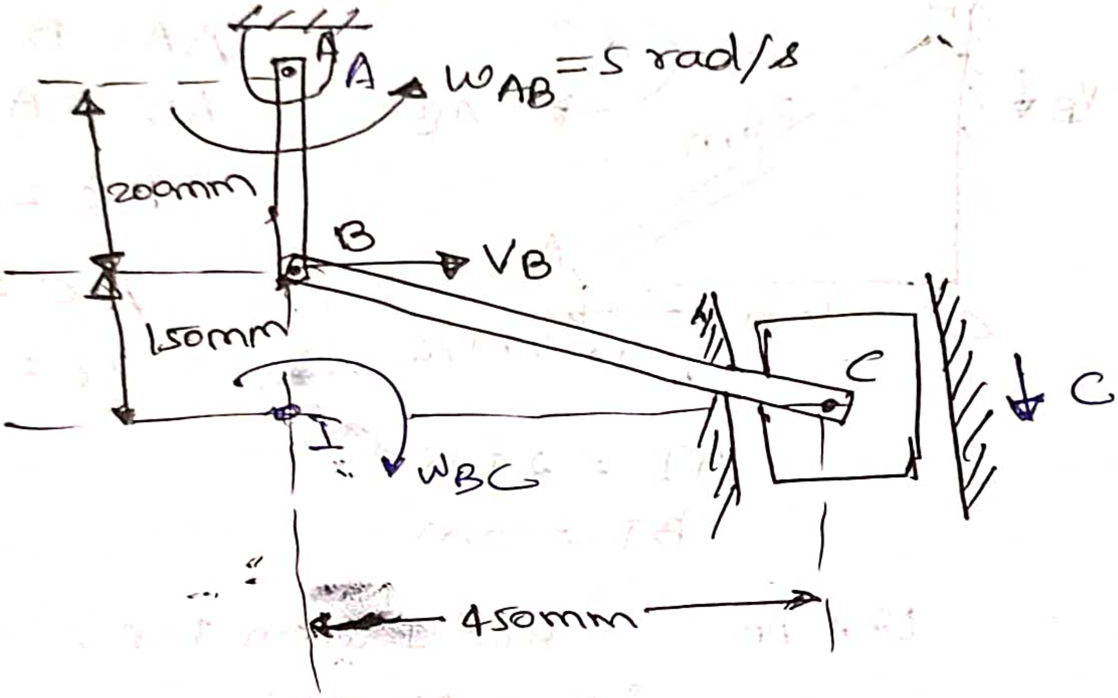
$V_A = 24\ m/sec$

$24 = 24 \times W_{AB}$

$\boxed{W_{AB} = 1 rad/sec}$

One slider ... and one rotation ...

Q. In the mechanism shown the angular velocity of link AB is 5 rad/sec anticlockwise. At the instant shown determine the angular velocity of link BC & velocity of piston C.



No. of bodies = No. of ICR = No. of angular

Link AB

Pt. A is an ICR

$V_A = 0$

$\vec{V}_B = AB \times \omega_{AB}$

$V_B = 200 \times 5$

$V_B = 1000$ mm/s

Link BC

Point I is an ICR

$V_B = BI \times \omega_{BC}$

$1000 = 150 \times \omega_{BC}$

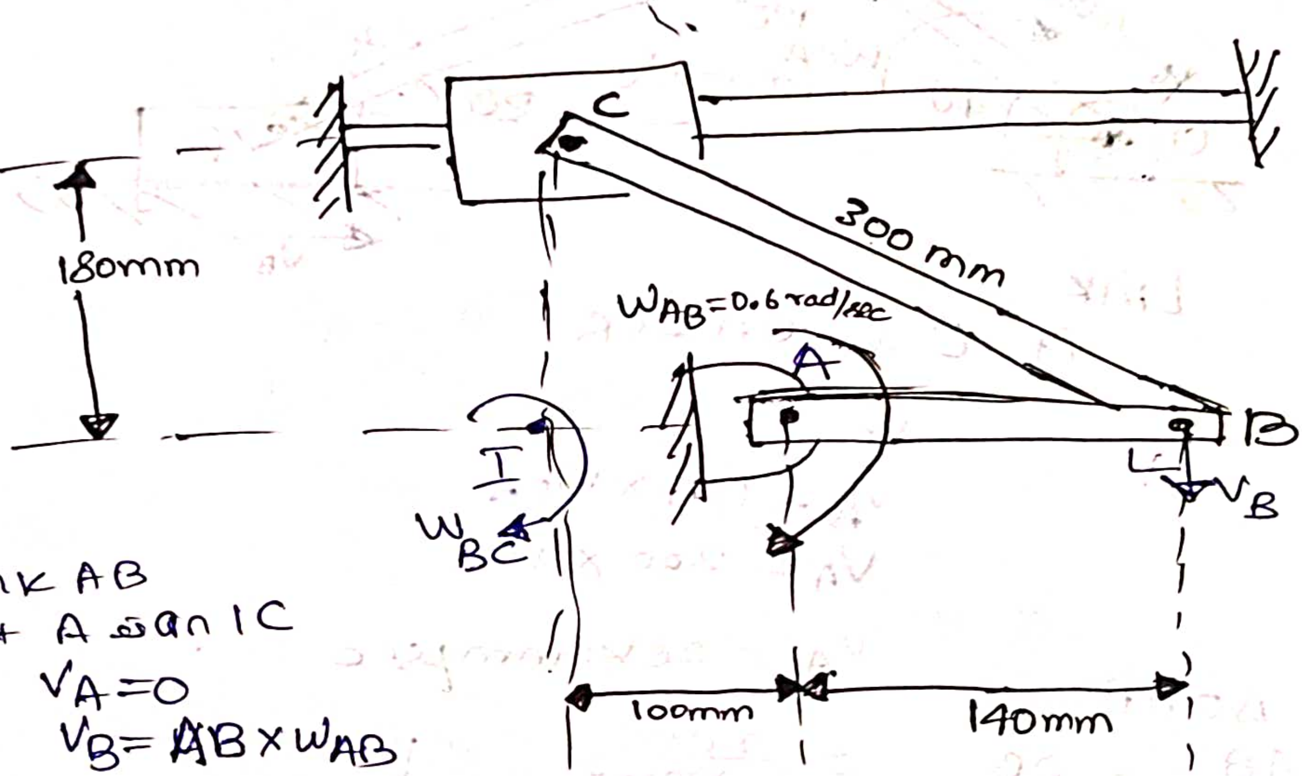
$\omega_{BC} = 6.66$ rad/sec

$V_C = IC \times \omega_{BC}$

$V_C = 450 \times 6.66$

$V_C = 3000$ mm/sec

Q. In fig. collar C slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine the angular velocity of link BC & velocity of collar C.



Link AB

Pt A is an IC

$V_A = 0$

$V_B = AB \times W_{AB}$

$V_B = 140 \times 0.6$

$V_B = 84 mm/s$

Link BC

Point I is an ICR

$V_B = BI \times W_{BC}$

$84 = 240 \times W_{BC}$

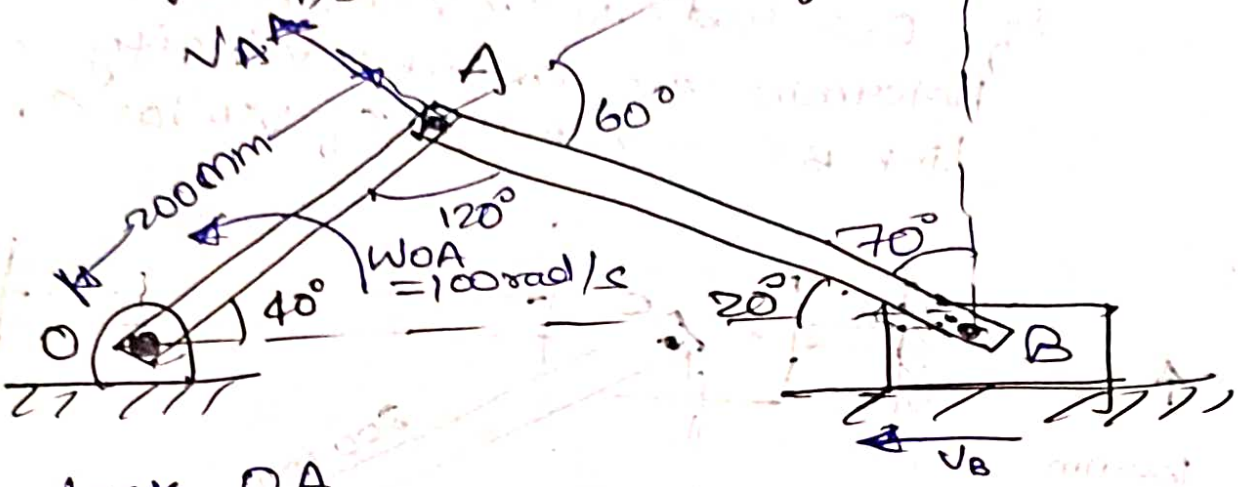
$W_{BC} = 0.35$ rad/sec

$V_C = IC \times W_{BC}$

$= 180 \times 0.35$

$V_C = 63 mm/s$

Q. A slider crank mechanism is shown in figure. The crank OA rotates anticlockwise at 100 rad/sec. Find the angular velocity (ω_{AB}) of rod AB & velocity of the slider at B.



Link OA

Pt O is an ICR

$$V_O = 0$$

$$V_A = OA \times W_{OA}$$

$$V_A = 200 \times 100$$

$$V_A = 20K \text{ mm/sec}$$

ΔOAB

$$\frac{AB}{\sin 40^\circ} = \frac{OB}{\sin 120^\circ} = \frac{200}{\sin 20^\circ}$$

$$AB = 200 \times \frac{\sin 40}{\sin 20}$$

$$\boxed{AB = 375.877 \text{ mm}}$$

$$OB = 200 \times \frac{\sin 120}{\sin 20}$$

$$\boxed{OB = 506.417 \text{ mm}}$$

ΔABI

$$\frac{AI}{\sin 70^\circ} = \frac{BI}{\sin 60^\circ} = \frac{375.877}{\sin 50^\circ}$$

$$AI = 375.877 \times \frac{\sin 70^\circ}{\sin 50^\circ}$$

$$AI = 461.081 \text{ mm}$$

$$BI = 375.877 \times \frac{\sin 60^\circ}{\sin 50^\circ}$$

$$BI = 424.934 \text{ mm}$$

Line AB

Point I is an ICR

$$V_A = AI \times W_{AB}$$

$$20000 = 461.081 \times W_{AB}$$

$$W_{AB} = 43.37 \text{ rad/sec}$$

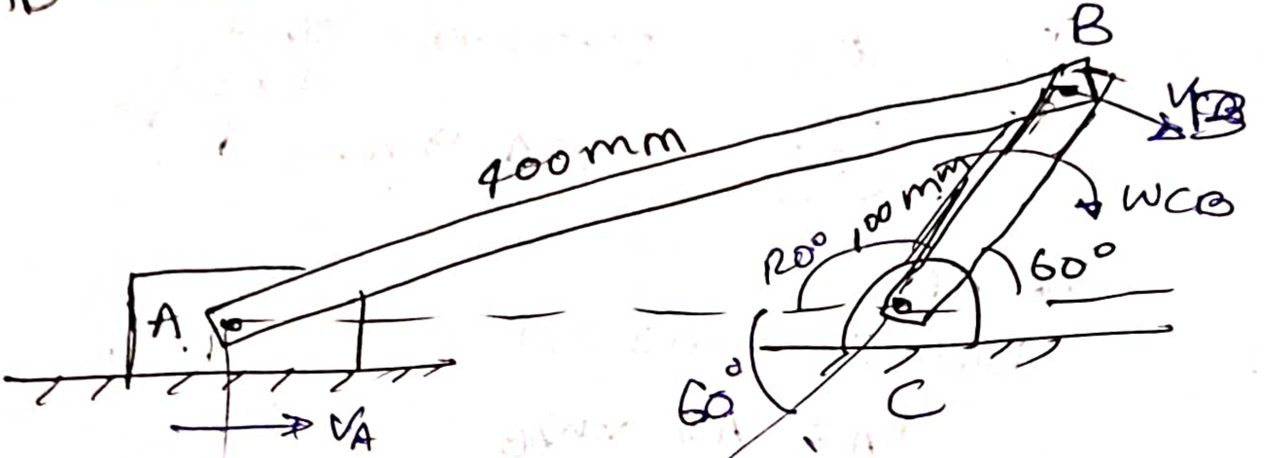
$$V_B = BI \times W_{AB}$$

$$V_B = 424.934 \times 43.37$$

$$V_B = 18429.38 \text{ mm/sec}$$

Q. The crank BC of a slider crank mechanism is rotating at a constant speed of 30 rpm clockwise. Determine the velocity of the piston A at the given instant.

AB = 400 mm & BC = 100 mm.



Link CB

$$\omega_{CB} = \frac{2\pi \times N}{60}$$

$$\omega_{CB} = \frac{2\pi \times 30}{60}$$

$$\omega_{CB} = 3.14 \text{ rad/sec}$$

Link CB

At C is an ICR

$V_C = 0$

$V_B = CB \times \omega_{CB}$

$V_B = 100 \times 3.14$

$V_B = 314 \text{ mm/s}$

In ΔACB

$$AB = \sqrt{AC^2 + BC^2 - 2AC \times BC \times \cos 120^\circ}$$

$$400 = \sqrt{AC^2 + 100^2 - 2 \times AC \times 100 \times \cos 120^\circ}$$

$$AC = 340.512 \text{ mm}$$

In ΔAIC

$$\frac{AI}{\sin 60^\circ} = \frac{CI}{\sin 90^\circ} = \frac{340.512}{\sin 30^\circ}$$

$$AI = 340.512 \cdot \frac{\sin 60^\circ}{\sin 30^\circ}$$

$$AI = 589.784 \text{ mm}$$

$$CI = \frac{340512 \sin 90^\circ}{\sin 30^\circ}$$

$$CI = 681.024 \text{ mm}$$

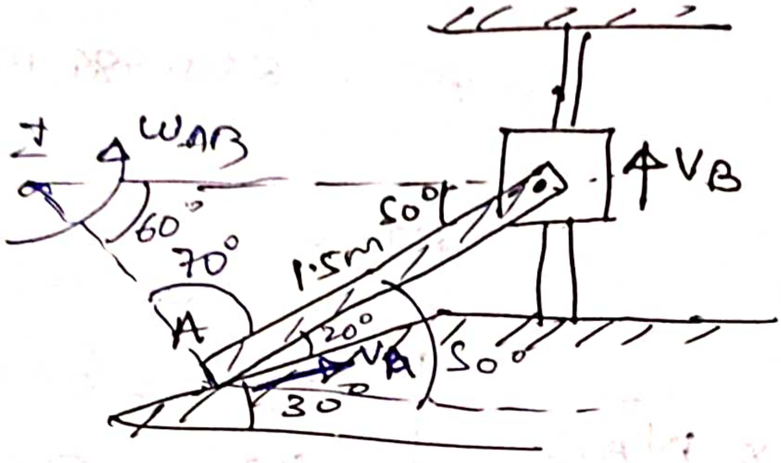
$$BI = BC + CI$$
$$= 100 + 681.024$$
$$= 781.024$$

Link AB

$$V_B = BI \times W_{AB}$$
$$314 = 781.024 \times W_{AB}$$
$$W_{AB} = 0.402 \text{ rad/s}$$

$$V_A = AI \times W_{AB}$$
$$V_A = 589.784 \times 0.402$$
$$V_A = 237.093 \text{ mm/sec}$$ Ans

Q. End B moves up with constant velocity of 2m/s. Find out the angular velocity of rod AB & velocity of end A. Where length of rod AB is 1.5 m



Link AB

At I is an ICR

$V_B = IB \times \omega_{AB}$

$2 m/s = 1.627 \times \omega_{AB}$

$\omega_{AB} = \frac{2m}{1.627}$

$\omega_{AB} = 1.229$ rad/s

$V_A = AI \times \omega_{AB}$

$= 1.326 \times 1.229$

$V_A = 1.629$ m/s

In $\triangle$AIB

$\frac{AI}{\sin 50°} = \frac{BI}{\sin 70°} = \frac{1.5}{\sin 60°}$

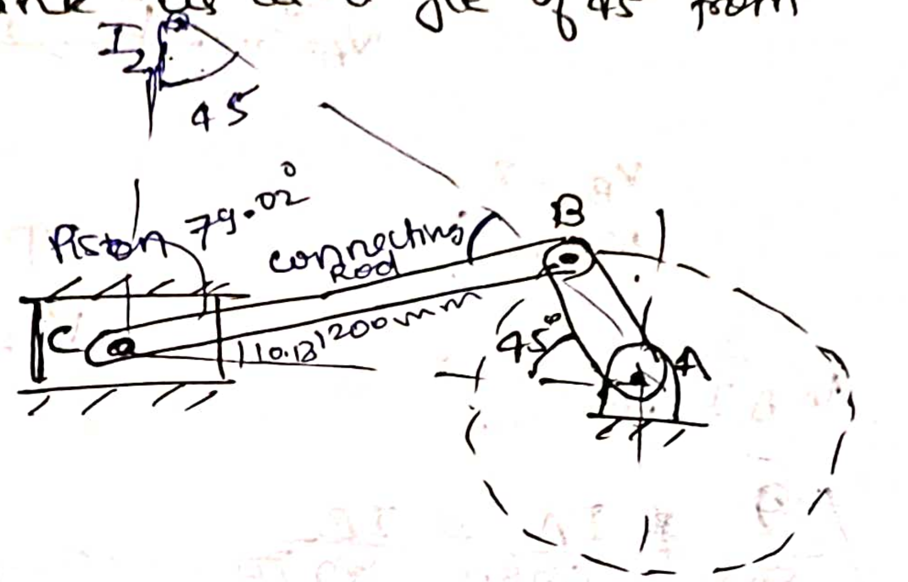
$AI = \frac{1.5 \sin 50°}{\sin 60°}$

$AI = 1.326m$

$BI = \frac{1.5 \sin 70°}{\sin 60°}$

$BI = 1.627m$

Q. In a crank & connecting rod mechanism the length of crank and connecting rod are 300 mm & 1200 mm respectively. The crank is rotating at 180 rpm. Find the velocity of piston, when the crank is at angle of 45° from Horizontal.



$N = 180\ rpm$

$\omega_{AB} = \frac{2\pi N}{60}$

$= \frac{2\pi \times 180}{60}$

$\boxed{\omega_{AB} = 18.85}$

In ΔBCA

$\frac{1.2}{\sin 45} = \frac{0.3}{\sin C}$

$C = 10.18$

$I_2 BC$

$\frac{1.2}{\sin 45} = \frac{I_2 C}{\sin 55.98} = \frac{I_2 B}{\sin 79}$

$I_2 C = \frac{1.2 \times \sin 56}{\sin 45}$

$I_2 C = 1.393$

$I_2 B = 1.67$

Link AB ICR = A

Pt A

$V_A = 0$

$V_b = AB \times \omega_{AB}$

$V_b = 300 \times 18.85$

$V_b = 5655\ mm/s$

$V_b = 5.655\ m/s$

Link BC ($ICR \to I_2$)

$V_b = I_2 B \times \omega_{bc}$

$5.655 = 1.67 \times \omega_{BC}$

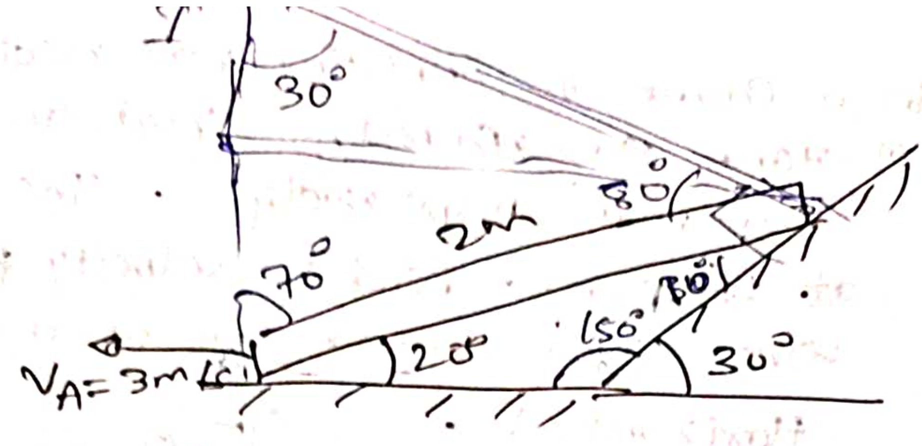
$\omega_{BC} = 3.385\ m/s$

Pt C

$V_C = I_2 C \times \omega_{bc}$

$V_C = 1.393 \times 3.385$

$V_C = 4.7168\ m/s$



$V_B = ?$

$\omega_{AB} = ?$

In $\Delta I, BC$

$$\frac{2}{\sin 30} = \frac{IA}{\sin 80} = \frac{IB}{\sin 70}$$

$IA = 3.939$ m

$IB = 3.758$

Pt A

$V_a = IA \times \omega_{ab}$ $V_b = IB \times \omega_{ab}$

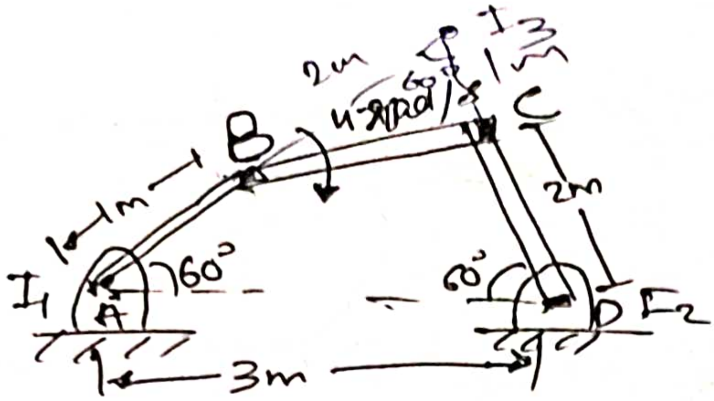
$3 = 3.939 \times \omega_{ab}$

$\omega_{ab} = 0.7616$ rad/s

$V_b = IB \times \omega_{ab}$

$= 3.75 \times 0.7616$

$V_b = 2.862$ m/s



Link AB

pt A → ICR

$V_a = 0$ $V_b = AB \times \omega_{ab}$

$8 = 1 \times \omega_{ab} \Rightarrow \omega_{ab} = 8\ r/s$

Link BC

pt C.

$V_c = IC \times \omega_{bc}$

$V_c = 1 \times 4$

$V_c = 4\ m/s$

pt B

$V_b = IB \times \omega_{bc}$

$V_b = 2 \times 4$

$V_b = 8\ m/s$

Link CD

D → ICR

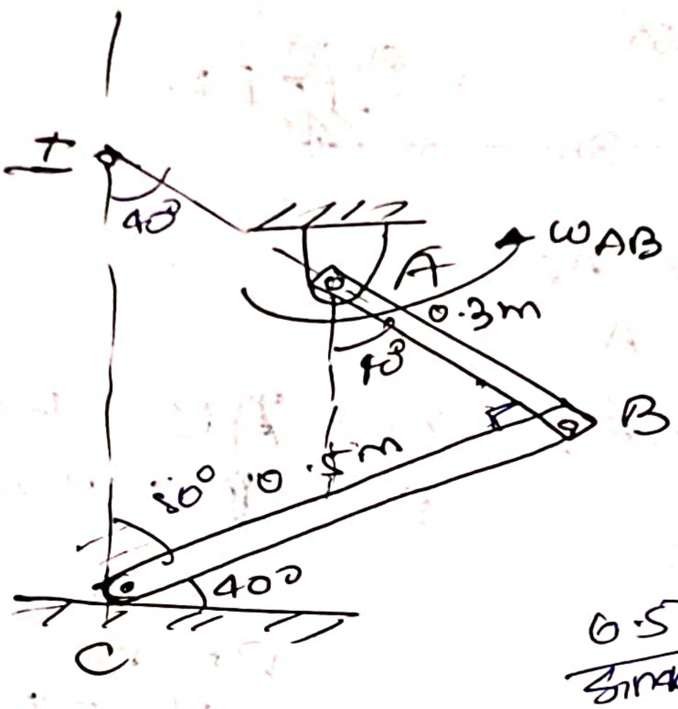
$V_d = 0$

$V_C = DC \times \omega_{DC}$

$4 = 2 \times \omega_{DC}$

$\omega_{DC} = 2\ r/s$

Q.



Link AB

$V_a = 0$

$V_b = AB \times \omega_{ab}$

$V_b = 0.3 \times 2$

$V_b = 0.6\,m/s$

Link BC ICR → I

$V_b = IB \times \omega_{bc}$

$0.6 = 0.5959 \times \omega_{bc}$

$\omega_{bc} = 1\,r/s$

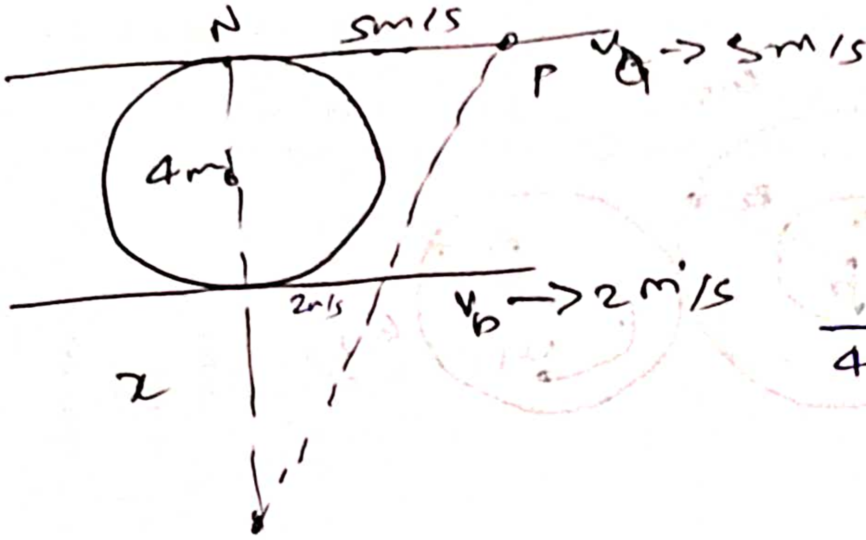
$\frac{0.5}{\sin 40°} = \frac{IB}{\sin 50} = \frac{IC}{\sin 90°}$

$IB = 0.5959$

$IC = 0.7779$

$V_c = IC \times \omega_{bc}$

$= 0.7779\,m/s$



$$\frac{5}{4+x} = \frac{2}{x}$$

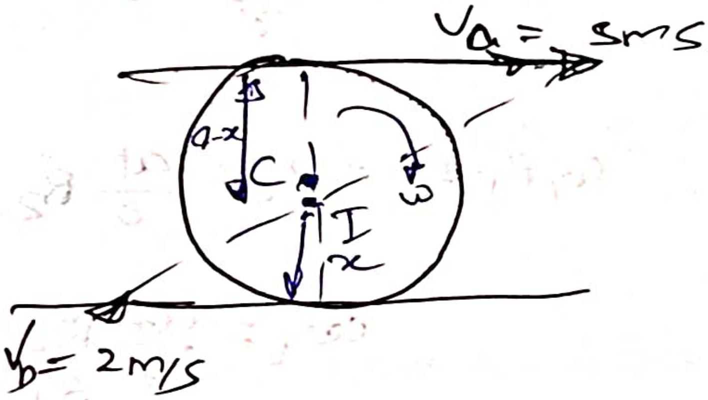
$x = 2.66 m$

$v = r \times \omega$

$V_a = IA \times \omega$

$5 = 6.66 \times \omega$

$\omega = 0.751 rad/s$



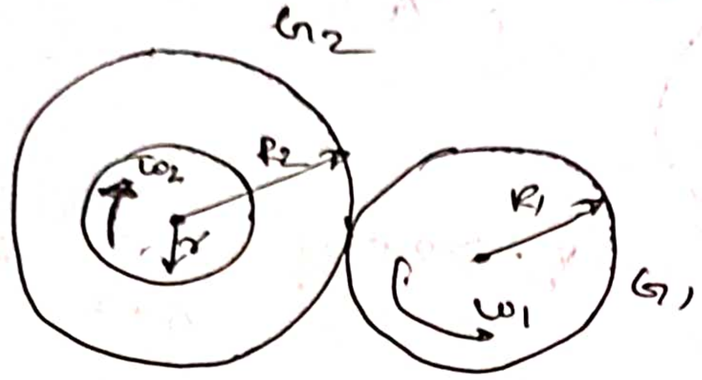
$$\frac{5}{4-x} = \frac{2}{x}$$

$x = 1.142$

$V_b = r_{I_b} \times \omega$

$2 = 1.142 \times \omega$

$\omega = 1.75 rad/s$



For G1

$$v = u + at$$

$$\omega_2 r = at$$

$$\omega_2 = \frac{at}{r}$$

$$\omega_2 R_2 = \frac{at}{r} R_2$$

$$\omega_1 R_1 = \omega_2 R_2 = \frac{at}{r} R_2$$

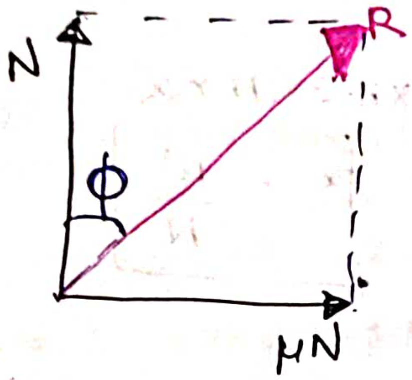
$$\omega_1 = \frac{at}{r} \frac{R_2}{R_1}$$

$$\omega_1 = \frac{d\theta_1}{dt} = \frac{at}{r} \frac{R_2}{R_1}$$

$$\theta_1 = \frac{at^2}{2r} \frac{R_2}{R_1}$$

Friction Angle (ϕ)

It is the angle b/w normal Force & resultant of normal & Friction force when the body is on the verge of moving.

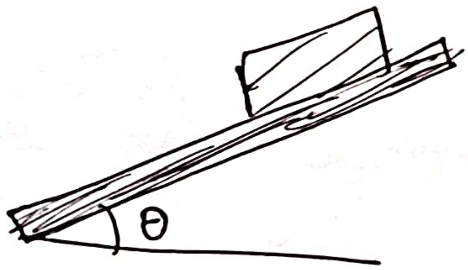


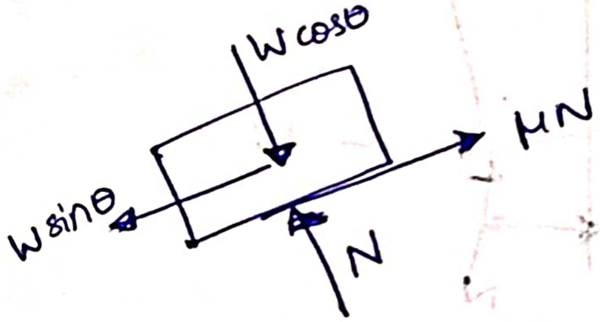
$$\tan\phi = \frac{\mu N}{N}$$

$$\boxed{\phi = \tan^{-1}\mu}$$

Angle of Friction (θ)

If a body is resting on an inclined surface then the angle of repose is the maximum angle at which the body can be at rest without slipping down.





$$W\sin\theta = \mu N$$

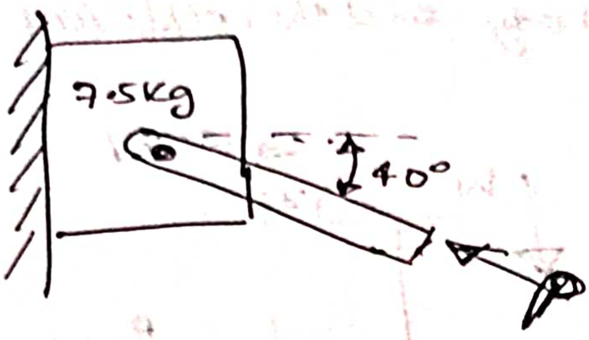
$$W\cos\theta = N$$

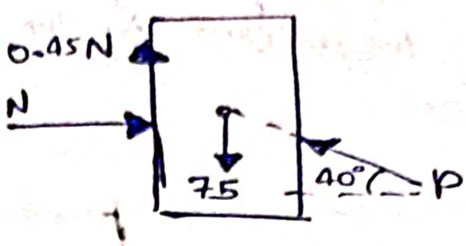
$$\tan\theta = \mu$$

$$\tan\theta = \tan\phi$$

$$\boxed{\theta = \phi}$$

Q. If the coefficient of friction b/w surfaces is 0.45, the smallest force P required to keep the block from falling down is ___ N. ($g = 10\ m/s^2$)





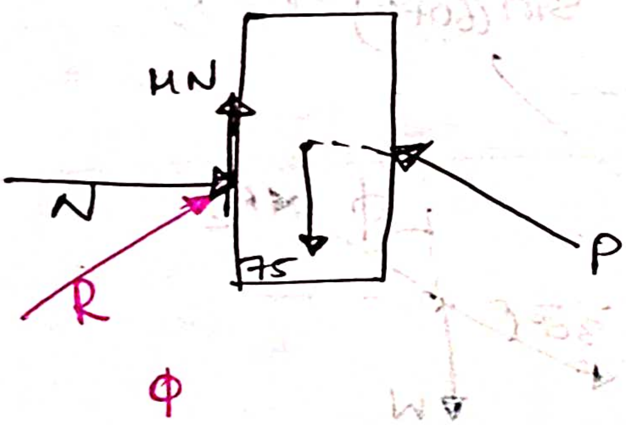
$\Sigma F_x = 0$

$N - P\cos 40° = 0$

$0.45N + P\sin 40° - 75 = 0$

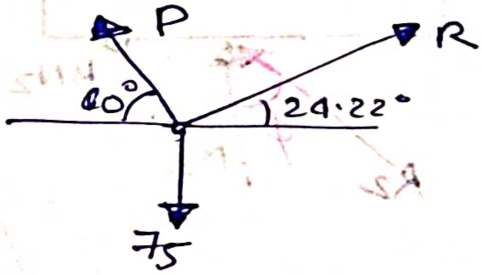
$P = 75.96$

Alternatively.



$\phi = \tan^{-1}(0.45)$

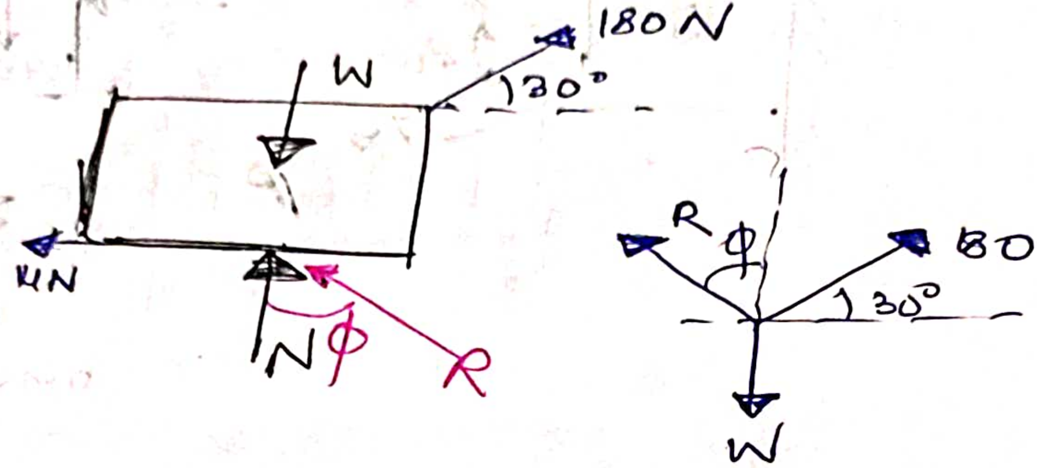
$\phi = 24.22°$



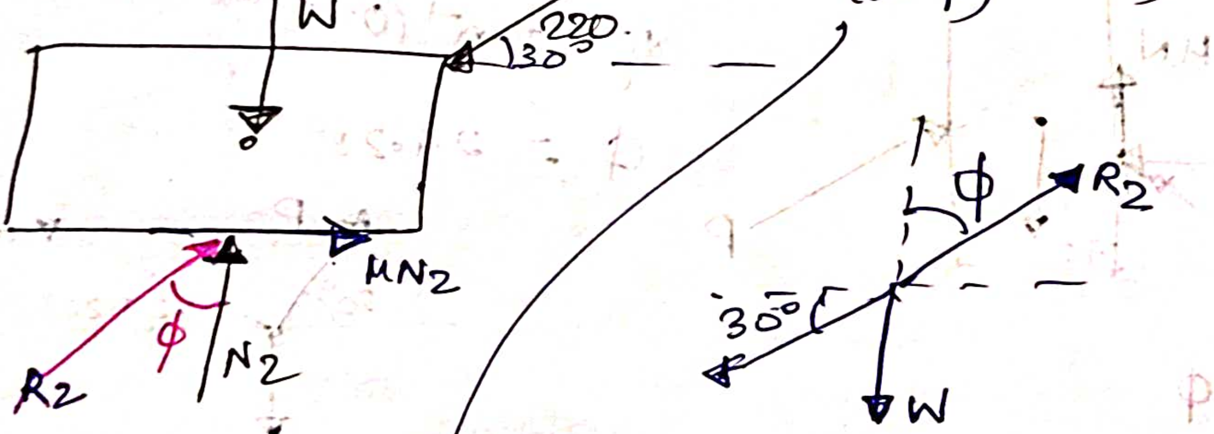
$$\frac{P}{\sin 114.22°} = \frac{75}{\sin 115.78} =$$

$P = 75.96$

A body, resting on a rough horizontal plane, required a pull of 180N inclined at 30° to the plane just to move it. It was found that a push of 220N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.



$$\frac{W}{\sin(60+\phi)} = \frac{180}{\sin(180-\phi)} = \frac{R}{\sin 120}$$



$$\frac{W}{\sin(120+\phi)} = \frac{220}{\sin(180-\phi)} = \frac{R_2}{\sin 60^\circ}$$

$$W = \frac{180 \sin(60+\phi)}{\sin\phi}$$

$$W = \frac{220 \sin(120+\phi)}{\sin\phi}$$

$$220 \sin(120 + \phi) = 180 \sin(60 + \phi)$$

$$220 [\sin 120 \cos\phi + \cos 120 \sin\phi]$$

$$= 180 [\sin 60 \cos\phi + \cos 60^\circ \sin\phi$$

$$1.22 [\sin 120^\circ + \cos 120 \tan\phi] = \sin 60^\circ + \cos 60^\circ \tan\phi$$

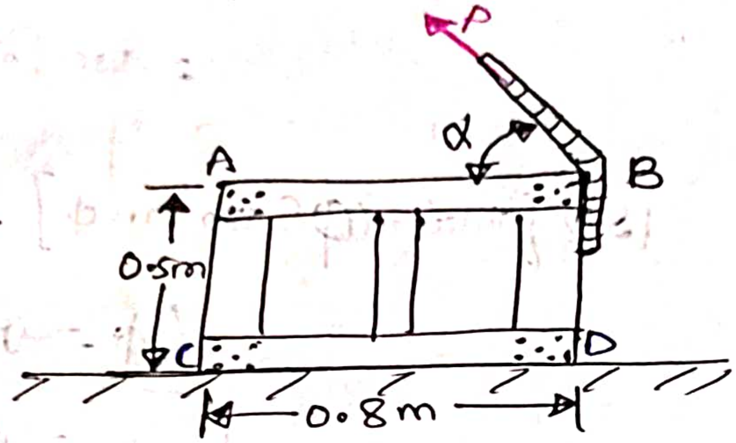
$$\boxed{\phi = 9.71}$$

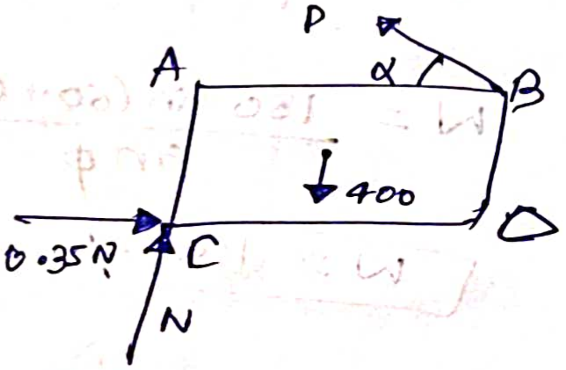
$$\tan\phi = \mu = 0.17$$

$$W = \frac{180 \sin(60 + \phi)}{\sin\phi}$$

$$\boxed{W = 1001 N}$$

Q. A. 40kg packing crate must be moved to the left along the floor without tipping. Knowing that the coefficient of static friction b/w the crate & the floor is 0.35. determine magnitude of the force P (in N) assuming crate does not tip.





$\Sigma M_B = 0$

$N \times 0.8 - 0.35 N \times 0.5 - 400 \times 0.4 = 0$

$N = 256N$

$\Sigma F_x = 0$

$P\cos\alpha = 0.35N = 89.6$ —— (i)

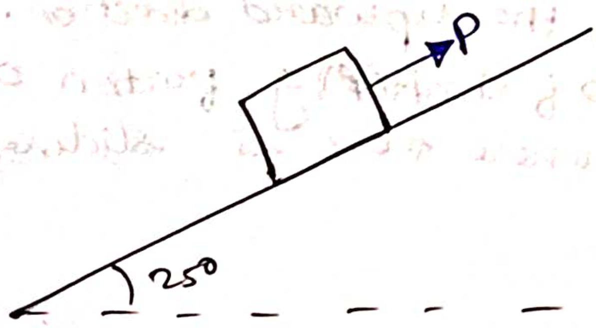
$\Sigma F_y = 0$

$P\sin\alpha = 400 - N = 144N$ —— (ii)

Eqn (i)^2 + Eqn (ii)^2

$P^2 = 89.6^2 + 144^2 \Rightarrow P = 169.6N$

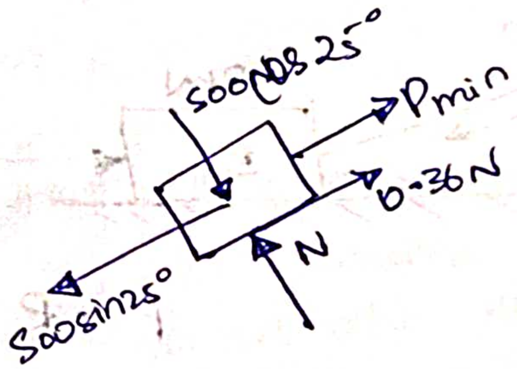
Q. A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in fig. Determine the minimum & maximum values of P for which the equilibrium can exist, if the angle of friction is 20°.



$\phi = 20°$

$\mu = \tan 20°$

$\mu = 0.36$

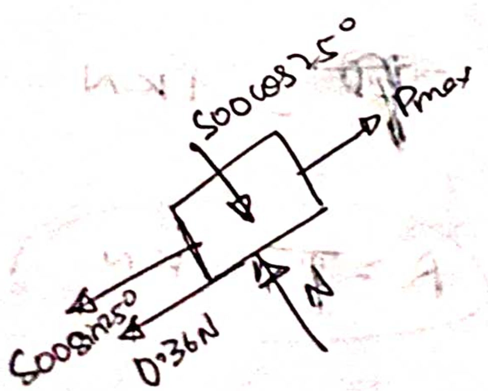


$N = 500\cos 25°$

$P_{min} = 500\sin 25° - 0.36N$

$P_{min} = 500\sin 25° - 0.36 \times 500\cos 25°$

$P_{min} = 48.17\ N$



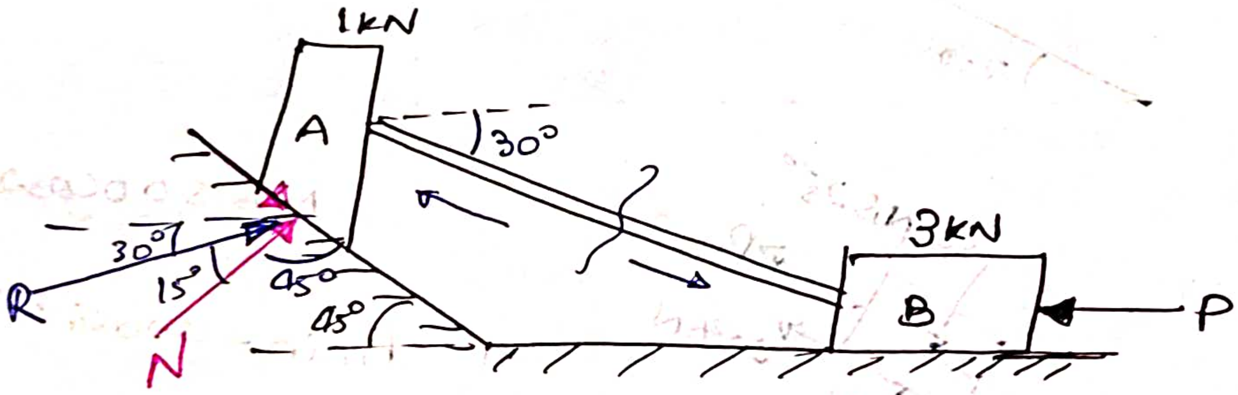
$N = 500\cos 25°$

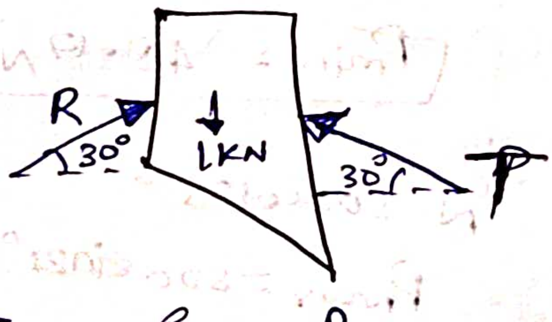
$P_{max} = 500\sin 25° + 0.36N$

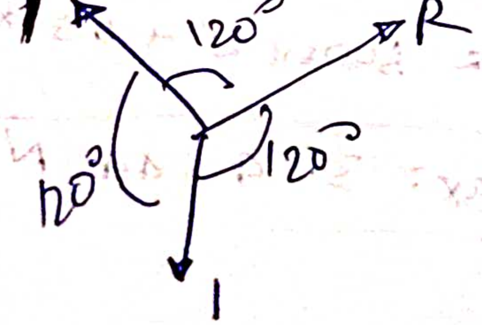
$P_{max} = 500\sin 25° + 0.36 \times 500\cos 25°$

$P_{max} = 374.44\ N$

Q. A block (A) weighing 1 kN rests on a rough inclined plane whose inclination to the horizontal is 45°. This block is connected to another block (B) weighing 3 kN rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal as shown in fig. Find horizontal force (P) required to be applied to the block (B) just to move the block (A) in the upward direction. Assume angle of limiting friction as 15° at all surface where there is sliding.





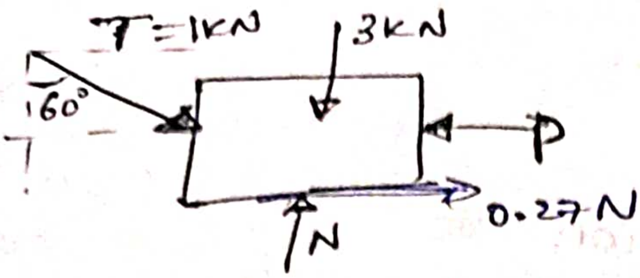


$P = R$

$2P \sin 30° = 1kN$

$P = 1KN$

$R = P = 1KN$



$\mu = \tan 15° = 0.27$

$\Sigma F_y = 0$

$N = 3KN + T\cos 60°$

$N = 3.5KN$

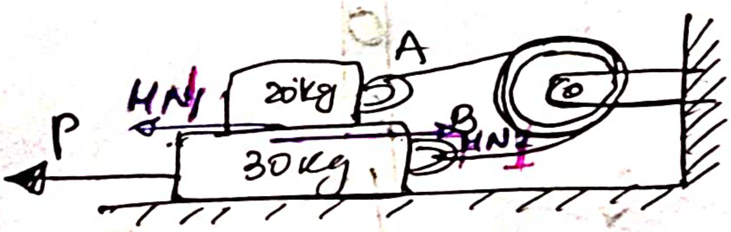
$\Sigma F_x = 0$

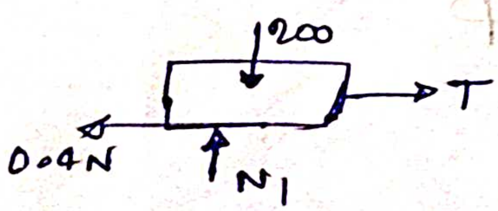
$P = 1KN \sin 60° + 0.27N$

$P = 1KN \times \frac{\sqrt{3}}{2} + 0.27 \times 3.5$

$P = 1.811 KN$

Q. Determine the smallest force P required to move the 30 kg block - Assume pulley to be frictionless & $\mu = 0.4$ for all the other surfaces.

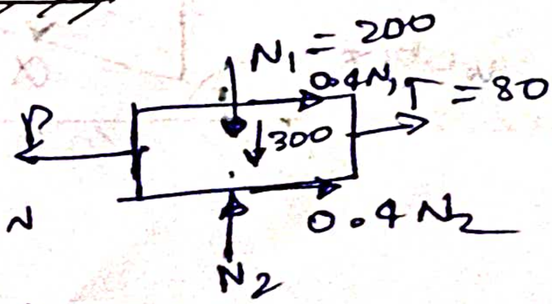




$N_1 = 200N$

$T = 0.4N_1 = 80N$

$N_2 = 200 + 300 = 500N$



$P = 0.4 \times 200 + 0.4 \times 500 + 80$

$P = 360N$

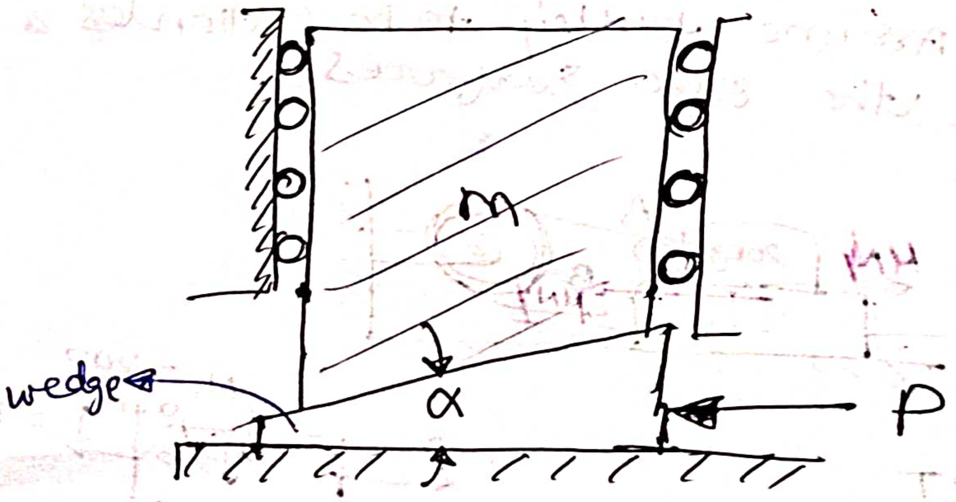
Application of Friction :-

1. ladder
2. Pulley
3. Rolling friction
4. wedge
5. screw jack
6. Brakes
7. Clutch
8. Bearing

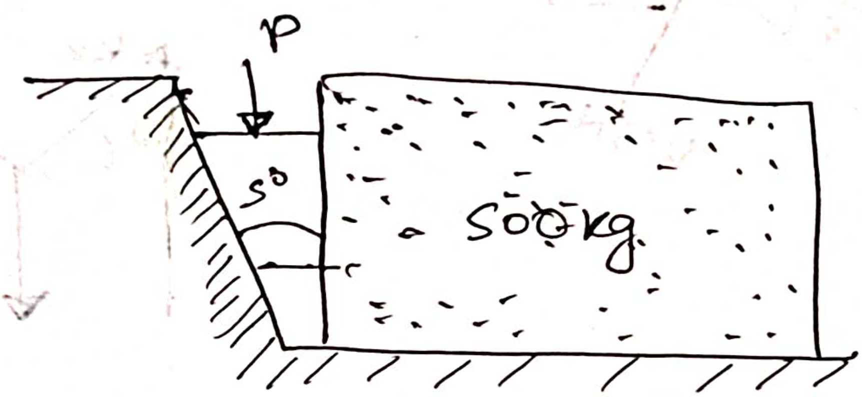
---

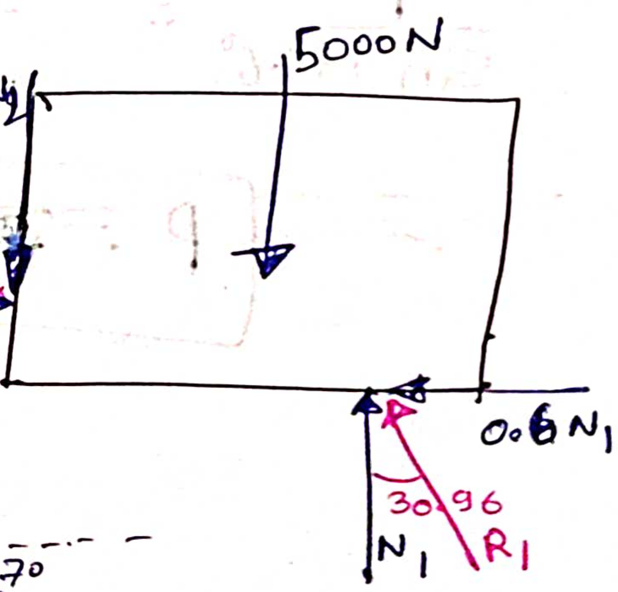
wedge

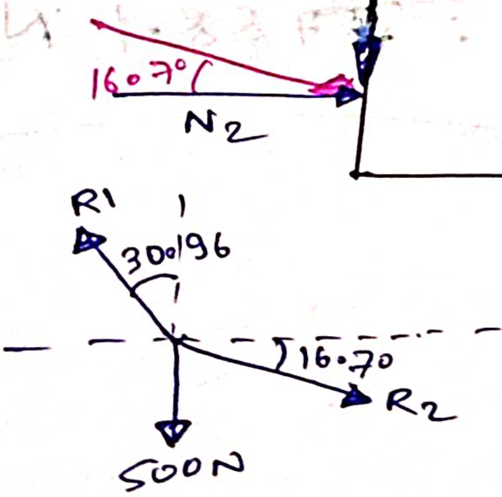
wedge are simple machines used to raise large stone blocks & other heavy loads (by applying minimum effort).



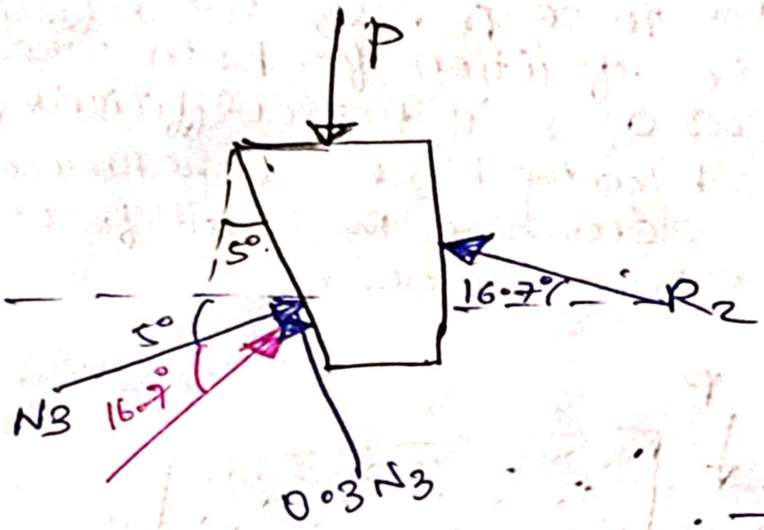
Q. The horizontal position of 500kg rectangular block of concrete is adjusted by the 5° wedge under the action of the force P. If the coefficient of ~~action~~ static friction for both wedge surfaces is 0.30 & if the coefficient of static friction b/w the block & the horizontal surface is 0.6 determine the least force P required to move the block.

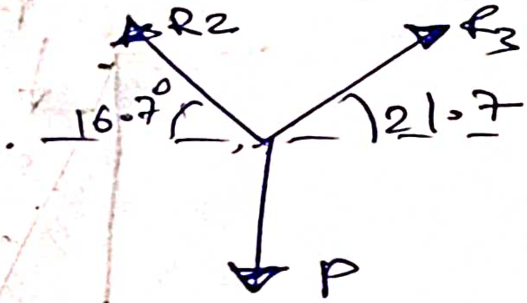






$$\frac{R_2}{\sin 149.04} = \frac{5000}{\sin 137.66} \Rightarrow \boxed{R_2 = 3818.98\,N}$$

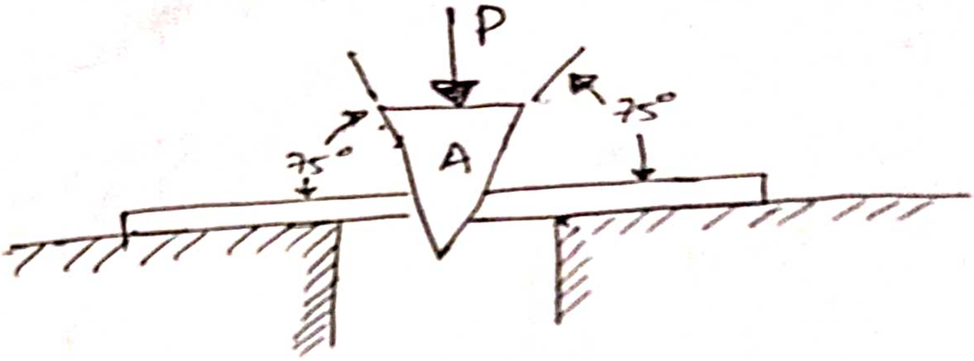


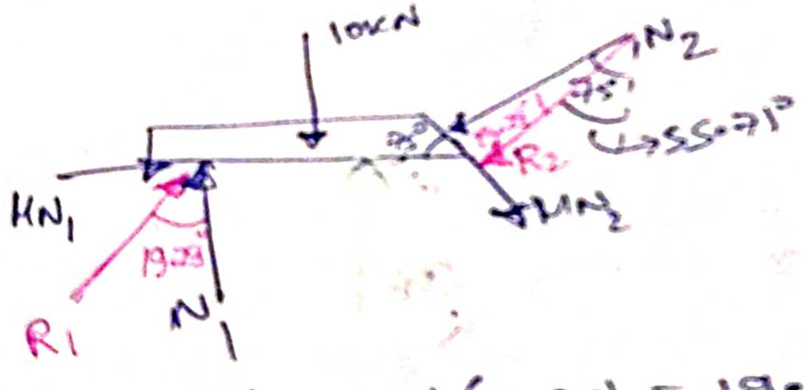


$$\frac{P}{\sin 146.6^\circ} = \frac{3818.98}{\sin 121.7^\circ}$$

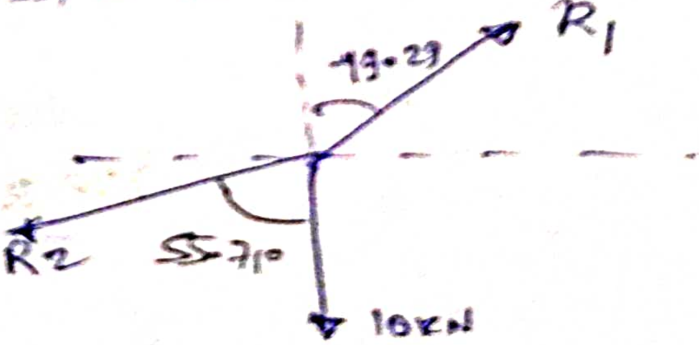
$$P = 2788.1\ N$$

Q. A wedge A of negligible weight is to be driven b/w two 10 kN plates B & C. The coefficient of static friction b/w all surfaces of contact is 0.35. Determine the magnitude of the Force P (in N) required to start moving the wedge.



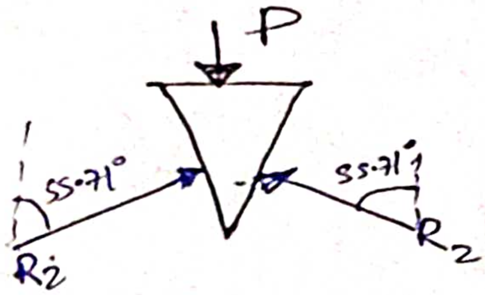


$$\phi = \tan^{-1}(0.35) = 19.29^\circ$$



$$\frac{R_2}{\sin(60.71} = \frac{10}{\sin 43.58^\circ}$$

$$\boxed{R_2 = 5.56\ kN}$$



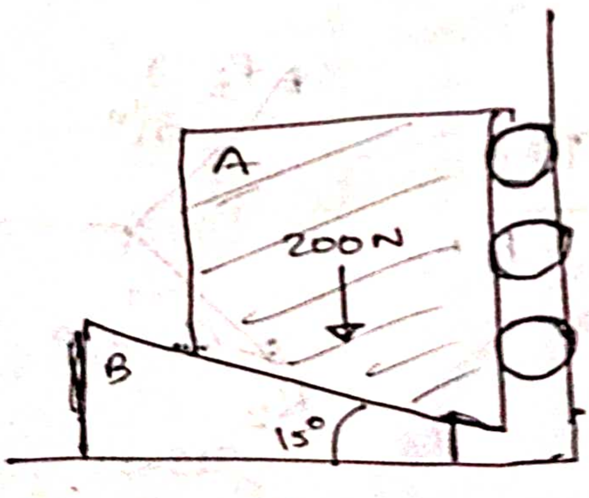
$\Sigma F_y = 0$

$P = 2 R_2 \cos 55.71°$

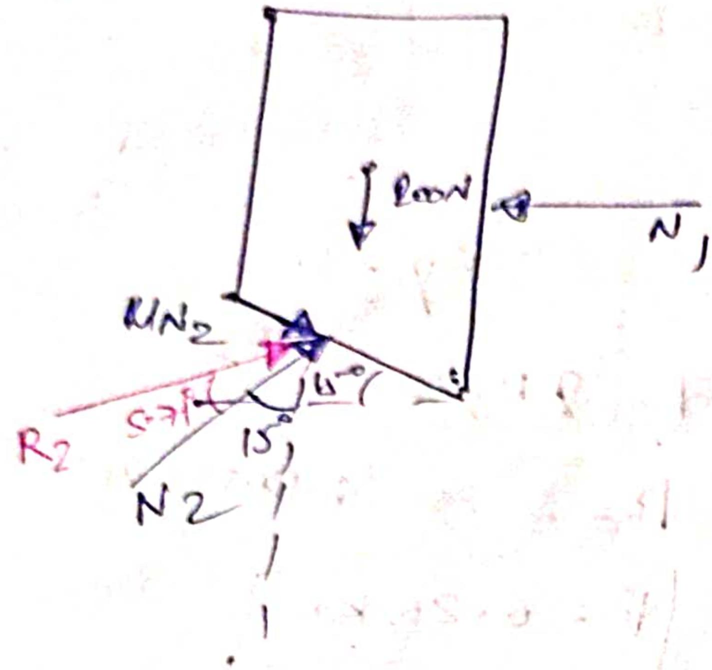
$P = 2 \times 5.56 \cos 55.71°$

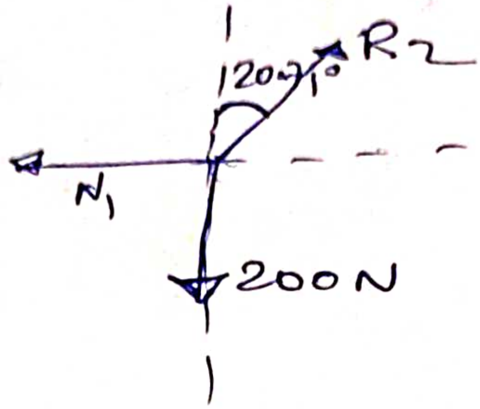
$\boxed{P = 6.26 \text{ KN}}$

Q. A 200N block rests as shown a wedge of negligible weight. The coefficient of static friction is 0.1 at both surfaces of the wedge, and friction b/w the block & the vertical wall may be neglected. Find the minimum force P required to move the block.



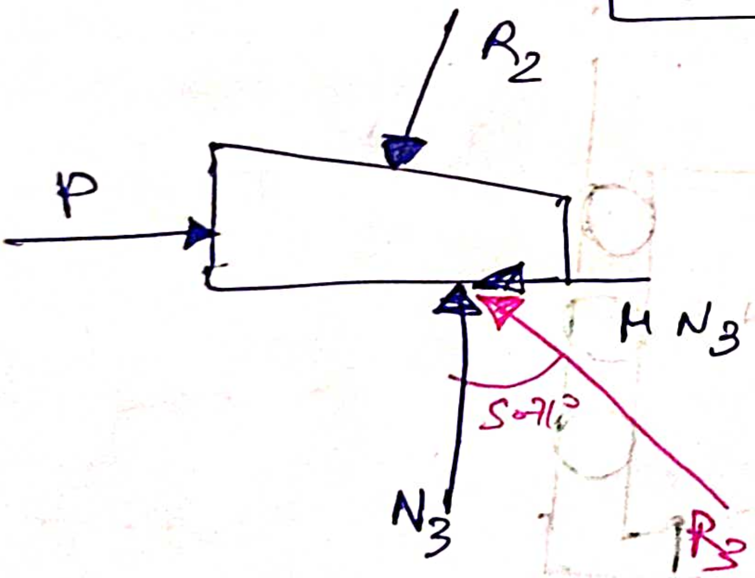
Soln:-

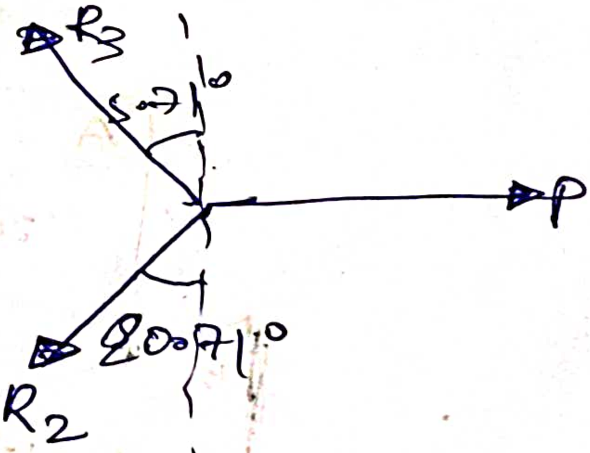




$$\frac{R_2}{\sin 90^\circ} = \frac{200}{\sin 110.71^\circ}$$

$$\boxed{R_2 = 213.81\ N}$$

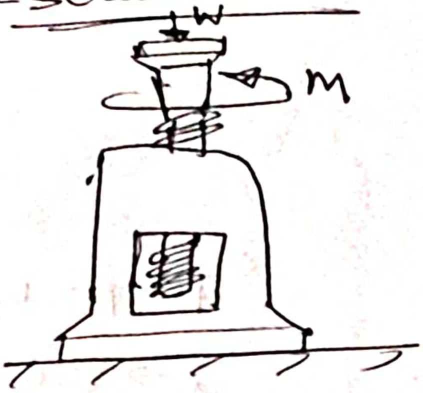


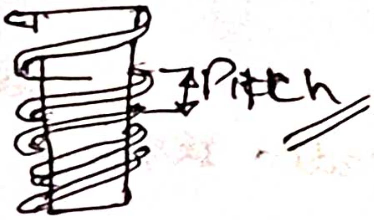


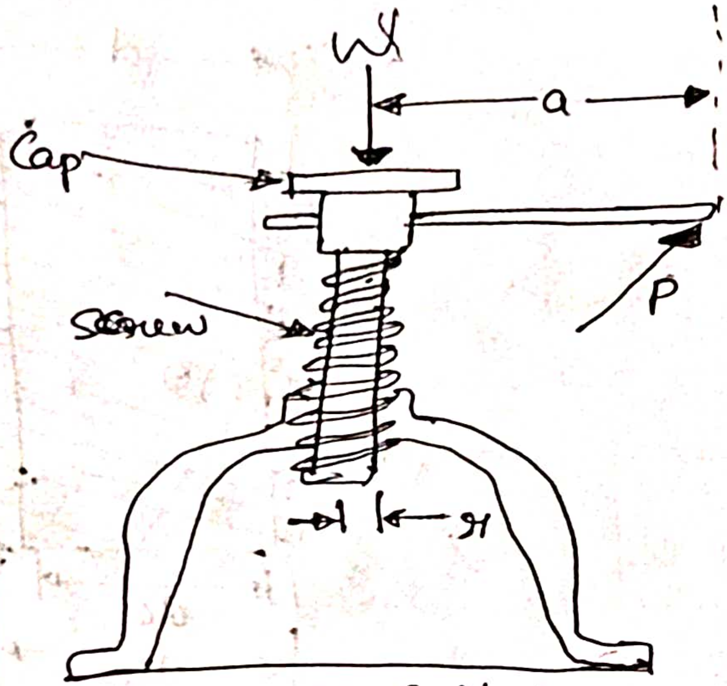
$$\frac{\overset{213.81}{\cancel{R_2}}}{\sin 95.71^\circ} = \frac{P}{\sin 153.58}$$

$$\boxed{P = 95.6\ N} \Leftarrow P = 213.81 \times \frac{\sin 153.58}{\sin 95.71^\circ}$$

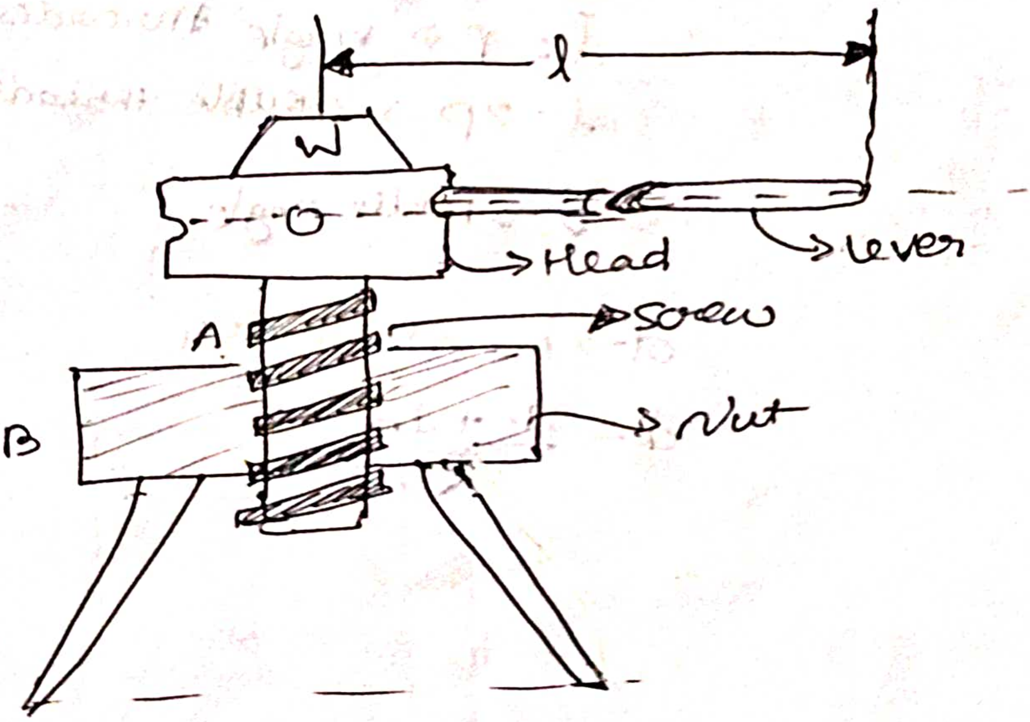
Screw Jack

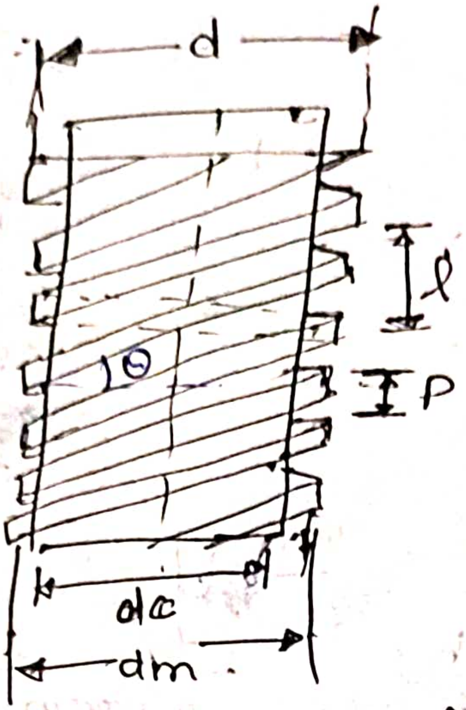






A screw as part of a jack carrying a load W.





(Double threaded screw)

p → pitch
l → lead

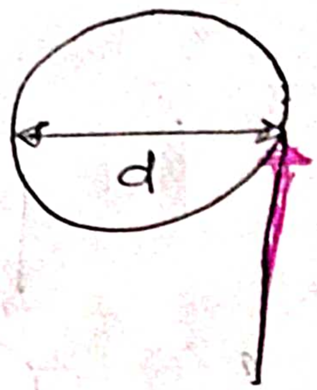
$l = p$ → single threaded screw
$l = 2p$ → double threaded screw

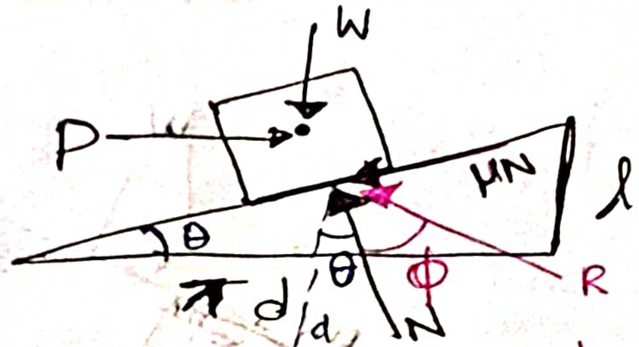
θ ⇒ Helix angle.

d → mean diameter

$$d = \frac{d_c + d_m}{2}$$

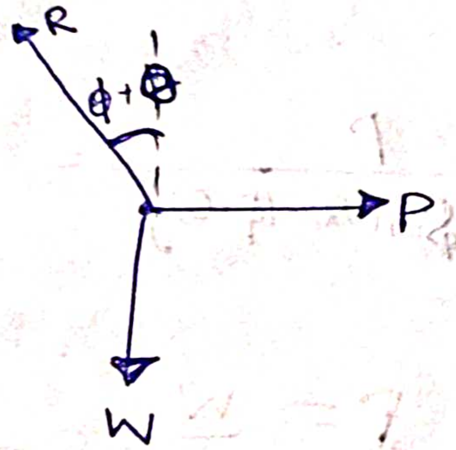
Effort to raise the load :-





$$\theta = \tan^{-1} \frac{l}{\pi d}$$

$$\phi = \tan^{-1} \mu$$

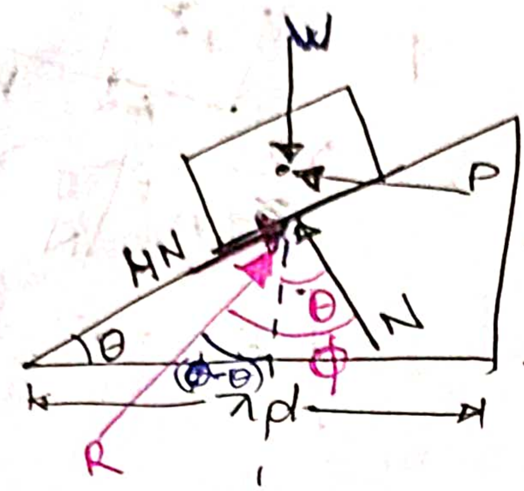


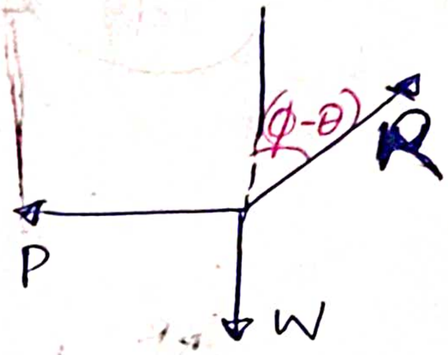
$$\frac{P}{\sin[180-(\theta+\phi)]} = \frac{W}{\sin[90+(\phi+\theta)]}$$

$$\frac{P}{\sin(\theta+\phi)} = \frac{W}{\cos(\theta+\phi)}$$

$$P = W \cdot \tan(\phi+\theta)$$

Effort required to lower the load :-





$$\frac{P}{\sin(180-(\phi-\theta))} = \frac{W}{\sin(90+(\phi-\theta))}$$

$$\frac{P}{\sin(\phi-\theta)} = \frac{W}{\cos(\phi-\theta)}$$

$$P = W \cdot \tan(\phi-\theta)$$

If $\tan(\phi-\theta) = 0$

or

$\phi = \theta$

then $P = 0$

Weight W is being lowered without any effort.

should

For self-locking

$P > 0$ or $\phi > \theta$

$$\eta = \frac{\text{Ideal effort to raise the load}}{\text{Actual effort to raise the load.}}$$

$$\eta = \frac{W \tan\theta}{W \tan(\phi+\theta)}$$

$$\boxed{\eta = \frac{\tan\theta}{\tan(\phi+\theta)}}$$

Q. A screw jack has mean diameter of 50 mm and pitch 10 mm. If the coefficient of friction b/w its screw & nut is 0.15, find the effort required at the end of 700 mm long handle to raise a load of 10 kN.

$$\phi = \tan^{-1} 0.15$$

$$\phi = 8.53$$

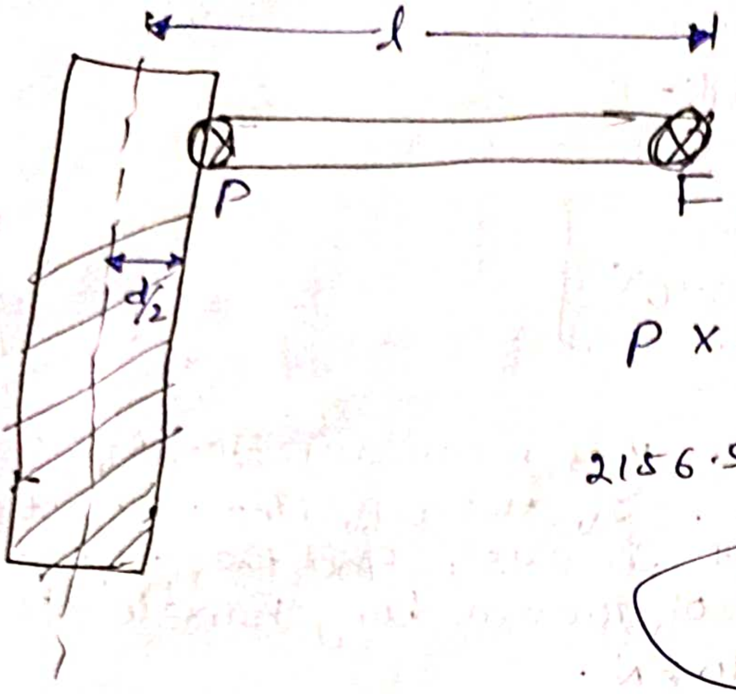
$$\theta = \tan^{-1}\left(\frac{10}{\pi \times 50}\right)$$

$$\theta = 3.64^\circ$$

$$P = W \tan(\phi + \theta)$$

$$P = 10 \times 10^3 (8.53 + 3.64)$$

$$\boxed{P = 2156.59 \text{ N}}$$



$$P \times \frac{d}{2} = F \times l$$

$$2156.59 \times \frac{50}{2} = F \times 700$$

$$F = 77\ N$$

Q. A load of 2.5 kN is to be raised by a screw jack with mean diameter of 75 mm & pitch of 12 mm. Find the efficiency of the screw jack, if the coefficient of friction b/w the screw & nut is 0.075.

$$\phi = \tan^{-1}(0.075) = 4.289^\circ$$

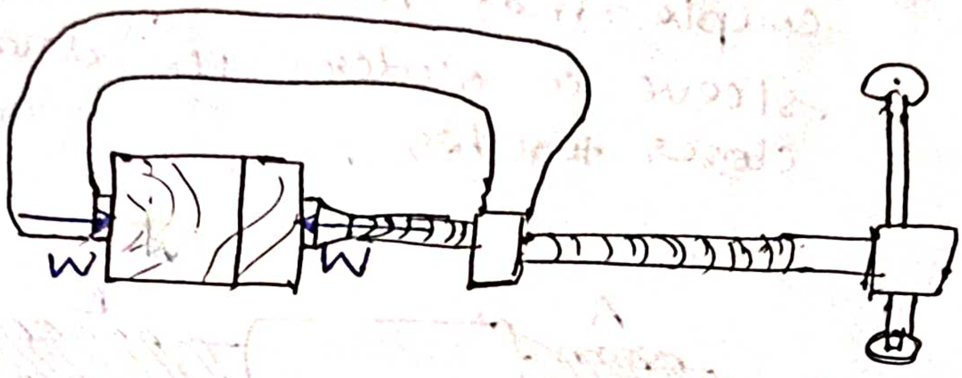
$$\theta = \tan^{-1}\left(\frac{12}{\pi \times 75}\right) = 2.910$$

$$\eta = \frac{\tan\theta}{\tan(\phi+\theta)} = \frac{\tan(2.91)}{\tan(4.289+2.91)}$$

$$\eta = 0.4024$$

$$\boxed{\eta = 40.24\%}$$

Q. A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction b/w threads is 0.30. If a maximum couple of 40 N·m is applied in tightening the clamp then the force exerted on the pieces of wood is ______ KN.

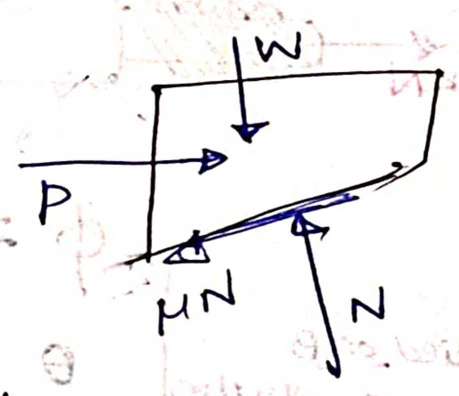


$\phi = \tan^{-1}(0.3)$

$\phi = 16.7^\circ$

$\theta = \tan^{-1}\left(\frac{4}{\pi \times 10}\right)$

$\theta = 7.256^\circ$



$P = W \tan(\theta + \phi)$

$8000 = W \tan(7.256 + 16.7)$

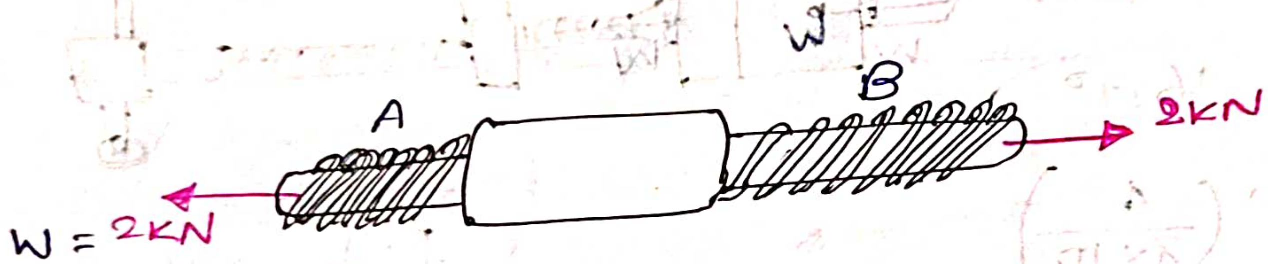
$W = 18005.5 \text{ N}$

Torque applied $= P \times \frac{d}{2} = 40$

$P \times \frac{0.01}{2} = 40$

$P = 8000 \text{ N}$

Q. The ends of two fixed rods A & B are each made in the form of a single threaded screw of mean radius 6mm & pitch 2mm. Rod A has a right handed thread & rod B has a left handed thread. The coefficient of static friction b/w the rods & the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.



$\phi = \tan^{-1}(0.12) = 6.84°$

$\theta = \tan^{-1}\left(\frac{2}{\pi \times 12}\right) = 3.036°$

$P = W\tan(\theta + \phi)$

$= 2\times10^3 \tan(3.036 + 6.84)$

$P = 348.193 \text{ N}$

Couple applied on A $= P \times \frac{d}{2}$

$= 348.193 \times 6\times10^{-3}$

$= 2.089$ N-m

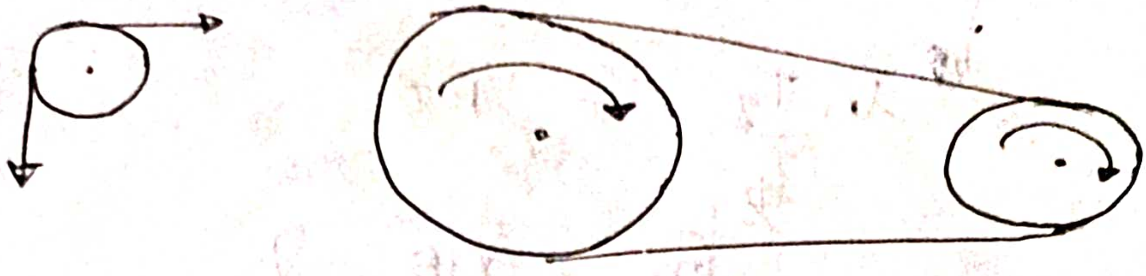
Couple applied on B $= 2.089$ Nm

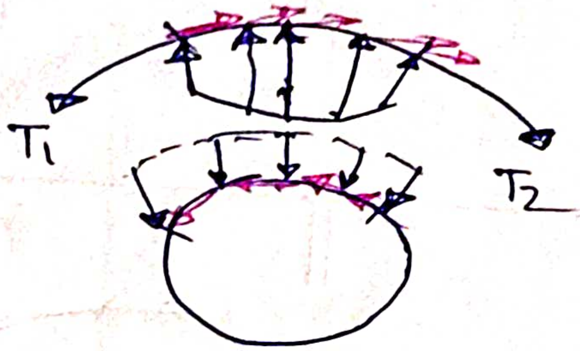
Total couple applied $= 2.089 + 2.089$

$= 4.178$ Nm

Ans

Belt Friction

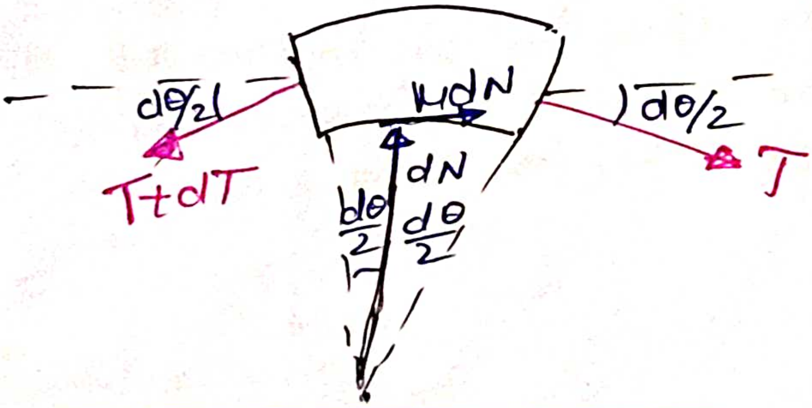




$\theta \rightarrow$ Angle of contact.

$T_1 \rightarrow$ Maxm tension in belt

$T_2 \rightarrow$ Minm tension in belt



$\Sigma F_y = 0$

$dN = T\sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2}$

$dN = T\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2} + dT\sin\frac{d\theta}{2}$

$\sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$

$dN = T\frac{d\theta}{2} + T\frac{d\theta}{2} + dT\frac{d\theta}{2}$

Negligible

$dN = T\,d\theta$

$\Sigma F_x = 0$

$T\cos\frac{d\theta}{2} + \mu dN = (T+dT)\cos\frac{d\theta}{2}$

~~$T\cos\frac{d\theta}{2}$~~ $+ \mu dN =$ ~~$T\cos\frac{d\theta}{2}$~~ $+ dT\cos\frac{d\theta}{2}$

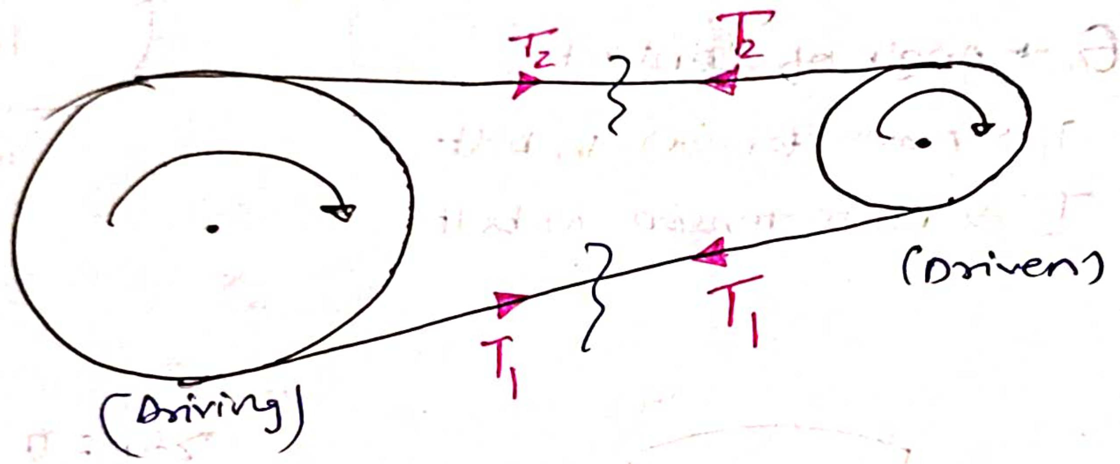
$\cos\frac{d\theta}{2} \approx 1$

$\mu T\,d\theta = dT \Rightarrow \frac{dT}{T} = \mu\,d\theta$

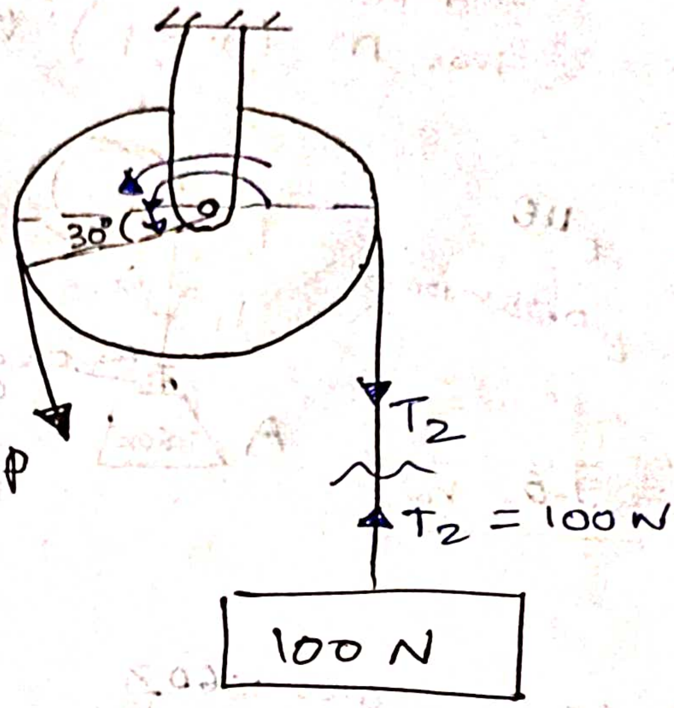
$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\theta} \mu \, d\theta$$

$$\ln \frac{T_2}{T_1} = \mu \theta$$

$$\boxed{\frac{T_2}{T_1} = e^{\mu\theta}}$$



Q. A rope is used to lift a 100N weight using a pulley as shown in the figure below. If the coefficient of friction is 0.3 the force necessary to begin lifting the load is ______.



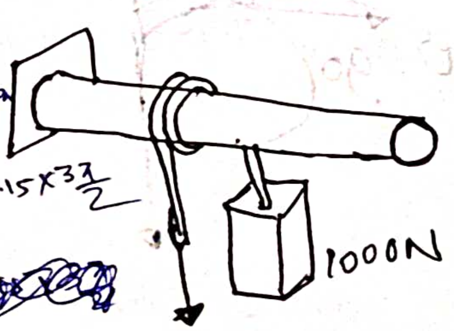
$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$T_1 = P$

$$\frac{P}{100} = e^{0.3 \times 210 \cdot \frac{\pi}{180}}$$

$$P = 300.28\ N$$

Q. A 1000N block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. If the coefficient of static friction b/w the rope & the rod is 0.15, the range of values of P for which the block will be in equilibrium is ______ N.



$T_1 = P_{min}$ $T_2 = 1000N$

$$\frac{P_{min}}{1000} = e^{0.15 \times 3\frac{\pi}{2}}$$

$$P_{min} = 243.2\ N$$

~~$P_{max} \ge 1000$~~

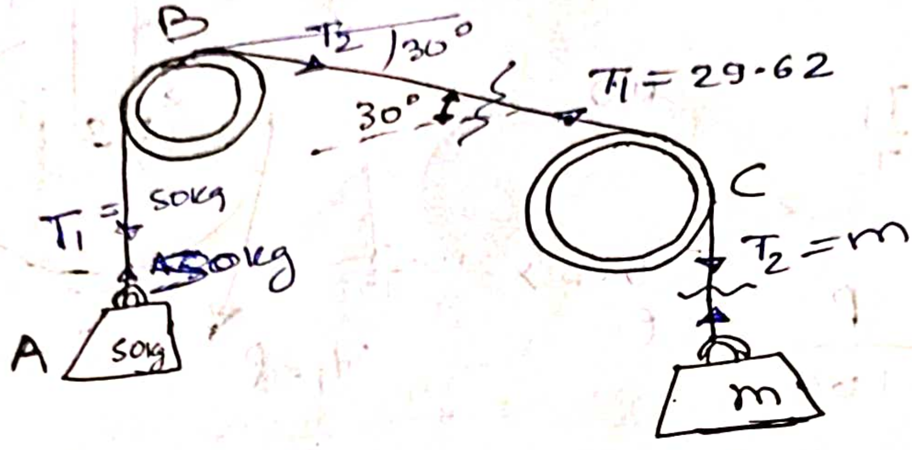
~~$P_{min} \le 1000$~~

$T_1 = 1000N$ $T_2 = P_{max}$

$$\frac{1000}{P_{max}} = e^{0.15 \times \frac{3\pi}{2}}$$

$$P_{max} = 4111.2$$

Q. A rope ABCD is looped over two pipes as shown. Known that the coefficient of static friction is 0.25 find the smallest value of the mass m (in kg) for which eqm is possible.



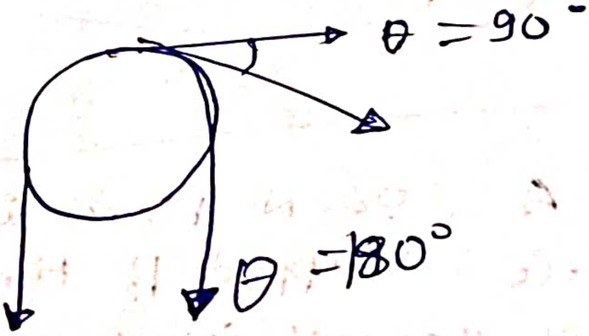
$$\frac{T_1}{T_2} = e^{\mu\theta}$$

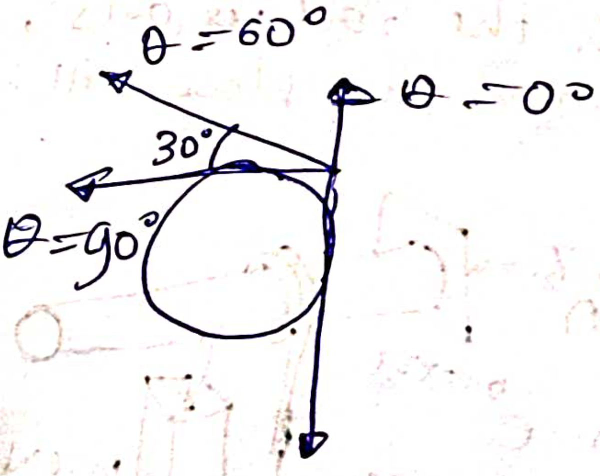
$$\frac{50}{T_2} = e^{0.25 \times 120 \times \frac{\pi}{180}}$$

$$\boxed{T_2 = 29.62 \text{ kg}}$$

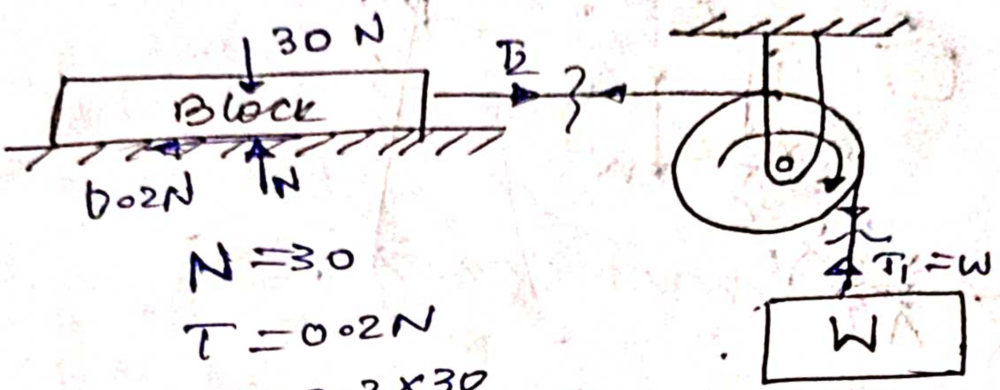
$$\frac{29.62}{m} = e^{0.25 \times 60 \frac{\pi}{180}}$$

$$\boxed{m = 22.79 \text{ kg}}$$





Q. Determine the minimum value of weight required to cause motion of a block which rests on a horizontal plane. The block weights 30N & the coefficient of friction b/w the block & surface is 0.2, whereas b/w pulley & rope is 0.3.



$N = 30$

$T = 0.2N$

$T = 0.2 \times 30$

$T = 6N$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{W}{6} = e^{0.3 \times \frac{\pi}{2}}$$

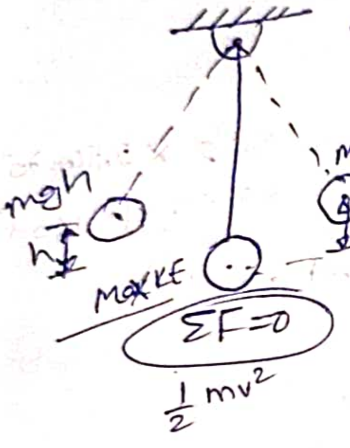
$W = 9.61\ N$ Ans

Vibration

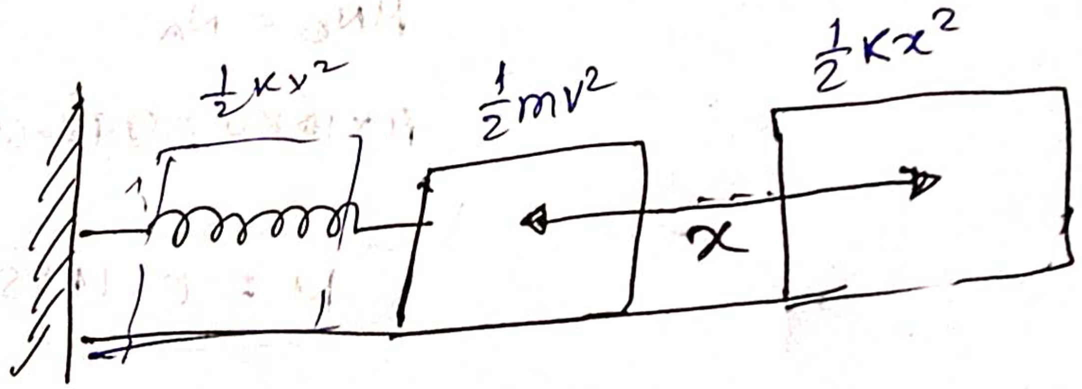
Q. What is vibration.

Ans → • A body is said to vibrate if it has a to & fro motion

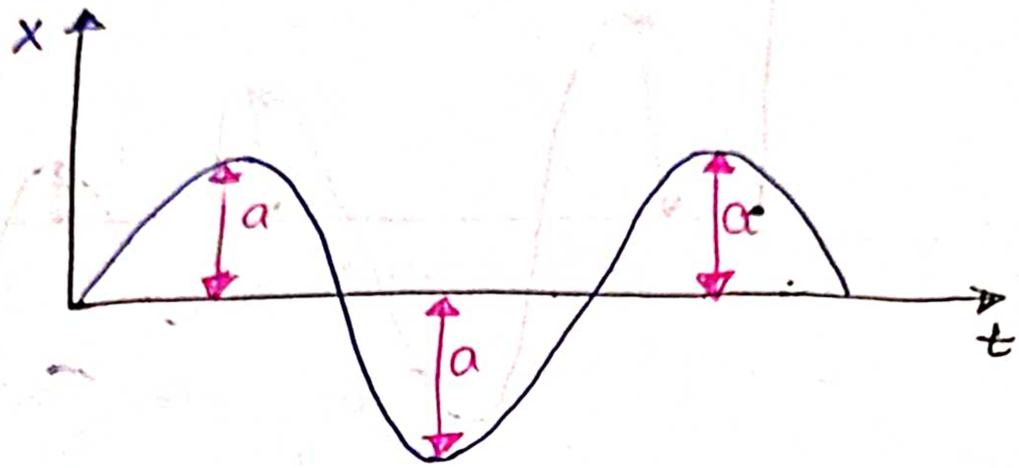
• when elastic bodies such as a spring, a beam or a shaft are displaced from the equilibrium position by the application of external forces & then released, they execute a vibratory motion.

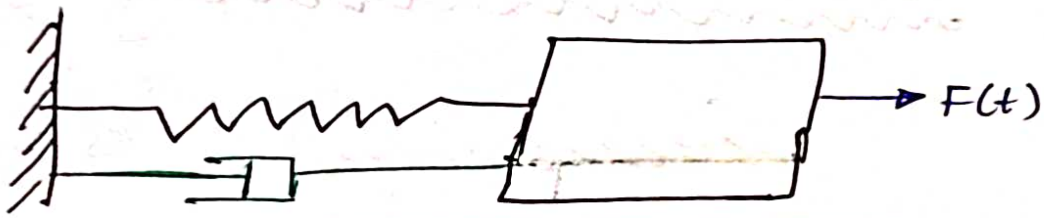


• when a body is displaced the internal forces in the form of elastic or strain energy are present in the body. At release, these forces bring the body to its original position. When the body reaches the equilibrium position the whole of the elastic or strain energy is converted into kinetic energy due to which the body continues to move in the opposite direction. The whole of the kinetic energy is again converted into strain energy due to which the body again returns to the equilibrium position



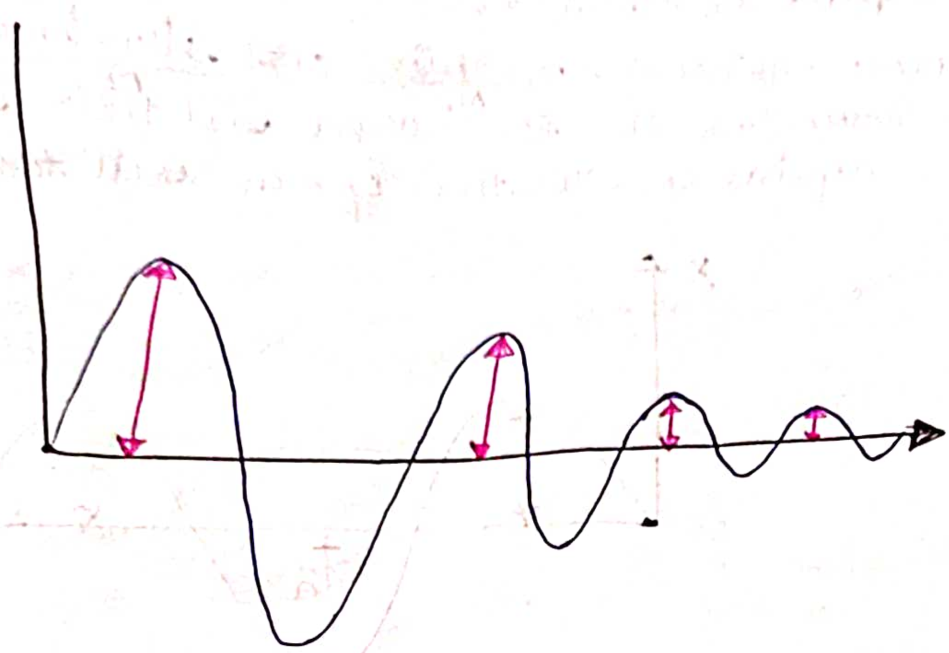
- All bodies possessing mass & elasticity are capable of vibrations
- Most engineering machines (ME) and structures (CE) experience vibrations to some degree and their design generally requires consideration of their oscillatory motions.



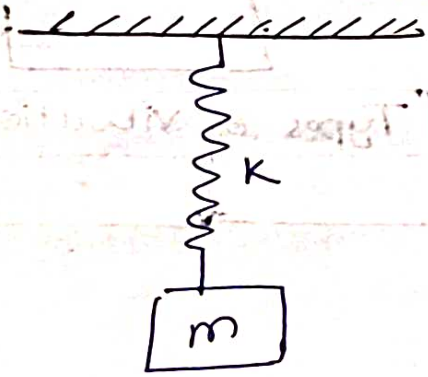


Types of Vibrations

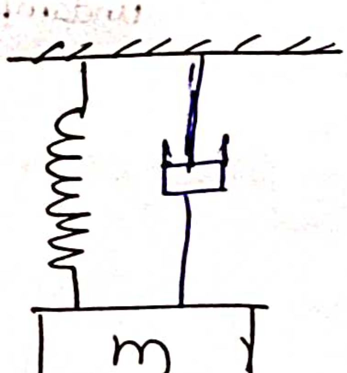
- Free
  - undamped
  - Damped
- Forced
  - undamped
  - Damped.



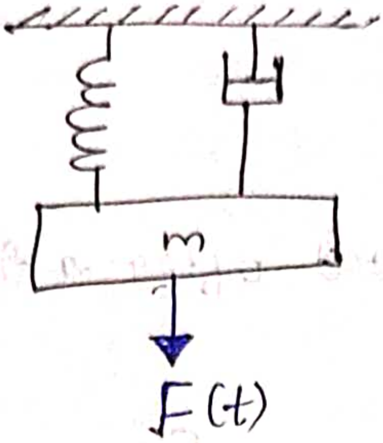
Free undamped oscillation :-



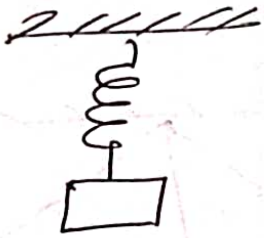
Free damped oscillation :-



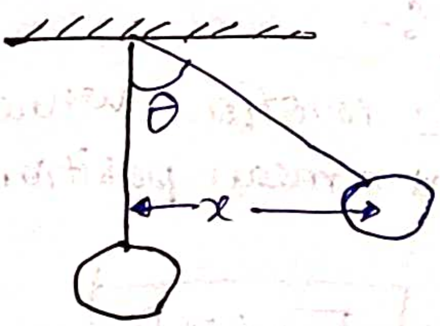
Forced Damped

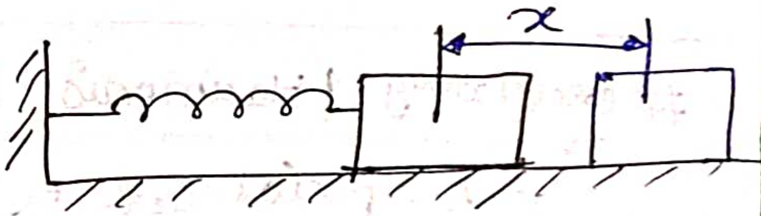


Free Vibration of undamped single degree of Freedom Systems



Single Degree of Freedom

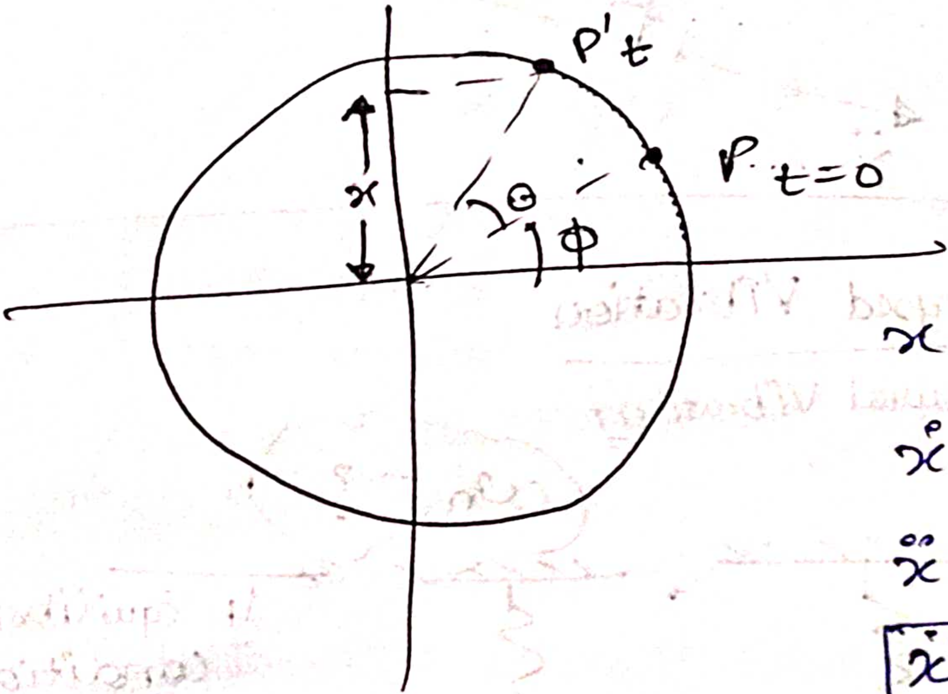




$$\boxed{f = \frac{1}{T}} \quad (Hz)$$

Angular frequency (ω) :-

$$\omega = 2\pi f \quad (rad/s)$$



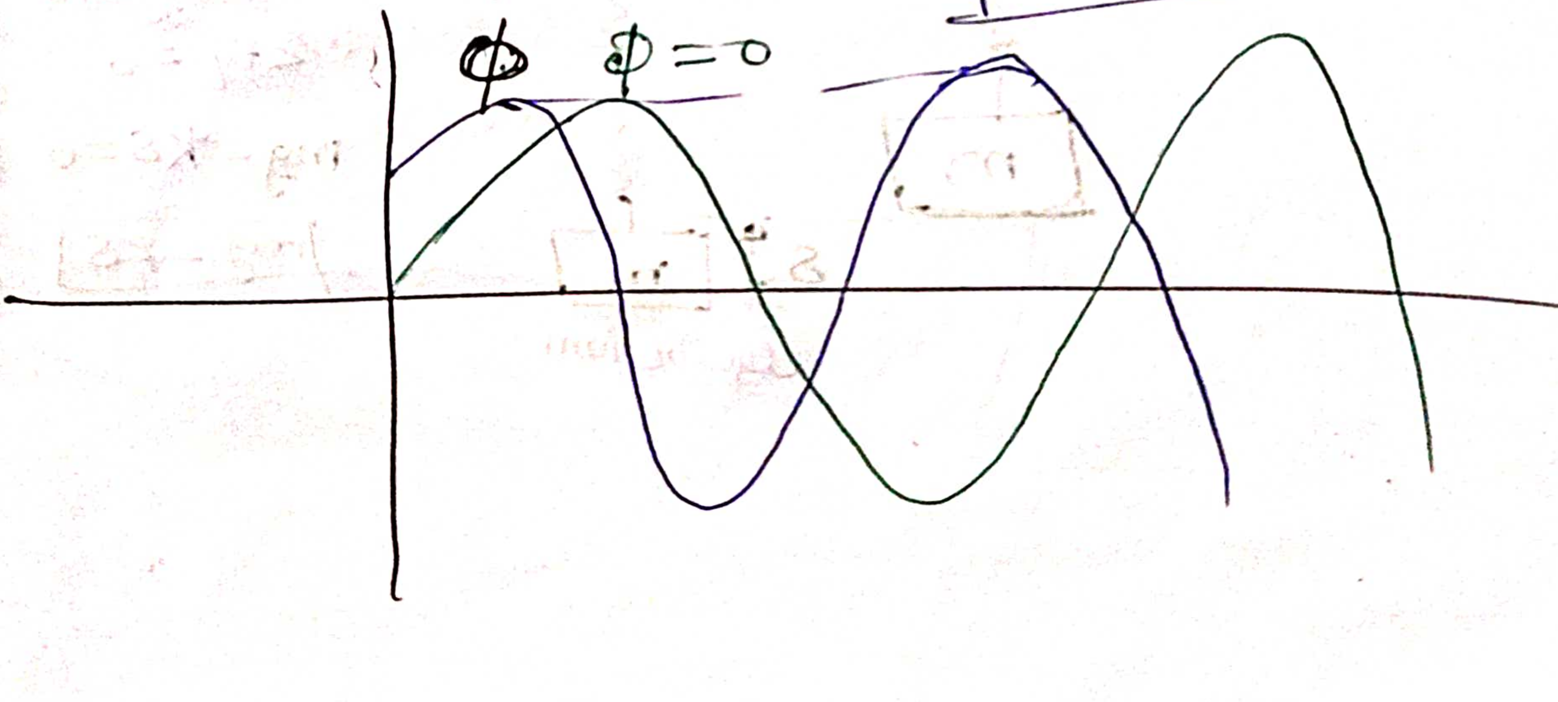
$$x = r \sin(\theta + \phi)$$

$$x = r \sin(\omega t + \phi)$$

$$\dot{x} = \omega r \cos(\omega t + \phi)$$

$$\ddot{x} = -\omega^2 r \sin(\omega t + \phi)$$

$$\boxed{\ddot{x} = -\omega^2 x}$$

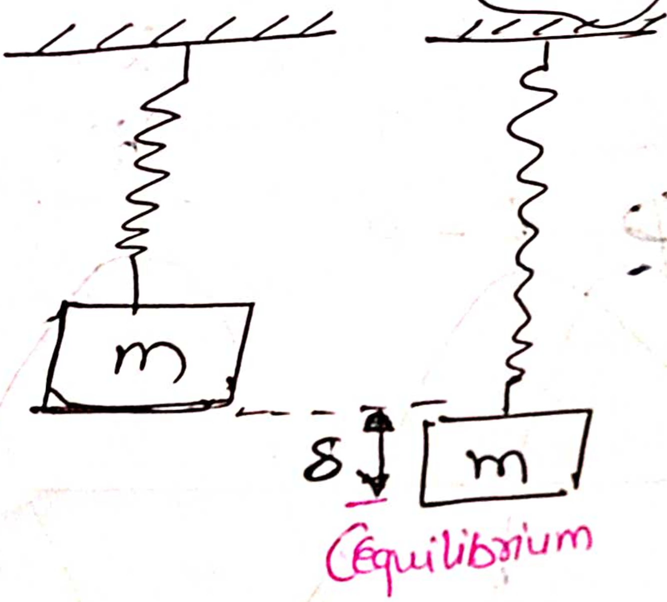


$\ddot{x}$

Free undamped Vibration

[Natural Vibration]

$\omega_n = ?$

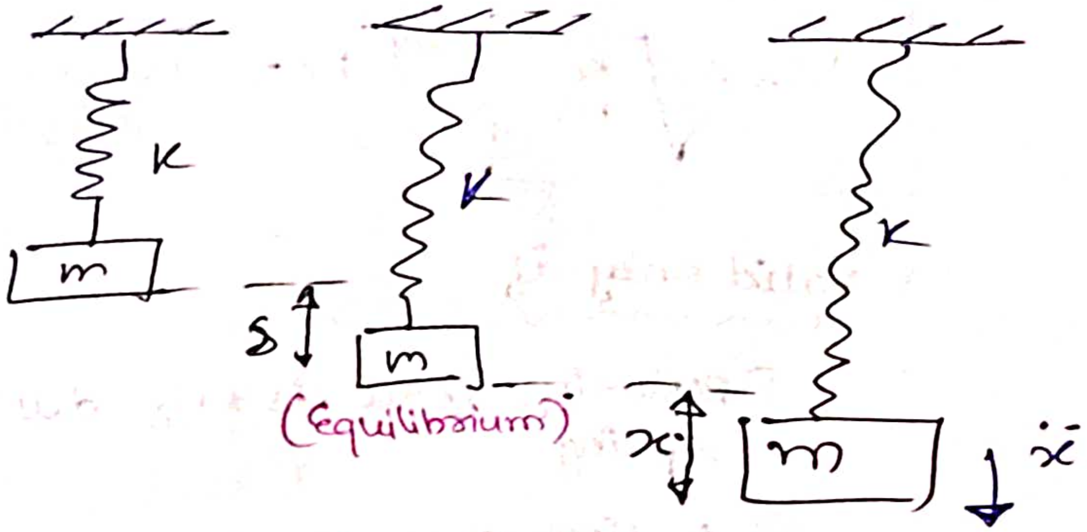


In Equilibrium condition:-

$\Sigma F_y = 0$

$mg - k\delta = 0$

$$\boxed{mg = k\delta}$$



After initial displacement x :-

$$\Sigma F_y = ma$$

$$mg - k(x+\delta) = m\ddot{x}$$

$$mg - kx - k\delta = m\ddot{x}$$

$$m\ddot{x} + kx = 0$$

$$\Rightarrow \ddot{x} + \frac{k}{m}x = 0$$

$\ddot{x} + \omega^2 x = 0$ — SHM

$$\omega_n^2 = \frac{k}{m}$$

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\omega_n = \sqrt{\frac{k_e}{m}}$$ [$k_e \rightarrow$ Equivalent stiffness]

* Valid only if

① Restoring force is only due to spring

② A single mass m is concentrated at a point.

$$mg = k\delta$$

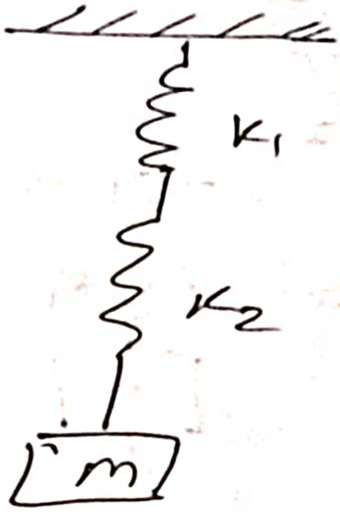
$$\therefore \frac{k}{m} = \frac{g}{\delta}$$

$$\omega_n = \sqrt{\frac{g}{\delta}}$$

* δ is the static deflection of the point where mass m is concentrated.

Spring in series

(1)



$$F = k\delta$$

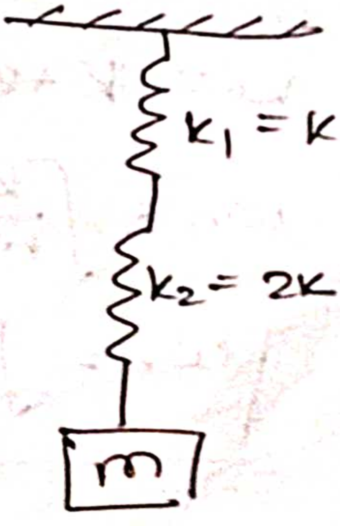
$$\boxed{\delta = \frac{F}{k}}$$

$$\delta = \delta_1 + \delta_2$$

$$\frac{F}{k_e} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\boxed{\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Q.



$\omega_n = ?$

$$\frac{1}{k_e} = \frac{1}{k} + \frac{1}{2k}$$

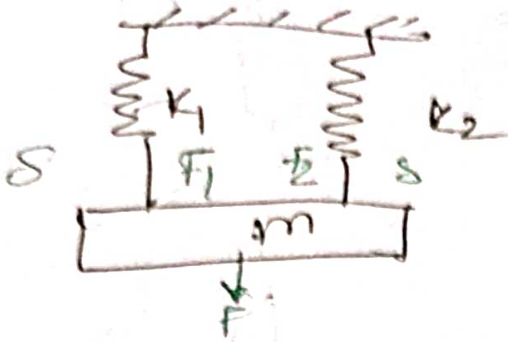
$$\frac{1}{k_e} = \frac{2+1}{2k} = \frac{3}{2k}$$

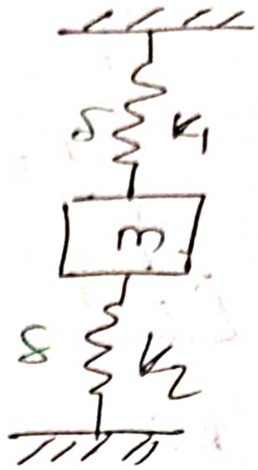
$$\boxed{k_e = \frac{2}{3}k}$$

$$\omega_n = \sqrt{\frac{k_e}{m}}$$

$$\boxed{\omega_n = \sqrt{\frac{2k}{3m}}}$$

Spring in Parallel :-



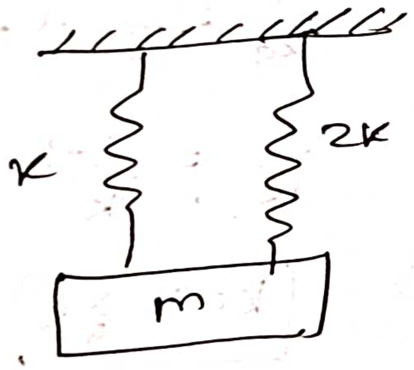


$$F = F_1 + F_2$$

$$K_e \delta = K_1 \delta + K_2 \delta$$

$$\boxed{K_e = K_1 + K_2}$$

Q.

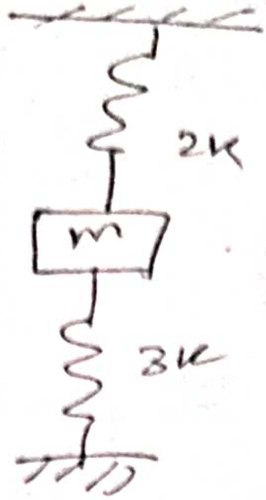


$$K_e = K + 2K = 3K$$

$$\omega_n = \sqrt{\frac{K_e}{m}}$$

$$\boxed{\omega_n = \sqrt{\frac{3K}{m}}}$$

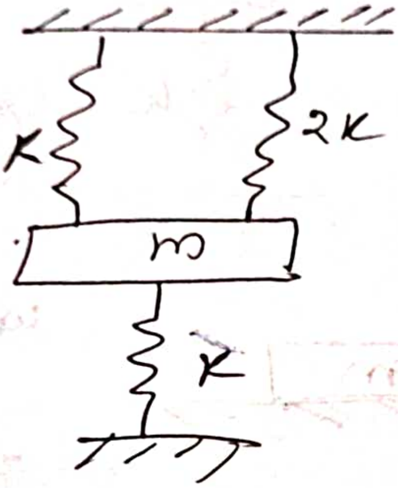
Q.



$k_e = 2k + 3k = 5k$

$$\omega_n = \sqrt{\frac{5k}{m}}$$

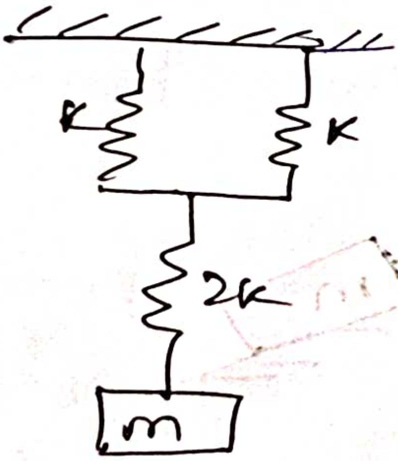
Q.

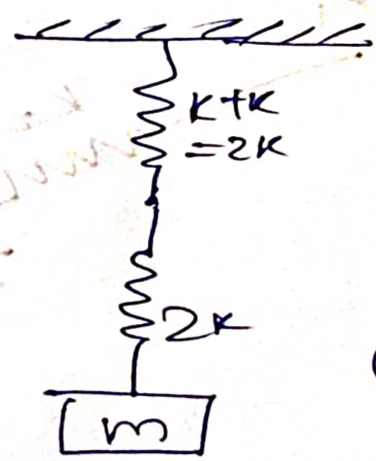


$k_e = k + 2k + k = 4k$

$$\omega_n = \sqrt{\frac{4k}{m}}$$

Q.



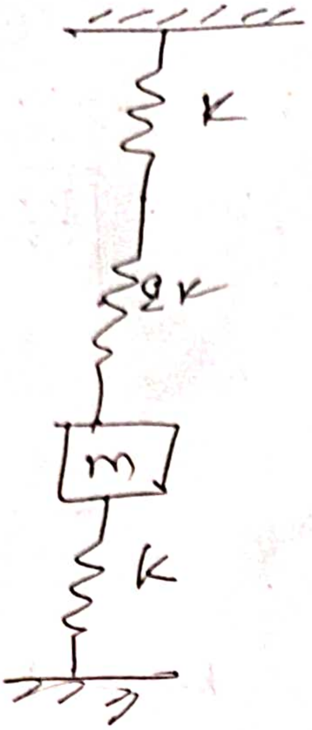


$$\frac{1}{k_e} = \frac{1}{2k} + \frac{1}{2k}$$

$k_e = k$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Q.

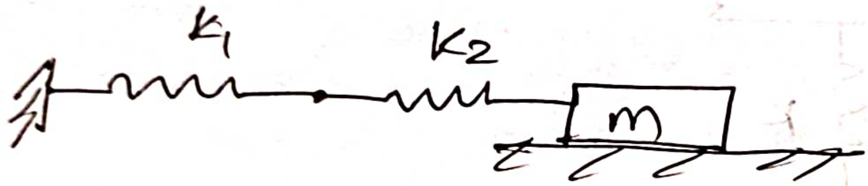


$$\frac{1}{k_e} = \frac{1}{k} + \frac{1}{2k}$$

$$k_e = \frac{2k}{3}$$

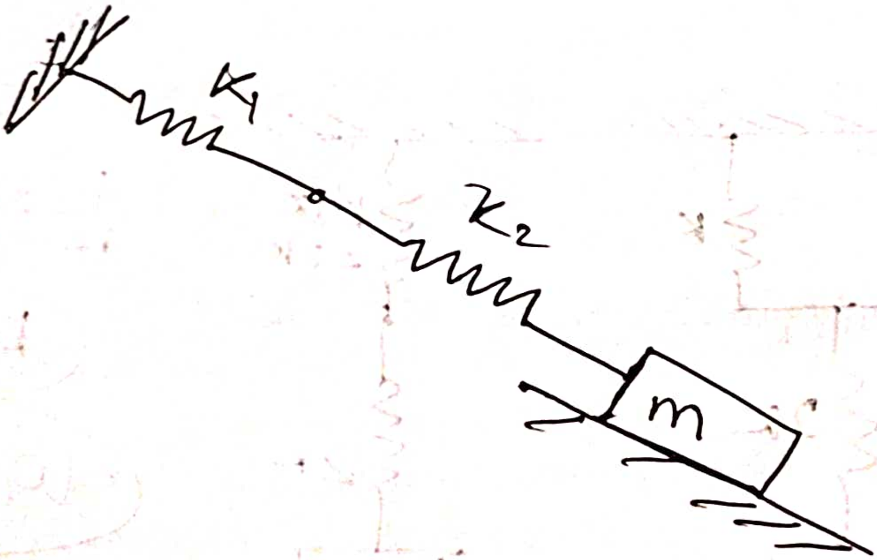
$$k_e = \frac{2k}{3} + k = \frac{5k}{3}$$

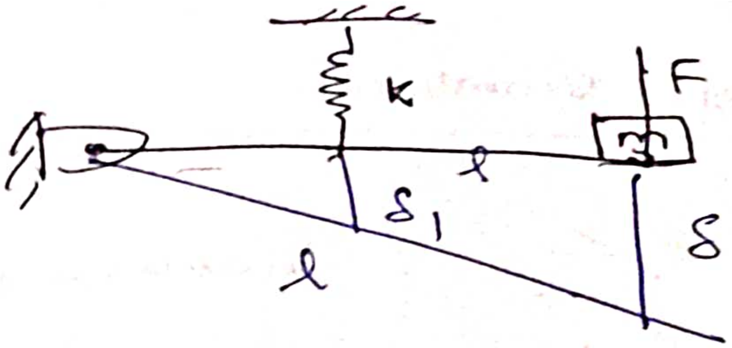
$$\omega_n = \sqrt{\frac{5k}{3m}}$$



$$\omega_n = \sqrt{\frac{k_e}{m}}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

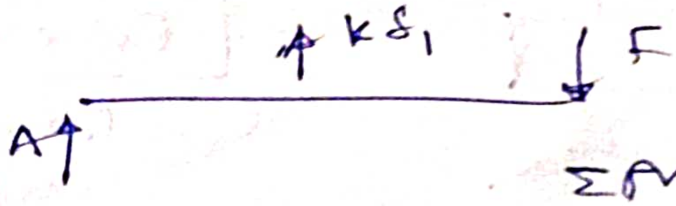




$$k_e = \frac{F}{\delta}$$

$$\frac{\delta}{2l} = \frac{\delta_1}{l}$$

$$\delta_1 = \frac{\delta}{2}$$



$$\Sigma M_A = 0$$

$$F \times 2l = k\delta_1 \times l$$

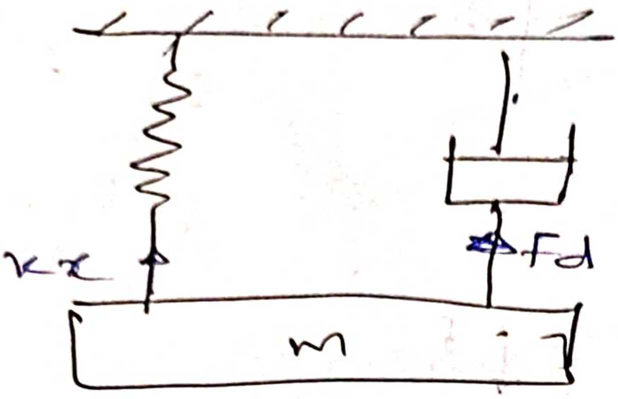
$$2F = \frac{k\delta}{2}$$

$$\frac{F}{\delta} = \frac{k}{4} = k_e$$

$$\omega_n = \sqrt{\frac{k_e}{m}}$$

$$\omega_n = \sqrt{\frac{k}{4m}}$$

Free damped Vibration



Viscous damping:–

$F_d \propto \dot{x}$

$$F_d = c\dot{x}$$

c = Damping coefficient

Energy method ✗

$$\sum f = m\ddot{x}$$

$$-kx - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

Let $x = Ae^{\alpha t}$

$$\dot{x} = \frac{dx}{dt} = A\alpha e^{\alpha t}$$

$$\ddot{x} = \frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t}$$

$$A\alpha^2 e^{\alpha t} + \frac{c}{m} A\alpha e^{\alpha t} + \frac{k}{m} A e^{\alpha t} = 0$$

$$\alpha^2 + \frac{c}{m}\alpha + \frac{k}{m} = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha = \frac{-\left(\frac{c}{m}\right) \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}}{2}$$

$$\alpha = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$\left(\frac{\left(\frac{c}{2m}\right)}{\left(\frac{k}{m}\right)}\right)^2 = \text{Degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\frac{k}{m}}} = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_n} = \zeta$$

= Damping factor

$$\frac{c}{2m} = \zeta\omega_n$$

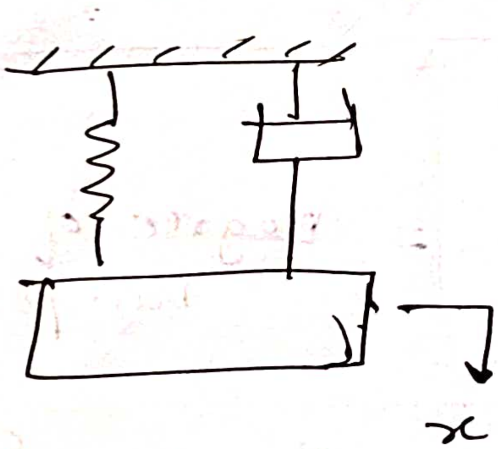
$$\alpha = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2}$$

$$\alpha = \omega_n \left[-\zeta \pm \sqrt{(\zeta^2 - 1)}\right]$$

$\zeta < 1$ (underdamping

$\zeta = 1$ Critical damping

$\zeta > 1$ over damping



Case-1 $\tau < 1$ (underdamping)

$$\alpha = \omega_n \left[-\tau \pm \sqrt{\tau^2 - 1} \right]$$

$$\alpha = \left[-\tau\omega_n \pm \sqrt{(1-\tau^2)}\,\omega_n \right]$$

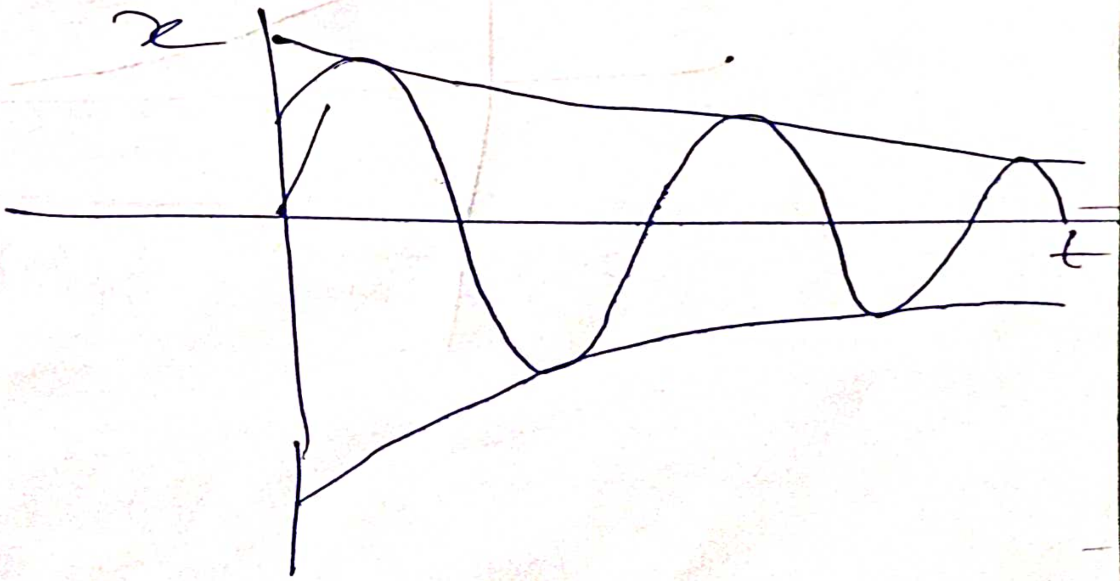
$$x = e^{-at} \sin(bt + \phi)$$

$$\boxed{x = r\, e^{-\tau\omega_n t} \sin\left(\sqrt{1-\tau^2}\,\omega_n + \phi\right)}$$

$$x = r \sin(\omega t + \phi)$$

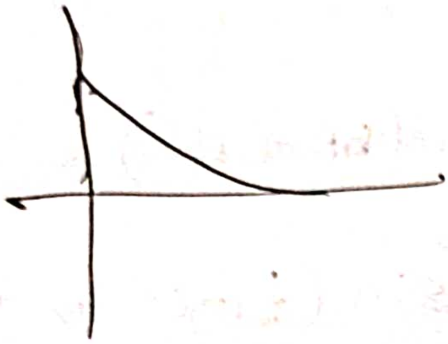
$$\boxed{\omega_d = \omega_n \sqrt{1-\tau^2}}$$

$$x = r\, e^{-\tau\omega_n t} \sin(\omega_d t + \phi)$$



Case - II $\quad \tau = 1$ (critical damping)

$$\tau = \frac{c}{2m\omega_n}$$

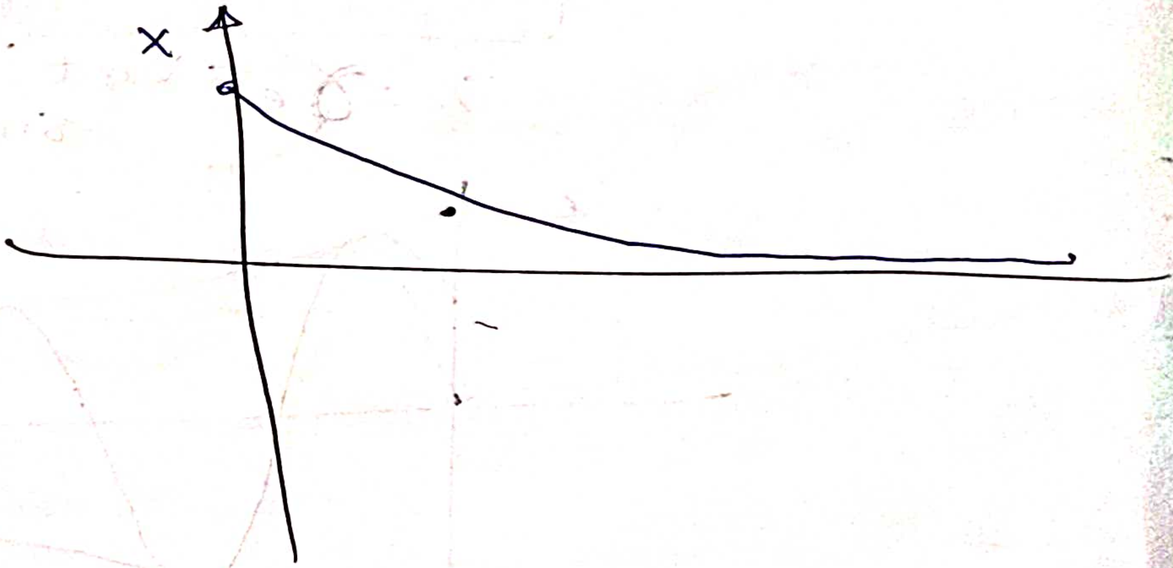


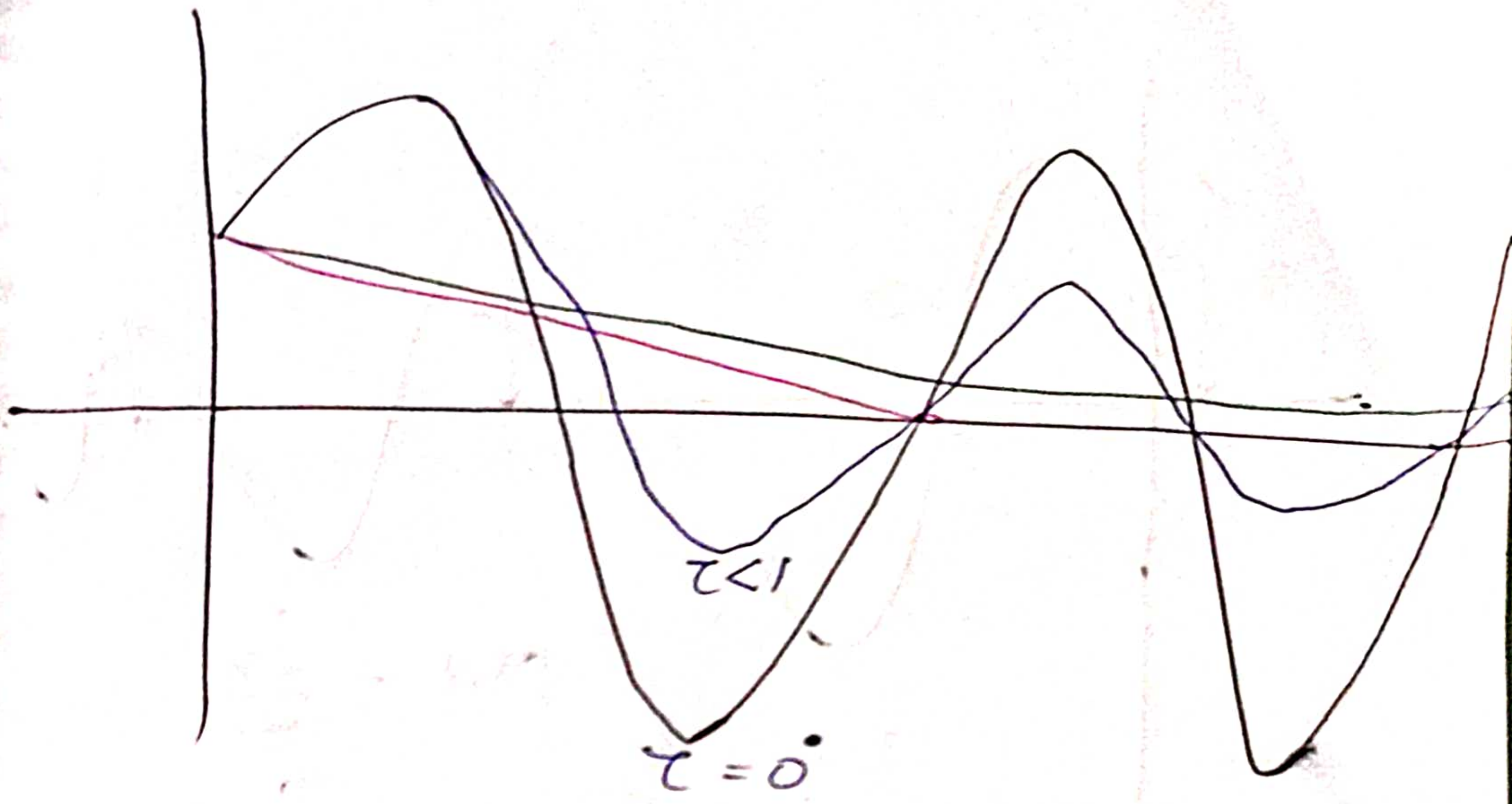
$$1 = \frac{C_c}{2m\omega_n}$$

C_c = critical Damping coefficient

$$\boxed{C_c = 2m\omega_n}$$

Case - III $\quad \tau > 1$ (over Damping





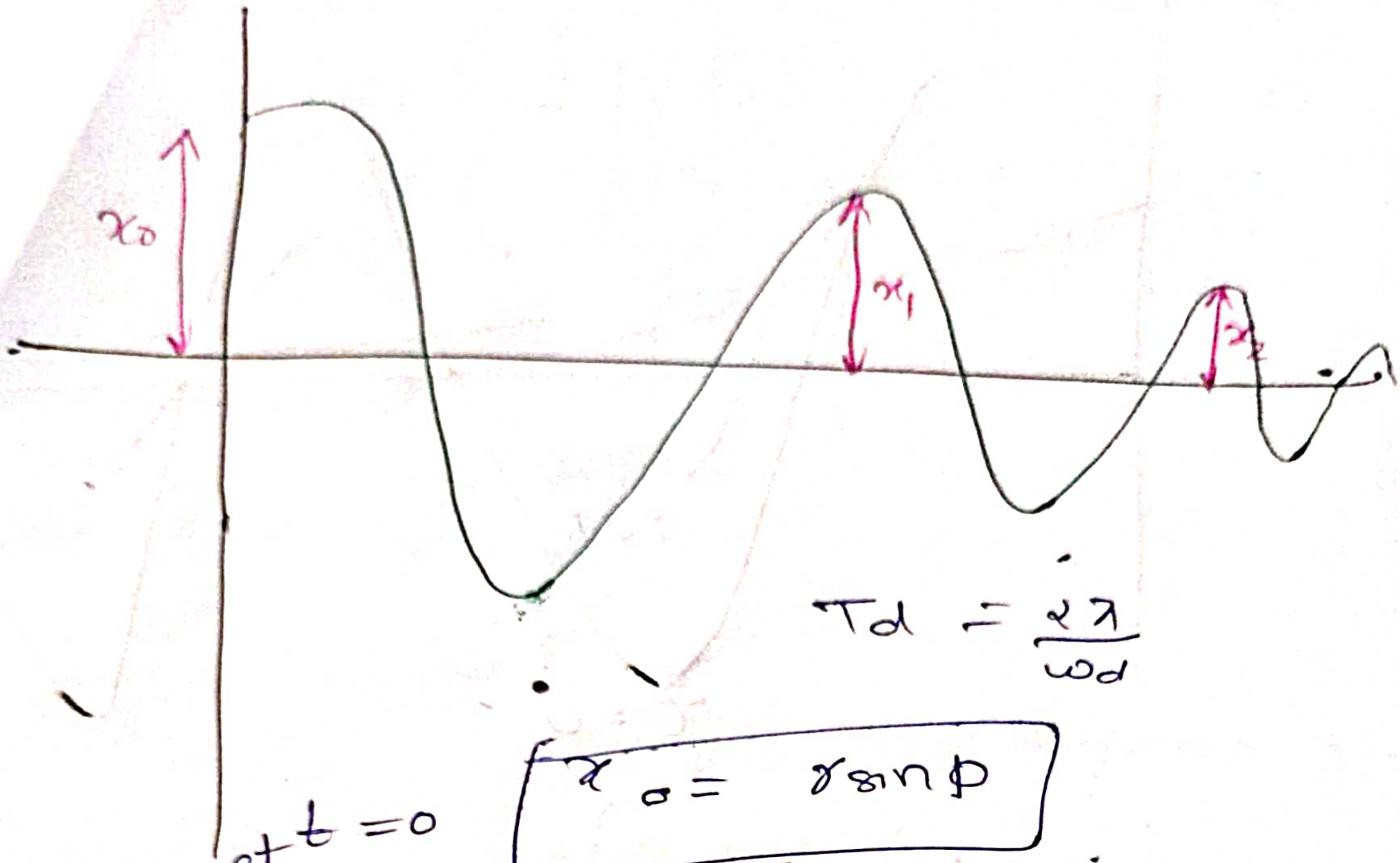
$$DOD = \tau^2$$

$$\tau = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_n} = \frac{c}{c_c}$$

$$c_c = 2m\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\tau^2}$$

$\omega_d < \omega_n$



$$T_d = \frac{2\pi}{\omega_d}$$

at $t = 0$

$$x_0 = r \sin\phi$$

at $t = T_d$

$$x_1 = r e^{-\zeta\omega_n T_d} \sin(\omega_d T_d + \phi)$$

$$= r e^{-\zeta\omega_n T_d} \sin(2\pi + \phi)$$

$$x_1 = r e^{-\zeta\omega_n T_d} \sin\phi$$

at $t = 2T_d$

$$x_2 = r e^{-\zeta\omega_n 2T_d} \sin\phi$$

$$x_N = r e^{-\zeta\omega_n N T_d} \sin\phi$$

$$\frac{x_0}{x_1} = \frac{r \sin\phi}{r e^{-\zeta \omega_n T_d} \sin\phi} = e^{\zeta \omega_n T_d}$$

$$\frac{x_1}{x_2} = \frac{r e^{-\zeta \omega_n T_d} \sin\phi}{r e^{-\zeta \omega_n 2T_d} \sin\phi} = e^{\zeta \omega_n T_d}$$

Ratio of amplitude of two successive vibration is same

$$\boxed{\frac{x_N}{x_{N+1}} = e^{\zeta \omega_n T_d}}$$

Logarithmic Decrement :-

$$\delta = \ln\left(\frac{x_N}{x_{N+1}}\right) = \zeta \omega_n T_d$$

$$= \zeta \omega_n \frac{2\pi}{\omega_d}$$

$$= \zeta \omega_n \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$\boxed{\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}}$$

Q. A vibrating system consists of mass of 200 kg a spring of stiffness 80 N/mm & a damper with damping coefficient 800 N/m/s Determine frequency of vibration of system (in Hz)

$m = 200 kg$

$k = 80 N/mm$

$800 N/m/s$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{800}{2 \times \sqrt{80 \times 10^3 \times 200}}$$

$$\boxed{\zeta = 0.1}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = \sqrt{\frac{80 \times 10^3}{200}} \sqrt{1-0.1^2}$$

$$\omega_d = 19.9 \text{ rad/s}$$

$$\omega_d = 2\pi f_d$$

$$f_d = \frac{\omega_d}{2\pi} = \frac{19.9}{2\pi}$$

$$\boxed{f_d = 3.167 \text{ Hz}}$$